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Light-Front Quantization of Chern-Simons Systems †

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Abstract

Light-front quantization of the Chern-Simons theory coupled to complex scalars is performed in the local light-cone gauge following the Dirac procedure. The light-front Hamiltonian turns out to be simple one and the framework may be useful to construct renormalized field theory of anyons. The theory is shown to be relativistic inspite of the unconventional transformations of the matter and the gauge field, in the non-covariant gauge adopted, under space rotations.

Key-words: Light-front; Anyons; Gauge theory; Quantization.

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1. Introduction

Chern-Simons (CS) gauge theories[1,2] coupled to matter field have been proposed to describe excitations with fractional statistics[3,4], *anyons*, and suggested to be relevant for describing the quantized Hall effect and possibly the high-T_c superconductivity[5] where the dynamics is effectively confined to a plane. There are, however, controversies related to the quantized field theoretical formulation. The Lagrangian (path integral) formulation[6], for example, seems to give result which disagree with the canonical Hamiltonian formulation[7−10]. It is claimed that the theory though shown relativistic has angular momentum anomaly[11] or shows anyonicity only in some nonlocal gauges[10,7]. Internal algebraic inconsistency[10] of using two *local* gauge conditions[12] in the context of the Coulomb gauge has also been stressed. The anomaly is also found absent in some recent works[13,14] and doubts have been raised about the anyonicity being gauge artefact[9]. We clarify here some of the points by performing the light-front (*l.f.*) quantization[15] of the CS theory coupled to the complex scalar field in the light-cone gauge. The *l.f.* vacuum[16,17] is known to be simpler than the conventional one and the anyonic excitations and possibly some non-perturbative effects may be studied more conveniently. In the descripton of the spontaneous symmetry breaking on the *l.f.*, for example, it was found[18] that we do obtain the same physical result as that in the equal-time quantization, although achieved through a different mechanism. The conventional description requires *additional external constraints* in the theory based on physical considerations while the analogous ones on the *l.f.* were shown[18] to arise from the *self-consistency requirements* in the Hamiltonian theory itself. We conclude from our study that the abovementioned rotational anomaly should rather be interpreted as gauge artefact, that even in the present theory the application of two local gauge-fixing conditions on the phase space is totally consistent, and that the *l.f.* Hamiltonian is simpler when compared to that found in the local or nonlocal Coulomb gauge and it may be useful for constructing a renormalized theory.

2. Light-front Quantization of Chern-Simons Theory

The CS gauge theory we discuss is described by

$$\mathcal{L} = (\mathcal{D}^{\mu}\phi)(\widetilde{\mathcal{D}}_{\mu}\phi^*) + \frac{\kappa}{4\pi}\epsilon^{\mu\nu\rho}A_{\mu}\partial_{\nu}A_{\rho} \tag{1}$$

Here ϕ is a complex scalar filed, A_{μ} is the gauge field, $\mathcal{D}_{\mu} = (\partial_{\mu}+ieA_{\mu})$, $\widetilde{\mathcal{D}}_{\mu} = (\partial_{\mu}-ieA_{\mu})$, $\epsilon^{\mu\nu\rho}$ is the Levi-Civita tensor needed to construct the Chern-Simons kinetic term. For the coordinates x^{μ}, and for all other vector or tensor quantities, we define the *light-front* $\pm$ *components* by $x^{\pm} = (x^0 \pm x^2)/\sqrt{2} = x_{\mp}$. We take $x^+ \equiv \tau$ to indicate the *light-front time coordinate*, x^- is the *longitudinal space coordinate* and x^1 is the *transverse* one. The conjugate momenta are $\pi = \widetilde{\mathcal{D}}_{-}\phi^*, \pi^* = \mathcal{D}_{-}\phi, \pi^{\mu} = a\epsilon^{+\mu\nu}A_{\nu}$ where $\kappa = 4\pi a$. The conserved current $j^{\mu} = ie(\phi^*\mathcal{D}^{\mu}\phi - \phi\widetilde{\mathcal{D}}^{\mu}\phi^*)$ is gauge invariant and its contravariant vector property must remain intact if the theory constructed is relativistic.

