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Light-Front Quantization of Field Theory †

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Abstract

Some basic topics in Light-Front (LF) quantized field theory are reviewed. Poincarè algebra and the *LF Spin operator* are discussed. The local scalar field theory of the conventional framework is shown to correspond to a *non-local Hamiltonian* theory on the LF in view of the *constraint equations* on the phase space, which relate the bosonic condensates to the non-zero modes. This new ingredient is useful to describe the *spontaneous symmetry breaking* on the LF. The instability of the symmetric phase in two dimensional scalar theory when the coupling constant grows is shown in the LF theory renormalized to one loop order. *Chern-Simons gauge theory*, regarded to describe excitations with fractional statistics, is quantized in the light-cone gauge and a simple LF Hamiltonian obtained which may allow us to construct renormalized theory of *anyons*.

Key-words: Light-front; Quantization; Gauge theory; Phase Transition.

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1 Introduction

Dirac[1] in 1949 pointed out the advantage of studying the relativistic quantum dynamics of physical system on the hyperplanes of the LF: $x^0 + x^3 = const.$, *front form*. Seven out of the ten Poincaré generators are here *kinematical* while in the conventional formulation on the hyperplanes $x^0 = const.$, *instant form*, only six have this property. Latter in 1966 the LF field theory was rediscovered by Weinberg[2] in his Feynman rules adapted for infinite momentum frame. Kogut e Soper[3] demonstrated in 1970 that the rules correspond to the quantization on the LF.

The LF vacuum is simpler than the conventional theory vacuum and in many cases the interacting theory vacuum may coincide with the perturbation theory one. This results from the fact that momentum four-vector is now given by $(k^-, k^+, k^\perp)$ where $k^\pm = (k^0 \pm k^3)/\sqrt{2}$. Here k^- is the LF energy while $k^\perp$ and k^+ indicate the transverse and the longitudinal components of the momentum. For a massive particle on the mass shell $k^\pm$ are positive definite and the conservation of the total longitudinal momentum does not permit the excitation of these quanta by the LF vacuum. The recent revival[4,5,6] of the interest in LF quantization owes to the difficulties encountered in the computation of nonperturbative effects, say, in the *instant form* QCD. In the conventional framework QCD vacuum state is quite complex due to the *infrared slavery* and it contains also gluonic and fermionic condensates. There seems to exist contradiction between the Standard Quark Model and the QCD containing quark and and gluon fields. Also in the Lattice gauge theory there is the well known difficulty in handling light fermions. LF quantization may throw some light to clarify this and other issues. In the context of the String theories it has been used, for example, in the case of the heterotic strings[7].

LF coordinates corresponding to (x^0, x^1, x^2, x^3) are defined by $(x^+, x^-, x^\perp)$ where $x^\pm = (x^0 \pm x^3)/\sqrt{2} = x_\mp$ and $x^\perp \equiv \bar{x} : (x^1 = -x_1,\ x^2 = -x_2)$. We will treat $x^+ \equiv \tau$ as the LF *time* coordinate and $x^- \equiv x$ as the longitudinal spatial coordinate. The LF components of any four-vector or any tensor are similarly defined. The metric tensor for the indices $\mu = (+, -, 1, 2)$ is $g^{++} = g^{--} = g^{12} = g^{21} = 0$; $g^{+-} = g^{-+} = -g^{11} =$

$-g^{22} = 1$. The transformation from the conventional to LF coordinates is seen *not* to be a Lorentz transformation.

Any two non-coincident points on the hyperplane $x^0 = const.$ have a spacelike separation: $(x-y)^2|_{x^0=y^0} = -(\vec{x}-\vec{y})^2 < 0$ and it becomes lightlike when the points coincide. The points on the LF hyperplane $x^+ = const.$ also have a spacelike separation: $(x-y)^2|_{x^+=y^+} = -(x^\perp - y^\perp)^2 < 0$ which reduces to lightlike when $x^\perp = y^\perp$, *but* with the important difference that now the points need not be necessarily coincident since $(x^- - y^-)$ may take arbitrary value. Admitting also the validity of the *microscopic causality* principle it can be shown that the appearence of nonlocality in the LF field theory along the longitudinal direction x^- is not necessarily unexpected. Consider, for example, the commutator $[A(x^+, x^-, x^\perp), B(0,0,0^\perp)]_{x^+=0}$ of two scalar observables A and B. The *microcausality* would require it to vanish for $x^\perp \neq 0$ when $x^2|_{x^+=0}$ is spacelike. Consequently it is proportional to $\delta^2(\bar{x})$ and its derivatives which implies locality in $x^\perp$; however, no restriction on the x^- dependence follows. Similar arguments in the equal-time case lead to the locality in all the three space coordinates. We note also that in view of the *microcausality* both $[A(x), B(0]_{x^+=0}$ and $[A(x), B(0)]_{x^0=0}$ may be nonvanishing only on the light cone $x^2 = 0$.

It is interesting to consider the Lehman spectral representation[8] for the scalar field

$$\langle|[\phi(x), \phi(0)]|\rangle_0 = \int_0^\infty d\sigma^2\, \rho(\sigma^2)\, \triangle(x;\sigma^2), \;\; \triangle(x;\sigma^2) = \int_{-\infty}^\infty \frac{d^4k}{(2\pi)^3} \epsilon(k^0)\delta(k^2-\sigma^2)\, e^{-ik.x}$$

Here the spectral function $\rho(\sigma^2)$ is Lorentz invariant and positive definite and $\triangle(x;\sigma^2)$ is the vacuum expectation value (v.e.v.) of the commutator of the free field and $\epsilon(y) = -\epsilon(-y) = \theta(y) - \theta(-y) = 1$ for $y > 0$. For the field theory with a local Lagrangian it can be shown in the equal-time framework that $\int_0^\infty d\sigma^2\, \rho(\sigma^2) = 1$. On the LF, $d^4k = d^2\bar{k}dk^+dk^-$, $k^2 = 2k^+k^- - {k^\perp}^2$, $k.x = k^+x^- + k^-x^+ - k^\perp.x^\perp$, and $(2|k^+|)\delta(k^2-\sigma^2) = \delta(k^- - [\bar{k}^2+\sigma^2]/(2k^+))$. Hence we show that $\triangle(x^+, x^-, \bar{x}; \sigma^2)|_{x^+=0} = -\frac{i}{4}\delta^2(\bar{x})\epsilon(x^-)$ and it follows that on the LF $[\phi(x^+, x^-, \bar{x}), \phi(0)]|_{x^+=0} = -\frac{i}{4}\delta^2(\bar{x})\epsilon(x^-)$ where v.e.v. of the expression is understood. In contrast to the equal-time case the equal-τ commutator is not vanishing and it has a nonlocal dependence on x^-. The same result will be shown to follow also in the canonical quantization on the LF when we use the Dirac procedure[9] in order to construct the Hamiltonian framework. We remind that any field theory written in terms of the LF coordinates describes necessarily a constrained dynamical system with a singular Lagrangian.

