Analysis of Chiral Symmetry Breaking Mechanism

by

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ABSTRACT

The renormalization group invariant quark condensate $\hat{\mu}$ is determined both from the consistent equation for quark condensate in the chiral limit and from the Schwinger-Dyson (SD) equation improved by the intermediate range QCD force singular like $\delta(q)$ which is associated with the gluon condensate. The solutions of $\hat{\mu}$ in these two equations are consistent. We also obtain the critical strong coupling constant α_c above which chiral symmetry breaks in these two approaches. The nonperturbative kernel of the SD equation makes α_c smaller and $\hat{\mu}$ bigger. An intuitive picture of the condensation above α_c is discussed. In addition, with the help of the Slavnov-Taylor-Ward (STW) identity we derive the equations for the nonperturbative quark propagator from the SD equation in the presence of the intermediate range force and find that the intermediate-range force is also responsible for dynamical chiral symmetry breaking.

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1   Introduction

Two of the most important features of nonperturbative quantum chromodynamics (QCD) are quark and gluon confinement and dynamical chiral symmetry breaking. The latter leads to the pion as a Nambu-Goldstone boson after the breakdown of an approximate chiral symmetry $SU(2) \times SU(2)$. Before QCD was developed, the famous current algerbra and PCAC relation

$$(m_u + m_d) < 0 \mid u\bar{u} + d\bar{d} \mid 0 > = f_\pi^2 \, m_\pi^2 (1 + O(m_\pi^2)) \,, \tag{1}$$

had indicated the non-zero vacuum expectation value of the quark and anti-quark field which is the order parameter of chiral symmetry breaking. Computer simulations of lattice gauge theory also suggest the spontaneous breakdown of chiral symmetry[1]. Furthermore, it is believed that the successful explanation of hadron spectroscopy and nucleon magnetic moments are due to the large constituent quark mass[2] for which chiral symmetry breaking is also responsible. Actually, the quark condensate generates a large dynamical quark mass[3,4] even in the case of the explicit chiral symmetry so that quarks acquire constituent masses with the order of Λ which is the QCD scale parameter.

It has been shown that in QCD with massless fermions the chiral symmetry invariance remains to any finite order expansion around the perturbative vacuum because of the manifest chiral invariance of the original lagrangian. Therefore chiral symmetry breaking must be associated with the physical vacuum instead of the perturbative one. The fact that the physical vacuum replaces the perturbative one is essential to all the nonperturbative QCD effects.

In fact, if the strong coupling constant exceeds some critical point the vacuum will become unstable. This leads to the rearrangement of the vacuum so that quarks acquire masses in order to cure this instability. When g is increased to some scale the number of fermion pairs will be indefinite in the ground state. As a result, an operator which destroys a fermion pair will have a non-zero expectation value $< 0 \mid \psi\bar{\psi} \mid 0 >$ with the vacuum as the physical one. Therefore, the existence of a critical value for the strong coupling constant is essential for dynamical symmetry bearing. This fact has been confirmed by many phenomenological models for chiral symmetry breaking.

Although it has been about three decades since the beginning of the chiral symmetry breaking study[5,6] the fully explanation of the chiral symmetry breaking mechanism still remains as an open problem in QCD. One of the problems is the determination of quark condensate. As mentioned above, the order parameter for chiral symmetry breaking is $< \psi\bar{\psi} >$. When $< \psi\bar{\psi} >$ is not zero, we have the Nambu-Goldstone realization[7] instead of the Wigner-Weyl realization[8]. In principle, QCD lagrangian describes not only the perturbative but also the nonperturbative effects so that $< \psi\bar{\psi} >$ should be determined theoretically. However, because in the physical vacuum there are a lot of nonperturbative phenomena with which we do not have effective ways to deal, one introduces the condensates $< \psi\bar{\psi} >$, $< G^2 >, \cdots$ as phenomenological parameters at present in theoretical calculations such as in QCD sum rule[9]. It is one of the motivations of the present paper

to give a semi- phenomenological determination of the quark condensate. We will do it both from the consistent equation for quark condensate and from the SD equation. The results of these two approaches are reasonable and in agreement with each other.

Another unsolved but interesting problem is the range of fores which may be responsible for chiral symmetry breaking. The present ideas about this problem may be divided into two categories. One of them assumes that perturbative force is the cause of chiral symmetry breaking[10] while the other claims that chiral symmetry breaking is due to the confinement mechanism[11,12]. Based on the lattice gauge result that T_F (the chiral symmetry restoration temperature) is bigger than T_c (the deconfinement temperature)[13] the author of Ref.[10] assumed that the range of force responsible for chiral symmetry breaking is very short and irrelevant to confinement so that in the kernel of the SD equation the one- gluon-exchange approximation was used to get the linearized SD equation. It was found that when the short-distance potential is strong enough it can break chiral symmetry. On the other hand, the authors of Refs.[11] and [12] proposed the confinement mechanism for chiral symmetry breaking and also obtained reasonable results.

We will present in this paper the idea that chiral symmetry breaking may be both due to the short-distance force and due to the long-distance force. This is motivated from the following reasons: (i) chiral symmetry breaking takes place in the QCD physical vacuum in which all kinds of forces in QCD theory may appear in the SD kernel so that both the long-distance and the short-distance forces could contribute; (ii) The lattice computer simulation results about T_c and T_F at present are not in agreement with each other[14]. Actually the later computation suggests that T_F is almost the same as $T_c^{[14,15]}$. In this case the long-distance force may also contribute to chiral symmetry breaking. On the other hand, even if T_F is bigger than T_c, confinement force may also plays a role since chiral symmetry breaking occurs in the presence of confinement in QCD theory at zero temperature; (iii) All the present models including the effective potential model[16,17,18], Higashijima approach[10] and the analysis which will be presented in this paper find that the critical strong coupling constant is of the order O(1) inspite of some differences in numerical values. This fact indicates that the force responsible for chiral symmetry breaking has been extrapolated into the long-distance region; (iv) In order to form the Goldstone boson, one should also need confinement force so that quarks acquire masses. Generally the forces to form a light meson ought to include the long-distance force. Considering the above reasons, we improve the SD equation by adding the gluon condensate kernel singular like $\delta(q)$ corresponding to intermediate range force so that the nonperturbative effects are also taken into account. We find that the nonperturbative modification decreases the critical coupling constant value α_c and increases the value of the quark condensate.

In Refs.[12] and [19] the generally accepted infrared gluon propagator $1/q^4$ corresponding to linear confinement potential is assumed so that the equations for the nonperturbative quark propagator are derived using the SD equation and the STW identity. In the present paper we also follow the above method to derive the equations for the nonperturbative quark propagator in the presence of the gluon propagator singular like $\delta(q)$ as mentioned above. We find that in two cases of b = 0 and b = −1 (b is the ghost self-

energy) the equations for the nonperturbative quark propagator can be solved analytically and in the real case b = -1 chiral symmetry breaking may occur. As we know the free gluon propagator corresponding to the Coulomb potential plays an important role at short distances while the confinement potential $1/q^4$ is essential at large distances. The gluon propagator $\delta(q)$ is just what shows its importance at the intermediate distances. Hence the intermediate-range QCD force may also be responsible for chiral symmetry breaking.

