Hadronic Correction to Coulomb Potential Between Quarks and Diquark Structure

by

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ABSTRACT

We have studied the hadronic correction from the background pion fields due to the chiral symmetry breaking to the Coulomb potential that governs the short-distance behavior of the interactions between the bound quarks. The background fields are associated with the constituent quark mass. We find a modified potential form which favors the diquark structure. We also roughly estimate an influence of this correction on the phase shifts in nucleon-nucleon scattering and find that it may cause an extra middle range attraction between nucleons which is expected.

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I. Introduction

When the transferred momentum between quarks becomes sufficiently low, the system reaches the non-perturbative regime which is characterized by two important properties. The first is the confinement of quarks and gluons inside hadrons and the second is the chiral symmetry breaking. The earlier studies on the hadronic properties indicate that a reasonable explanation of the hadronic spectra demands large masses for u and d quarks [1] and as well as the nucleon magnetic moments [2], the so-called constituent quark mass is about 330 MeV which is much larger than the current quark mass of about $4 \sim 10$ MeV. It is believed that the constituent quark mass comes from a completely different source compared to the current mass. The lattice field theory calculation also confirms the existence of the chiral symmetry breaking [3]. The chiral phase transition is still not fully understood so far, but many investigations of its significance and consequences have been carried out. Diakonov, Petrov and Pobylitsa [4,5] showed that in a nucleon, due to the chiral symmetry breaking the single quark effective Lagrangian is described as

$$L_{QCD}^{low\ momentum} = \overline{\Psi}_{\alpha}(i\slashed{\partial} - m - Me^{i\pi^a\tau^a\gamma_5})\Psi_{\alpha}, \tag{1}$$

where α is the color index and $\pi^a(x)$ is the pion field associated with the nucleon, namely the quarks submerge in the pion background field atmosphere. In this work, we only discuss the SU(2) case, so τ^a is the common Pauli matrices. M is the constituent mass of quarks while m is the current mass which is much smaller compared to M and can be neglected in our later calculations.

More precisely, the $L_{QCD}^{low\ momentum}$ gives rise to an equation of motion for a single quark, but the quark moves in a background field of pions and therefore not indeed free. It is true that the background pion field $\pi^a(x)$ is caused by a collective effect of all quarks inside the baryon, therefore the description is phenomenological, but correctly manifests the chiral properties of the bound quarks. Meanwhile in this scenario, the quark wavefunction $\Psi_{\alpha}(x)$ is a function of coordinates.

Turning to investigate the interaction between the bound quarks, the underlying theory is QCD. Generally speaking, the interaction can be described by the potential model where there are the Coulomb and confinement terms. It is believed that the Coulomb potential is due to the one-gluon-exchange interaction and can be derived in the framework of perturbative QCD, so it governs the short-distance behavior of the potential, while the confinement term comes from the non-perturbative QCD and is still not clear nowadays. Therefore, in this work, we only deal with a correction to the Coulomb part, but not the whole potential.

The traditional method for deriving such a potential is standard. One first writes down the two-quark scattering amplitude via one-gluon-exchange diagrams [6,7] and carries out the Breit expansion with the spinor of free quark in the momentum space [8] where the instantaneous approximation is taken to obtain a potential form in momentum space, finally, a Fourier transformation converts it into a potential in the coordinate space. The procedure is simple and the derived Coulomb potential has a form as

$$V^{Coul}(r) = <\lambda_i^a\lambda_j^a> \frac{\alpha_s}{4r} \qquad\qquad (i \neq j), \tag{2}$$

where α_s is the QCD coupling constant $(\alpha_s = g_s^2/4\pi)$ and $<\lambda_i^a\lambda_j^a>$ is the color average

in a color-singlet hadron as

$$< \lambda_i^a \lambda_j^a > = \begin{cases} -\frac{16}{3} & for \quad mesons \\ \\ -\frac{8}{3} & for \quad baryons \end{cases}.$$ (3)

Instead, it is obvious that if the quarks are not free, but move in a background pion field $\pi^a(x)$, one cannot write a Dirac equation in momentum space as in the case of free quark where $(i\not{p} - m)u_\alpha(p) = 0$, because p_μ does not commute with the Hamiltonian and is no longer a good quantum number. Therefore we are not able to follow the traditional procedure for the Breit expansion, but need to carry out all derivations completely in the coordinate space. Thus one needs to employ the new wavefunction $\Psi_\alpha(x)$.

The equation of motion of quarks which submerge in the background pion field can be reduced to a coupled differential equation group [4,5]. It is very complicated and can only be solved numerically. To gain more understanding of the whole picture, we prefer an analytic form even though need to invoke some approximations. Therefore later in this work, we try to achieve a compact form for the modified Coulomb potential under a reasonable approximation. Furthermore when we consider the spin status of quarks, the situation becomes more complicated than for free quarks, because the regular spin projection operator or helicity operator does not apply here, so one has to seek for a proper expression for the spin projection which is consistent with the Lagrangian (1). We follow Ternov et al. [9] to define an appropriate spin projection operator.

The basis of this work is that we employ the quark wavefunction which is the solution of the modified Dirac equation in coordinate space, in the expression $(-ig_s)^2 \Psi_q \frac{\lambda^a}{2} \gamma_\mu \Psi_q \frac{-i}{q^2} \Psi_{q'} \frac{\lambda^a}{2} \gamma^\mu \Psi_{q'}$ to replace the free-quark wavefunction in the traditional approaches. This correction reflects a hadronic effect on the short distance behavior between quarks inside baryons. Since this effect is purely non-perturbative, so far there is no reliable way to determine the form of the pion field and one has to invoke for reasonable ansatz whose validity should be tested by comparing the obtained results with experimental data. For the pion field, there is various ansatz, in the present work we will apply two popular ansatz to give corresponding modified Coulomb potential.

Since this correction may have some phenomenological consequences in phenomenology, we apply the results to make a rough evaluation on the nucleon-nucleon scattering phase shifts. The preliminary calculation shows a qualitative improvement for the middle range behavior (about 0.5 fm) of the nuclear force and furthermore this correction can modify the baryon spectra by a few tens of MeV, if the potential model is applied [10].