Local light-cone gauge (*l.c.g.*), $A_- = 0$, is easily shown to be accessible in the Lagrangian theory; it will be shown below to be accessible also in the Hamiltonian theory on the phase space of the CS gauge theory. Since a *self-consistent Hamiltonian theory*[19] must not contradict the Lagrangian theory we may start by deriving first the necessary boundary conditions from the Lagrange eqs. written in the *l.c.g.*. For example, from $2a\partial_- A_1 = j^+$ we derive that the electric charge is given by $Q = \int d^2x\, j^+ = 2a\int dx^1\,[A_1(x^- = \infty, x^1) - A_1(x^- = -\infty, x^1)]$. For nonvanishing charge, A_1 may thus *not* be taken to satisfy the periodic or the vanishing boundary conditions at infinity along x^-. We will assume the *anti-periodic* boundary conditions for the gauge fields along x^- while the vanishing ones along x^1. For the scalar fields following similar arguments we assume vanishing boundary conditions. The canonical Hamiltonian may then be written as

$$H_c = \int d^2x \left[\, (\mathcal{D}_1\phi)(\widetilde{\mathcal{D}}_1\phi^*) - A_+\Omega \,\right] \tag{2}$$

where $\Omega = ie(\pi\phi - \pi^*\phi^*) + a\epsilon^{+ij}\partial_i A_j + \partial_i\pi^i$ and $i = -, 1$.

We follow the Dirac method[19] to construct an Hamiltonian for the constrained dynamical system (1). From the definitions of the canonical momenta we find the primary constraints: $\pi^+ \approx 0, \top^i \equiv \pi^i - a\epsilon^{+ij}A_j \approx 0, \top \equiv \pi - \widetilde{\mathcal{D}}_-\phi^* \approx 0, \top^* \equiv \pi^* - \mathcal{D}_-\phi \approx 0$ where $\approx$ indicates *weak equality*[19]. The preliminary Hamiltonian is taken to be $H' = H_c + \int d^2x\,[\,u\top + u^*\top^* + u_i\top^i + u_+\pi^+\,]$ where u, u^*, u^i, u_+ are Lagrange multiplier fields. We postulate initially the standard *equal-τ* Poisson brackets, and require the *persistency in τ* of the constraints making use of $df(x,\tau)/d\tau = \{f(x,\tau), H'(\tau)\} + \partial f/\partial\tau$. We find a *secondary constraint* $\Omega \approx 0$. The Hamiltonian is then extended to include this one as well and the step repeated and we find that no new constraint is generated.

The Ω and π^+ can be shown to generate gauge transformations and the constraints $\pi^+ \approx 0$ and $\Omega \approx 0$ are first class[19] while the remaining ones are second class[19]. From the Hamilton's eqs. of motion we verify that there does exist a choice of the Lagrange multiplier fields for which $A_- \approx 0$ and $dA_-/d\tau \approx 0$. The light-cone gauge $A_- \approx 0$ is thus accessible on the phase space (for a fixed τ). We add in the theory this gauge-fixing constraints so that now the set of second class constraints may be checked to be: $\top_m, \quad m = 1, 2..6$: $\top_1 \equiv \top^-, \top_2 \equiv \top^1, \top_3 \equiv \top, \top_4 \equiv \top^*, \top_5 \equiv A_-, \top_6 \equiv \Omega$ while $\pi^+ \approx 0$ stays first class. Next the Poisson brackets are modified to define the Dirac brackets $\{f,g\}_D$ such that the second class constraints may be written as *strong equalities*[19], $\top_m = 0$ and eqs. of motion given by $df(x,\tau)/d\tau = \{f(x,\tau), H_{(extended)}(\tau)\}_D + \partial f/\partial\tau$. The Dirac brackets are constructed to be

$$\{f,g\}_D = \{f,g\} - \int d^2u d^2v\, \{f, \top_m(u)\}\, C^{-1}_{mn}(u,v)\, \{\top_n(v), g\} \qquad (3)$$