The plan of the paper is as follows. In Section 2 we present the consistent equation for quark condensate and discuss its general features. In Section 3 we give the form of the SD equation improved by the $\delta(q)$ kernel associated with the gluon condensate. Numerical results of the renomatization group invariant quark condensate $\hat{\mu}$ solved from the above two equations are given in Section 4. Section 5 is devoted to the discussions on the effects of the intermediate-range force on chiral symmetry breaking. Conclusions and discussions are presented in the last section.

2   General discussions on the consistent equation

It has been shown by analyzing the generating functional in path integral[16−18] that in the effective potential model the composite operator

$$S(x,y) = < \psi(x)\bar{\psi}(y) >, \tag{2}$$

where we have suppressed the spinor and color indices, can be expressed as

$$S^{-1} = S_0^{-1} + \Sigma, \tag{3}$$

when the external source corresponding to $\psi(x)\bar{\psi}(y)$ is turned off. S_0 is the free propagator of fermions and

$$\Sigma = -\frac{\delta\Gamma_2}{\delta S}, \tag{4}$$

represents the quark self-energy when the external source is tuned off and Γ_2 is the sum of all the two-particle- irreducible vacuum diagrams evaluated with the fermion propagator replaced by S. Consequently, S coincides with the complete quark propagator when the source is set to zero. Thus we have the consistent equation for quark condensate in the chiral limit

$$< \psi\bar{\psi} >= \lim_{x\to 0,m\to 0} Tr < 0 \mid T(\psi_\alpha^i(x)\bar{\psi}_\beta^j(0)) \mid 0 >, \tag{5}$$

where α, β and i,j are spinor and color indices respectively and the trace operator is with respect to these two kinds of indices. As discussed in Introduction the vacuum is understood as the physical one.

After Fourier transformation, Eq.(5) becomes

$$
\begin{aligned}
< \psi\bar{\psi} > &= Tr \int \frac{d^4p}{(2\pi)^4} S(p, <\psi\bar{\psi}>, \cdots) \\
&= Tr \int \frac{d^4p}{(2\pi)^4} \frac{i}{\not{p} + \Sigma(p, <\psi\bar{\psi}>, \cdots)},
\end{aligned}
\tag{6}
$$

where the fermionic self-energy Σ contains the contributions not only from the perturbative region but also from the nonperturbative region. For example, the operator product expansion includes the higher dimension contribution,

$$
\int d^4x e^{iqx} < 0 \mid T\psi(x)\bar{\psi}(0) \mid 0 > = \sum_n C_n(q) < 0 \mid \hat{O}^n \mid 0 >
$$
$$
= C_0(q) < 0 \mid 1 \mid 0 > + C_1(q) < 0 \mid \psi\bar{\psi} \mid 0 > + \cdots,
\tag{7}
$$

where $C_0(q)$ is the perturbative coefficient and the second term is the quark condensate contribution. Other terms are from higher dimension operators which appear in the Wilson expansion such as $< G^2 >$ and $< \psi\bar{\psi}\ \psi\bar{\psi} >$.

In writing Eq.(6) we have assumed $Z(p^2) = 1$ in the following general form,

$$
S(p) = \frac{iZ(p^2)}{\not{p} - \Sigma(p^2)}.
\tag{8}
$$

This assumption is shown to be true in the Landau gauge when the nonperturbative force is not introduced[20]. However, when we consider the nonperturbative force contributions to chiral symmetry breaking as discussed in Section 1 $Z(p^2)$ may not be one in the infrared region, which will be seen clearly in the next section.

The OPE expansion (7) is available in the short-distance range $p^2 >> \Lambda^2$ where Λ is the QCD scale parameter. It has been extended to the region $p \sim 1\ GeV$ and has been calculated in Refs.[3] and [21]. At the order $O(\alpha)$ where α is the strong coupling constant $\Sigma(p)$ has the form improved by the renormalization group in the Landau gauge[25]

$$
\Sigma(p) = \frac{4\pi}{3p^2} \alpha(p) < \psi\bar{\psi} > (p),
\tag{9}
$$

where

$$
\alpha(p) = \frac{4\pi}{\beta_0 ln(p^2/\Lambda^2)},
\tag{10}
$$

$$
< \psi\bar{\psi} > (p) = \hat{\mu}^3 (\frac{1}{2} ln \frac{p^2}{\Lambda^2})^d.
\tag{11}
$$

In the above equations $\hat{\mu}$ is the renormalization group invariant quantity which will be determined phenomenologically later, $\beta_0 = 11 - \frac{2}{3}n_f$ and d is the anomalous dimension associated with the quark condensate.

To evaluate the consistent equation (6) we divide the integral into two regimes with the separating scale λ beyond which perturbative calculations are permitted. This method will also be adopted in the analysis of the SD equation. After integrating the angular coordinates Eq.(6) becomes

$$< \psi\bar\psi >_M = \frac{N}{2\pi^2} \left\{ \int_0^\lambda dp \frac{p^3 \Sigma(p)}{p^2 + \Sigma^2(p)} + \int_\lambda^M dp \frac{p^3 \Sigma(p)}{p^2 + \Sigma^2(p)} \right\}, \tag{12}$$

where we have introduced the ultra-violet cut-off M which will be canceled since QCD is renormalizable, N is the color number. λ is usually taken as the typical hadronic scale 1 GeV. The second part of Eq.(12) can be integrated out since when $p < \lambda$, the expression of $\Sigma(p)$ in the perturbative region can be used and $\Sigma^2(p)$ in the denominator can be neglected because of the fact that $\Sigma^2(p) << p^2$ as $\lambda < p$ (This can be seen from the phenomenological value of $< \psi\bar\psi >$ as $(250\ MeV)^{3[9]}$). Then the second integral in Eq.(12) is just the difference between the quark condensates at the scales of M and λ. Hence we have

$$\frac{N}{2\pi^2} \int_0^\lambda dp \frac{p^3 \Sigma(p)}{p^2 + \Sigma^2(p)} = < \psi\bar\psi > (\lambda). \tag{13}$$

In canceling the cut-off M we have used the $\Sigma(p)$ at $O(\alpha)$. Actually it does not lose generality since M could be sufficiently large so that the expression of $\Sigma(p)$ at $O(\alpha)$ can be used to cancel M. As will be seen later when the corrections of $\Sigma(p)$ at $O(\alpha^2)$ is taken into account Eq.(13) is slightly modified.