The paper is organized as following. After this introduction, we present our formulation about deriving the modified Coulomb potential where the spin projection operator is properly defined. In Sec.III, we give numerical results graphically corresponding to two different ansatz for $\pi^a(x)$. In Sec.VI, we analyze and discuss the obtained results in some detail and finally in the last section we draw our conclusion.

II. Formulation

From the Lagrangian (1), the equation of motion for a single quark in the background pion field is given as

$$(i\not{\partial} - M e^{i\pi^a \tau^a \gamma_5})\Psi_\alpha(x) = 0,$$ (4)

where the current quark mass is omitted.

(i) The spin projection operator.

Before going on to derive the potential with the new spinor $\Psi_\alpha(x)$, first let us define the spin projection operator. The Hamiltonian operator corresponding to the Lagrangian (1) can be written down directly

$$\hat{H} = -i\vec{\alpha} \cdot \vec{\nabla} + M\beta e^{i\pi^a(x)\tau^a\gamma_5}, \tag{5}$$

where $\vec{\alpha} = \gamma_0\vec{\gamma}$ and $\beta = \gamma_0$ as the usual convention. According to ref.[9], an operator can be decomposed into two parts as

$$\hat{F}\Psi = ([F] + \{F\})\Psi, \tag{6}$$

where

$$[F] = \frac{1}{2}(F\Lambda + \Lambda F)\Lambda, \tag{7}$$

$$\{F\} = \frac{1}{2}(F\Lambda - \Lambda F)\Lambda, \tag{8}$$

and $\Lambda \equiv \frac{\hat{H}}{(H^2)^{1/2}} = \frac{\hat{H}}{\epsilon}$, and the ϵ is an absolute value of the eigen-energy of $\hat{H}$, it is definitely positive and non-zero. So $[F]$ denotes the part of $\hat{F}$ which commutes with $\hat{H}$ whereas $\{F\}$ anticommutes with $\hat{H}$. The authors of ref.[9] pointed out that only the part $[F]$ has physical significance. For example, a free fermion has Hamiltonian operator as $(\vec{\alpha} \cdot \vec{p} + m\beta)$, so its velocity operator is obtained $\frac{d}{dt}\vec{r} = c\vec{\alpha}$, while $[\frac{d}{dt}\vec{r}] = \frac{\vec{p}}{m}$ has very clear meaning [9].

The spin projection operator is defined as $[P(n, s)] = \frac{1}{2}(P(n, s)\Lambda + \Lambda P(n, s))\Lambda$ where $s = \pm\frac{1}{2}$ and $P(n, s) = \frac{1}{2}(1 - \frac{2sW\cdot n}{M})$. The W has a well-known form

$$W_\mu = -\frac{1}{4i}\epsilon_{\mu\nu\rho\sigma}\sigma^{\nu\rho}\partial^\sigma.$$

With the Hamiltonian (5), we have obtained the projection operator as

$$\begin{aligned}
[P(n,s)] &= \frac{1}{2}[1 + \frac{s}{2}(\frac{1}{M}(i\gamma_0\vec{\alpha} \cdot \vec{\nabla}) + (1 - i\gamma_5\tau^a\pi^a(x))\gamma_5\slashed{n} \\
&+ \gamma_5\slashed{n}(\frac{i}{M}\gamma_0\vec{\alpha} \cdot \vec{\nabla} + 1 + i\gamma_5\tau^a\pi^a(x))], \tag{9}
\end{aligned}$$

where n_ν denotes an arbitrary direction with normalization $n^2 = -1$ and s is the defined projection eigen-value ($\pm 1/2$). For convenience, we choose n_ν along the z-axis as $n_\nu = (0, 0, 0, 1)$.

(ii) The potential.

The one-gluon-exchange diagram is depicted in Fig.1, and the amplitude can be written as

$$V = < q'_1 q'_2 | g_s^2 \overline{\Psi}(x)\gamma^\mu\frac{\lambda^a}{2}\Psi(x)D_{\mu\nu}^{ab}(x - y)\overline{\Psi}(y)\gamma^\nu\frac{\lambda^b}{2}\Psi(y)|q_1 q_2 >, \tag{10}$$

where g_s is the QCD coupling constant, λ^a is the $SU(3)_c$ generator, and the gluon propagator $D_{\mu\nu}^{ab}(x-y)$ in the unitary gauge is

$$D_{\mu\nu}^{ab} = \delta^{ab} \int \frac{d^4q}{(2\pi)^4} \frac{-i(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2})}{q^2 - i\epsilon} e^{-iq(x-y)}. \tag{11}$$

Due to the on-shell condition of quarks, even though this "on-mass-shell" is different from the free quark mass shell, but given in eq.(4), $q_\mu q_\nu$ term vanishes.

As aforementioned, if we substitute the numerical solutions for $\Psi(x)$ and $\Psi(y)$ to the expression of V, it would be hard to observe the analytic properties of the modified potential. Therefore we employ the tricks provided by Li, Yan and Liu [11] who studied the proton spin puzzle in this scenario. In form (10), let us rewrite the expression of the vertex as

$$< q_1' | \overline{\Psi}(x)\gamma^\mu \Psi(x) | q_1 > \longrightarrow < q_1' | T\overline{\Psi}(x)\gamma^\mu \Psi(x') | q_1 > |_{x' \to x}, \tag{12}$$

where the color indices and λ^a are omitted. In this form, we deliberately add a time-order operator T to guarantee the limitation order for $x' \to x$. Then

$$< q_1' | T(\overline{\Psi}(x)\gamma^\mu \Psi(x') | q_1 > |_{x' \to x} = < q_1' | : \overline{\Psi}(x)\gamma^\mu \Psi(x') : | q_1 > + Tr(S_F(x,x')\gamma^\mu)|_{x' \to x}. \tag{13}$$