where $C^{-1}(x,y)$ is the inverse of the constraint matrix with the elements $C_{mn} = \{\top_m, \top_n\}$ and given by

$$\begin{pmatrix} 0 & -4a\partial^x{}_- & 0 & 0 & 0 & 0 \\ 4a\partial^x{}_- & [\phi^*(x)\phi(y)+\phi(x)\phi^*(y)] & 2ai\phi(x) & -2ai\phi(x)^* & 0 & -4a \\ 0 & 2ai\phi(y) & 0 & (2a)^2 & 0 & 0 \\ 0 & -2ai\phi^*(y) & (2a)^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(2a)^2 \\ 0 & -4a & 0 & 0 & 2(2a)^2 & 0 \end{pmatrix} \frac{K(x-y)}{(2a)^2} \tag{4}$$

with $K(x-y) = -(1/4)\,\epsilon(x^- - y^-)\,\delta(x^1 - y^1)$, $\partial_-{}^x K(x,y) = (-1/2)\delta^2(x-y)$, and $\epsilon(x) = 1$ for $x > 0$, -1 for $x < 0$, $\epsilon(0) = 0$. The Dirac brackets have the property $\{f, \top_m\}_D = \{\top_m, f\}_D = 0$ for arbitrary variable f. We find that A_+ which is already absent in $\top_m$, drops out also from H_c since $\Omega = 0$. The $\pi^+ \approx 0$ stays first class even with respect to the Dirac brackets and the multiplier u_+ is left undetermined. The variable π^+ decouples and we may choose $u_+ = 0$ so that π^+ and A_+ are eliminated. The light-front Hamiltonian then simplifies to

$$H^{l.f.}(\tau) = \int d^2x\ (\mathcal{D}_1\phi)(\widetilde{\mathcal{D}}_1\phi^*) \tag{5}$$

There is still a $U(1)$ *global* gauge symmetry generated by Q. The scalar fields transform under this symmetry but they are left invariant under the local gauge transformations since, $\{\Omega, f\}_D = 0$. The only *independent variables* left are ϕ and ϕ^* which satisfy the well known equal-τ *l.f. Dirac brackets*

$$\{\phi, \phi\}_D = 0,\ \{\phi^*, \phi^*\}_D = 0, \quad \{\phi(x,\tau), \phi^*(y,\tau)\}_D = K(x,y) \tag{6}$$

We remark that we could alternatively eliminate π^+ by introducing *another local gauge-fixing weak condition* $A_+ \approx 0$ (and $dA_+/d\tau \approx 0$) which is shown to be accessible. The additional modification of brackets does not alter the Dirac brackets of the scalar field already obtained. There is thus *no inconsistency in choosing the two local and weak gauge-fixing conditions* $A_\pm \approx 0$ *on the phase space at one fixed time* τ *in the CS gauge theory*; that they are accessibile follows from the Hamilton's eqs. of motion. Analogous conclusion holds also for the local Coulomb gauge in the equal-time formulation where we require $A^0 \approx 0$ and $div\vec{A} \approx 0$.