The crucial point now is how to deal with $\Sigma(p)$ in the nonperturbative region $p < \lambda$. As we know chiral symmetry breaking is associated with the strong coupling constant α. In the perturbative range α is determined by the one-gluon-exchange approximation. However, when Q^2 approaches to low energy region multi-gluon-exchange diagrams have to be considered and this effect may lead to the effective gluon mass which freezes the strong coupling constant at low energy[22,23]. Based on this analysis we assume that $\alpha(p)$ is fixed below a fixing scale μ. This also leads to the fixing of $\Sigma(p)$ below μ as $\Sigma(p)$ is associated with α. Then Eq.(13) becomes

$$\frac{3}{8\pi^2}\mu^2 \Sigma(\mu) - \frac{3}{4\pi^2}\Sigma^3(\mu) ln\left(1 + \frac{\mu^2}{\Sigma^3(\mu)}\right) = < \psi\bar\psi > (\lambda) - \int_\mu^\lambda dp \frac{p^3 \Sigma(p)}{p^2 + \Sigma^2(p)}. \tag{14}$$

Hence we have

$$\Sigma(\mu) \geq \frac{2}{\lambda^2} < \psi\bar\psi >_\lambda . \tag{15}$$

As we know, Σ is a function of α and $\hat\mu^3$. After the cancellation of $\hat\mu^3$ in the two sides of Eq.(15) we will get a constraint on α for chiral symmetry breaking. This will lead to the critical coupling constant α_c. Eq.(14) is just the equation which is used to determine the quantity $\hat\mu$. The solutions of this equation depend on our phenomenological assumptions about $\Sigma(p)$ in the energy region between μ and λ. They will be given in Section 4.

3   General discussions on the improved SD equation

As was discussed in introduction the one-gluon- exchange approximation of the SD equation should be modified by introducing the nonperturbative force. QCD sum rule is one of the most effective way at present to deal with the phenomena in the intermediate energy region. Motivated by the idea of QCD sum rule we start from the perturbative region, and then extrapolate to the intermediate energy range. This leads to the nonperturbative kernel of the SD equation shown in fig.1(b) apart from the perturbative kernel Fig.1(a).

Generally the SD equation has the following form

$$S^{-1}(p) = S_0^{-1}(p) + g^2 C_F \int \frac{d^4 q}{(2\pi)^4} \Gamma_\mu(p, q) S(p - q) \gamma_\nu D^{\mu\nu}(q), \tag{16}$$

where $\Gamma_\mu(p, q)$ is the quark-gluon-quark vertex and C_F is the casimir invariant of the quark representation. In the one-gluon-exchange approximation only the kernel Fig.1(a) contributes and $\Gamma_\mu = \gamma_\mu$, therefore we have

$$Z^{-1}(p^2)\Sigma(p^2) = \frac{3C_F}{4\pi^2 i} \int d^4 k \frac{\alpha(p, k)}{(p - k)^2} \frac{Z(k^2)\Sigma(k^2)}{k^2 - \Sigma^2(k^2)} \tag{17}$$

where the complete quark propagator is parameterized as Eq.(8). It was shown $Z(p^2)$ equals to one in the Landau gauge in the one-gluon-exchange approximation[20]. It will be seen later that when the nonperturbative kernel is included $Z(p^2)$ does no longer equal to one in the infrared region.

The kernel Fig.1(b) can be obtained by replacing the kernel Fig.1(a)

$$_p < 0 \mid T A_\mu^a(x_1) A_\nu^b(x_2) \mid 0 >_p, \tag{18}$$

where $\mid 0 >_p$ is the perturbative vacuum, with

$$< 0 \mid A_\mu^a(x_1) A_\nu^b(x_2) \mid 0 >, \tag{19}$$

without contraction of the two gluon fields.

In order to treat the quantity (19) we follow the paper by Cenlenza and Shakin[24] to decompose $A_\mu^a(x)$ into two parts

$$A_\mu^a(x) = \tilde{A}_\mu^a(x) + \mathcal{A}_\mu^a(x), \tag{20}$$

where $\tilde{A}_\mu^a(x)$ contains the momentum only lower than $1/\xi_0^{QCD}$ and $\mathcal{A}_\mu^a(x)$ is only nonzero for $k > 1/\xi_0^{QCD}$. ξ_0^{QCD} is the coherence length associated with the gluon condensate. $\tilde{A}_\mu^a(x)$ is treated as a classical field just as in Ref.[25]. To simplify the calculations we take the limit $\xi_0^{QCD} \to \infty$ so that only the k = 0 mode contributes to $\tilde{A}_\mu^a(x)$. In this

large ξ_0^{QCD} approximation the expression (19) may be written as the following form on the ground of Lorentz invariance

$$< 0 \mid g^2 A_\mu^a(x_1) A_\nu^b(x_2) \mid 0 > = \frac{1}{32} \delta^{ab} g_{\mu\nu} R, \tag{21}$$

where

$$\begin{aligned} R &= < 0 \mid g^2 \tilde{A}_\mu^a(x_1) \tilde{A}_\nu^b(x_2) \mid 0 > \\ &= -\{\frac{32\pi^2}{3} < 0 \mid \frac{\alpha}{\pi} G_{\mu\nu}^a(0) G_a^{\mu\nu}(0) \mid 0 >\}^{1/2}, \end{aligned} \tag{22}$$

which indicates that in the low-momentum approximation R is related to the gauge invariant gluon condensate parameter. Numerically $R = -1.12 \ GeV^2$ from the phenomenological value of the gluon condensate[9].

Adding the nonperturbative kernel to the SD equation (16) and parameterizing the quark propagator as

$$S^{-1}(p) = i[A(p^2) + \gamma^\mu \partial_\mu B(p^2)], \tag{23}$$

one derive two equatiions (the kernel (21) in momentum space is $\frac{(2\pi)^4}{32} \delta^{ab} g_{\mu\nu} R\delta(q)$)

$$\begin{aligned} iZ^{-1}(p^2)\Sigma(p^2) &= 3g^2 C_F \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(p-k)^2} \frac{Z(k^2)\Sigma(k^2)}{k^2 - \Sigma^2(k^2)} \\ &+ i\frac{RC_F}{8} \frac{Z(p^2)\Sigma(p^2)}{p^2 - \Sigma^2(p^2)} \end{aligned} \tag{24}$$

and

$$1 - Z^{-1}(p^2) = -\frac{RC_F}{16} \frac{Z(p^2)}{p^2 - \Sigma^2(p^2)} \tag{25}$$

Eq.(24) is just the modified form of Eq.(17) and it can be seen from Eq.(25) that when R = 0 we have $Z(p^2) = 1$ which is just the result of Ref.[20].