We remark that in the LF quantization we (time) order with respect to x^+ rather than x^0. The *microcausality*, however, ensures that the retarded commutators $[A(x), B(0)]\theta(x^0)$ and $[A(x), B(0)]\theta(x^+)$ do not lead to disagreement in the two formulations. In fact in the regions $x^0 > 0, x^+ < 0$ and $x^0 < 0, x^+ > 0$, where the commutators appear to give different values the x^2 is spacelike and consequently both of them vanish. Such (retarded) commutators in fact appear in the S-matrix elements when we use the Lehmann, Symanzik and Zimmerman (LSZ)[10] reduction formulae.

2. Poincare Generators on the LF

The Poincaré generators in coordinate system (x^0, x^1, x^2, x^3), satisfy $[M_{\mu\nu}, P_\sigma] = -i(P_\mu g_{\nu\sigma} - P_\nu g_{\mu\sigma})$ and $[M_{\mu\nu}, M_{\rho\sigma}] = i(M_{\mu\rho} g_{\nu\sigma} + M_{\nu\sigma} g_{\mu\rho} - M_{\nu\rho} g_{\mu\sigma} - M_{\mu\sigma} g_{\nu\rho})$ where the metric is $g_{\mu\nu} = diag\,(1, -1, -1, -1)$, $\mu = (0, 1, 2, 3)$ and we take $\epsilon_{0123} = \epsilon_{-+12} = 1$. If we define $J_i = -(1/2)\epsilon_{ikl} M^{kl}$ and $K_i = M_{0i}$, where $i, j, k, l = 1, 2, 3$, we find $[J_i, F_j] = i\epsilon_{ijk} F_k$ for $F_l = J_l, P_l$ or K_l while $[K_i, K_j] = -i\epsilon_{ijk} J_k$, $[K_i, P_l] = -iP_0 g_{il}$, $[K_i, P_0] = iP_i$, and $[J_i, P_0] = 0$.

The LF generators are P_+, P_-, P_1, P_2 , $M_{12} = -J_3$, $M_{+-} = -K_3$, $M_{1-} = -(K_1 + J_2)/\sqrt{2} \equiv -B_1$, $M_{2-} = -(K_2 - J_1)/\sqrt{2} \equiv -B_2$, $M_{1+} = -(K_1 - J_2)/\sqrt{2} \equiv -S_1$, and $M_{2+} = -(K_2 + J_1)/\sqrt{2} \equiv -S_2$. We find $[B_1, B_2] = 0, [B_a, J_3] = -i\epsilon_{ab} B_b, [B_a, K_3] = iB_a, [J_3, K_3] = 0, [S_1, S_2] = 0, [S_a, J_3] = -i\epsilon_{ab} S_b, [S_a, K_3] = -iS_a$ where $a, b = 1, 2$ and $\epsilon_{12} = -\epsilon_{21} = 1$. Also $[B_1, P_1] = [B_2, P_2] = iP^+, [B_1, P_2] = [B_2, P_1] = 0, [B_a, P^-] = iP_a, [B_a, P^+] = 0, [\mathrm{S}_1, P_1] = [S_2, P_2] = iP^-, [S_1, P_2] = [S_2, P_1] = 0, [S_a, P^+] = iP_a, [S_a, P^-] = 0, [B_1, S_2] = -[B_2, S_2] = -iJ_3, [B_1, S_1] = [B_2, S_2] = -iK_3$. For $P_\mu = i\partial_\mu$, and $M_{\mu\nu} \to L_{\mu\nu} = i(x_\mu \partial_\nu - x_\nu \partial_\mu)$ we find $B_a = (x^+ P^a - x^a P^+), S_a = (x^- P^a - x^a P^-), K_3 = (x^- P^+ - x^+ P^-)$ and $J_3 = (x^1 P^2 - x^2 P^1)$. Under the conventional *parity* operation $\mathcal{P}$: ($x^{\pm} \leftrightarrow x^{\mp}, x^{1,2} \to -x^{1,2}$) and ($p^{\pm} \leftrightarrow p^{\mp}, p^{1,2} \to -p^{1,2}$), we find $\vec{J} \to \vec{J}$, $\vec{K} \to -\vec{K}$, $B_a \to -S_a$ etc.. The six generators P_l, M_{kl} leave $x^0 = 0$ hyperplane invariant and are called[1] *kinematical* while the remaining P_0, M_{0k} the *dynamical* ones. On the LF there are *seven* kinematical generators : $P^+, P^1, P^2, B_1, B_2, J_3$ and K_3 which leave the LF hyperplane, $x^0 + x^3 = 0$, invariant and the three *dynamical* ones S_1, S_2 and P^- form a mutually commuting set. We note that each of the set $\{B_1, B_2, J_3\}$ and $\{S_1, S_2, J_3\}$ generates an $E_2 \simeq SO(2) \otimes T_2$ algebra; this will be shown below to be relevant for defining the *spin* for massless particle. Including K_3 in each set we find two subalgebras each with four elements. Some useful identities are $e^{i\omega K_3}\, P^{\pm}\, e^{-i\omega K_3} = e^{\pm\omega}\, P^{\pm}$, $e^{i\omega K_3}\, P^{\perp}\, e^{-i\omega K_3} = P^{\perp}, e^{i\bar{v}.\bar{B}}\, P^-\, e^{-i\bar{v}.\bar{B}} = P^- + \bar{v}.\bar{P} + \frac{1}{2}\bar{v}^2 P^+, e^{i\bar{v}.\bar{B}}\, P^+\, e^{-i\bar{v}.\bar{B}} = P^+, e^{i\bar{v}.\bar{B}}\, P^{\perp}\, e^{-i\bar{v}.\bar{B}} = P^{\perp} + v^{\perp} P^+, e^{i\bar{u}.\bar{S}}\, P^+\, e^{-i\bar{u}.\bar{S}} = P^+ + \bar{u}.\bar{P} + \frac{1}{2}\bar{u}^2 P^-, e^{i\bar{u}.\bar{S}}\, P^-\, e^{-i\bar{u}.\bar{S}} = P^-, e^{i\bar{u}.\bar{S}}\, P^{\perp}\, e^{-i\bar{u}.\bar{S}} = P^{\perp} + u^{\perp} P^-$ where $P^{\perp} \equiv \bar{P} = (P^1, P^2)$, $v^{\perp} \equiv \bar{v} = (v_1, v_2)$ and $(v^{\perp}.P^{\perp}) \equiv (\bar{v}.\bar{P}) = v_1 P^1 + v_2 P^2$ etc. Analogous expressions with P^{μ} replaced by X^{μ} can be obtained if we use $[P^{\mu}, X_{\nu}] \equiv [i\partial^{\mu}, x_{\nu}] = i\delta^{\mu}_{\nu}$.