Our goal is to study the correction to the Coulomb potential from the background pion field at the lowest order, so when we deal with the expansion (13), we have to take approximations. In this treatment we try to single out the correction term at the leading order, and it is easy to note that the second term in (13) represents a static correction from the pion field (see below). Obviously, it does not vanish only due to existence of a non-trivial pion field $\pi^a(x)$. In fact if $\pi^a(x)$ is zero, namely the Lagrangian converts back to the free quark Lagrangian, under the limit $x' \to x$ the second term in (13) vanishes due to the trace structure, and the first term just recovers the regular vertex for free quarks. Obviously, if the first term were the vertex in the complete theory under the limit, the second term must be zero. Therefore, (12) is an approximation we take, which is trivially true for the free quark case. But for a lagrangian with the pion field, as long as a non-trivial $\pi^a(x)$ exists, the second term gives rise to a non-vanishing contribution. Therefore, one can be convinced that to the leading order, the first term of (13) corresponds to the free quark contribution where the mass is the constituent quark mass and $\pi^a(x)$ is ignored, thus causes the regular Coulomb potential. Meanwhile the second term is the correction term caused by the background pion field $\pi^a(x)$. In other words, we attribute the correction from the pion field to the second term of (13) at this leading order, so (13) simply gives a modified form, the regular Coulomb term plus a correction. Furthermore it is noted that this expression is valid only if $< q_1' | q_1 > = 1$ [1], ignoring different color indices of the two quarks, it means that this approximation is a static modification to the color current and charge. The authors of ref. [11] showed that for the deep inelastic scattering, this approximation gives results consistent with data, so we have reason to believe that the second term of (13) indeed includes the majority of the correction caused by the pion background field.

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[1] Here we thank Prof. Chang for calling us to notice this point

This modification is very similar to that in the multipole expansion method to approach hadronic transitions [12], there

$$L_Q = \int d^3x \overline{\Psi}(i\not{\partial} - g\not{A} - m)\Psi - \frac{1}{2}\frac{g^2}{4\pi}\sum_a \int d^3x d^3y \rho_a(\vec{x}, t)\frac{1}{|\vec{x} - \vec{y}|}\rho_a(\vec{y}, t), \qquad (14)$$

where $\rho_a(\vec{x}, t)$ and $\rho_a(\vec{y}, t)$ are color density and the first term in L_Q is the normal one. Therefore our correction term in (13) corresponds to the second term of eq.(14) which applies in a different physical field.

The first term of (13) contributes to the regular Coulomb potential $\frac{<\lambda^a\lambda^a>\alpha_s}{4r}$, and the second term is the subject of our present study.

(iii) Derivations

An explicit expression of $S_F(x, x')$ was given in ref. [11] and it is noted that in this case $S_F(x, x')$ cannot be written as $S_F(x - x')$ which is true for free fermions. Thus

$$S_F(x, x') = \frac{1}{(2\pi)^4}\int d^4p S_F(x, p)e^{ip(x-x')}, \qquad (15)$$

and

$$S_F(x, p) = S_F^{(0)}(x, p)\sum_{n=0}^{\infty}[(-i\not{\partial}_x)S_F^{(0)}(x, p)]^n, \qquad (16)$$

$$S_F^{(0)} = -\frac{\not{p} - M\hat{U}}{p^2 - M^2}, \qquad (17)$$

where

$$\hat{U} = exp(-i\gamma_5\pi^a(x)\tau^a). \qquad (18)$$

As a usually adopted approach, one needs to take the hedgehog ansatz [13] as

$$\tau^a\pi^a(x) = \phi(r)\hat{r} \cdot \vec{\tau}, \qquad (19)$$

where $\hat{r} = \frac{\vec{r}}{r}$ is a unit radial vector.

The integration over p in eq.(15) is divergent, so one can use the commonly adopted dimensional regularization approach and take an identity to get rid of the divergence [11]

$$\frac{N_c}{(4\pi)^2}M\frac{2}{\epsilon} = \frac{F_\pi^2}{16}, \qquad (20)$$

where F_π is the pion decay constant.

Thus substitute all the results into eq.(10), and as the spin projections of quarks are under consideration, we also need to insert in the spin projection operator defined in (9).

$$\begin{aligned}
V &= <\lambda^a\lambda^a> \{\frac{\alpha_s}{4r} \\
&+ g_s^2 \int [Tr([P(n, s_1)]^\dagger S_F(x, x')[P(n, s_1')]\gamma_\mu)(\int \frac{d^4q}{(2\pi)^4}\frac{e^{iq(x-y)}}{\vec{q}^2}) \\
&\cdot Tr([P(n, s_2)]^\dagger S_F(y, y')[P(n, s_2')]\gamma^\mu)|_{\substack{x' \to x \\ y' \to y}}]d^3x d^3y\},
\end{aligned} \qquad (21)$$

where $r \equiv |\vec{x} - \vec{y}|$.

It is noted that there are no cross terms between the first and second terms of (13), because if the momentum carried by the intermediate gluon is not zero, the second term vanishes for $< q'_1|q_1 >=< q'_2|q_2 >= 0$, but if $\vec{p}_1 = \vec{p}'_1$ and $\vec{p}_2 = \vec{p}'_2$, in the free quark piece (the first term of (13)) $\overline{u}(\vec{p}_1)\gamma_\mu u(\vec{p}_1)$ can be calculated explicitly and is proportional to $\vec{p}_1$ in the instantaneous approximation, then one can have a cross term as $\vec{p}_1 \cdot \vec{\nabla}\phi(r_2)$, an average over $\vec{p}_1$ would give a null result.

The integration over $\vec{x}$ and $\vec{y}$ means that the two quarks are anywhere and this average effect could be a global correction. Later our derived formula shows that this overall effect vanishes. In fact, carrying out this overall integration is meaningless because we hope to study interaction between quarks as a function of their distance. Therefore in later expressions we keep the quark 1 at position $\vec{r}_1$ and quark 2 at position $\vec{r}_2$, to match the dimension, one needs to use a parameter I_N of dimension of volume to replace the spatial integration.