Eq.(24) can also be derived from the Bethe-Salpeter (BS) equation[26] which describes bound states. The BS equation for a fermion-antifermion bound state is

$$\left[S^{-1}(q + \frac{1}{2}P)\chi(q,P)S^{-1}(q - \frac{1}{2}P)\right]_{\alpha\beta} = \int \frac{d^4 q'}{(2\pi)^4} K_{\alpha\beta;\alpha'\beta'} \chi_{\alpha'\beta'}(q',P), \tag{26}$$

where P is the total momentum of the bound state and $\chi_{\alpha\beta}(q,P)$ is the BS wavefunction; S is the fermion propagator and K denotes the two-particle-irreducible kernel. α, β, α' and β' are Dirac spinor indices. Taking only the one-gluon-exchange kernel and its corresponding nonperturbative one similar to Fig.1(b) into account we have

$$K_{\alpha,\beta;\alpha',\beta'}(q,q';P) = -C_F(\gamma^\mu)_{\alpha\alpha'}(\gamma^\nu)_{\beta\beta'}[g^2 D_{\mu\nu}(q-q') + \frac{(2\pi)^4}{32}g_{\mu\nu}R\delta(q-q')], \qquad (27)$$

where $D_{\mu\nu}(q)$ is the free gluon propagator. In the case $P_\mu = 0$,[10]

$$\chi_{\alpha\beta}(q,0) = (\gamma_5)_{\alpha\beta}F(q^2), \qquad (28)$$

where $F(q^2)$ is some function of q^2. Substituting Eq.(27) into Eq.(26) and considering Eq.(28), we have Eq.(24).

In Ref.[10] the author solved the SD equation numerically by considering two boundary conditions (i) $\Sigma(p) \to \Sigma(0)$ as $p \to 0$; (ii) $\Sigma(p)$ runs as the current quark mass when $p \to \infty$ since in this limit nonperturbative effects disappear. However, the improved SD equation (24) and (25) have more complicated structure. In order to get the numerical solutions we use the same assumptions for $\alpha(p)$ and $\Sigma(p)$ adapted in the discussions about the consistent equation for quark condensate, i.e., in the region $p < \mu$, $\alpha(p)$ and $\Sigma(p)$ freeze to constants. Let $p = \mu$ in Eqs.(24) (25) and rotating these two equations to Euclidean space we have

$$\Sigma(\mu^2)\left[Z^{-1}(\mu^2) + \frac{RC_F}{8}\frac{Z(\mu^2)}{\mu^2 + \Sigma^2(\mu^2)}\right]$$
$$= \frac{3C_F}{2\pi}\left\{\frac{\alpha(\mu)}{\mu^2}\int_0^\mu dk\frac{k^3\Sigma(\mu^2)Z(k^2)}{k^2 + \Sigma^2(\mu^2)}\int_\mu^\infty dk\frac{\alpha(k)k\Sigma(k^2)Z(k^2)}{k^2 + \Sigma^2(k^2)}\right\} \qquad (29)$$

where we have integrated the angular coordinates and

$$Z(p^2) = \frac{1}{2}(-u(p^2) + \sqrt{u^2(p^2) + 4u(p^2)}), \qquad (30)$$

with

$$u(p^2) = -\frac{16}{RC_F}(p^2 + \Sigma^2(p^2)). \qquad (31)$$

It can be seen from Eq.(30) that when $p^2 >, \Lambda^2, Z(p^2)$ equals to one. It is reasonable since in this energy range nonperturbative kernel does no longer play a role.

With the aid of Eq.(30) we are able to solve Eq.(29) numerically. Since the nonperturbative kernel associated with the gluon condensate is an attractive potential it is expected that this potential will help the quark condensate happen. Therefore the critical coupling constant for chiral symmetring breakdown will decrease and the value of the gauge-invariant quark condensate will become bigger.

4   Numerical results and some comments

In this section we present numerical solutions of the consistent equation for quark condensate and the improved SD equation. Some comments on these solutions will also be made.

4.1 Models in the nonperturbative region

The nonperturbative range $p < \lambda$ $(\sim 1\ GeV)$ may be divided into two region $p < \mu$ and $\mu < p < \lambda$. In the region $p < \mu$ we have assumed that $\alpha(p)$ is frozen and $\Sigma(p)$ is also fixed to be the constituent quark mass. The problem now is how to describe the nonperturbative effects in the range $\mu < p < \lambda$. This is just what we do not know at present because of our lack of knowledge about the long-distance phenomena. Some phenomenological models will be employed. To carry out calculations we use two models in this paper. One of them is the so-called QCD like model (which will be called Model A) where $\alpha(p)$ has the same form[10] as in the perturbative region

$$\alpha(p^2) = \frac{4\pi}{\beta_0 ln \frac{p^2}{\Lambda^2}}. \tag{32}$$

From the renormalization group analysis there is a relation between $< \psi\bar{\psi} > (p)$ and $\alpha(p)$ in the perturbative region[25]

$$< \psi\bar{\psi} > (p) =< \psi\bar{\psi} > (p_0) \left(\frac{\alpha(p)}{\alpha(p_0)} \right)^{-d}, \tag{33}$$

where p_0 is a fixing point which is renormalization group invariant. If we adapt the QCD-like model for $\alpha(p)$ Eq.(33) also implies the same behavior of $< \psi\bar{\psi} > (p)$ when $\mu < p < \lambda$ as that when $p > \lambda$. Hence the expression of $\Sigma(p^2)$ in Eq.(9) remains unchanged when $\mu < p < \lambda$. This ansatz was also used in Ref.[18].

Another model (which will be called Model B) is based on the Richardson potential[27] in which $\alpha(p)$ depends on a single parameter

$$\alpha(p^2) = \frac{4\pi}{\beta_0 ln\left(1 + \frac{p^2}{\Lambda^2}\right)}. \tag{34}$$

The above expression for $\alpha(p)$ can lead to a linear confining potential after the Fourier transformation. This form has been used to obtain the excellent fit to the data. Implied by Eq.(33) we have the following form for $\Sigma(p^2)$ in the range $p > \mu$ (to the order $O(\alpha)$),

$$\Sigma(p) = \frac{16\pi^2}{3\beta_0} \frac{\hat{\mu}^3}{p^2} \frac{\left[\frac{1}{2}ln(1 + p^2/\Lambda^2) \right]^d}{ln(1 + p^2/\Lambda^2)}. \tag{35}$$

4.2 Solutions for the consistent equation

With the aid of the phenomenological assumptions of $\alpha(p)$ and $\Sigma(p)$ between the points μ and λ Eq.(14) can be solved numerically. In Model B implied by the Richardson potential after substituting Eq.(35) into Eq.(14) we find a critical point $\mu_c \approx 0.2\ GeV$ corresponding to $\alpha_c = \alpha(\mu_c) = 0.66\pi$. $\hat{\mu}$ is zero below α_c, i.e., chiral symmetry remains. When $\alpha > \alpha_c, \hat{\mu}$ has nontrivial solutions. Numerical results are listed in Table 1 where we have used $\Lambda = 200\ GeV$.