We check now the *self-consistency*[19]. From the Hamilton's eq. for ϕ we derive ($e = 1$, $\pi^* = \partial_- \phi$): $\partial_- \partial_+ \phi(x, \tau) = \{\pi^*(x, \tau), H(\tau)\}_D = \frac{1}{2} \mathcal{D}_1 \mathcal{D}_1 \phi - i \mathcal{A}_+ \partial_- \phi - \frac{i}{2}(\partial_- \mathcal{A}_+)\phi$ where $-2a\, \partial_- \mathcal{A}_+ = j^1 = -ie(\phi^* \mathcal{D}_1 \phi - \phi \widetilde{\mathcal{D}}_1 \phi^*)$. On comparing this with the corresponding Lagrange eq. $\partial_+ \partial_- \phi = \frac{1}{2} \mathcal{D}_1 \mathcal{D}_1 \phi - i A_+ \partial_- \phi - \frac{i}{2}(\partial_- A_+)\phi$ in the light-cone gauge it is suggested for convenience to rename the expression $\mathcal{A}_+$ on the phase space by (the above eliminated) A_+. We thus obtain agreement also with the other Lagrange eq. $-2a\, \partial_- A_+ = j^1 = -ie(\phi^* \mathcal{D}_1 \phi - \phi \widetilde{\mathcal{D}}_1 \phi^*)$. The Gauss' law eq. is seen to correspond to $\Omega = 0$ and the remaining Lagrange eq. is also shown to be recovered. The Hamiltonian theory in the light-cone gauge constructed here is thus shown self-consistent. The variable A_+ has *reappeared* on the phase space and we have *effectively* $A_- = 0$ (and not $A_\pm = 0$). Similar discussion can be made in the Coulomb gauge in relation to A^0 and there is *no inconsistency on using the non-covariant local gauges* for the CS system. That only the nonlocal gauges may describe[10] the fractional statistics consistently for the Lagrangian (1) is not true; it should also arise in the quantum dynamics of the simpler Hamiltonian theory on the *l.f.* or in the local Coulomb gauge. In the latter case or in the nonlocal gauges the Hamiltonian is complicated and renormalized theory seems difficult to construct. A *dual description*[7,10] may also be constructed on the *l.f.*. We can rewrite the Hamiltonian density as $\mathcal{H} = (\partial_1 \hat{\phi})(\partial_1 \hat{\phi}^*)$ if we use $A_1 = \partial_1 \Lambda$ where $8a\, \Lambda(x^-, x^1) = \int d^2y\, \epsilon(x^- - y^-)\, \epsilon(x^1 - y^1)\, j^+(y)$ and define $\hat{\phi} = e^{i\Lambda} \phi$, $\hat{\phi}^* = e^{-i\Lambda} \phi^*$. The field $\hat{\phi}$ clearly does not have the vanishing Dirac bracket (or commutator) with itself and it gives rise to the manifest fractional statistics. The theory is quantized via the correspondence of $i\{f, g\}_D$ with the commutator $[f, g]$ of the corresponding field operators. Any ambiguity in the operator ordering is resolved by the Weyl ordering.

3. Relativistic Covariance and Absence of Anomaly

The relativistic invariance of the theory above is shown by checking the Poincaré algebra of the field theory space time symmetry generators. The canonical energymomentum tensor derived from (1) is given by

$$\theta_c{}^{\mu\nu} = (\widetilde{\mathcal{D}}^\mu \phi^*)(\partial^\nu \phi) + (\mathcal{D}^\mu \phi)(\partial^\nu \phi^*) + a\epsilon^{\sigma\mu\rho} A_\sigma \partial^\nu A_\rho - \eta^{\mu\nu}\mathcal{L} \tag{7}$$

where $\partial_\mu \theta_c{}^{\mu\nu} = 0$ by construction. In the light-cone gauge they get simplified, for example, $\theta_c{}^{++} = 2\pi\pi^*$, $\theta_c{}^{+1} = -(\pi\partial_1\phi + \pi^*\partial_1\phi^*)$, $\theta_c{}^{+-} = (\mathcal{D}_1\phi)(\widetilde{\mathcal{D}}_1\phi^*) = \mathcal{H}$. The momentum generators defined by $P^\mu = \int d^2x\, \theta_c{}^{+\mu}$ are conserved and shown to generate the translations, e.g., $\{\phi, P_\mu\}_D = \partial_\mu\phi$, $\{\phi^*, P_\mu\}_D = \partial_\mu\phi^*$. The Noether current following from the Lorentz invariance of (1) is $J^{\mu\rho\sigma} = -J^{\mu\sigma\rho} = x^\rho\theta_c{}^{\mu\sigma} - x^\sigma\theta_c{}^{\mu\rho} - i(\partial\mathcal{L}/\partial(\partial_\mu A_\alpha))(\Sigma^{\rho\sigma})^\alpha{}_\beta A^\beta$ where $(\Sigma_{\rho\sigma})_{\alpha\beta} = i(\eta_{\rho\alpha}\eta_{\sigma\beta} - \eta_{\rho\beta}\eta_{\sigma\alpha})$ and $\partial_\mu J^{\mu\rho\sigma} = 0$. The Lorentz generators are $M^{\mu\nu} = -M^{\nu\mu} = \int d^2x\, J^{+\mu\nu} = \int d^2x\, [x^\mu\theta_c{}^{+\nu} - x^\nu\theta_c{}^{+\mu} - (A^\mu\pi^\nu - A^\nu\pi^\mu)]$ and in the *l.c.g.* they simplify to $M^{-1} = \int d^2x\, [x^-\theta_c{}^{+1} - x^1\theta_c{}^{+-} - aA_1^2]$, $M^{+1} = x^+P^1 - \int d^2x\, x^1\theta_c{}^{++}$, $M^{+-} = x^+P^- - \int d^2x\, x^-\theta_c{}^{++}$.