3. LF Spin Operator. Hadrons in LF Fock Basis

The Casimir generators of the Poincaré group are : $P^2 \equiv P^{\mu} P_{\mu}$ and W^2, where $W_{\mu} = (-1/2)\epsilon_{\lambda\rho\nu\mu} M^{\lambda\rho} P^{\nu}$ defines the Pauli-Lubanski pseudovector. It follows from $[W_{\mu}, W_{\nu}] = i\epsilon_{\mu\nu\lambda\rho} W^{\lambda} P^{\rho}$, $[W_{\mu}, P_{\rho}] = 0$ and $W.P = 0$ that in a representation charactarized by particualr eigenvalues of the two Casimir operators we may simultaneously diagonalize P^{μ} along with just one component of W^{μ}. We have $W^+ = -[J_3 P^+ + B_1 P^2 - B_2 P^1], W^- = J_3 P^- + S_1 P^2 - S_2 P^1, W^1 = K_3 P^2 + B_2 P^- - S_2 P^+$, and $W^2 = -[K_3 P^1 + B_1 P^- - S_1 P^+]$ and it shows that W^+ has a special place since it contains only the kinematical generators. On the LF we define $\mathcal{J}_3 = -W^+/P^+$ as the *spin operator*[11]. It may be shown to commute with P_{μ}, B_1, B_2, J_3, and K_3. For $m \neq 0$ we may use the parametrizations $p^{\mu} : (p^- = (m^2 + {p^{\perp}}^2)/(2p^+), p^+ = (m/\sqrt{2})e^{\omega}, p^1 = -v_1 p^+, p^2 = -v_2 p^+)$ and $\tilde{p}^{\mu} : (1, 1, 0, 0)(m/\sqrt{2})$ in the rest frame. We have $P^2(p) = m^2 I$ and $W(p)^2 = W(\tilde{p})^2 = -m^2[J_1^2 + J_2^2 + J_3^2] = -m^2 s(s+1) I$ where s assumes half-integer values. Starting from the rest state $|\tilde{p}; m, s, \lambda, ..\rangle$ with $J_3\,|\tilde{p}; m, s, \lambda, ..\rangle = \lambda\,|\tilde{p}; m, s, \lambda, ..\rangle$ we may build an arbitrary eigenstate of $P^+, P^\perp, \mathcal{J}_3$ (and P^-) on the LF by

$$|p^+, p^\perp; m, s, \lambda, ..\rangle = e^{i(\bar{v}.\bar{B})} e^{-i\omega K_3}\,|\tilde{p}; m, s, \lambda, ..\rangle \tag{1}$$

If we make use of the following *identity* for the spin operator

$$\mathcal{J}_3(p) = \; J_3 + v_1 B_2 - v_2 B_1 = \quad e^{i(\bar{v}.\bar{B})}\; J_3\; e^{-i(\bar{v}.\bar{B})} \tag{2}$$

we find $\mathcal{J}_3\,|p^+, p^\perp; m, s, \lambda, ..\rangle = \lambda\,|p^+, p^\perp; m, s, \lambda, ..\rangle$. Introducing also $\mathcal{J}_a = -(\mathcal{J}_3 P^a + W^a)/\sqrt{P^\mu P_\mu}$, $a = 1, 2$, which contain dynamical generators we verify that $[\mathcal{J}_i, \mathcal{J}_j] = i\epsilon_{ijk}\mathcal{J}_k$.

For $m = 0$ case when $p^+ \neq 0$ a convenient parametrization is $p^\mu : (p^- = p^+ {v^\perp}^2/2,\; p^+, p^1 = -v_1 p^+, p^2 = -v_2 p^+)$ and $\tilde{p} : (0, p^+, 0^\perp)$. We have $W^2(\tilde{p}) = -(S_1^2 + S_2^2){p^+}^2$ and $[W_1, W_2](\tilde{p}) = 0$, $[W^+, W_1](\tilde{p}) = -ip^+ W_2(\tilde{p})$, $[W^+, W_2](\tilde{p}) = ip^+ W_1(\tilde{p})$ showing that W_1, W_2 and W^+ generate the algebra $SO(2) \otimes T_2$. The eigenvalues of W^2 are hence not quantized and they vary continuously. This is contrary to the experience so we impose that the physical states satisfy in addition $W_{1,2}|\,\tilde{p};\, m = 0, ..\rangle = 0$. Hence $W_\mu = -\lambda P_\mu$ and the invariant parameter λ is taken to define as the *spin* of the massless particle. From $-W^+(\tilde{p})/\tilde{p}^+ = J_3$ we conclude that λ assumes half-integer values as well. We note that $W^\mu W_\mu = \lambda^2 P^\mu P_\mu = 0$ and that on the LF the definition of the spin operator appears unified for massless and massive particles. A parallel discussion based on $p^- \neq 0$ may also be given.