As in the Breit equation, one has to take the spontaneous approximation to obtain interquark potential, and we do the same here. With formulae (15) through (21), one may obtain the correction term to the Coulomb potential and all calculations are standard.

$$
\begin{aligned}
V^{corr} &= \frac{< \lambda^a\lambda^a > \alpha_s}{4}[-iTr([P(n,\sigma'_1)^\dagger]\gamma_\mu[P(n,\sigma_1)]S_F(x,x'))(\frac{1}{r}) \\
&\cdot \quad (-i)Tr([P(n,\sigma'_2)^\dagger]\gamma^\mu[P(n,\sigma_2)]S_F(y,y'))]|_{\substack{x'\to x \\ y'\to y}} \cdot (\frac{F_\pi^2}{8})^2 \cdot I_N^2 \\
&= \frac{< \lambda^a\lambda^a > \alpha_s}{4}\frac{}{r}\phi(r_1)\phi(r_2)\frac{d}{dr_1}\phi(r_1)\frac{d}{dr_2}\phi(r_2)[(1 + \sigma + \sigma' + \sigma\sigma')cos\theta \\
&- \quad 2(\sigma + \sigma')cos\theta_1 cos\theta_2](\frac{F_\pi^2}{8})^2 \cdot I_N^2,
\end{aligned}
\tag{22}
$$

where for simplicity, we take $\sigma_i = 2s_i$ as the projections which have eigen values ± 1 and $\sigma \equiv \sigma_1 \cdot \sigma'_1$ and $\sigma' \equiv \sigma_2 \cdot \sigma'_2$, $r = |\vec{r}_1 - \vec{r}_2|$.

In this expression, there are $cos\theta$, $cos\theta_1$ and $cos\theta_2$, θ_1 and θ_2 are the polar angles of $\vec{r}_1$ and $\vec{r}_2$ respectively and $\theta = \theta_1 - \theta_2$. Their existence is due to the hedgehog ansatz as $\pi^a(x) = \phi(x)\hat{r}^a$, so the background pion field has a spatial direction. The term proportional to $cos\theta_1 cos\theta_2$ is owing to our special choice of the spin projection axis. For a physical picture which makes a common sense, there is no any special direction is the space, so that one needs to integrate out θ_1 and θ_2, while keeping $\theta_1 - \theta_2$ fixed.

Thus the total potential is

$$
\begin{aligned}
V_{eff}^{mod} &= \frac{\alpha_s < \lambda^a\lambda^a >}{4r} + \frac{< \lambda^a\lambda^a > \alpha_s}{4}\frac{}{r}\phi(r_1)\phi(r_2)\frac{d}{dr_1}\phi(r_1)\frac{d}{dr_2}\phi(r_2) \cdot \\
&[(1 + \sigma + \sigma' + \sigma\sigma')cos\theta - 2(\sigma + \sigma')cos\theta_1 cos\theta_2](\frac{F_\pi^2}{8})^2 \cdot I_N^2 + V',
\end{aligned}
\tag{23}
$$

where V' comes from the relativistic correction in the first term of Breit expansion (13), since in this scenario M is the constituent quark mass, so that the relativistic correction is not large. Writing $dcos\theta_1 dcos\theta_2$ into $dcos(\theta_1 - \theta_2)dcos\theta_2$ and integrating over $dcos\theta_2$ the second term in the square bracket of (23) vanishes. This leads to V^{mod} which will be plotted in Figs.3 and 4.

In next section, we will employ two widely accepted ansatz for $\phi(r)$ to evaluate the correction from the background pion field.

III. The numerical calculation of V^{corr}.

The pion field $\pi^a(x)$ principally is a consequence of QCD, and should be derived if we knew how to handle the non-perturbative QCD, but due to lack of such knowledge, generally one has to take the hedgehog ansatz and meanwhile $\phi(r)$ may have different forms. In this section, we adopt two widely accepted ansatz for $\phi(r)$, (1) is that suggested by the authors of refs.[4,5], and (2), the Skyrme form [13].

(1) The profile function [4,5]

$$\phi(r) = 2arctan(\frac{r_0}{r})^2, \tag{24}$$

where $r_0 = 0.57 \ fm$ and in the expressions the dimensional parameter I_N is taken as $\frac{4\pi}{3}r_0^3$. According to ref.[4,5], this r_0 can give $g_A = 1.15$, $g_{\pi NN} = 13.6$, so it indeed convinces us that the valence quark picture is right. This $\phi(r)$ has a property that

$$\frac{d}{dr}\phi(r) = \frac{-4r_0^2 r}{r_0^4 + r^4} \xrightarrow{r \to 0} 0,$$

which is important and we will see it later.

The numerical results are shown in Fig.3. In this figure, we set r_1 as 0.3 fm and 0.5 fm and let r_2 move around, then we graphically demonstrate the dependence of the modified Coulomb potential as a function of $r = |\vec{r_1} - \vec{r_2}|$. In this graph we let $\vec{r_1}$ and $\vec{r_2}$ be along the same line, i.e., $cos\theta$ can be 0 and 1.

The curve 1 corresponds to the pure Coulomb potential in the free quark case, and curve 2 and 3 for the $r_1 = 0.3$ fm and 0.5 fm respectively. It is observed that there is a bump at the modified potential curves as r gets to certain values and its appearance will be explained in next section.

(2) The Skyrme form.

In principle, the function $\phi(r)$ is obtained numerically by solving a very complicated non-linear differential equation, however, Atiyah and Manton suggested an ansatz [14]

$$\phi(r) = \pi[1 - \frac{1}{\sqrt{1 + \lambda^2/r^2}}]. \tag{25}$$

With this profile function form, they achieved $\lambda^2 = 2.11$ and the dimensionless energy is $1.243 \times 12\pi^2$ compared to the numerical result $1.232 \times 12\pi^2$. Since this form is analytic, so our calculation can be greatly simplified by adopting this function. However, with this function

$$\frac{d}{dr}\phi(r) = \frac{-2\pi\lambda^2}{(r^2 + \lambda^2)^{3/2}}$$

does not go to zero as $r \to 0$.

The numerical results are shown in Fig.4, all parameters are the same as adopted for Fig.3. There a jump is observed at the curves and this discontinuity is due to the fact $\frac{d}{dr}\phi(r)$ does not approach to zero when $r \to 0$ and will be discussed in next section.

