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Finite Temperature One-Loop Renormalization of $\lambda\varphi^4$ and Gross-Neveu Model – The Search for Triviality

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Abstract

We investigate the temperature dependent one-loop effective potential associated with two different models in a generic D dimensional spacetime. The first one is the massive self-interacting $\lambda\varphi^4$ model. The second is the Gross-Neveu model. We analize the possibility of triviality in these models at some temperature and also calculed the $\mathcal{B}$ coefficient of the Callan-Zymanzik equations in the situations where triviality can be achieved. In the $\lambda\varphi^4$ model for $D = 4$ the renormalized coupling constant attains its maximum at zero temperature and decreases monotonically as the temperature increases, so there is no triviality in this case. The situation is quite different for $D < 4$, where the renormalized coupling constant vanishes at some finite temperature β_T^{-1}. Above this triviality temperature, the effective potential becomes unbounded bellow, corresponding to a metastable vacuum. In the massless Gross-Neveu model for $D = 3$ the thermal contribution to the renormalized coupling constant is zero, which means at least in the one-loop aproximation, triviality cannot be obtained.

Key-words: Finite temperature; Renormalization.

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1 Introduction

In a recent work the one-loop effective potential at finite temperature of a neutral scalar field defined in a $D = 4$ dimensional spacetime with trivial and non-trivial topology of the spacelike sections was discussed by Ford and Svaiter[1]. Using the analytic regularization of Speer[2] and Bollini, Giambiagi and Domingues [3], and the Bogoliubov, Parasiuk and Heep renormalization procedure [4], an exact expression of the temperature and topological dependent renormalized mass and coupling constant were derived. The possibility of triviality in these models was also discussed. In the present paper our aim is twofold. First, to extend the discussion of the massive self-interacting $\lambda\varphi^4$ model to an arbitrary D dimensional spacetime assuming trivial topology of the spacelike sections. The study of triviality in the $\lambda\varphi^4$ model is not only of academic interest, since the scalar field appears in the Higgs sector of gauge theories. Second, analize the possibility of triviality in a model with asymptotic freedom. Besides Yang-Mills theories in $D = 4$, the other known perturbative renormalizable asymptotically free theories are the Nambu-Jona-Lasinio and the Gross-Neveu model [5]. In the latter, a N component fermion field with a quartic self-interaction is assumed. The model is renormalizable for $D = 2$ and develop asymptotic freedom. The interest of the Gross-Neveu model has been increasing in the literature in the last years since it was proved that the solid state antiferromagnetic Heisenberg model in the continuum limit reduce to the Gross-Neveu model [6]. We will show that in this model, since the dimensionality of the order parameter is bigger than one, the model can develop triviality at some temperature β_T^{-1}, even in $D = 2$. Note that the situation is different from the massive self-interacting $\lambda\varphi^4$ model where since the dimensionality of the order parameter is one, triviality is achieved only for $D < 4$.

Our procedure to regularize the models will be the following: instead of introducing a momentum cutoff in the Euclidean region, we prefer to regularize the theory using the Principle of Analytic Extension. Renormalization is done in the usual way of introducing counterterms to cancel the polar part of the analytic extensions. It is straighforward to show that the counterterms are temperature independent. This express the fact that if a model at zero temperature is renormalizable up to one-loop with some counterterms, it is also renormalizable at finite temperature up to one-loop by the same counterterms. Working in a generic D dimensional spacetime, we present a closed expression of the renormalized mass and coupling constant and the $\mathcal{B}$ coefficient of the Callan-Zymanzik equation [7][8]. Although it is widdely believed that non-renormalizable models are meaningless in the context of perturbation theory, they allow new insight in the study of critical phenomena. It was shown that for some non-renormalizable models (in spacetime dimension D_0) it is possible to compute the Green's functions of the non-renormalizable theory studying the model in a smaller spacetime dimension ($D < D_0$) where the model becomes super-renormalizable. As was discussed by Parisi, some non-renormalizable models are super-renormalizable models computed at the infrared stable fixed point. The renormalized Green's functions of a super-renormalizable model at the infrared unstable fixed point are also the renormalizable Green's functions of the non-renormalizable model [9]. It is clear that this approach may present some problems, for example if the infrared fixed point does not exist or if the $\mathcal{B}$ function does not have any nontrivial zero in the one-loop aproximation and higher order loops must be take into account. Although some models as for example a current-current interaction belongs to this last class we would like to stress that it is not necessary to disregard non-renormalizable models. Here we accept the idea that at one-loop it is possible to extract information on the physical behavior of the models, even if they are non-renormalizable. A standard example of a non-renormalizable interaction is gravity where it was shown that at one-loop level all the divergences of the Green's functions can be absorbed in the counterterms and the theory is finite at one-loop level [10]. A quite recent discussion about non-renormalizable interactions was given by Barvinsky, Kamenshchik and Kamazin, where the renormalization group was developed for non-renormalizable theories involving the gravitational field coupled with scalar fields [11]. Indeed, using the methods of Constructive Field Theory [12], it has been shown that perturbatively non-renormalizable models do exist, and are constructed: its Schwinger(Green) functions are expressible as phase space expansions which become *convergent* series in the thermodynamic limit. For instance the $D = 3$ dimensional Gross-Neveu model which is not renormalizable in the context of perturbation theory was recently constructed [13]. Perturbative non-renormalizability should not be a definitive criterion to discard a model in Field Theory. This will be the case when we will treat $\lambda\varphi^4$ model even in $D > 4$ at finite temperature or the Gross-Neveu model also at finite temperature for $D > 2$.

