CBPF-NF-047/98

A Remark on the Anti-BRS Structure of the Topological Yang-Mills Field Theory

M.W. de Oliveira

Centro Brasileiro de Pesquisas Físicas – CBPF
Departamento de Campos e Partículas – DCP
Rua Dr. Xavier Sigaud, 150
22290-180 – Rio de Janeiro, RJ – BRAZIL
E-mail: mwerneck@cbpfsu1.cat.cbpf.br and

H. Zerrouki

Institut für Theoretische Physik,
Technische Universität Wien,
Wiedner Hauptstraße 8-10,
A-1040 Wien, AUSTRIA
E-mail: zerrouki@tph15.tuwien.ac.at

Abstract. We build up and analyze the constrained cohomology set-up for the anti-BRS structure of the Witten's topological Yang-Mills field theory. We also discuss a conjecture on the existence of a novel class of topological observables arising within the context of the anti-BRS differential algebra. The anti-BRS counterparts of the Witten's observables are then identifyed in terms of the method of the spectral sequences.

Key-words: Topological Field Theory.

1 Introduction

For one decade now, Witten's topological Yang-Mills field theory remains outstanding in both domains of theoretical high-energy physics and algebraic geometry [1].

From a physical viewpoint, this class of models have been conjectured to describe gravity in a rather unusual regime, the so-called topological phase, in which the metric tensor would appear as an expectation value of a more fundamental set of quantum fields. Moreover, due to the existence of a certain topological twist algorithm, the topological theories display an intimate connection with N=2 supersymmetric gauge models which, in turn, are known to possess a wide range of applications in both two- and four-dimensional field theory (see e.g. [2]).

In mathematics, the topological field models of the Witten's type are also interesting as an useful tool in the solution of thorny problems in intersection number theory. As a matter of fact, these models were originally conceived to provide a lagrangian framework for the evaluation of the Donaldson's polynomials in terms of specific field correlators of certain topological observables. More specifically, it can be shown that to each correlation function of that type, there corresponds some intersection number of homology cycles belonging to the moduli space of the instantonic configurations of the gauge potentials. Indeed, it is precisely because of the topological nature of the Witten's theory that these correlators turn out to be integer numbers, allowing thus for such a remarkable correspondence.

Mainly for this last reason, several authors addressed the issue of providing an accurate characterization framework for the aforementioned topological observables in terms of a geometrical and (or) cohomological-algebraic formalism [3, 4, 5, 6, 7, 8, 9, 10]. Furthermore, in a more recent effort, Delduc, Maggiore, Piguet and Wolf succeded in specifying the topological observables in a fully BRS-algebraic context, by making use of the so-called constrained cohomology set-up [11]. Roughly speaking, the Witten's observables are seen to belong not to the ordinary functional space of the BRS cohomology classes, but rather to the more restricted set of the gauge observables which remain invariant under the action of the topological shift part of the BRS transformations.

On the other hand, the anti-BRS structure of the topological Yang-Mills field theories has attracted a good deal of interest due, in part, to its complementary role with respect to the better understood BRS symmetry transformations, as well as to the hope of revealing itself as a further invariance of the theory [12, 13]. It turns out moreover that, in order to be consistently implemented, the anti-BRS differential algebra demands the introduction of extra fields to complete the Witten's original set of variables, entailing thus a more complicated pattern of transformation laws. In fact, thanks to the uncommon mixing of both gauge-free and gauge-fixing sectors, the anti-BRS differential algebra of the topological theory may eventually tempt someone to enquire about the existence of a novel class of topological observables. Actually, such a conjecture has been proposed by Perry and Teo in ref.[12] and, in view of the cohomological techniques now at our disposal, deserves a closer investigation.

This is the aim of the present letter: to identify all possible classes of topological observables corresponding to the anti-BRS structure of the topological Yang-Mills field theory. The method employed is the constrained cohomology technique of [11] adapted to the case of the anti-BRS differential algebra. We shall then be able to give a precise – and negative – answer to the conjecture of Perry and Teo: the pure anti-BRS observables are essentially the barred counterparts of the BRS ones, despite of the presence of new scalar field variables in the formalism.

The work is organized as follows: in Section 2 we describe the role of each field variable of the theory, including the new ones introduced in [12]; we show afterwards how to obtain the specific constrained cohomology set-up for the anti-BRS structure, solving the respective cohomological problem by means of the by now standard method of the spectral sequences; in Section 3 we interpret our results and present our concluding remarks.