This corrected Coulomb potential form favors the diquark structure of baryons, which we will discuss in next section. Besides, this modification may have phenomenological consequences in other fields.

First, in the nucleon-nucleon scattering evaluation, the study on the phase shifts may determine the nuclear force behavior. In fact the nuclear force stands as the Van der Waals force induced by QCD. However the derivation is somewhat different from that given in this work, because the interacting two quarks are from two different clusters (nucleons), so in the expression (1) the Ψ_q and $\Psi_{q'}$ are solutions corresponding to two different modeified Dirac equations where the centers are at different coordinates. But the procedure for deriving effective potential is the same. In the traditional approaches, the contribution from the pure Coulomb potential is proportional to $1/r^6$ while from the linear confinement is to $1/r^3$ [15]. Usually they fail to result a middle range nuclear force which can meet experimental data, i.e. the obtained middle range attraction is not sufficiently strong. This correction would cause a new term proportional to $1/r^\alpha$ where $3 \leq \alpha \leq 6$. Our rough estimation indicates that this value α is close to 6, i.e. reflects a modification of the Coulomb potential, but can strengthen the middle range attraction. However a careful evaluation of the scattering phase shifts is a very difficult work and we will present all more accurate evaluations in our next work [16].

Besides as some authors estimated the baryon spectra in terms of the potential model [10], if we replace the regular Coulomb potential with this corrected one, a few tens of MeV difference should be resulted. Because the spectrum is an overall effect and the obtained correction term is proportional to $cos\theta$ where θ is the angle between the radial vectors of the concerned two quarks which is caused by the hedgehog mechanism, for spectrum evaluation one needs to average over the angle, thus $\int d\Omega_1 d\Omega_2 cos(\theta_1 - \theta_2) = \pi^4$ gives an additional factor to the spectrum.

IV. Discussions.

In this work, we employ two different ansatz for the background pion field. The analytic and numerical results all indicate that the pion field indeed gives rise to a hadronic correction to the interaction between quarks (or quark-antiquark). There are some features to be noted.

(i) The effective potential including the hadronic correction depends on the positions of two quarks, because the pion field is a function of coordinates, besides due to hedgehog ansatz, the pion field is along the radial direction, and it causes the dependence of the interquark potential on the angle between two quarks. Namely, quarks interact differently while being at different positions.

(ii) Because $\phi(r)$ is a decreasing function of r and acts as an external field for the effective potential between quarks, the two quarks at different positions feel different interactions. As discussed above, this is a modification to the effective color currents carried by the two quarks and the color current density is a function of coordinates.

(iii) The angle $\theta = \theta_1 - \theta_2$ plays a crucial role in the hadronic correction. If $r = |\vec{r}_1 - \vec{r}_2|$ is small when $|\vec{r}_1|$ is fixed, the angle between two quark-radial directions is less than $\pi/2$, (see Fig.2), the dependence of the modified potential on the distance is a smooth curve which is only a bit shifted to the larger distance side compared to the pure Coulomb potential. However, if r is larger than a certain value, the angle between two quarks along radial directions becomes larger than $\pi/2$, a bump at the curve appears. The reason is

obvious. Fixing $\vec{r}_1$ and letting $\vec{r}_2$ move, without losing generality, and to elucidate the whole picture, we assume $\vec{r}_1$ and $\vec{r}_2$ remain along the same axis. As quark 1 is at position $\vec{r}_1$, and quark 2 moves towards the origin, one can observe that the curve is shifted to right side, but as the quark-2 position crosses the origin, the polar angle $\theta = \theta_1 - \theta_2$ immediately changes into $\pi - \theta$. In Figs.3 and 4, $\theta = 0$ when two quarks are on th same side of the origin, whereas when the quark-2 position crosses the origin, i.e. the distance $r = |r_1 - r_2|$ is larger than a certain value, $\theta = -\pi$. This induces $cos\theta$ to change a sign, and a bump at the curve appears. There would be another case that, quark 1 and quark 2 reside on the opposite sides with respect to the origin, and are very close to each other, then separating them to larger distance (r gets larger), a corresponding graph which is similar to Fig.3 or 4, shows a somewhat sharper increase near $r \to 0$ region and then tends to the curve of the regular Coulomb potential. In fact all these modified forms have the same asymptotic large-r behavior as the regular Coulomb potential, and it is expected.

Let us study the spin factor in the correction term.

$$A = (1 + \sigma + \sigma' + \sigma\sigma')cos\theta \tag{26}$$

(a) if $\sigma = \sigma' = 1$, $A \equiv A_1 = 4cos\theta$;
(b) if $\sigma = 1$, $\sigma' = -1$ (or $\sigma = -1$, $\sigma' = 1$), $A \equiv A_2 = 0$;
(c) if $\sigma = \sigma' = -1$, $A \equiv A_3 = 0$.

In (a), the two quarks (or a quark, an antiquark) do not flip spin projections during the interaction with gluon; (b) one quark (antiquark) flips while another does not; (c) the two quarks flip simultaneously. As $\theta = 0$ and π, $|A_1|$ gets to the maximum. Generally, both the flip and non-flip amplitudes exist with a proper probability. One may also see that the correction term vanishes at some special angles, for example $\theta = \pi/2$, namely, $\vec{r}_1$ is perpendicular to $\vec{r}_2$. As $\vec{r}_2$ crosses the origin, A_1 changes sign and the bump in Fig.(3) and (4) would be resulted.

The dependence of the potential on the distance between quarks is illustrated in Fig.3 and 4, where Fig.3 corresponds to ansatz (1) and Fig.4 to ansatz (2) respectively, and when the two quarks reside on the same side with respect to the origin the angle is 0, while the distance gets larger and they reside on opposite sides of the angle is π.

(iv) The second term in eq.(13) contributes only in the case that the momentum carried by the interchange gluon is zero. In ref.[17], based on the QCD sum rules, the authors also derived a QCD force which is singular like $\delta(q)$ where q is the gluon momentum. This force plays an important role in the intermediate momentum range. At very short and very long distance regions, the Coulomb and linear pieces of the whole potential take dominance respectively, therefore the modification to the Coulomb potential in our present work may correspond to the force derived in ref. [17].