We wish to show that if we take into account the thermal dependence of the renormalized coupling constant, some models develop triviality at some temperature. In the majority of the papers in the literature the temperature dependence of the coupling constant is neglected. Parisi claims that this approach is reasonable since if we are interested in critical phenomena the variation of the mass with the temperature is the most important fact. Therefore, it is sufficient to consider the renormalized coupling constant as constant and the sign of the mass drives the phase transition. It can be shown that the error involved in this approximation affects only terms that are not singular near the critical temperature β_c^{-1} [14]. The situation is different in our case, since the goal of our investigation is not the criticality but the possibility of triviality. Then let us consider that the thermal dependence of the coupling constant is not neglected. We will show that in the same way as in some models at some spacetime dimension D there is a temperature of criticality, also in some situations, for some D it may be obtained a temperature where the model becomes free (triviality temperature β_T^{-1}). Recently Loewe and Rojas studied QED in the presence of an external constant eletromagnetic field assuming that the system is in thermal equilibrium with a reservoir at temperature β^{-1} [15]. Using the real-time formalism the effective action was calculed and also the temperature dependent coupling constant, but the possibility of triviality was not raised. In our treatament, we will meet a basic difficulty: for temperatures above the temperature of triviality the renormalized coupling constant becomes negative. This fact shows that the vacuum is a metastable state. There is no way to avoid this difficulty since the thermal dependence of the coupling constant is naturally present in QFT at finite temperature. Perhaps a way of circunvecting this dificulty would be to modify the perturbative expansion. A well known example of this procedure occurs at the criticality were we must change the perturbative expansion in order to deal with infrared divergences. For massless fields, in superrenormalizable models, in the perturbative expansion we meet infrared divergences, even in off-mass-shell amplitudes and modifications must be done in order to avoid them. This case was investigated by Jackiw and Templeton [16]. In the case of triviality we don't know yet how to deal with the problem. One way to avoid this difficulty would be to consider the coupling constant as independent of the temperature, but in this case we would have an artificial situation where we are treating mass and coupling constant in different manners, since the temperature dependent mass idea is widely used to explain spontaneous broken symmetry. The outline of the paper is the following. In section II we sketch the formalism of the effective potential. In section III, the massive self-interacting $\lambda\varphi^4$ model is analised. In section IV we repeated the calculations in the Gross-Neveu model. Conclusions are given in section V . In this paper we use $\frac{h}{2\pi} = c = k_B = 1$.

2 The effective action and the effective potential at zero temperature

In this chapter we will briefly review the basic features of the effective potential associated with a real massive self-interacting scalar field at zero temperature. Although the formalism of this section may be found in standard texbooks, we recall here its main results for completeness. Let us supose a real massive scalar field $\varphi(x)$ with the usual $\lambda\varphi^4(x)$ self-interaction, defined in a static spacetime. Because the manifold is static, there is a global timelike killing vector field orthogonal to the spacelike sections. Owing to this, energy and thermal equilibrium has a precise meaning. For the sake of simplicity let us supose that the manifold is flat. In the path integral approach, the basic object is the generating functional,