2 The Anti-BRS Structure and its Cohomology

We begin our discussion by recalling the relevant field variables appearing in the set-up introduced by Perry and Teo in ref.[12]. One introduces[1] a one-form gauge connection A, a couple of one-forms consisting of a topological ghost and its respective antighost (to be regarded here as an anti-BRS partner with opposite ghost charge) $(\psi,\ \bar{\psi})$, a pair of zero-forms standing for the Faddeev-Popov ghost-antighost system $(c,\ \bar{c})$ and a further pair of zero-forms $(\varphi,\ \bar{\varphi})$, assuming the role of gauge ghosts for the topological ghost and its anti-BRS counterpart. We need moreover a set of Lagrange multiplier fields: the one-form[2] B (which eventually gives rise to the well-known two-form multiplier of the anti-self-duality condition), the usual gauge-multiplier zero-form b and the zero-form $\bar{\eta}$, the multiplier of the covariant gauge-fixing condition upon the topological ghost. To complete the anti-BRS framework consistently, Perry and Teo defined other two zero-forms: the field[3] $\bar{b}$, which has to be understood as the barred partner of the gauge-multiplier, and η, the non-barred companion of the topological multiplier.

We attribute here a grading to each p-form which is the sum of its degree (or rank) plus the ghost number of the corresponding component tensor. According to this attribution, a p-form will be regarded as a commuting (resp. anticommuting) object provided its grading is even (resp. odd), as may inferred from Table 1 below:

	A	ψ	c	φ	$\bar{\psi}$	B	$\bar{c}$	b	$\bar{\varphi}$	$\bar{\eta}$	$\bar{b}$	η
$deg.$	1	1	0	0	1	1	0	0	0	0	0	0
$\mathcal{N}_g$	0	1	1	2	-1	0	-1	0	-2	-1	0	1
$gr.$	o	e	o	e	e	o	o	e	e	o	e	o

Table 1: Degrees, ghost numbers and gradings (even or odd) of the p-form field variables.

It is a long standing fact that the gauge-fixed action functional of the topological

[1] Unless otherwise stated in the course of the paper, we will work in a notation in which a generic differential p-form ω_p writes as:

$$\omega_p(x) = \frac{1}{p!}\ \omega_{\mu_1\ldots\mu_p}(x)\ dx^{\mu_1}\wedge\ldots\wedge dx^{\mu_p},$$

in $D=4$ Euclidean space dimensions. All the p-form fields employed here are Lie algebra valued.

[2] This form is named κ in ref.[12]; we, however, use a capital letter to stress its one-form character.

[3] This scalar is called λ by the authors of [12]; we interpret it as the Lagrange multiplier's partner.

Yang-Mills field theory[4] is left invariant by a set of infinitesimal field transformations described by a differential operator s, the BRS operator, which is nilpotent, i.e. $s^2 = 0$ (see refs.[14, 15, 16]). Indeed, as explained in detail in [12], the BRS operation is a symmetry of the model when the above introduced set of variables is required to vary as[5]:

$$\begin{array}{ll} sA = \nabla c + \psi, & \\ s\psi = [c,\ \psi] + \nabla\varphi, & \\ sc = \frac{1}{2}\{c,\ c\} + \varphi, & \\ s\varphi = [c,\ \varphi], & \\ s\bar{\psi} = -B, & sB = 0, \\ s\bar{c} = b, & sb = 0, \\ s\bar{\varphi} = \bar{\eta}, & s\bar{\eta} = 0, \\ s\bar{b} = \eta, & s\eta = 0, \end{array} \tag{1}$$

where the differential operator ∇ is the graded covariant exterior derivative acting upon the p-forms[6]:

$$\nabla\omega \ = \ d\omega \ + \ [A,\ \omega\}, \tag{2}$$

the commutator (resp. anticommutator) being understood whether the p-form ω is even (resp. odd) with respect to the grading. Now, to obtain the full anti-BRS structure of the theory, one defines a further infinitesimal operation $\bar{s}$ onto the field variables, possessing the following properties:

$$\{s,\ \bar{s}\} \ = \ \bar{s}^2 \ = \ 0. \tag{3}$$

---

[4]The topological action principle is assumed to be the same as the one obtained by Perry and Teo.

[5]Apart from minor minus sign modifications, the BRS transformations presented here are essentially the same as the ones employed in [12].

[6]The operator d is the usual exterior derivative with $d = dx^{\mu}\partial_{\mu}$ and, as a consequence, with $d^2 = 0$.

As an immediate consequence of (3), the anti-BRS transformations will write as:

$$\begin{aligned}
&\bar{s}A = \nabla\bar{c} + \bar{\psi}, \\
&\bar{s}\bar{\psi} = [\bar{c},\ \bar{\psi}] + \nabla\bar{\varphi}, \\
&\bar{s}\bar{c} = \tfrac{1}{2}\{\bar{c},\ \bar{c}\} + \bar{\varphi}, \\
&\bar{s}\bar{\varphi} = [\bar{c},\ \bar{\varphi}], \\
&\bar{s}\psi = [\bar{c},\ \psi] - [\bar{\psi},\ c] + \nabla\bar{b} + B, \\
&\bar{s}B = \{\bar{c},\ B\} - [\bar{\psi},\ b] - \nabla\bar{\eta} - [\bar{\varphi},\ \nabla c + \psi], \\
&\bar{s}c = \{\bar{c},\ c\} - b + \bar{b}, \\
&\bar{s}\bar{b} = [\bar{c},\ \bar{b}] - \bar{\eta} - [\bar{\varphi},\ c], \\
&\bar{s}\varphi = [\bar{c},\ \varphi] - [\bar{b},\ c] - \eta, \\
&\bar{s}\eta = \{\bar{c},\ \eta\} + [\bar{b},\ b] + \{\bar{\eta},\ c\} + \left[\bar{\varphi},\ \tfrac{1}{2}\{c,\ c\} + \varphi\right], \\
&\bar{s}b = [\bar{c},\ b] - \bar{\eta}, \\
&\bar{s}\bar{\eta} = \{\bar{c},\ \bar{\eta}\} + [\bar{\varphi},\ b].
\end{aligned} \tag{4}$$

We are now ready to start our cohomology analysis along the lines envisioned by Delduc et al. in ref.[11]. In that paper, a suitable global ghost (supplemented with a counting operation) was introduced into the BRS differential algebra in order to rephrase an usual cohomology problem into a constrained cohomology one. With this same purpose in mind, we decompose the $\bar{s}$ operator at hand in terms of three new operators ($\bar{D}_{(0)},\ \bar{D}_{(1)},\ \bar{D}_{(2)}$) as given below:

$$\bar{s} \ = \ \bar{D}_{(0)} \ + \ \bar{\varepsilon}\bar{D}_{(1)} \ + \ \bar{\varepsilon}^2\bar{D}_{(2)}, \tag{5}$$

where $\bar{\varepsilon}$ is a bosonic parameter[7]. The counting operator is given accordingly by:

$$\mathcal{N} \ = \ \bar{\varepsilon}\frac{\partial}{\partial\bar{\varepsilon}}. \tag{6}$$

[7]Here, we refrain from assigning any ghost number to the parameter $\bar{\varepsilon}$ and insist in maintaining the (anti-)ghosty interpretation of the barred p-form fields. One will see shortly that this choice will not modify the essence of the reasoning followed in the solution of our constrained cohomology problem.

The action of the three operators defined above is then found to be:

$$
\begin{array}{lll}
\bar{D}_{(0)}A = \nabla\bar{c}, & \bar{D}_{(1)}A = \bar{\psi}, & \bar{D}_{(2)}A = 0, \\
\bar{D}_{(0)}\bar{\psi} = [\bar{c},\ \bar{\psi}], & \bar{D}_{(1)}\bar{\psi} = \nabla\bar{\varphi}, & \bar{D}_{(2)}\bar{\psi} = 0, \\
\bar{D}_{(0)}\bar{c} = \frac{1}{2}\{\bar{c},\ \bar{c}\}, & \bar{D}_{(1)}\bar{c} = 0, & \bar{D}_{(2)}\bar{c} = \bar{\varphi}, \\
\bar{D}_{(0)}\bar{\varphi} = [\bar{c},\ \bar{\varphi}], & \bar{D}_{(1)}\bar{\varphi} = 0, & \bar{D}_{(2)}\bar{\varphi} = 0, \\
\bar{D}_{(0)}\psi = [\bar{c},\ \psi], & \bar{D}_{(1)}\psi = -[\bar{\psi},\ c] + \nabla\bar{b} + B, & \bar{D}_{(2)}\psi = 0, \\
\bar{D}_{(0)}B = \{\bar{c},\ B\} - [\bar{\psi},\ b] - \nabla\bar{\eta}, & \bar{D}_{(1)}B = -\left[\bar{\varphi},\ \nabla c + \psi\right], & \bar{D}_{(2)}B = 0, \\
\bar{D}_{(0)}c = \{\bar{c},\ c\} - b, & \bar{D}_{(1)}c = \bar{b}, & \bar{D}_{(2)}c = 0, \\
\bar{D}_{(0)}\bar{b} = [\bar{c},\ \bar{b}] - \bar{\eta}, & \bar{D}_{(1)}\bar{b} = -[\bar{\varphi},\ c] & \bar{D}_{(2)}\bar{b} = 0, \\
\bar{D}_{(0)}\varphi = [\bar{c},\ \varphi], & \bar{D}_{(1)}\varphi = -[\bar{b},\ c] - \eta, & \bar{D}_{(2)}\varphi = 0, \\
\bar{D}_{(0)}\eta = \{\bar{c},\ \eta\} + [\bar{b},\ b] + \{\bar{\eta},\ c\}, & \bar{D}_{(1)}\eta = \left[\bar{\varphi},\ \frac{1}{2}\{c,\ c\} + \varphi\right], & \bar{D}_{(2)}\eta = 0, \\
\bar{D}_{(0)}b = [\bar{c},\ b], & \bar{D}_{(1)}b = -\bar{\eta}, & \bar{D}_{(2)}b = 0, \\
\bar{D}_{(0)}\bar{\eta} = \{\bar{c},\ \bar{\eta}\}, & \bar{D}_{(1)}\bar{\eta} = [\bar{\varphi},\ b], & \bar{D}_{(2)}\bar{\eta} = 0.
\end{array}
\tag{7}
$$