As an illustration consider the three particle state on the LF with the total eigenvalues p^+, λ and $p^\perp$. In the *standard frame* with $p^\perp = 0$ it may be written as $(|x_1 p^+, k_1^\perp; \lambda_1\rangle |x_2 p^+, k_2^\perp; \lambda_2\rangle |x_3 p^+, k_3^\perp; \lambda_3\rangle\;)$ with $\sum_{i=1}^3 x_i = 1$, $\sum_{i=1}^3 k_i^\perp = 0$, and $\lambda = \sum_{i=1}^3 \lambda_i$. Aplying $e^{-i(\bar{p}.\bar{B})/p^+}$ on it we obtain $(|x_1 p^+, k_1^\perp + x_1 p^\perp; \lambda_1\rangle |x_2 p^+, k_2^\perp + x_2 p^\perp; \lambda_2\rangle |x_3 p^+, k_3^\perp + x_3 p^\perp; \lambda_3\rangle\;)$ now with $p^\perp \neq 0$. The x_i and $k_i^\perp$ indicate relative (invariant) parameters and do not depend upon the reference frame. The x_i is the fraction of the total longitudinal momentum carried by the i^{th} particle while $k_i^\perp$ its transverse momentum. The state of a pion with momentum $(p^+, p^\perp)$, for example, may be expressed as an expansion over the LF Fock states constituted by the different number of partons

$$|\pi : p^+, p^\perp\rangle = \sum_{n,\lambda} \int \bar{\Pi}_i \frac{dx_i d^2 k^\perp{}_i}{\sqrt{x_i}\, 16\pi^3} |n : x_i p^+, x_i p^\perp + k^\perp{}_i, \lambda_i\rangle \; \psi_{n/\pi}(x_1, k^\perp{}_1, \lambda_1; x_2, ...) \tag{3}$$

where the summation is over all the Fock states n and spin projections λ_i, with $\bar{\Pi}_i dx_i = \Pi_i dx_i \;\delta(\sum x_i - 1)$, and $\bar{\Pi}_i d^2 k_i^\perp = \Pi_i dk_i^\perp \;\delta^2(\sum k_i^\perp)$. The wave function of the parton $\psi_{n/\pi}(x, k^\perp)$ indicates the probability amplitue for finding inside the pion the partons in the Fock state n carrying the 3-momenta $(x_i p^+, x_i p^\perp + k_i^\perp)$. The Fock state of the pion is also off the energy shell : $\sum k_i^- > p^-$.

The *discrete symmetry* transformations may also be defined on the LF Fock states. For example, under the conventional parity $\mathcal{P}$ the spin operator $\mathcal{J}_3$ is not left invariant. We may rectify this by defining *LF Parity operation* by $\mathcal{P}^{lf} = e^{-i\pi J_1}\mathcal{P}$. We find then $B_1 \to -B_1, B_2 \to B_2, P^\pm \to P^\pm, P^1 \to -P^1, P^2 \to P^2$ etc. such that $\mathcal{P}^{lf}|p^+, p^\perp; m, s, \lambda, ..\rangle \simeq |p^+, -p^1, p^2; m, s, -\lambda, ..\rangle$. Similar considerations apply for charge conjugation and time inversion. For example, it is straightforward to construct the free *LF Dirac spinor* $\chi(p) = [\sqrt{2}p^+\Lambda^+ + (m - \gamma^a p^a)\,\Lambda^-]\tilde{\chi}/\sqrt{\sqrt{2}p^+ m}$ which is also an eigenstate os $\mathcal{J}_3$ with eigenvalues $\pm 1/2$. Here $\Lambda^\pm = \gamma^0\gamma^\pm/\sqrt{2} = \gamma^\mp\gamma^\pm/2 = (\Lambda^\pm)^\dagger$, $(\Lambda^\pm)^2 = \Lambda^\pm$, and $\chi(\tilde{p}) \equiv \tilde{\chi}$ with $\gamma^0\tilde{\chi} = \tilde{\chi}$. The conventional (equal-time) spinor can also be constructed by the procedure analogous to that followed for the LF spinor and it has the well known form $\chi_{con}(p) = (m + \gamma.p)\tilde{\chi}/\sqrt{2m(p^0 + m)}$. Under the conventional parity operation $\mathcal{P} : \chi'(p') = c\gamma^0\chi(p)$ (since we must require $\gamma^\mu = L^\mu{}_\nu\, S(L)\gamma^\nu S^{-1}(L)$ etc.). We find $\chi'(p) = c[\sqrt{2}p^-\Lambda^- + (m - \gamma^a p^a)\,\Lambda^+]\,\tilde{\chi}/\sqrt{\sqrt{2}p^- m}$. For $p \neq \tilde{p}$ it is not proportional to $\chi(p)$ in contrast to the result in the case of the usual spinor where $\gamma^0\chi_{con}(p^0, -\vec{p}) = \chi_{con}(p)$ for $E > 0$ (and $\gamma^0\eta_{con}(p^0, -\vec{p}) = -\eta_{con}(p)$ for $E < 0$). However, applying parity operator twice we do show $\chi''(p) = c^2\chi(p)$ hence leading to the usual result $c^2 = \pm 1$. The LF parity operator over spin 1/2 Dirac spinor is

$\mathcal{P}^{lf} = c(2J_1)\gamma^0$ and the corresponding transform of χ is shown to be an eigenstate of $\mathcal{J}_3$.

4. Spontaneous Symmetry Breaking (SSB) Mechanism. Continuum Limit of Discretized LF Quantized Theory. Nonlocality of LF Hamiltonian

The quantization of scalar theory in equal-time framework is found in the text books but the existence of the continuum limit of the Discretized Light Cone Quantized (DLCQ)[12] theory, the nonlocal nature of the LF Hamiltonian, and the description of the SSB on the LF were clarified only recently.