$$Z[J] = < 0, out|0, in >= \int \mathcal{D}[\varphi] \exp\{i[S[\varphi] + \int d^4x J(x)\varphi(x)]\} \quad (1)$$

where $\mathcal{D}[\varphi]$ is an apropriate measure and $S[\varphi]$ is the classical action associated with the scalar field. The quantity $Z[J]$ gives the transition amplitude from the initial vacuum $|0, in >$ to the final vacuum $|0, out >$ in the presence of some source $J(x)$, which is zero outside some interval $[-T, T]$ and inside this interval was switched on and off adiabatically. Since we are interested in the connected part of the time ordered products of the fields, we take as usual the connected generating functional $W[J]$. This quantity is defined in terms of the vacuum persistente amplitude by

$$e^{iW[J]} = < 0, out|0, in > . \quad (2)$$

The connected n point function $G_c(x_1, x_2, .., x_n)$ is defined by

$$G_c(x_1, x_2, .., x_n) = \frac{\delta^n W[J]}{\delta J(x_1)...\delta J(x_n)}|_{J=0}. \tag{3}$$

Expanding $W[J]$ in a functional Taylor series, the n-order coefficient of this series will be the sum of all connected Feynman diagrams with n external legs, i.e. the connected Green's functions defined by eq.(3). Then

$$W[J] = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^4x_1..d^4x_n \; G_c^{(n)}(x_1, x_2....x_n) J(x_1)J(x_2)..J(x_n). \tag{4}$$

The classical field φ_0 is given by the normalized vacuum expectation value of the field

$$\varphi_0(x) = \frac{\delta W}{\delta J(x)} = \frac{< 0, out|\varphi(x)|0, in >_J}{< 0, out|0, in >_J}, \tag{5}$$

and the effective action $\Gamma[\varphi_0]$ is obtained by performing a functional Legendre transformation

$$\Gamma[\varphi_0] = W[J] - \int d^4x J(x)\varphi_0(x). \tag{6}$$

Using the functional chain rule and the definition of φ_0 given by eq.(5) we have

$$\frac{\delta\Gamma[\varphi_0]}{\delta\varphi_0} = -J(x). \tag{7}$$

Just as $W[J]$ generate the connected Green's functions via a functional Taylor expansion, the effective action can be represented as a functional power series around the value $\varphi_0 = 0$, where the coeficients are just the proper n point functions $\Gamma^{(n)}(x_1, x_2, .., x_n)$ i.e

$$\Gamma[\varphi_0] = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^4x_1 d^4x_2...d^4x_n \; \Gamma^{(n)}(x_1, x_2, .., x_n) \; \varphi_0(x_1)\varphi_0(x_2)..\varphi_0(x_n). \tag{8}$$

The coefficients of the above functional expansion are the connected 1 particle irreducible diagrams ($1PI$). Actually $\Gamma^{(n)}(x_1, x_2, ..., x_n)$ is the sum of all $1PI$ Feynman diagrams with n external legs. Writting the effective action in powers of momentum (around the point where all external momenta vanish) we have

$$\Gamma[\varphi_0] = \int d^4x \Big(-V(\varphi_0) + \frac{1}{2}(\partial_\mu\varphi)^2 Z[\varphi_0] \; + \; .. \Big). \tag{9}$$

The term $V(\varphi_0)$ is called the effective potential[17][18] .To express $V(\varphi_0)$ in terms of the $1PI$ Green's functions, let us write $\Gamma^{(n)}(x_1, x_2, .., x_n)$ in the momentum space:

$$\Gamma^{(n)}(x_1, x_2, .., x_n) = \frac{1}{(2\pi)^n} \int d^4k_1 d^4k_2..d^4k_n (2\pi)^4 \delta(k_1+k_2+..k_n)\, e^{i(k_1x_1+..k_nx_n)} \tilde{\Gamma}^{(n)}(x_1, x_2, .., x_n). \tag{10}$$

Assuming that the model is translationally invariant, i.e. φ_0 is constant over the manifold, we have

$$\Gamma[\varphi_0] = \int d^4x \sum_{n=1}^{\infty} \frac{1}{n!} \Big(\tilde{\Gamma}^{(n)}(0, 0, ...)(\varphi_0)^n + ... \Big). \tag{11}$$

If we compare eq.(9) with eq.(11) we obtain that

$$V(\varphi_0) = -\sum_n \frac{1}{n!} \tilde{\Gamma}^{(n)}(0, 0, ..)(\varphi_0)^n, \tag{12}$$

then $\frac{d^nV}{d\varphi_0^n}$ is the sum of the all $1PI$ diagrams carring zero external momenta. Assuming that the fields are in equilibrium with a thermal reservoir at temperature β^{-1}, in the Euclidean time formalism, the effective potential $V(\beta, \varphi_0)$ can be identified with the free energy density and can be calculated by imposing periodic (antiperiodic) boundary conditions on the bosonic (fermionic) fields.