Here, due to the nilpotency of $\bar{s}$, the following algebra emerges:

$$
\left(\bar{D}_{(0)}\right)^2 \;=\; 0, \qquad \left\{\bar{D}_{(0)},\ \bar{D}_{(1)}\right\} \;=\; 0, \qquad \left(\bar{D}_{(1)}\right)^2 \;+\; \left\{\bar{D}_{(0)},\ \bar{D}_{(2)}\right\} \;=\; 0. \tag{8}
$$

The structure in (8) is the anti-BRS analogue of the constrained cohomology algebra obtained by Delduc et al. and, in much the same way as in [11], will enable us to completely evaluate the $\bar{s}$-cohomology. Still, the set of transformations in (7) are not in the most convenient form to develop our analysis. At this point, it proves useful to introduce a second counting operator $\widetilde{\mathcal{N}}$:

$$
\widetilde{\mathcal{N}}\ \widetilde{\omega} = \widetilde{\omega}, \qquad\qquad \widetilde{\omega} = \left\{\ c,\ \bar{b},\ \varphi,\ \eta,\ b,\ \bar{\eta}\ \right\}, \tag{9}
$$

according to which, the $\bar{D}_{(0)}$ and the $\bar{D}_{(1)}$ operators in (8) decompose as follows:

$$
\begin{array}{ll}
\bar{D}_{(0)} \;=\; \bar{d}_{(0)} \;+\; \bar{d}_{(1)}, & \bar{D}_{(1)} \;=\; \bar{\delta}_{(0)} \;+\; \bar{\delta}_{(1)}, \\[2ex]
\left[\widetilde{\mathcal{N}},\ \bar{d}_{(0)}\right] \;=\; 0, & \left[\widetilde{\mathcal{N}},\ \bar{\delta}_{(0)}\right] \;=\; 0, \\[2ex]
\left[\widetilde{\mathcal{N}},\ \bar{d}_{(1)}\right] \;=\; \bar{d}_{(1)}, & \left[\widetilde{\mathcal{N}},\ \bar{\delta}_{(1)}\right] \;=\; \bar{\delta}_{(1)},
\end{array}
\tag{10}
$$

with the corresponding transformation laws:

$$
\begin{array}{llll}
\bar{d}_{(0)}A = \nabla\bar{c}, & \bar{d}_{(1)}A = 0, & \bar{\delta}_{(0)}A = \bar{\psi}, & \bar{\delta}_{(1)}A = 0, \\
\bar{d}_{(0)}\bar{\psi} = [\bar{c},\ \bar{\psi}], & \bar{d}_{(1)}\bar{\psi} = 0, & \bar{\delta}_{(0)}\bar{\psi} = \nabla\bar{\varphi}, & \bar{\delta}_{(1)}\bar{\psi} = 0, \\
\bar{d}_{(0)}\bar{c} = \frac{1}{2}\{\bar{c},\ \bar{c}\}, & \bar{d}_{(1)}\bar{c} = 0, & \bar{\delta}_{(0)}\bar{c} = 0, & \bar{\delta}_{(1)}\bar{c} = 0, \\
\bar{d}_{(0)}\bar{\varphi} = [\bar{c},\ \bar{\varphi}], & \bar{d}_{(1)}\bar{\varphi} = 0, & \bar{\delta}_{(0)}\bar{\varphi} = 0, & \bar{\delta}_{(1)}\bar{\varphi} = 0, \\
\bar{d}_{(0)}\psi = [\bar{c},\ \psi], & \bar{d}_{(1)}\psi = 0, & \bar{\delta}_{(0)}\psi = B, & \bar{\delta}_{(1)}\psi = -[\bar{\psi},\ c] + \nabla\bar{b}, \\
\bar{d}_{(0)}B = \{\bar{c},\ B\}, & \bar{d}_{(1)}B = -[\bar{\psi},\ b] - \nabla\bar{\eta}, & \bar{\delta}_{(0)}B = -[\bar{\varphi},\ \psi], & \bar{\delta}_{(1)}B = -[\bar{\varphi},\ \nabla c], \\
\bar{d}_{(0)}c = \{\bar{c},\ c\} - b, & \bar{d}_{(1)}c = 0, & \bar{\delta}_{(0)}c = \bar{b}, & \bar{\delta}_{(1)}c = 0, \\
\bar{d}_{(0)}\bar{b} = [\bar{c},\ \bar{b}] - \bar{\eta}, & \bar{d}_{(1)}\bar{b} = 0, & \bar{\delta}_{(0)}\bar{b} = -[\bar{\varphi},\ c], & \bar{\delta}_{(1)}\bar{b} = 0, \\
\bar{d}_{(0)}\varphi = [\bar{c},\ \varphi], & \bar{d}_{(1)}\varphi = 0, & \bar{\delta}_{(0)}\varphi = -\eta, & \bar{\delta}_{(1)}\varphi = -[\bar{b},\ c], \\
\bar{d}_{(0)}\eta = \{\bar{c},\ \eta\}, & \bar{d}_{(1)}\eta = [\bar{b},\ b] + \{\bar{\eta},\ c\}, & \bar{\delta}_{(0)}\eta = [\bar{\varphi},\ \varphi], & \bar{\delta}_{(1)}\eta = \left[\bar{\varphi},\ \frac{1}{2}\{c,\ c\}\right], \\
\bar{d}_{(0)}b = [\bar{c},\ b], & \bar{d}_{(1)}b = 0, & \bar{\delta}_{(0)}b = -\bar{\eta}, & \bar{\delta}_{(1)}b = 0, \\
\bar{d}_{(0)}\bar{\eta} = \{\bar{c},\ \bar{\eta}\}, & \bar{d}_{(1)}\bar{\eta} = 0, & \bar{\delta}_{(0)}\bar{\eta} = [\bar{\varphi},\ b], & \bar{\delta}_{(1)}\bar{\eta} = 0.
\end{array}
\tag{11}
$$