It has been shown from Table 1 that with the decrease of the fixing point μ from the critical point there is a sharp increase of $\hat{\mu}$ and then it changes slowly. $\Sigma(\mu)$ increases with

the decrease of μ. Σ is the dynamical mass and is responsible for generating the constituent quark mass which is about 300 MeV for light quarks. Consequently one should choose such a fixing point μ that $\Sigma(\mu)$ is about the constituent quark mass. Table 1 shows that $\mu = 0.10 \ GeV$ is such a point at which $\Sigma(\mu)$ is about 300 MeV. Therefore $\hat{\mu} = 69 \ MeV$ is the solution of the gauge-invariant quark condensate at $O(\alpha)$. Table 1 also shows that $\hat{\mu}$ does not change a lot in the range of $\Sigma(\mu)$ from 50 MeV~300 MeV. Hence $\hat{\mu}$ should be around 70 MeV at $O(\alpha)$. This value coincides with the upper bound $\hat{\mu} < (170 \pm 50)MeV$ in Ref.[28] which follows from the combination of the spectral representation properties of the hadronic axial currents two-point functions and their behavior in the deep euclidean region. However, it is too small to compare with the QCD parameter Λ. This is because we have neglected the $O(\alpha^2)$ modifications to Σ which is big in the $\mu < p < \lambda$ region. In fact, $\Sigma(p)$ has the following expression at $O(\alpha^2)$

$$\Sigma^{(2)}(p) = \Sigma(p)(1 + \Delta(p)), \tag{36}$$

where $\Delta(p)$ is the effect of loop corrections which has the renormalization group improved form in the Landau gauge[21]

$$\Delta(p) = \frac{4}{\beta_0 ln(p^2/\Lambda^2)} \left\{ -\frac{1}{3} - \frac{2}{\beta_0}(\gamma_2 + \frac{2\gamma_1\beta_2}{\beta_0}) + \frac{2\beta_2}{\beta_0}(1 - \frac{2\gamma_1}{\beta_0}) \ lnlnp^2/\Lambda^2 \right.$$
$$\left. + \frac{313}{16} - \frac{5}{6}n_f \right\}, \tag{37}$$

where $\beta_2 = -\frac{51}{4} + \frac{19}{12}n_f$, $\gamma_1 = 2$, $\gamma_2 = \frac{101}{121} - \frac{5}{18}n_f$. The above equation is obtained in the perturbative region. $\Delta(1GeV) = 0.45$, $\Delta(50GeV) = 0.08$. Hence the correction can be ignored as p is large enough, say beyond 50 GeV. It becomes very large in the low momentum area such as 1 GeV. It becomes even more important in the region $p < \lambda$. Just as the case $O(\alpha)$ it is helpful to adapt some phenomenological assumptions. To get an estimate of the quark condensate at $O(\alpha^2)$ we use the same prescription as at $O(\alpha)$ in Model B, i.e., p^2/Λ^2 in the logarithms in Eq.(36) is replaced by $1 + p^2/\Lambda^2$ when $p < 1GeV$. In order not to drop the $O(\alpha^2)$ contributions in the perturbative range we use the expression of $\Sigma(p)$ at $O(\alpha)$ only when $p > 50GeV$. The numerical results are shown in Table 2.

The behaviors of $\hat{\mu}$ and $\Sigma^{(2)}(\mu)$ are just like the $O(\alpha)$ case, but with a large enhancement of $\hat{\mu}$. The critical point for chiral symmetry breaking to takes place is $\mu_c = 0.43 \ GeV$ corresponding $\alpha_c = 0.20\pi$ which is compatible with the value of Ref.[17]. $\Sigma(\mu)$ does not change a lot in the region between 162 MeV and 375 MeV. Hence $\hat{\mu} \simeq 150MeV$ at the order $O(\alpha^2)$. This value is compatible with the QCD parameter Λ and consistent with the constraint $\hat{\mu} < (170 \pm 50)MeV$. The important feature that quarks condense rapidly beyond α_c is the same as the case $O(\alpha)$.

Apart from Model B calculations are also carried in Model A. The results at $O(\alpha)$ and $O(\alpha^2)$ are shown in Table 3 and Table 4 respectively.

It can be seen that $\alpha_c = \pi$, $\hat{\mu} \simeq 85MeV$ at $O(\alpha)$ and $\alpha_c = 0.20\pi, \hat{\mu} \simeq 155MeV$

at $O(\alpha^2)$ in Model A respectively. The picture that beyond α_c quark condensation takes place quickly and then changes slowly remains in Model A. Our solutions are consistent in these two models. It is expected so since the picture for chiral symmetry breaking should be model-independent in spite of numerical differences among the different models. It should be emphasized here that in the approach of the consistent equation it does not matter what kinds of forces contribute to chiral symmetry meaking if one knows an exact form of $\Sigma(p, < \bar{\psi}\psi >, \cdots)$. In principle an exact form of $\Sigma(p, < \bar{\psi}\psi, \cdots)$ should include contributions from all the possible forces.

4.3 Solutions for the improved SD equation

First we solve the SD equation in Model A. As a first step we do not take the nonperturbative kernel associated with the gluon condensate into account. Substituting the expressions of $\alpha(p)$ and $\Sigma(p)$ in Model A into Eqs.(24) and (25) and letting R = 0 we find numerical results in Table 5.

The critical coupling constant α_c is 0.73π. This value is just the same as the result of Ref.[18] where it was obtained from the effective potential method. It is not surprising since we have assumed the same behaviors of $\alpha(p)$ and $\Sigma(p^2)$ in the nonperturbative region and one-gluon-exchange diagram contributes to the kernel.

The value of $\hat{\mu}$ charges slowly between $100MeV < \Sigma(\mu) < 300MeV$. $\hat{\mu} \simeq 100MeV$ is the solution which also satisfies the limit $\hat{\mu} < (170 \pm 50)MeV$.

Now we turn to the solutions of the improved SD equation with the value $R = -1.12GeV^2$. The results from Eq.(24) and (25) are listed in Table 6.

It can be seen from Table 6 that the critical coupling constant α_c is decreased to 0.42π when the intermediate -range force is concerned. This coincides with our general discussions in Section 3 since the involvement of the nonperturbative kernel associated with the gluon condensate should help quark condensate take place. This fact is also illuminated by the value of $\hat{\mu} \simeq 110MeV$ from the improved SD equation which is bigger than that from the SD equation. It can be also seen that when the fixing point μ exceeds μ_c there is a sharp increase of $\hat{\mu}$ and then it changes slowly. This behavior is in agreement with that obtained from the consistent equation. In addition to Model A we also repeated the calculations in Model B. Although there are some changes in the numerical results, the general feature for the picture of dynamical symmetry breaking and the influence of the nonperturbative kernel remain unchanged.