The dependence on the interquark distance is quite different from the regular Coulomb potential obtained for the free quark case where the potential is only a function of the distance between two quarks. The modified potential not only depends on the distance, but also on the polar angle spanned between the two-quark radial vectors and the position of the two quark system . This is just because of $\pi^a(x) = \hat{r}^a \phi(r)$.

For the ansatz (1), since $\frac{d}{dr}\phi(r)|_{r\to 0} = 0$, a continuous shoulder (bump) appears at a certain value of r, and the curve is smooth. In contrary, for ansatz (2), the curve is

discontinuous, i.e. there is a finite jump at the corresponding value, that is proportional to the discontinuity $\frac{d}{dr}\phi(r)|_{r\to 0} \neq 0$, which is a finite number.

A direct consequence of this modification is that the modified potential has a shape which is wider than the regular Coulomb potential at short distance region, and while the distance becomes larger, i.e. the two quarks reside on opposite sides with respect to the origin, a shoulder appears at the potential curves. By the Quantum Mechanics, it is well known that for a Coulomb potential, this modification corresponds to a larger effective charge which makes the Coulomb well wider, then the binding energy becomes more negative, namely if there is a bound state, the binding is stronger and the state is more stable. Furthermore, the upwards shoulder which jumps up about a few tens of MeV, effectively increases the well wall and it definitely makes the bound state even more stable.

Therefore based on a simple analysis, this modified form indicates that the additional pion field inclines to strengthen the binding effect between quarks as long as they are in $\bar{3}$ (for quarks) and a singlet (for quark-antiquark). This is especially important for the baryon case. If two quarks which submerge in the background pion field, move to certain positions, i.e. exist on opposite sides with respect to the origin, they intend to be bound together, so this modification from the pion field indeed favors the diquark structure of baryons. In particular, the diquark, if it exists, according to our analysis, should reside near the origin of the baryon, while the third quark moves around. As this diquark breaks, one of the quarks may join the other one or the third quark to constitute a new diquark, all these structures exist with certain probabilities.

This modification supports our understanding of the diquark structure.

V. Conclusion.

In this work we consider the effects from the background pion field for baryons which characterizes the chiral symmetry breaking and causes the constituent quark mass.

For the valence constituent quark mass scenario, the potential model is a reasonable description of the real physics for baryons, because the constituent quark masses are not small and the relativistic correction is not very important, but if quarks only possess current quark masses, the relativistic effects would never be negligible. Namely, due to the chiral symmetry breaking, quarks gain constituent masses and the potential model makes sense.

To derive the potential, the traditional way is to employ the Breit equation, where the free quark spinors are used and their masses are constituent masses. However as many authors pointed out that the constituent quark mass is associated with the background pion field, therefore an obvious modification to the traditional potential is to include the effects from the additional pion field. To realize it, instead of the free quark spinors, one should use the spinors which are solutions of the modified Dirac equation with mass term $Me^{\pi^a(x)\tau^a\gamma_5}$. To elucidate the physical picture, instead of purely numerical solution, we follow the way suggested by some theoreticians to separate the correction term from the total potential whose zero-th order is exactly the regular Coulomb potential.

The authors of ref.[9] considered the influence of the pion field in the study of proton spin problem, and found that the contribution of the pion field may be the key to solve the the proton spin crisis. Therefore, it motivates us to investigate effects of the pion field

to other subjects.

Since the confinement part of the potential comes from non-perturbative QCD effects which are still obscure, we avoid such ambiguity in this work and only try to investigate modifying the Coulomb part of the potential which is caused by the one-gluon-exchange diagram and can be calculated perturbatively. We derive a modified Coulomb potential as given in preceeding sections and observe an obvious correction to the regular Coulomb potential. Especially, when the two quarks reside on opposite sides with respect to the origin, the effective potential well wall increases and it makes a bound state of the two quarks more stable. This conclusion supports the theory that diquark structure exists inside baryons.

In the derivations we take some approximation, so the positions and heights of the shoulders on the curves shown in Fig.3 and 4 may be not very accurate, but as discussed above, the validity of the approach was discussed and a convincing result was given in ref.[9], so one can believe that the qualitative behavior of the modified Coulomb potential that we have found in this work should be right.

In fact, the background pion field may be a consequence of a collective effects of the quarks and is due to the non-perturbative QCD, therefore, in our treatment we partly include some non-perturbative QCD, and it is also consistent with the conclusion of ref. [17]. As we know from ref.[4,5] that due to the chiral symmetry breaking the quarks become constituent quarks "dressed" with the pion field, therefore derivation of mutual interaction between quarks must include such "dresses", but not the bare quarks. In fact, as the zero-th order approximation, $\pi^a(x) \to 0$ or $F_\pi \to 0$, we just recover the free quark case, namely obtain the regular Coulomb potential. Obviously, this picture also causes modification to the confinement part of the potential and we may study it later when we have more knowledge on the sources of the confinement.

We also employ the modified Coulomb potential to evaluate the phase shifts of nucleon-nucleon scattering from which one may determine the nuclear force, we have found that this correction cause a Van der Waals force which is proportional to $1/r^\alpha$ and $3 \le \alpha \le 6$. This change can increase the middle range attraction of nuclear force which is expected. Moreover, we also estimate a possible change of the evaluation of the baryon spectrum due to the correction and leave all uncertainties to the parameters by fitting data. It indicates that such a correction to the Coulomb part of a potential may bring some substantial effects to certain processes and we will study them in more detail and publish the results elsewhere.

Figure caption

Fig.1. The Feynman diagram of the one-gluon-exchange interaction between two quarks (quark-antiquark).

Fig.2. The geometric diagram where two quarks exist inside the hadron, r_0 is a parameter which characterizes the pion field $\phi(x)$. r_1, r_2, θ_1 and θ_2 correspond to the positions of quark 1 and quark 2 where the azimuthal angles are irrelevant so omitted.