The above introduced operators obey to the algebraic relations:

$$
\begin{array}{rclrcl}
\left(\bar{d}_{(0)}\right)^2 & = & 0, & \left(\bar{\delta}_{(0)}\right)^2 & = & \delta_{(\bar{\varphi})}, \\
\left\{\bar{d}_{(0)},\ \bar{d}_{(1)}\right\} & = & 0, & \left\{\bar{\delta}_{(0)},\ \bar{\delta}_{(1)}\right\} & = & 0, \\
\left(\bar{d}_{(1)}\right)^2 & = & 0, & \left(\bar{\delta}_{(1)}\right)^2 & = & 0,
\end{array}
\tag{12}
$$

with the bosonic operator $\delta_{(\bar{\varphi})}$ denoting a gauge transformation with parameter $\bar{\varphi}$, and also to:

$$
\left\{\bar{d}_{(0)},\ \bar{\delta}_{(0)}\right\} = 0, \qquad \left\{\bar{d}_{(0)},\ \bar{\delta}_{(1)}\right\} + \left\{\bar{d}_{(1)},\ \bar{\delta}_{(0)}\right\} = 0, \qquad \left\{\bar{d}_{(1)},\ \bar{\delta}_{(1)}\right\} = 0.
\tag{13}
$$

By observing the first line in (12) and the first anticommutator in (13), one notices that the operators $\bar{d}_{(0)}$ and $\bar{\delta}_{(0)}$ characterize a set of algebraic relations which suggest once more a constrained cohomology study. In fact, one can easily convince himself that $\bar{\delta}_{(0)}$ is indeed nilpotent when applied upon gauge invariant monomials. Actually, from the theory of the spectral sequences [18], it can be shown that the cohomologies $\mathcal{H}^*(\bar{D}_{(0)})$ and $\mathcal{H}^*_c(\bar{D}_{(1)})$ are respectively isomorphic to subspaces of the cohomologies $\mathcal{H}^*(\bar{d}_{(0)})$ and $\mathcal{H}^*_c(\bar{\delta}_{(0)})$, i.e. the usual cohomology of the nilpotent operator $\bar{d}_{(0)}$ and the constrained cohomology of $\bar{\delta}_{(0)}$. We here take advantage of these facts to simplify our task, focusing our attention firstly on the identification of the larger space $\mathcal{H}^*_c(\bar{\delta}_{(0)})$ instead of $\mathcal{H}^*_c(\bar{D}_{(1)})$.

Later on, we will be able to sharpen our results to the specific case of that latter (see also ref.[17], page 202).

Following [11], we find it useful to perform a change of basis for the above introduced fields. Actually, as in the more familiar case of the Yang-Mills theory, our present cohomology problem enforces us to consider the space of the tensor field components together with their respective symmetric covariant derivatives, instead of the smaller space of p-form fields and their covariant exterior derivatives, where antisymmetrization is of course implicit. To this aim, we switch to the tensorial notation and choose a new set of independent field variables as given below[8]:

$$\{\ X,\ \bar{\delta}_{(0)}X,\ \bar{c},\ \bar{\varphi},\ \psi_\mu,\ B_\mu,\ \varphi,\ \eta\ \}\ , \tag{14}$$

with X standing for

$$X \equiv \begin{cases} F_{\mu\nu}, \\ \bar{\psi}_\mu, \\ \nabla_\mu\bar{\psi}_\nu + \nabla_\nu\bar{\psi}_\mu, \\ \nabla_\mu\psi_\nu,\ \nabla_\mu B_\nu,\ \nabla_\mu\varphi,\ \nabla_\mu\eta, \end{cases} \tag{15}$$