Another interesting result is the behavior of $Z(p^2)$, which is determined by Eqs.(30) and (31). Fig.2 shows the shape of $Z(p^2)$ with $\mu = 0.22GeV$. It can be seen that $Z(p^2) < 1$ in the infrared region and $Z(p^2)$ almost equals to one as $p > 2$ GeV. This fact shows that the nonperturbative kernel singular like $\delta(q)$ plays its role only in the range $p \leq \lambda$. When the large-distance force which is more singular than $\delta(q)$ (for instance, $1/p^4$ potential) is considered, the most infrared behavior of $Z(p^2)$ may be changed. However, the shape of $Z(p^2)$ in the region $p \sim \lambda$ can be believed since $\delta(q)$ is more important than

the infrared forces which are more singular than $\delta(q)$ in this region. When carrying out calculations in the case of the consistent equation we have assumed $Z(p^2) = 1$ which is correct in the perturbative range. From Fig.2 it seems that in the area $p \sim 1 GeV$ where the intermediate range force plays a more important role than the more singular forces this assumption is also not bad. What is not clear is the behavior of $Z(p^2)$ when $p << \lambda$. This is a very complicated problem since the nonperturbative forces in this region are very difficult to be dealt with.

4.4 Some comments

From the numerical solutions of the consistent equation for quark condensate and the improved SD equation in two models (Model A and Model B) a picture for chiral symmetry breaking can be given, i.e., below a critical coupling constant α_c there is no chiral symmetry breaking; as α exceeds α_c there is a sharp increase of the quark condensate and then the condensation changes very slowly until the constituent quark mass is reached. This picture should be model-independent and our solutions support this point.

The numerical value of $\hat{\mu}$ is reasonable and coincides with the present upper bound. The advantage of the consistent equation is that it includes all range forces automatically after we adopt some phenomenological model of $\Sigma(p^2)$. Therefore it is more suitable to give the value for $\hat{\mu}$. The SD equation is more useful in discussing the forces which may be responsible for chiral symmetry breaking. Our results indicate that the intermediate range force also contributes to chiral symmetry breaking. It can be expected that the influence of the nonperturbative kernel is about $5\% \sim 35\%$ of the perturbative contribution[9] since we have followed the QCD sum rule's idea, i.e., to start from the asymptotic region and then to add nonperturbative effects as modifications.

5   Intermediate-range force effects on chiral symmetry breaking

As pointed out earlier the kernel (21) plays its role mainly in the intermediate range. In this section we will analyse its effects on chiral symmetry breaking. The condition that chiral symmetry breaking exists is

$$\{S(p), \gamma_5\} = 2i\gamma_5 B(p^2) \neq 0, \tag{38}$$

if we decompose S(p) as

$$S(p) = -i(A(p^2)\not{p} + B(p^2)). \tag{39}$$

Substituting the kernel (21) into the SD equation (16) we have

$$i\Sigma(p) = \frac{RC_F}{32}\Gamma_\mu(p, 0)S(p)\gamma^\mu. \tag{40}$$

Actually the potential with the singularity of $\delta(q)$ was first assumed by Munczek and Nemirovsky[29] by decomposing the gluon propagator into two part. The first part is $\delta(q)$

which represents the potential in the region of average quark- antiquark separation while the second part determines the potential at the origin and at large distances. The term $\delta(q)$ was used as the zeroth-order potential to obtain the meson mass spectrum[29,30].

In order to cancel $\Gamma_\mu(p,0)$ in Eq.(40) we just follow the method in Ref.[12] to make use of the STW identity in the Landau gauge,

$$-ik^\mu \Gamma_\mu(p,k)[1+b(k^2)] = [1-B(k,p)]S^{-1}(p+k) - S^{-1}(p)[1-B(k,p)], \qquad (41)$$

where $B(k,p)$ is the ghost-quark scattering kernel and $b(k^2)$ is the ghost self-energy which obeys the following SD equation

$$k^2 b(k^2) = -ig^2 C_A \int \frac{d^4q}{(2\pi)^4} G_\mu(k,q) G(k-q)(k-q)_\nu D^{\mu\nu}(q), \qquad (42)$$

where $G_\mu(k,q) = k^\alpha G_{\alpha\mu}(k,q)$ is the ghost-gluon vertex function, C_A is the gauge field casimir invariant and $G(k)$ is the ghost propagator,

$$G(k) = \frac{i}{k^2(1+b(k^2))} \qquad (43)$$

Substituting Eqs.(21) and (43) into Eq.(42) the SD equation for $b(k^2)$ can be obtained,

$$b(k^2)k^2(1+b(k^2)) = \frac{(2\pi)^4}{32} R C_F C_A k^\mu G_\mu(k,0). \qquad (44)$$

With the aid of Eqs.(41)-(44) we obtain a final expression of the STW identity

$$\left(1+\frac{b}{2}\right)\Gamma_\mu(p,0) = i\frac{\partial S^{-1}(p+k)}{\partial k_\mu}\Big|_{k_\mu=0} - \frac{1}{2}bS(p)\Gamma_\mu(p,0)S^{-1}(p), \qquad (45)$$

where $b = b(0)$, which is treated as a parameter.

Using the decomposition of the quark propagator (39) Eqs.(40) and (45) are reduced to the following forms in Euclidean space

$$(1+b)B^4 - \frac{4+3b}{1+b}tA^4 - \frac{2}{1+b}t^2A^3A' - 2tAB^2A' - \frac{2b}{1+b}tA^2BB' - 4A^2B^2 = 0,$$

$$tA^2B + (1+2b)B^3 + \frac{2}{1+b}tA^2B' + 2B^2B' - \frac{2b}{1+b}tABA' - \frac{b}{1+b}A^2B = 0, \quad (46)$$

where $A \equiv A(t), B \equiv B(t)$ which are dimensionless variables,

$$A(p^2) = -\frac{32}{RC_F}A(t), \ B(p^2) = \sqrt{-\frac{32}{RC_F}}B(t), \ (t = -\frac{32}{RC_F}p^2). \qquad (47)$$

The derivatives in Eq.(46) are with respect to t.

In principle, the nonperturbtative quark propagator, determined by the intermediate range force $\delta(q)$, can be solved from Eq.(46). We have not found its analytic solutions

since it is nonlinear. However, Eq.(46) can be solved exactly in the following two specific cases.
(i) $1 + b = 0$, we have

$$A(t) = c_1 t^{-1/2}, \ B(t) = c_2, \tag{48}$$

where c_1 and c_2 are integral constants.
(ii) $b = 0$, we have

$$A^2(t) = \frac{c_1^2}{t^4} - c_2^2 e^{-t} \left(\frac{1}{t} + \frac{3}{t^2} + \frac{6}{t^3} + \frac{6}{t^4} \right), \ B(t) = c_2 e^{-t/2}. \tag{49}$$

It can be seen the intermediate-range force may also lead to dynamical chiral symmetry breaking in the above two cases since the condition (38) is satisfied.