Consider first the two dimensional case with $\mathcal{L} = [\dot{\phi}\phi' - V(\phi)]$. Here $\tau \equiv x^+ = (x^0 + x^1)/\sqrt{2}$, $x \equiv x^- = (x^0 - x^1)/\sqrt{2}$, $\partial_\tau \phi = \dot{\phi}, \partial_x \phi = \phi'$, and $d^2x = d\tau dx$. The eq. of motion, $\dot{\phi}' = (-1/2)\delta V(\phi)/\delta\phi$, shows that $\phi = const.$ is a possible solution. We write[13] $\phi(x,\tau) = \omega(\tau) + \varphi(x,\tau)$ where $\omega(\tau)$ corresponds to the *bosonic condensate* and $\varphi(\tau,x)$ describes (quantum) *fluctuations* above it. The value of $\omega (= \langle 0|\phi|0\rangle)$ will be seen to characterize the corresponding vacuum state. The translational invariance of the ground state requires that ω be a constant so that $\mathcal{L} = \dot{\varphi}\varphi' - V(\phi)$. Dirac procedure[9] is applied now to construct Hamiltonian theory which would permit[1] us to to construct a quantized relativistic field theory. We may avoid using distribuitions if we restrict x to a finite interval from $-L/2$ to $L/2$. The *physical limit to the continuum ($L \to \infty$)*, however, must be taken latter to remove the spurious finite volume effects. Expanding φ by Fourier series we obtain $\phi(\tau,x) \equiv \omega + \varphi(\tau,x) = \omega + \frac{1}{\sqrt{L}} q_0(\tau) + \frac{1}{\sqrt{L}} \sum'_{n\neq 0} \; q_n(\tau)\, e^{-ik_n x}$ where $k_n = n(2\pi/L)$, $n = 0, \pm 1, \pm 2, \ldots$ and the *discretized theory* Lagrangian becomes $i\sum_n \; k_n\, q_{-n} \; \dot{q}_n - \int dx \;\; V(\phi)$. The momenta conjugate to q_n are $p_n = ik_n q_{-n}$ and the canonical LF Hamiltonian is found to be $\int dx\, V(\omega + \varphi(\tau,x))$. The primary constraints are thus $p_0 \approx 0$ and $\Phi_n \equiv p_n - ik_n q_{-n} \approx 0$ for $n \neq 0$. We follow[14] the standard Dirac procedure[9] and find three *weak constraints*[9] $p_0 \approx 0$, $\beta \equiv \int dx\, V'(\phi) \approx 0$, and $\Phi_n \approx 0$ for $n \neq 0$ on the phase space and they are shown to be *second class*[9]. We find $(n, m \neq 0)$ $\{\Phi_n, p_0\} = 0$, $\{\Phi_n, \Phi_m\} = -2ik_n\delta_{m+n,0}$, $\{\Phi_n, \beta\} = \{p_n, \beta\} = -(1/\sqrt{L}) \int dx \; [V''(\phi) - V''([\omega + q_0]/\sqrt{L})]\, e^{-ik_n x} \equiv -\alpha_n/\sqrt{L}$,

$\{p_0, \beta\} = -(1/\sqrt{L})\int dx\ V''(\phi) \equiv -\alpha/\sqrt{L}$, $\{p_0, p_0\} = \{\beta, \beta\} = 0$. Implement first the pair of constraints $p_0 \approx 0$, $\beta \approx 0$ by modifying the Poisson brackets to the star bracket $\{\}^*$ defined by $\{f,g\}^* = \{f,g\} - [\{f,p_0\}\ \{\beta,g\} - (p_0 \leftrightarrow \beta)]\,(\alpha/\sqrt{L})^{-1}$. We may then set $p_0 = 0$ and $\beta = 0$ as *strong relations*[9]. We find by inspection that the brackets $\{\}^*$ of the remaining variables coincide with the standard Poisson brackets except for the ones involving q_0 and p_n $(n \neq 0)$: $\{q_0, p_n\}^* = \{q_0, \Phi_n\}^* = -(\alpha^{-1}\alpha_n)$. For example, if $V(\phi) = (\lambda/4)(\phi^2 - m^2/\lambda)^2$, $\lambda \geq 0, m \neq 0$ we find $\{q_0, p_n\}^*\ [\{3\lambda(\omega + q_0/\sqrt{L})^2 - m^2\}L + 6\lambda(\omega + q_0/\sqrt{L})\int dx\varphi + 3\lambda\int dx\,\varphi^2] = -3\lambda\,[2(\omega + q_0/\sqrt{L})\sqrt{L}q_{-n} + \int dx\,\varphi^2\, e^{-ik_n x}]$.

We next implement the constraints $\Phi_n \approx 0$ $(n \neq 0)$. We have $C_{nm} = \{\Phi_n, \Phi_m\}^* = -2ik_n\delta_{n+m,0}$ and its inverse is given by $C^{-1}{}_{nm} = (1/2ik_n)\delta_{n+m,0}$. The *final* Dirac bracket which taking care of all the constraints is then given by

$$\{f,g\}_D = \{f,g\}^* - \sum_n{}' \frac{1}{2ik_n}\{f,\,\Phi_n\}^*\,\{\Phi_{-n},\,g\}^*. \tag{4}$$

where we may now in addition write $p_n = ik_n q_{-n}$. It is easily shown that $\{q_0, q_0\}_D = 0, \{q_0, p_n\}_D = \{q_0,\, ik_n q_{-n}\}_D = \frac{1}{2}\{q_0, p_n\}^*, \{q_n, p_m\}_D = \frac{1}{2}\delta_{nm}$.

The limit to the continuum[14], $L \to \infty$ is taken as usual: $\Delta = 2(\pi/L) \to dk$, $k_n = n\Delta \to k$, $\sqrt{L}\,q_{-n} \to lim_{L\to\infty}\int_{-L/2}^{L/2} dx\,\varphi(x)\,e^{ik_n x} \equiv \int_{-\infty}^{\infty} dx\,\varphi(x)\,e^{ikx} = \sqrt{2\pi}\tilde{\varphi}(k)$ for all n, $\sqrt{2\pi}\varphi(x) = \int_{-\infty}^{\infty} dk\,\tilde{\varphi}(k)\,e^{-ikx}$, and $(q_0/\sqrt{L}) \to 0$. From $\{\sqrt{L}q_m, \sqrt{L}q_{-n}\}_D = L\,\delta_{nm}/(2ik_n)$ following from $\{q_n, p_m\}_D$ for $n, m \neq 0$ we derive, on using $L\delta_{nm} \to \int_{-\infty}^{\infty} dx e^{i(k-k')x} = 2\pi\delta(k-k')$, that $\{\tilde{\varphi}(k), \tilde{\varphi}(-k')\}_D = \delta(k-k')/(2ik)$ where $k, k' \neq 0$. If we use the integral representation of the sgn function the well known LF Dirac bracket $\{\varphi(x,\tau), \varphi(y,\tau)\}_D = -\frac{1}{4}\epsilon(x-y)$ is obtained. The expressions of $\{q_0, p_n\}_D$ (or $\{q_0, \varphi'\}_D$) show that the DLCQ is harder to work with here. The continuum limit of $\beta = 0$ is