together with all the symmetric covariant derivatives of the following type:

$$\nabla_{\mu_1}...\nabla_{\mu_n}F_{\mu\nu},\quad \nabla_{\mu_1}...\nabla_{\mu_n}\left(\nabla_\mu\bar{\psi}_\nu + \nabla_\nu\bar{\psi}_\mu\right),\quad \nabla_{\mu_1}...\nabla_{\mu_n}\{\ \nabla_\mu\psi_\nu,\ \nabla_\mu B_\nu,\ \nabla_\mu\varphi,\ \nabla_\mu\eta\ \}\ , \tag{16}$$

where we notice the presence of the covariant derivatives of $\bar{\psi}_\mu$ through a symmetrization (i.e. the third line of (15)) and through $\bar{\delta}_{(0)}F_{\mu\nu}$. With respect to (14), $\bar{d}_{(0)}$ and $\bar{\delta}_{(0)}$ act as:

$$\begin{aligned} \bar{d}_{(0)} &= \sum_X \left\{ [\bar{c},\ X\} \frac{\partial}{\partial X} + [\bar{c},\ \bar{\delta}_{(0)}X\} \frac{\partial}{\partial\left(\bar{\delta}_{(0)}X\right)} \right\} + \tfrac{1}{2}\{\bar{c},\ \bar{c}\}\frac{\partial}{\partial\bar{c}} + [\bar{c},\ \bar{\varphi}]\frac{\partial}{\partial\bar{\varphi}} + \\ &\qquad + \{\bar{c},\ \psi_\mu\}\frac{\partial}{\partial\psi_\mu} + [\bar{c},\ B_\mu]\frac{\partial}{\partial B_\mu} + [\bar{c},\ \varphi]\frac{\partial}{\partial\varphi} + \{\bar{c},\ \eta\}\frac{\partial}{\partial\eta}, \end{aligned} \tag{17}$$

and

$$\begin{aligned} \bar{\delta}_{(0)} &= \sum_X \left\{ \left(\bar{\delta}_{(0)}X\right)\frac{\partial}{\partial X} - [\bar{\varphi},\ X]\frac{\partial}{\partial\left(\bar{\delta}_{(0)}X\right)} \right\} + B_\mu\frac{\partial}{\partial\psi_\mu} - [\bar{\varphi},\ \psi_\mu]\frac{\partial}{\partial B_\mu} + \\ &\qquad - \eta\frac{\partial}{\partial\varphi} + [\bar{\varphi},\ \varphi]\frac{\partial}{\partial\eta}, \end{aligned} \tag{18}$$

[8]The fields c, $\bar{b}$, b, $\bar{\eta}$ are naturally excluded from (14) as they can be shown to constitute doublets. To see this, one defines a counting assigning weight one to the antighost $\bar{c}$ and zero to the other fields: the $\bar{d}_{(0)}$ transformations of (11) will thus separate in such a way that the couples $(c,\ b)$ and $(\bar{b},\ \bar{\eta})$ define the trivial pattern. Clearly, the doublets cannot give rise to monomials belonging to $\mathcal{H}^*(\bar{d}_{(0)})$.

In order to study the constrained cohomology $\mathcal{H}^*_c(\bar{\delta}_{(0)})$, we introduce a further operator:

$$\bar{\delta}'_{(0)} \;=\; \sum_X X \frac{\partial}{\partial\left(\bar{\delta}_{(0)}X\right)} + \psi_\mu \frac{\partial}{\partial B_\mu} - \varphi \frac{\partial}{\partial \eta}, \tag{19}$$

which entails the following crucial property:

$$\left\{ \bar{\delta}_{(0)},\ \bar{\delta}'_{(0)} \right\} = \bar{\mathcal{F}}, \tag{20}$$

with the filtering operator, $\bar{\mathcal{F}}$, reading as below

$$\bar{\mathcal{F}} = \sum_X \left\{ X \frac{\partial}{\partial X} + \left(\bar{\delta}_{(0)}X\right) \frac{\partial}{\partial\left(\bar{\delta}_{(0)}X\right)} \right\} + \psi_\mu \frac{\partial}{\partial \psi_\mu} + B_\mu \frac{\partial}{\partial B_\mu} + \varphi \frac{\partial}{\partial \varphi} + \eta \frac{\partial}{\partial \eta}. \tag{21}$$

One has moreover the two algebraic relations:

$$\left[\bar{d}_{(0)},\ \bar{\mathcal{F}} \right] = 0, \qquad\qquad \left[\bar{\delta}_{(0)},\ \bar{\mathcal{F}} \right] = 0. \tag{22}$$