In Ref.[31] the authors analysed the gauge-ghost field proper vertices and indicated that $b = -1$ in Pagels' case where the confinement force is linear. It is also found that $b = -1$ in our case if we follow the calculations in Ref.[31].

6   Conclusions and discussions

In summary we present the calculations of the gauge invariant quark condensate $\hat{\mu}$ and the analysis on the range of forces for chiral symmetry breaking. The calculations of $\hat{\mu}$ are carried out both in the consistent equation for quark condensate and in the improved SD equation. The value of $\hat{\mu}$ is reasonable. The solutions in these two approaches suggest an intuitive picture for the mechanism of chiral symmetry breaking which is in agreement with studies before. This picture is independent of the phenomenological models we adopted in the nonperturbative region. We also find that besides confinement and short-distance force the intermediate-range QCD force also contributes to chiral symmetry breaking. Hence we can conclude that forces in difference ranges are all responsible for chiral symmetry breaking.

In our calculations we have neglected the possible contributions to $\Sigma(p)$ from higher dimension condensates. The most familiar one is the gluon condensate $< G^2 >$. This term is proportional to the current quark mass, hence it has no corrections to $\Sigma(p)$ in the chiral limit. Other condensates have higher dimensions. For instance the four quark condensate $< \psi\bar{\psi}\psi\bar{\psi} >$ may be factorized to $< \psi\bar{\psi} >^2$, so one may expect its correction to be small.

In the case of the consistent equation, the $O(\alpha^2)$ corrections to $\hat{\mu}$ is very large since $\Delta(p)$ is considered. If there is an enhancement of Σ at the order $O(\alpha^2)$ comparing to $\Sigma^{(2)}$, μ_c will be bigger. Thus the contributions to $\Sigma(p)$ from nonperturbative regions become smaller. Therefore it is expected that the corrections from $O(\alpha^3)$ will be less important than that at $O(\alpha^2)$. This discussion is also valid for higher order corrections.

In the discussions of the improved SD equation we do not take higher order modifications into account. This is because the kernel will also be modified and it is very complicated. Therefore the consistent equation is more useful in determining the value of the quark condensate.

Another problem is the choice of gauge which is also met elsewhere. We have worked in the Landau gauge. However, the quark condensate is a gauge independent quantity. In Ref.[10] the author pointed out that the Landau gauge seems to be the favorite gauge in some processes. This problem may be overcome by introducing the gauge invariant fermion propagator by means of the Schwinger connector[17,32].

Acknowledgment

Table 1. Values of $\hat{\mu}$ and $\Sigma(\mu)$ at $O(\alpha)$ (Model B)

$\mu(GeV)$	0.20	0.19	0.18	0.17	0.16	0.15	0.14	0.13	0.10
$\hat{\mu}(MeV)$	0	55.4	66.2	70.2	71.8	72.2	71.9	71.1	68.5
$\Sigma(\mu)(MeV)$	0	24	51	72	92	113	136	163	318

Table 2. Values of $\hat{\mu}$ and $\Sigma(\mu)$ at $O(\alpha^2)$ (Model B)

$\mu(GeV)$	0.43	0.40	0.38	0.36	0.34	0.32	0.30
$\hat{\mu}(MeV)$	0	138.0	146.7	150.5	151.3	150.5	148.7
$\Sigma^{(2)}(MeV)$	0	114	162	208	256	310	375

Table 3. Values of $\hat{\mu}$ and $\Sigma(\mu)$ at $O(\alpha)$ (Model A)

$\mu(GeV)$	0.25	0.24	0.23	0.22	0.21	0.205
$\hat{\mu}(MeV)$	0	76.5	84.3	87.2	86.2	83.1
$\Sigma(MeV)$	0	58	99	148	227	312

Table 4. Values of $\hat{\mu}$ and $\Sigma(\mu)$ at $O(\alpha^2)$ (Model A)

$\mu(GeV)$	0.46	0.45	0.43	0.41	0.39	0.37	0.35
$\hat{\mu}(MeV)$	0	120.1	143.1	153.3	156.2	156.2	153.5
$\Sigma^{(2)}(MeV)$	0	59	117	171	219	270	325

Table 5. Values of $\hat{\mu}$ and $\Sigma(\mu)$ from the SD equation (Model A)

| $\mu(GeV)$	0.27	0.26	0.25	0.24	0.23	0.22	0.21
$\hat{\mu}(MeV)$	0	86.7	95.5	99.7	101.1	100.8	98.4
$\Sigma(\mu)(MeV)$	0	59	94	129	171	229	338

Table 6. Values of $\hat{\mu}$ and $\Sigma(\mu)$ from the improved SD equation (Model A)

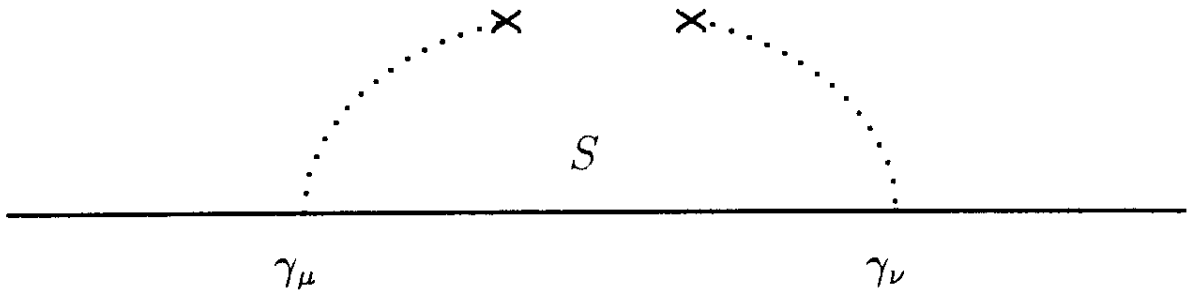| $\mu(GeV)$	0.31	0.30	0.29	0.28	0.27	0.26	0.25	0.24	0.23	0.22
$\hat{\mu}(MeV)$	0	112.2	120.1	124.4	124.4	124.4	122.0	118.7	114.0	108.7
$\Sigma(\mu)(MeV)$	0	76	104	131	150	175	196	219	245	287

Figure Captions:

Fig.1 Kernels of the SD equation. The dashed line denotes gluon. (a) Perturbative Kernel; (b) Nonperturbative kernel which is from gluon condensate.