$$
\begin{aligned}
&lim_{L\to\infty}\frac{1}{L}\int_{L/2}^{L/2} dx\, V'(\phi) \equiv \\
&\omega(\lambda\omega^2 - m^2) + lim_{L\to\infty}\frac{1}{L}\int_{-L/2}^{L/2} dx\Big[(3\lambda\omega^2 - m^2)\varphi + \lambda(3\omega\varphi^2 + \varphi^3)\Big] = 0
\end{aligned}
\tag{5}
$$

while that for the LF Hamiltonian is ($P^- \equiv H^{l.f.}$)

$$
P^- = \int dx\,\Big[\omega(\lambda\omega^2 - m^2)\varphi + \frac{1}{2}(3\lambda\omega^2 - m^2)\varphi^2 + \lambda\omega\varphi^3 + \frac{\lambda}{4}\varphi^4\Big] \tag{6}
$$

These results follow immediately if we worked directly in the continuum formulation[13]; we do have to handle generalized functions now. In the LF Hamiltonian theory we have an additional new ingredient in the form of the *constraint equation* (5). Elimination of ω using it would lead to a *nonlocal LF Hamiltonian*[14] in contrast to the corresponding local one in the equal-time formulation. At the tree level the integrals appearing in (5) are convergent from the theory of Fourier transform. When $L \to \infty$, it results in $V'(\omega) = 0$, which in the equal-time theory is essentially *added* to it as an external constraint. In the renormalized theory[15] the constraint equation describes the high order quantum corrections to the tree level value of the condensate.

The quantization is performed via the correspondence $i\{f, g\}_D \to [f, g]$. Hence $\varphi(x,\tau) = (1/\sqrt{2\pi})\int dk\;\; \theta(k)\;\; [a(k,\tau)\; e^{-ikx} + a^\dagger(k,\tau)e^{ikx}]/(\sqrt{2k})$, were $a(k,\tau)$ and $a^\dagger(k,\tau)$ satisfy the canonical equal-τ commutation relations, $[a(k,\tau), a(k',\tau)^\dagger] = \delta(k - k')$ etc.. The vacuum state is defined by $a(k,\tau)|vac\rangle = 0\,,\; k > 0$ and the tree level description of the *SSB* is given as follows. The values of $\omega = \langle|\phi|\rangle_{vac}$ obtained from $V'(\omega) = 0$ *characterize* the different vacua in the theory. Distinct Fock spaces corresponding to different values of ω are built as usual by applying the creation operators on the corresponding vacuum state. The $\omega = 0$ corresponds to a *symmetric phase* since the hamiltonian is then symmetric under $\varphi \to -\varphi$. For $\omega \neq 0$ this symmetry is violated and the system is in a *broken or asymmetric phase.*

The self-consistency[9] may also be checked. Hamilton's eq. gives $\dot\varphi(x,\tau) = -i\,[\varphi(x,\tau), H^{l.f.}(\tau)] = -\int dy\,\epsilon(x-y)\,V'(\phi(y,\tau))/4$ and we recover the Lagrange eq. $\dot{\varphi}'(x,\tau) = -V'(\phi(x,\tau))/2$. If we substitute the value of $V'(\phi)$ obtained from the latter in the former we find after an integration by parts $\dot{\varphi}(x,\tau) = \dot{\varphi}(x,\tau) - \Big[\dot{\varphi}(\infty,\tau)\epsilon(\infty - x) - \dot{\varphi}(-\infty,\tau)\epsilon(-\infty - x)\Big]/2$. For finite values of x this leads to $\dot{\varphi}(\infty,\tau) + \dot{\varphi}(-\infty,\tau) = 0$. On the other hand, if we integrate the momentum space expansion of $\varphi'(x,\tau)$ given above we may show that $\varphi(\infty,\tau) - \varphi(-\infty,\tau) = 0$. Hence we are led to $\partial_\tau\varphi(\pm\infty,\tau) = 0$ as a self-consistency condition. This is analogous to the condition $\partial_t\varphi(x^1 = \pm\infty, t) = 0$ which in contrast is *added* to the equal-time theory upon invoking physical considerations. The constraint eq. is then seen to follow also upon a space integration of the Lagrange eq.. A self-consistent Hamiltonian formulation can thus be built in the continuum which can also describe the *SSB*.

The extension[14] to 3 + 1 dimensions and to global continuous symmetry is straightforward. Consider real scalar fields $\phi_a (a = 1, 2, ..N)$ which form an isovector of global internal symmetry group $O(N)$. We now write $\phi_a(x,\bar{x},\tau) = \omega_a + \varphi_a(x,\bar{x},\tau)$ and the Lagrangian density is $\mathcal{L} = [\dot{\varphi}_a\varphi'_a - (1/2)(\partial_i\varphi_a)(\partial_i\varphi_a) - V(\phi)]$, where $i = 1, 2$ indicate the transverse space directions. The Taylor series expansion of the constraint equations $\beta_a = 0$ gives a set of coupled eqs. $L\,V'_a(\omega) + V''_{ab}(\omega)\int dx\varphi_b + V'''_{abc}(\omega)\int dx\varphi_b\varphi_c/2 + ... = 0$. Its discussion at the tree level leads to the conventional theory results. The LF symmetry generators are found to be $G_\alpha(\tau) = -i\int d^2\bar{x}dx\varphi'_c(t_\alpha)_{cd}\varphi_d = \int d^2\bar{k}\,dk\,\theta(k)a_c(k,\bar{k})^\dagger(t_\alpha)_{cd}a_d(k,\bar{k})$ where $\alpha,\beta = 1, 2, .., N(N-1)/2$, are the group indices, t_α are hermitian and antisymmetric generators of $O(N)$, and $a_c(k,\bar{k})^\dagger$ $(a_c(k,\bar{k}))$ is creation (destruction) operator contained in the momentum space expansion of φ_c. These are to be contrasted with the generators in the equal-time theory, $Q_\alpha(x^0) = \int d^3x\,J^0 = -i\int d^3x(\partial_0\varphi_a)(t_\alpha)_{ab}\varphi_b - i(t_\alpha\omega)_a\int d^3x(d\varphi_a/dx_0)$. Thus the generators on the LF always annihilate the LF vacuum and the SSB is now seen in the broken symmetry of the quantized theory Hamiltonian. The criterian for the counting of the number of Goldstone bosons on the LF follows to be the same as in the conventional theory. On the other hand, the first term on the right hand side of $Q_\alpha(x^0)$ does annihilate the conventional theory vacuum but the second term gives now non-vanishing contributions for some of the (broken) generators. The symmetry of the vacuum is thereby broken while the quantum Hamiltonian remains invariant. The physical content of SSB in the *instant form* and the *front form*, however, is the same though achieved by differnt descriptions. Alternative proofs[14] on the LF, in two dimensions, can be given of the Coleman's theorem related to the absence of Goldstone bosons and of the pathological nature of massless scalar theory; we are unable to implement the second class constraints over the phase space.