The expressions of the generators as obtained on using the symmetric Belinfante tensor $\theta_B{}^{\mu\nu} = [\theta_c{}^{\mu\nu} + a\epsilon^{\lambda\mu\beta}\partial_\lambda(A_\beta A^\nu)]$, or the symmetric gauge invariant one[7] differ from $\theta_c{}^{\mu\nu}$ only by a surface term whose contribuition to the Lorentz genertors vanishes. We remind that A_1 is now a dependent variable and the extra term in M^{-1} is sometimes called[11,7,9] *anomalous spin* induced on the scalar field due to the constrained dynamics generated by the C.S. term. A direct verification[20] of the closure of the Poincaré algebra on the mass shell is straightforward. The anomalous spin term does not break the relativistic invariance. We do find $\{\phi(x,\tau), M^{-1}(\tau)\}_D = [x^-\partial^1 - x^1\partial^-]\phi(x,\tau) - \frac{i}{2}\phi(x,\tau)\int d^2y\,\epsilon(x^- - y^-)\delta(x^1 - y^1)\, A_1(y,\tau)$, $\{\phi(x,\tau), M^{+1}(\tau)\}_D = [x^+\partial^1 - x^1\partial^+]\phi(x,\tau)$, $\{\phi(x,\tau), M^{+-}(\tau)\}_D = [x^+\partial^- - x^-\partial^+]\phi(x,\tau)$. The unusual term containing A_1 on the right hand side has been called[11,7] a *rotational anomaly* arising from the anomalous spin. Our discussion, however, shows that we may rather interpret the anomalous transformation of the scalar field in the *l.c.g.* (or in the Coulomb gauge[11,7]) as *gauge artefacts*. For example, the unusual commutators $\{M^{\mu\nu}, A_-\}_D = 0$ or $\{P^\mu, A_-\}_D = 0$ originate from the construction of the Dirac bracket on working in the *l.c.g.*. As a matter of fact A_1 also satisfies an unusual transformation, $\{A_1(x,\tau), M^{-1}(\tau)\}_D = (x^-\partial^1 - x^1\partial^-)A_1 - A_+ + (1/\partial_-)\,\partial_1 A_1$ but $\{\partial_- A_1(x,\tau), M^{-1}(\tau)\}_D = (x^-\partial^1 - x^1\partial^-)(\partial_- A_1) - (\partial_- A_+)$. Since $j^+ \sim \partial_- A_1$ and $j^1 \sim \partial_- A_+$ it follows that the gauge invariant current j^μ does preserve the property of a contravariant vector in the *l.c.g.* as it should. The *anyonicity* seems not to be related to the unusual behavior under rotations of the scalar or the gauge field in non-covariant gauges but rather to the (renormalized) quantum dynamics of CS system which is described, for example, on the *l.f.* by eqs. (5) and (6) *or* alternatively by the *dual description* above which is more difficult for constructing a renormalized theory. A parallel discussion in the Coulomb gauge can be clearly made and in the case of the fermionic field as well.

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