Fig.3. The modified Coulomb potential where the ansatz suggested by the authors of ref.[4,5] is adopted.

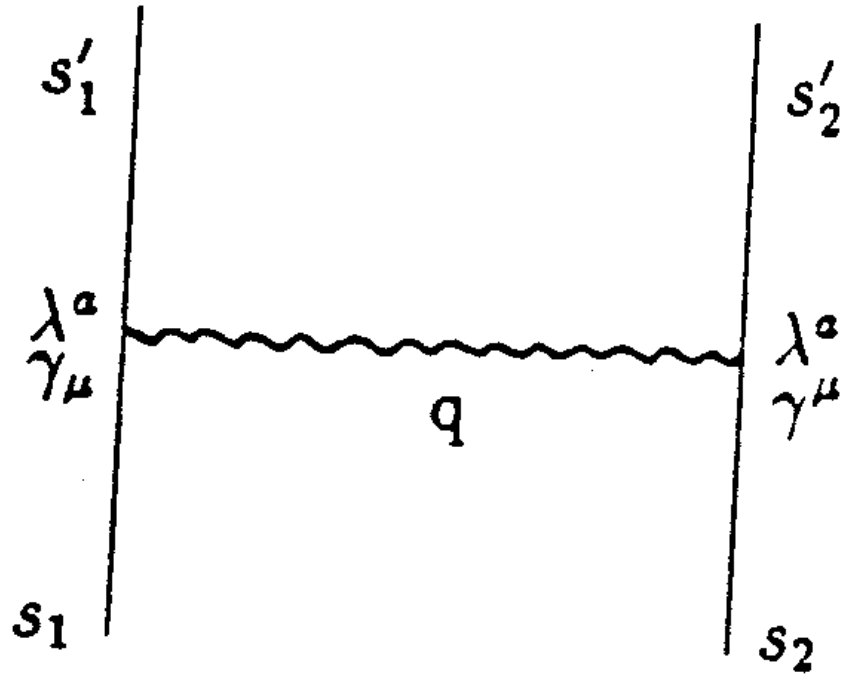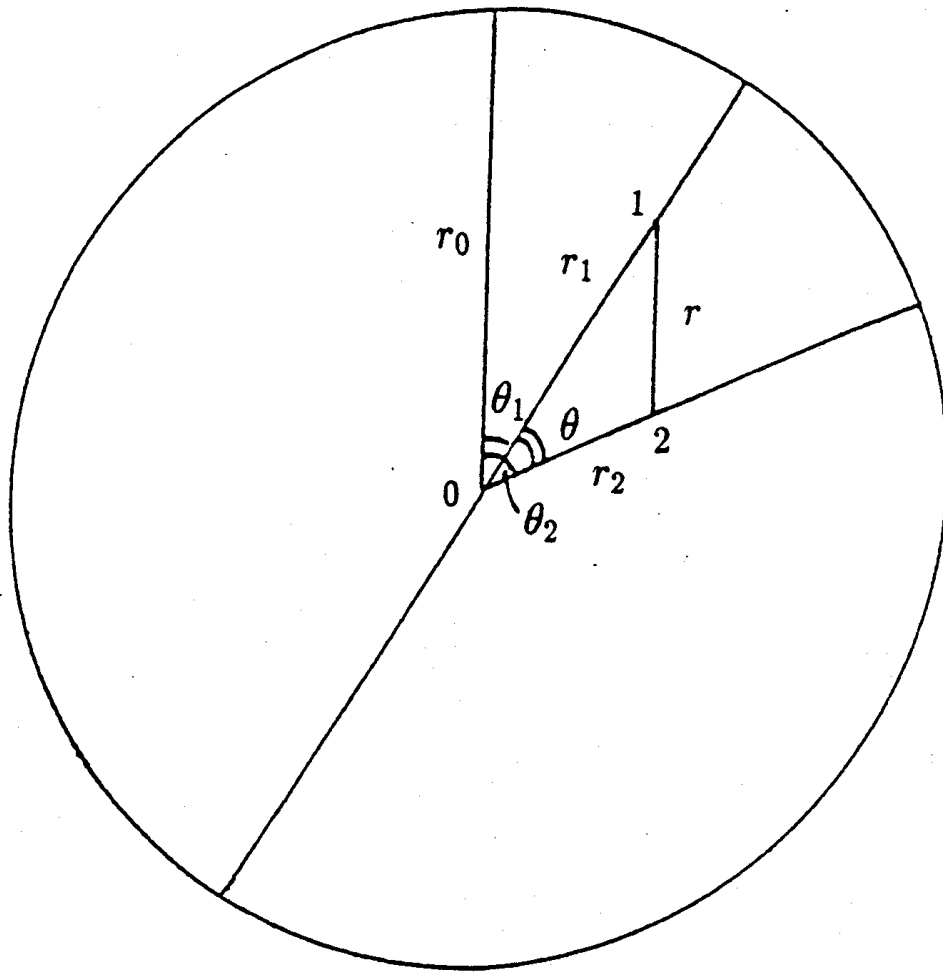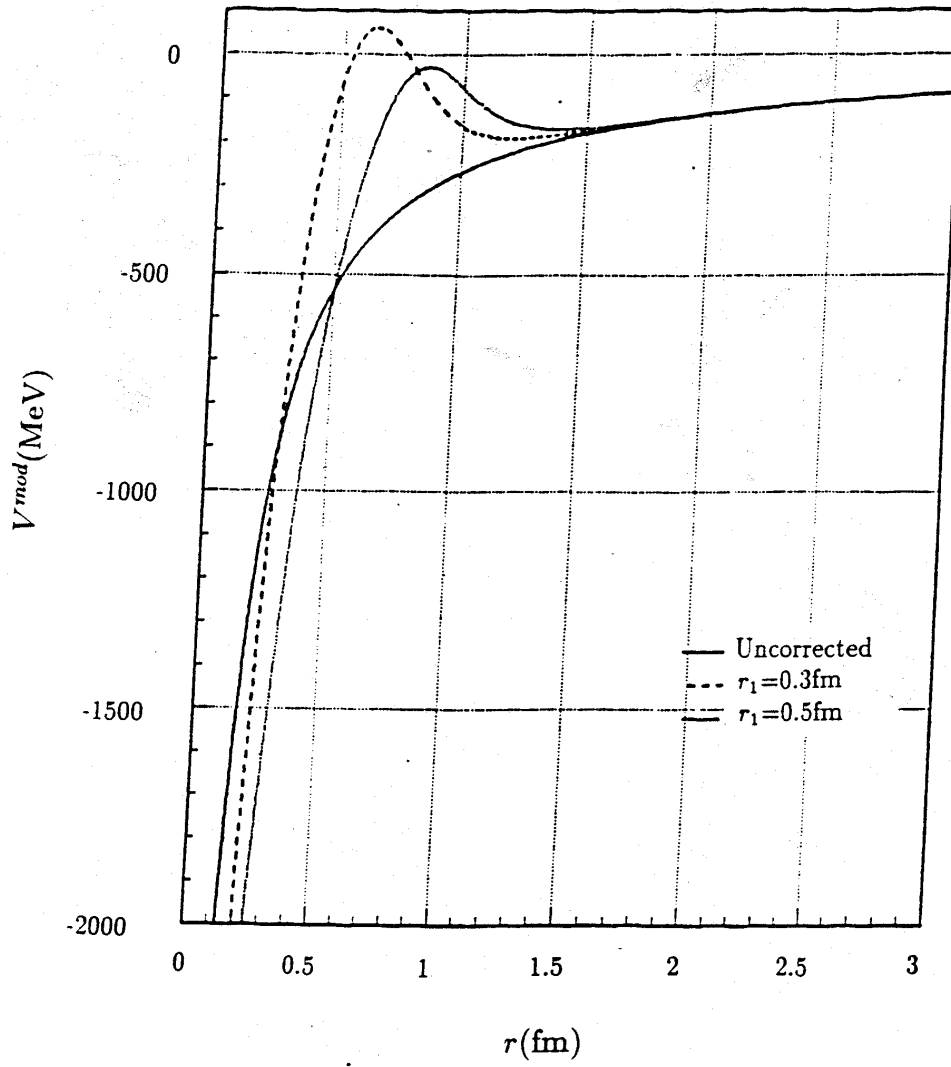Fig.4 