We now identify $\mathcal{H}^*_c(\bar{\delta}_{(0)})$. Let us expand a generic element $\bar{\Delta}$ of $\mathcal{H}^*_c(\bar{\delta}_{(0)})$ in terms of the eigenvalues of the filtering operator $\bar{\mathcal{F}}$:

$$\bar{\Delta} = \sum_{n \geq 0} \bar{\Delta}^{(n)}, \tag{23}$$

where

$$\bar{\mathcal{F}}\ \bar{\Delta}^{(n)} = n\ \bar{\Delta}^{(n)}. \tag{24}$$

From the property (20), one gets the relation

$$\bar{\delta}_{(0)}\ \bar{\delta}'_{(0)}\ \bar{\Delta}^{(n)} = n\ \bar{\Delta}^{(n)}, \tag{25}$$

for each level $n \geq 0$. This allows one to write:

$$\bar{\Delta}^{(n)} = \frac{1}{n}\ \bar{\delta}_{(0)}\ \bar{\delta}'_{(0)}\ \bar{\Delta}^{(n)}, \tag{26}$$

for the levels $n \geq 1$. As a consequence, $\bar{\Delta}$ in (23) can be written as:

$$\bar{\Delta} = \bar{\Delta}^{(0)} \ +\ \bar{\delta}_{(0)}\ \bar{\Delta}', \tag{27}$$

with

$$\bar{\Delta}' \equiv \sum_{n \geq 1} \frac{1}{n}\ \bar{\delta}'_{(0)}\ \bar{\Delta}^{(n)}. \tag{28}$$

Eq.(27) identifies the constrained cohomology $\mathcal{H}^*_c(\bar{\delta}_{(0)})$ by expressing it as the set of $\bar{d}_{(0)}$-invariant monomials which are independent of the following variables:

$$\left\{\ X,\ \bar{\delta}_{(0)}X,\ \psi_\mu,\ B_\mu,\ \varphi,\ \eta\ \right\}. \tag{29}$$

$\mathcal{H}^*_c(\bar{\delta}_{(0)})$ is hence the space of $\bar{d}_{(0)}$-cohomology classes given by functionals of the type:

$$\bar{\Phi}\left(\bar{\mathcal{P}}_{inv}(\bar{c}),\, \bar{\mathcal{Q}}_{inv}(\bar{\varphi})\right), \tag{30}$$

the functionals $\bar{\mathcal{P}}_{inv}(\bar{c})$ and $\bar{\mathcal{Q}}_{inv}(\bar{\varphi})$ being group invariant monomials depending on the (non-differentiated) antighosts $\bar{c}$ and $\bar{\varphi}$, respectively[9].
The subsequent and final step in our analysis corresponds to the identification of the constrained cohomology $\mathcal{H}^*_c(\bar{D}_{(1)})$. As it was mentioned previously, $\mathcal{H}^*_c(\bar{D}_{(1)}) \subset \mathcal{H}^*_c(\bar{\delta}_{(0)})$ and one is left with two mutually independent possibilities: either $\mathcal{H}^*_c(\bar{D}_{(1)})$ is empty, or some of the gauge invariant classes $\bar{\mathcal{P}}_{inv}(\bar{c})$ and $\bar{\mathcal{Q}}_{inv}(\bar{\varphi})$ do belong to $\mathcal{H}^*_c(\bar{D}_{(1)})$. A simple inspection of the set of $\bar{D}_{(1)}$-transformations in (7) reveals that the whole set of functional classes $\bar{\mathcal{P}}_{inv}(\bar{c})$ and $\bar{\mathcal{Q}}_{inv}(\bar{\varphi})$ – as well as their products – are indeed in the constrained cohomology of $\bar{D}_{(1)}$, implying that $\mathcal{H}^*_c(\bar{D}_{(1)}) \equiv \mathcal{H}^*_c(\bar{\delta}_{(0)})$. The functionals (30) are precisely the anti-BRS counterparts of the cohomology classes obtained in ref.[11] for the BRS formulation of the topological Yang-Mills model.

As regards the anti-BRS zero-form observables, $\bar{\mathcal{O}}_{(0)}$, we adopt here the definition given by Witten in [1] and discard from the analysis all the $\bar{c}$-dependent functionals appearing in the class $\bar{\Phi}$ above. Finally, the conclusion is that:

$$\left\{\bar{\mathcal{O}}_{(0)}\right\} \;=\; \left\{\bar{\mathcal{Q}}_{inv}(\bar{\varphi})\right\}. \tag{31}$$

To obtain the higher rank topological observables one may employ, for example, the vectorial transformation of Birmingham et al. [24], adapted to the anti-BRS case (see also ref.[25]). Therefore, the conjecture of ref.[12] is not confirmed in this pure anti-BRS study, the set observables in (31) being simply the anti-BRS counterparts of the ordinary BRS ones. One has to keep in mind, however, that new classes of topological observables may possibly arise in a more general algebraic framework, i.e. by combining both BRS and anti-BRS transformations into a unique nilpotent operator and by studying its constrained cohomology. This latter context is being considered in full detail in [26].