Fig.2 Behavior of $Z(p^2)$.

$$S$$

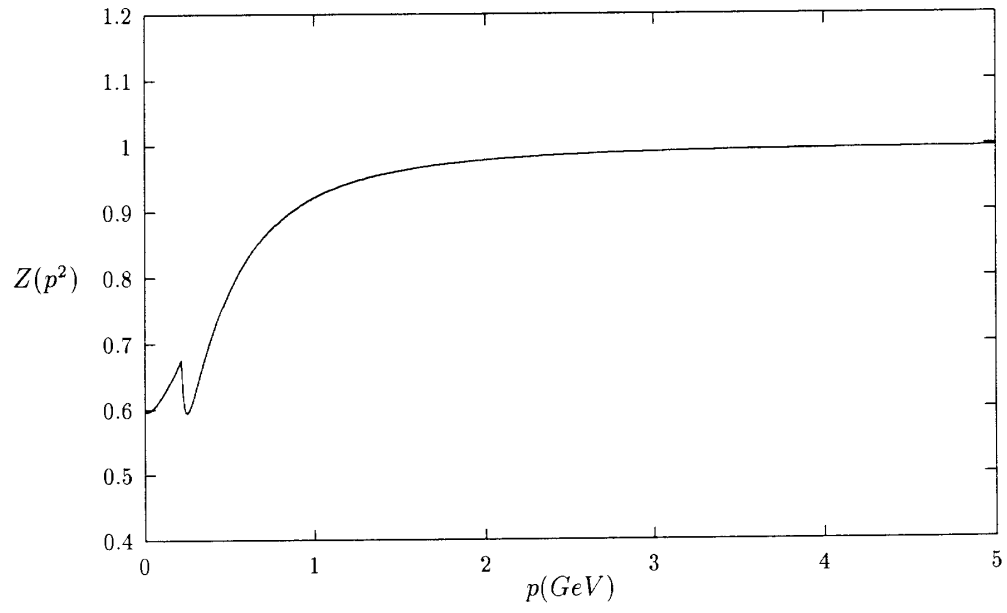$$\gamma_\mu \qquad\qquad \gamma_\nu$$
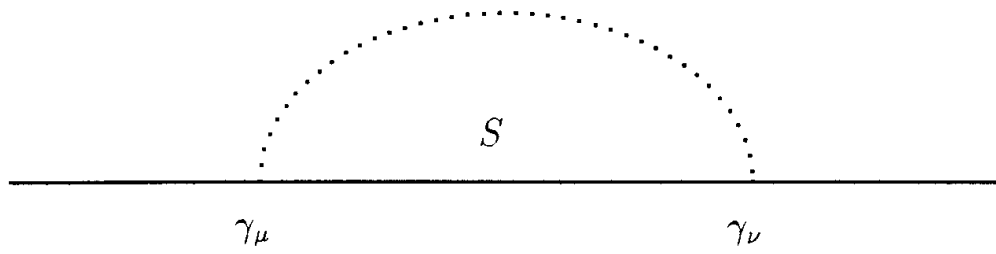
Fig.1(a)

Fig.1(b)

Fig.2

References

[1] H. Hamber and G. Parisi, Phys. Rev. Lett. 47(1981)1742;
    E. Marinari, G. Parisi and C. Rebbi, Phys. Rev. Lett. 47(1981)1795.

[2] D. Espriu, P. Pascual and R. Tarrach, Nucl. Phys. B214(1983)285;
    B. L. Ioffe and A. V. Smilga, Nucl. Phys. B232(1984)109;
    I. I. Balitsky and A. V. Yung, Phys. Lett. B129(1983)328.

[3] H. D. Politzer, Nucl. Phys. B117(1976)397.

[4] V. Elias, T. G. Steel and M. D. Scadron, Phys. Rev. D38(1988)1584.

[5] M. Gell-Mann and M. Levy, Nuovo Cimento 16(1960)705.

[6] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122(1961)345.

[7] J. Goldstone, Nuovo Cimento 19(1961)154.

[8] S. Coleman, J. Math. Phys. 7(1986)787.

[9] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B147(1978)385,
    448; For a general review see L. J. Reinders et al., Phys. Rep. 127(1985)1;
    V. L. Chernyak and A.R. Zhatnitsky, Phys. Rep. 112(1984)173.

[10] K. Higashijima, Phys. Rev. D29(1984)1228.

[11] J. M. Cornwall, Phys. Rev. D22(1980)1452;
     A. Casher, Phys. Lett. 83B(1979)395.

[12] H. Pagels, Phys. Rev. D15(1977)2991.

[13] J. Kogut, M. Stone, H. W. Wyld, J. Shigemitsu, S. H. Shenker and D. K. Sinclair,
     Phys. Rev. Lett. 48(1982)1140.

[14] F. Karsh, CERN-TH-5498/89, to appear in a review monograph on Quark-Gluon
     Plasma, edited by R. C. Hwa, to be published by World Scientific.

[15] J. Kogut, M. Stone, H. W. Wyld, W. R. Gibbs, J. Shigemitsu, S. H. Shenker and D.
     K. Sinclair, Phys. Rev. Lett. 50(1983)393.

[16] J. M. Cornwall, R. Jackiw and E. Tomboulis, Phys. Rev. D10(1974)2428.

[17] M. Peskin, in "Recent Advances in Field Theory and Statistical Mechanics", Pro-
     ceedings of the Houches Summer School, Session XXXIX, edited by J. B. Zuber and
     R. Stora (North-Holland, Amsterdam, 1984).

[18] A. Barducci, R. Casalbuoni, S. De curtis, D. Dominici and R. Gatto, Phys. Rev.
     D38(1988)238.

[19] V. Sh. Gogokhia and B. A. Magradze, Phys. Lett. 217B(1989)162;
V. Sh. Gogokhia, Phys. Lett. 224B(1989)177.

[20] K. Higashijima and A. Nishimura, Nucl. Phys. B113(1976)173.

[21] P. Pascual and E. de Rafeal, Z. Phys. C12(1982)127.

[22] G. Parisi and R. Petronzio, Phys. lett. 94B(1980)51; J. M. Cornwall, Phys. Rev. D26(1982)1453.

[23] Y. Ding, T. Huang and Z. Chen, Phys. Lett. 196(1987)191.

[24] L. S. Celenza and C. M. Shakin, Phys. Rev. D34(1986)1591.

[25] T. Huang and Z. Huang, Phys. Rev. D39(1989)1213.

[26] E. E. Salpeter and H. A. Bethe, Phys. Rev. 84(1951)1232.

[27] J. L. Richardson, Phys. Lett. 82B(1979)272.

[28] C. Becchi, S. Narison, E. de Rafeal and F. J. Yndurain, Z. Phys. C8(1981)335.

[29] H. J. Munczek and A. M. Nemirovsky, Rev. D28(1983)181.

[30] T. Huang and Z. Huang, Commun in Theor. Physics, 11(1989)479.

[31] Yu-ping Kuang and Yu-ping Yi, High Energy Phys. and Nucl. Phys. 4(1980)286.

[32] J. Liu, Z. Huang and T. Huang, BIHEP-TH-89-9 (unpublished).