In the next section we will investigate the one-loop renormalization of a massive self-interacting scalar field in thermal equilibrium with a reservoir at temperature β^{-1}. Instead of regularizing the model introducing a ultraviolet cut-off in the Euclidean momenta we prefer to use the Principle of Analytic Extension. The advantage of this method lies in the fact that the temperature dependence of the physical quantities, mass and coupling constant appear in a very straightforward way.

3 The one-loop effective potential in the $\lambda\varphi^4$ model at zero and finite temperature.

In this section we will generalize the results obtained in Ref.(1) to a generic D dimensional spacetime. Let us assume the following Lagrange density associated with a massive neutral scalar field:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi_u)^2 - \frac{1}{2}m_0^2\ \varphi_u^2 - \frac{\lambda_0}{4!}\varphi_u^4 \tag{13}$$

where $\varphi_u(x)$ is the unrenormalized field and m_0 and λ_0 are the bare mass and bare coupling constant respectively. We may rewrite the Lagrange density as the usual form where the counterterms will appear explicity. Defining the quantities

$$\varphi_u(x) = (1+\delta Z)^{\frac{1}{2}}\varphi(x) \tag{14}$$

$$m_0^2 = (m^2+\delta m^2)(1+\delta Z)^{-1} \tag{15}$$

$$\lambda_0 = (\lambda+\delta\lambda)(1+\delta Z)^{-2}, \tag{16}$$

and substituting eq.(14), eq.(15) and eq.(16) in eq.(13) we have

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{4!}\varphi^4 + \frac{1}{2}\delta Z(\partial_\mu\varphi)^2 - \frac{1}{2}\delta m^2\varphi^2 - \frac{1}{4!}\delta\lambda\varphi^4, \tag{17}$$

where δZ, δm^2, and $\delta\lambda$ are the wave function, mass and coupling constant counterterms of the model. After the Wick rotation, in the one-loop aproximation, the effective potential is given by [18]:

$$V(\varphi_0) = V_I(\varphi_0) + V_{II}(\varphi_0) \tag{18}$$

where,

$$V_I(\varphi_0) = \frac{1}{2}m^2\varphi_0^2 + \frac{\lambda}{4!}\varphi_0^4 - \frac{1}{2}\delta m^2\varphi_0^2 - \frac{1}{4!}\delta\lambda\varphi_0^4, \tag{19}$$

and

$$V_{II}(\varphi_0) = \sum_{s=1}^{\infty}\frac{(-1)^{s+1}}{2s}\left(\frac{1}{2}\lambda\varphi_0^2\right)^s\int\frac{d^Dq}{(2\pi)^D}\frac{1}{(\omega^2+\vec{q}^{\ 2}+m^2)^s}. \tag{20}$$

Before going on, we would like to discuss some important points. It is well known that the renormalizability of any model depends on the dimension D of the spacetime. In the

$\lambda\varphi^4$ model for $D > 4$ the model is nonrenormalizable and for $D < 4$ the model is superrenormalizable. Nevertheless we will do all the calculations in a generic D dimensional spacetime. Although one cannot handle in perturbation theory a non-renormalizable model, at the one-loop aproximation this problem is circunvented, since we have not to deal with the growth of the the degree of divergence of a graph with the order of perturbation theory and one would not need to fix infinite parameters in order to render the theory finite. As we discussed in the introduction it is sometimes possible to obtain results for a non-renormalizable model studying the behavior of its Green's functions of the superrenormalizable model at the infrared stable fixed point. Some non-renormalizable models are super-renormalizable ones computed at the infrared stable fixed point. Although there are some problems in this approach and many non-renormalizable models does not belong to this class, as gravity and the current-current interaction between fermions in $D > 2$, we continue to work in a generic D dimensional spacetime. Performing analytic or dimensional regularization, we must introduce a mass parameter μ, in terms of which, dimensional analysis give to the field a dimension $[\varphi] = \mu^{1/2(D-2)}$ and to the coupling constant a dimension $[\lambda] = \mu^{4-D}$. Mass has dimension of inverse length, μ i.e. $[\mu] = [m] = L^{-1}$, and the effective potential i.e. the energy density per unit volume has dimension of L^{-D}.