[9]It should be noticed that $\nabla_\mu \bar{c}$, together with all its symmetric covariant derivatives, are known to be excluded from the space of anti-BRS invariant operators $\mathcal{H}^*(\bar{d}_{(0)})$, in much the same way as it occurs for their counterparts in the BRS framework for the usual Yang-Mills field theory (see refs.[19, 20, 21, 22, 23]).

3 Concluding Remarks

We have analyzed the anti-BRS differential algebra of the topological Yang-Mills field theory in the context of the constrained cohomology set-up introduced by Delduc et al. in ref.[11]. In particular, we obtained all possible classes of anti-BRS topological observables in an explicitly algebraic manner, i.e. in terms of the method of the spectral sequences [18]. Unfortunately, our results do not confirm the conjectured feature of the existence of new topological invariants in the model [12]. As a matter of fact, despite of the presence of additional scalar variables nedeed for the consistent closure of the anti-BRS structure, the anti-BRS versions of the Witten's observables are given essentially by the barred counterparts of the originary observables identifyed by Witten in [1]. We remark, however, that new topological structures could possibly show up in a combined BRS–anti-BRS formalism where the constrained cohomology technique may be again of great usefulness.

Acknowledgements

The authors are thankful to Olivier Piguet for a remark on the constrained cohomology technique. M.W.O. is indebted to the head of the D.C.P. - C.B.P.F., José A. Helayël-Neto, for the kind hospitality and to Conselho Nacional de Desenvolvimento Científico e Tecnológico (C.N.Pq. - BRAZIL) for the financial support. H.Z. thanks the Fonds zur Förderung der Wissenschaftlichen Forschung (AUSTRIA) for a post-doctoral grant under contract number P11582-PHY.

References

[1] E. Witten, Comm. Math. Phys. **117** (1988) 353.

[2] D. Birmingham, M. Blau, M. Rakowski and G. Thompson, Phys. Rep. **209** (1991) 129.

[3] S. Ouvry, R. Stora and P. van Baal, Phys. Lett. **B220** (1989) 159.

[4] A. Blasi and R. Collina, Phys. Lett. **B222** (1989) 419.

[5] H. Kanno, Z. Physik C, **43** (1989) 477.

[6] P. van Baal, Acta Phys. Pol. **B21** (1990) 73.

[7] R. Myers, Int. J. Mod. Phys. **A5** (1990) 1369.

[8] J. Kalkman, Comm. Math. Phys. **153** (1993) 447.

[9] R. Stora, F. Thuillier and J.C. Wallet, "Algebraic Structure of Cohomological Field Theory Models and Equivariant Cohomology", proccedings of the 1st Caribbean School of Mathematics and Theoretical Physics, Saint François, Guadeloupe, 1993.

[10] R. Stora, "Equivariant Cohomology and Topological Theories", proccedings of the International Symposion on BRS Symmetry, Kyoto, Japan, 1995; preprint Enslapp-A-571/95.

[11] F. Delduc, N. Maggiore, O. Piguet and S. Wolf, Phys. Lett. **B385** (1996) 320.

[12] M.J. Perry and E. Teo, Nucl. Phys. **B392** (1993) 369.

[13] P. Fré and P. Soriani, "The N=2 Wonderland: from Calabi-Yau Manifolds to Topological Field Theories", World Scientific, Singapore, 1995.

[14] L. Baulieu and I.M. Singer, Nucl. Phys. (Proc. Suppl.) 5 (1998) 12.

[15] J.M.F. Labastida and M. Pernici, Phys. Lett. **B212** (1988) 91.

[16] R. Brooks, D. Montano and J. Sonnenschein, Phys. Lett. **B214** (1988) 91.

[17] O. Piguet and K. Sibold, "Renormalized Supersymmetry", Birkhauser, Basel, 1981.

[18] J.A. Dixon, "Cohomology and Renormalization of Gauge Theories I, II and III", Imperial College preprints, 1977.

[19] G. Bandelloni, J. Math. Phys. **27** (1986) 159.

[20] J. Dixon, Comm. Math. Phys. **139** (1991) 495.

[21] M. Dubois-Violette, M. Henneaux, M. Talon and C.M. Viallet, Phys. Lett. **B289** (1992) 361.

[22] F. Brandt, N. Dragon and M. Kreuzer, Phys. Lett. **231** (1989) 263; Nucl. Phys. **B332** (1990) 224, 250.

[23] F. Brandt, Ph.D. thesis, University of Hannover, 1991.

[24] D. Birmingham and M. Rakowski, Phys. Lett. **B272** (1992) 217.

[25] D. Birmingham and M. Rakowski, Phys. Lett. **B275** (1992) 289; J. Geom. Phys. **11** (1993) 129.

[26] M.W. de Oliveira and H. Zerrouki, in preparation.