We remark that the simplicity of the LF vacuum is in a sense compensated by the involved nonlocal Hamiltonian. The latter, however, may be treatable using advance computational techniques. In a recent work[15] it was also shown that renormalized theory may be constructed without the need of first solving the constraint eq. for ω. Instead we perform renormalization and obtain a renormalized constraint equation. For $(\phi^4)_2$ theory this along with the equation expressing mass renormalization condition are sufficient to describe the phase transition in the theory. It was found to be of the second order, which agrees with the conjecture of Simon and Griffiths[16], in contrast to the first order transition found if we follow the variational methods.

5. Chern-Simons (CS) Gauge Theory

LF quantization may turn out to be useful for nonperturbative computations in QCD and in the study of relativistic bound states of light fermions. To elucidate some general features in gauge theory quantized on the LF we consider[17] the CS theory described by the singular Lagrangian $\mathcal{L} = (\mathcal{D}^\mu\phi)(\widetilde{\mathcal{D}}_\mu\phi^*) + (\kappa/4\pi)\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho$, which is known to be relevant for the theory of *anyons*- excitations with fractional statistics. Here $\mathcal{D}_\mu = (\partial_\mu + ieA_\mu)$, $\widetilde{\mathcal{D}}_\mu = (\partial_\mu - ieA_\mu)$, and the theory a has a conserved and gauge invariant four-vector current $j^\mu = ie(\phi^*\mathcal{D}^\mu\phi - \phi\widetilde{\mathcal{D}}^\mu\phi^*)$. Its contravariant vector property must remain intact if the Hamiltonian theory constructed is relativistic.

On the LF the light cone gauge (l.c.g.), $A_- = 0$, is clearly accessible in the Lagangian formulation. It will be shown to be so also on the phase space. Before applying the Dirac method to construct an Hamiltonian we must consider the boundary conditions (bcs) on the fields involved in our non-covariant gauge. The self-consistency[9] requires that the Hamiltonian theory must not contradict the Lagrangian theory and we may thus examine first the Lagrange eqs. in l.c.g.. We find an expression of the electric charge Q on integrating (one of) the eq. of motion $2a\partial_- A_1 = j^+$, where $\kappa = 4\pi a$: $Q = \int d^2x\, j^+ = 2a \int dx^1 \left[A_1(x^- = \infty, x^1) - A_1(x^- = -\infty, x^1)\right]$. It follows that if the charge is nonvanishing A_1 can not satisfy the periodic or the vanishing bcs at infinity along x^-. We will assume the *anti-periodic* bcs for the gauge fields along x^- and the vanishing ones along x^1. For the scalar fields similar arguments allow us to assume vanishing bcs at infinity. The canonical Hamiltonian, after integration by parts using these bcs, may then be written as $H_c = \int d^2x \big[(\mathcal{D}_1\phi)(\widetilde{\mathcal{D}}_1\phi^*) - A_+\Omega \big]$ where $\Omega = ie(\pi\phi - \pi^*\phi^*) + a\epsilon^{+ij}\partial_i A_j + \partial_i \pi^i$ and $i = -, 1$. From this as the starting point[17] we apply the Dirac procedure[9] to construct a self-consistent Hamiltonian theory corresponding to the singular CS Lagrangian. We find two first class constraints $\pi^+ \approx 0$ and $\Omega \approx 0$ which generate gauge transformations and four second class ones, $\top \equiv \pi - \widetilde{\mathcal{D}}_-\phi^* \approx 0, \top^* \equiv \pi^* - \mathcal{D}_-\phi \approx 0$, and $\top^i \equiv \pi^i - a\epsilon^{+ij}A_j \approx 0$. The extended Hamiltonian is $H' = H_c + \int d^2x \big[u\top + u^*\top^* + u_i\top^i + u_+\pi^+ \big]$ where u, u^*, u^i, u_+, (and A_+) are Lagrange multiplier fields. The eqs. of motion are obtained from $df(x,\tau)/d\tau = \{f(x,\tau), H'(\tau)\} + \partial f/\partial\tau$ and from them we conclude that a set of multipliers may be chosen such that $A_- \approx 0$ and $dA_-/d\tau \approx 0$. The *local* l.c.g. $A_- \approx 0$ is thus also accessible on the phase space. We add in the theory this gauge-fixing constraints so that now the set of second class constraints becomes $\top_m, \quad m = 1, 2..6$: $\top_1 \equiv \top^-, \top_2 \equiv \top^1, \top_3 \equiv \top, \top_4 \equiv \top^*, \top_5 \equiv A_-, \top_6 \equiv \Omega$ while $\pi^+ \approx 0$ stays first class. The initial Poisson brackets are now modified to define the Dirac brackets $\{f, g\}_D$ such that the second class constraints may be written as *strong equalities*[9] $\top_m = 0$ and $df(x,\tau)/d\tau = \{f(x,\tau), H'(\tau)\}_D + \partial f/\partial\tau$. The Dirac brackets are constructed[17] to be

$$\{f, g\}_D = \{f, g\} - \int d^2u d^2v \, \{f, \top_m(u)\} \, C^{-1}_{mn}(u, v) \, \{\top_n(v), g\} \tag{7}$$

where $C^{-1}(x, y)$ is given by

$$\begin{pmatrix} 0 & -4a\partial^x{}_- & 0 & 0 & 0 & 0 \\ 4a\partial^x{}_- & [\phi^*(x)\phi(y)+\phi(x)\phi^*(y)] & 2ai\phi(x) & -2ai\phi(x)^* & 0 & -4a \\ 0 & 2ai\phi(y) & 0 & (2a)^2 & 0 & 0 \\ 0 & -2ai\phi^*(y) & (2a)^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(2a)^2 \\ 0 & -4a & 0 & 0 & 2(2a)^2 & 0 \end{pmatrix} \frac{K(x-y)}{(2a)^2} \tag{8}$$