There is no difficulty to extend the above results to finite temperature states. After a Wick rotation, the functional integral runs over the fields that satisfy periodic boundary conditions in Euclidean time. The effective action can be defined as in the zero temperature case by a functional Legendre transformation and regularization and renormalization procedures follows the same steps as in the zero temperature case. Although the counterterms introduced at finite temperature are the same as in the zero temperature case, i.e. the polar part of the effective potential is the same in both cases, as we will see, the assumption of the equilibrium with a thermal reservoir at finite temperature β^{-1} opens the possibility of triviality. In particular, as we will show, in the $\lambda\varphi^4$ model (for $D < 4$) a temperature β_T^{-1} where the model becomes free, may be found. To study temperature effects we perform as usual the following replacement in the Euclidean region:

$$\int \frac{d\omega}{2\pi} \to \frac{1}{\beta}\sum_n \tag{21}$$

and

$$\omega \to \frac{2\pi n}{\beta} \tag{22}$$

where $\omega_n = \frac{2\pi n}{\beta}$ are the Matsubara frequencies. Defining the dimensionless quantities:

$$c^2 = \frac{m^2}{4\pi^2\mu^2} \tag{23}$$

and

$$(\beta\mu)^2 = a^{-1}, \tag{24}$$

the Born terms plus one-loop terms contributing to the effective potential give,

$$V(\beta,\varphi_0) = V_I(\varphi_0) + V_{II}(\beta,\varphi_0)$$

where,

$$V_I(\beta,\varphi_0) = \frac{1}{2}m^2\varphi_0^2 + \frac{\lambda}{4!}\varphi_0^4 - \frac{1}{2}\delta m^2\varphi_0^2 - \frac{1}{4!}\delta\lambda\varphi_0^4, \tag{25}$$

and

$$V_{II}(\beta,\varphi_0) = \frac{1}{\beta}\sum_{s=1}^{\infty}\frac{(-1)^{s+1}}{2s}\left(\frac{\lambda}{8\pi^2}\right)^s\left(\frac{\varphi_0}{\mu}\right)^{2s}\int\frac{d^dq}{(2\pi)^d}A_1^{M^2}(s,a). \tag{26}$$

The function

$$A_N^{c^2}(s,a_1,a_2,..,a_N) = \sum_{n_1,n_2..n_N=-\infty}^{\infty}(a_1n_1^2 + a_2n_2^2 + ... + a_Nn_N^2 + c^2)^{-s} \tag{27}$$

is the inhomogeneous Epstein zeta function[19], and finally

$$M^2 = \frac{1}{4\pi^2\mu^2}(\vec{q}^{\,2}) + c^2.$$

Note that the mass parameter μ introduced in eq.(23) and eq.(24) will be used from now on, since we must have dimensionless functions in working with analytic extensions.

Let us define the modified inhomogeneous Epstein zeta function as

$$E_N^{c^2}(s,a_1,a_2,..a_N)=\sum_{n_1,n_2,..n_N=1}^{\infty}(a_1n_1^2+..+a_Nn_N^2+c^2)^{-s}. \qquad (28)$$

Defining the new coupling constant and a new vacuum expectation value of the field ϕ (dimensionless for $D=4$),

$$g=\frac{\lambda}{8\pi^2} \qquad (29)$$

$$\frac{\varphi_0}{\mu}=\phi \qquad (30)$$

$$k^i=\frac{q^i}{2\pi\mu} \qquad (31)$$

we rewrite eq.(26) as,

$$V_{II}(\beta,\phi)=\mu^D\sqrt{a}\sum_{s=1}^{\infty}\frac{(-1)^{s+1}}{2s}g^s\phi^{2s}\sum_{n=-\infty}^{\infty}\int d^dk\frac{1}{(an^2+c^2+\vec{k}^{\,2})^s}. \qquad (32)$$