The modified Coulomb potential where the Skyrme form for the pion field $\phi(x)$ is adopted.
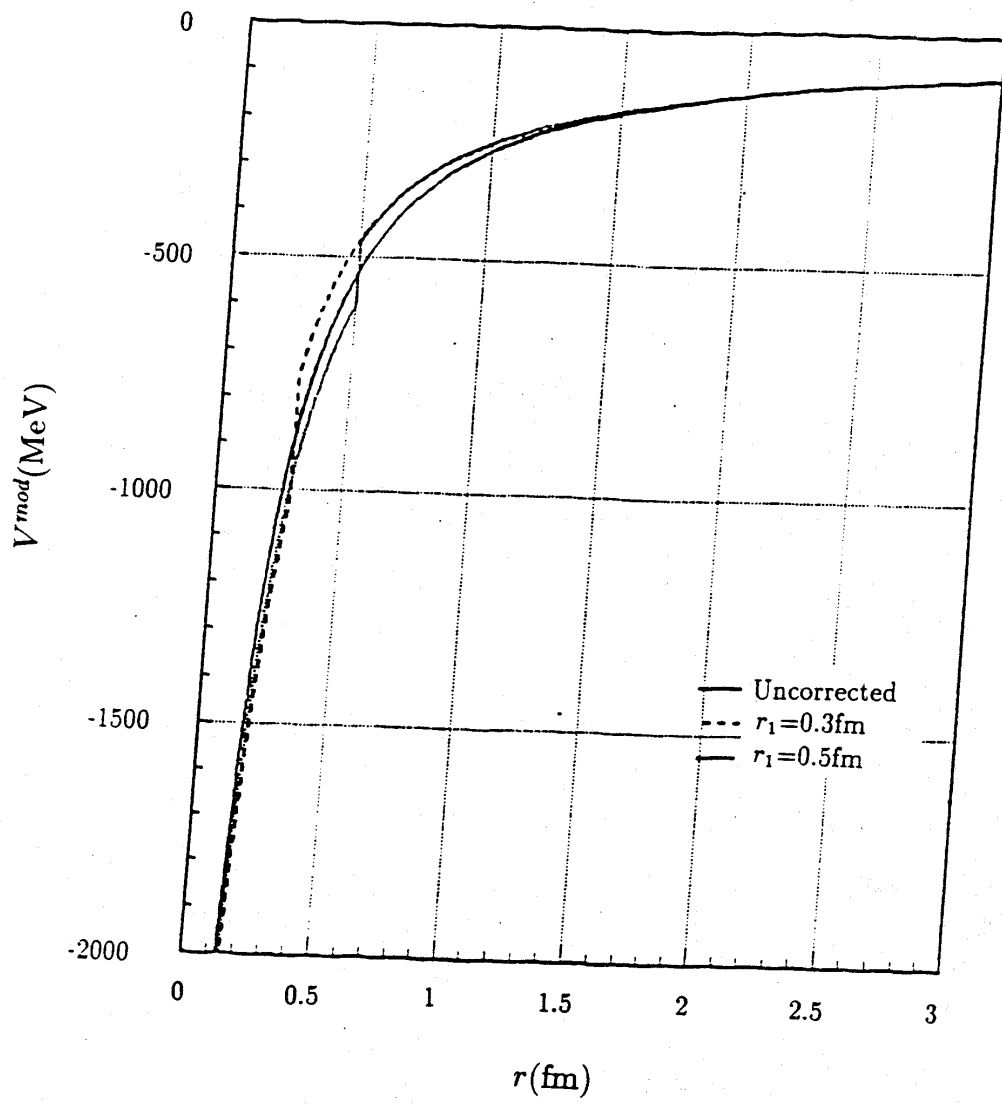
Fig.1

Fig.2

Fig.3

Fig.4

References

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