It is the inverse of the constraint matrix with the elements $C_{mn} = \{\top_m, \top_n\}$ and $K(x-y) = -(1/4)\,\epsilon(x^- - y^-)\,\delta(x^1 - y^1)$. We find that A_+ which is already absent in $\top_m$, drops out also from H_c since $\Omega = 0$. The $\pi^+ \approx 0$ stays first class even with respect to the Dirac brackets and the multiplier u_+ is left undetermined. The variable π^+ decouples and we may choose $u_+ = 0$ so that π^+ and A_+ are eliminated. The LF Hamiltonian then simplifies to

$$H^{l.f.}(\tau) = \int d^2x\ (\mathcal{D}_1\phi)(\widetilde{\mathcal{D}}_1\phi^*) \tag{9}$$

There is still a $U(1)$ *global* gauge symmetry generated by Q. The scalar fields transform under this symmetry but they are left invariant under the local gauge transformations since, $\{\Omega, f\}_D = 0$. The only *independent variables* left are ϕ and ϕ^* which satisfy the well known equal-τ *LF Dirac brackets*

$$\{\phi,\phi\}_D = 0,\ \{\phi^*,\phi^*\}_D = 0, \quad \{\phi(x,\tau),\phi^*(y,\tau)\}_D = K(x,y) \tag{10}$$

We remark that we could alternatively eliminate π^+ by introducing *another local gauge-fixing weak condition* $A_+ \approx 0$ (and $dA_+/d\tau \approx 0$) which is easily shown to be accessible. The additional modification of brackets does not alter the Dirac brackets of the scalar field already obtained. There is thus *no inconsistency in choosing the two local and weak gauge-fixing conditions* $A_\pm \approx 0$ *on the phase space at one fixed time* τ *in the CS gauge theory*; that they are accessibile follows from the Hamilton's eqs. of motion.

We check now the *self-consistency*. From the Hamilton's eq. for ϕ we derive ($e=1$, $\pi^* = \partial_-\phi$): $\partial_-\partial_+\phi(x,\tau) = \{\pi^*(x,\tau), H(\tau)\}_D = \frac{1}{2}\mathcal{D}_1\mathcal{D}_1\phi - i\mathcal{A}_+\partial_-\phi - \frac{i}{2}(\partial_-\mathcal{A}_+)\phi$ where $-2a\,\partial_-\mathcal{A}_+ = j^1 = -ie(\phi^*\mathcal{D}_1\phi - \phi\widetilde{\mathcal{D}}_1\phi^*)$. On comparing this with the corresponding Lagrange eq. $\partial_+\partial_-\phi = \frac{1}{2}\mathcal{D}_1\mathcal{D}_1\phi - iA_+\partial_-\phi - \frac{i}{2}(\partial_-A_+)\phi$ in the l.c.g. it is suggested for convenience to rename the expression $\mathcal{A}_+$ on the phase space by (the above eliminated) A_+. We thus obtain agreement also with the other Lagrange eq. $-2a\,\partial_-A_+ = j^1 = -ie(\phi^*\mathcal{D}_1\phi - \phi\widetilde{\mathcal{D}}_1\phi^*)$. The Gauss' law eq. is seen to correspond to $\Omega = 0$ and the remaining Lagrange eq. is also shown to be recovered. The Hamiltonian theory in the l.c.g. constructed here is thus shown self-consistent. The variable A_+ has *reappeared* on the phase space and we have *effectively* $A_- = 0$ (and *not* $A_\pm = 0$). Similar discussion can be made in the Coulomb gauge in relation to A^0 and there is *no inconsistency on using the non-covariant local gauges* for the CS system. That only the nonlocal gauges[18] may describe consistently the excitations with fractional statistics in the CS system does not agree with our conclusions. We find that it should also arise in the quantum dynamics of the simpler Hamiltonian theory described by (9) and (10) on the LF in the *local l.c.g.*, which possibly may be used to construct renormalized theory of *anyons*, or in the *local* Coulomb gauge in the conventional framework. In the latter case or in the nonlocal gauges the Hamiltonian is complicated and renormalized theory seems difficult to construct. A *dual description*[17,19] may also be constructed on the LF. We can rewrite the Hamiltonian density as $\mathcal{H} = (\partial_1\hat{\phi})(\partial_1\hat{\phi}^*)$ if we use $A_1 = \partial_1\Lambda$ where $8a\,\Lambda(x^-,x^1) = \int d^2y\,\epsilon(x^- - y^-)\,\epsilon(x^1 - y^1)\,j^+(y)$ and define $\hat{\phi} = e^{i\Lambda}\phi$, $\hat{\phi}^* = e^{-i\Lambda}\phi^*$. The field $\hat{\phi}$ clearly does not have the vanishing Dirac bracket (or commutator) with itself and leads to manifest fractional statistics.

The relativistic invariance of the theory above is shown[17] by checking the Poincaré algebra of the field theory space time symmetry generators. We also come to the conclusion that the *anyonicity* seems not to be related to the unusual (not unexpected[17] in non-covariant gauges) behavior under space rotations (sometimes referred to as rotational anomaly[20,19]) of the scalar or the gauge field but rather to the (renormalized) quantum dynamics of CS system, for example, described by (9) and (10).

6. Conclusions

The LF quantization seems useful and complementary to the conventional one and may be used with some advantage in the context of gauge theories like QCD and CS systems among others for stdying nonperturbative effects. The description of the physical observation (like the SSB, Higgs mechanism, Anyonicity, Phase transition etc.) on the LF may be somewhat different. The self-consistency conditions contained in the constrained dynamical system on the LF (phase space) seem to correspond to (at least some of) the external constraints we generally add in the conventional quantization on the basis of physical considerations. The local non-covariant gauges[21] which have been successfully used in Yang-Mills gauge theories may be used consistently also in the case of CS gauge theory.

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