To regularize the model we will use a mix between dimensional and zeta function analytic regularizations. Let us first use dimensional regularization[20]. Using the well known result,

$$\int\frac{d^dk}{(k^2+a^2)^s}=\frac{\pi^{\frac{d}{2}}}{\Gamma(s)}\Gamma(s-\frac{d}{2})\frac{1}{a^{2s-d}}, \qquad (33)$$

eq. (32) becomes

$$V_{II}(\beta,\phi)=\mu^D\sqrt{a}\sum_{s=1}^{\infty}\frac{(-1)^{s+1}}{2s}g^s\phi^{2s}\frac{\pi^{\frac{d}{2}}}{\Gamma(s)}\Gamma(s-\frac{d}{2})\sum_{n=-\infty}^{\infty}\frac{1}{(an^2+c^2)^{s-\frac{d}{2}}}. \qquad (34)$$

Defining,

$$f(D,s)=f(d+1,s)=\frac{(-1)^{s+1}}{2s}\pi^{\frac{d}{2}}\Gamma(s-\frac{d}{2})\frac{1}{\Gamma(s)} \qquad (35)$$

and substituting eq.(27) and eq.(35) in eq.(34) we obtain,

$$V_{II}(\beta,\phi)=\mu^D\sqrt{a}\sum_{s=1}^{\infty}f(D,s)g^s\phi^{2s}A_1^{c^2}(s-\frac{d}{2},a). \qquad (36)$$

As we will see soon, the terms $s \leq \frac{D}{2}$ are divergent and we will regularize the one-loop effective potential using the Principle of the Analytic Extension. Then, let us assume that the each term in the series of the one-loop effective potential $V(\beta,,\phi)$ is the analytic extension of these terms, defining in the begining in a open connected set. To render the discussion more general, let us discuss the process of the analytic continuation of the modified inhomogeneous Epstein zeta function given by eq.(27). For $Re(s) > \frac{N}{2}$, the $E_N^{c^2}(s, a_1, a_2, ..a_N)$ converges and represent an analytic function of s, so $Re(s) > \frac{N}{2}$ is the largest possible domain of the convergences of the series. This means that in eq (36) in the case $D = 4$ only the terms $s = 1$ and $s = 2$ are divergent. The term $s = 1$ is the divergent one-loop diagram of the connected two point function and it contributes with a quadratic divergence. The $s = 2$ term is the divergent one-loop diagram of the connected four point function, and it contributes to the effective potential with a logarithmic divergence. Using a Mellin transform it is possible to find the analytic extension of the modified inhomogeneous Epstein zeta function. After some calculations using the results of Ref.[21] we have:

$$V_{II}(\beta,\phi) = \mu^D \sum_{s=1}^{\infty} f(D,s) g^s \phi^{2s} \sqrt{\pi}\left(\frac{m}{2\pi\mu}\right)^{D-2s} \frac{1}{\Gamma(s-\frac{d}{2})}\left(\Gamma(s-\frac{D}{2})+4\sum_{n=1}^{\infty}\left(\frac{mn\beta}{2}\right)^{s-\frac{D}{2}} K_{\frac{D}{2}-s}(mn\beta)\right) \tag{37}$$

where $K_\mu(z)$ is the Kelvin function [22].

It is not difficult to show that:

$$V_{II}(\beta,\phi) = \mu^D \sum_{s=1}^{\infty} g^s \phi^{2s} h(D,s)\left(\frac{1}{2^{\frac{D}{2}-s+2}}\Gamma(s-\frac{D}{2})(\frac{m}{\mu})^{D-2s} + \sum_{n=1}^{\infty}\left(\frac{m}{\mu^2\beta n}\right)^{\frac{D}{2}-s} K_{\frac{D}{2}-s}(mn\beta)\right) \tag{38}$$

where:

$$h(D,s) = \frac{1}{2^{\frac{D}{2}-s-1}}\frac{1}{\pi^{\frac{D}{2}-2s}}\frac{(-1)^{s+1}}{s}\frac{1}{\Gamma(s)}. \tag{39}$$

For the sake of simplicity let us suppose $D = 4$. It is clear that this model is renormalizable and an appropriate choice of δm^2 and $\delta\lambda$ will render the analytic extension of the terms of the series in s in the effective potential analytic functions in the neighbourhood of the poles $s = 1$ and $s = 2$ respectivelly. The idea to continue analytically expressions and subtract poles was exploited by Speer, Bollini and others. In the method used by Bollini, Giambiagi and Domingues, a complex parameter s was introduced as an expoent of the denominator of the loop expressions and the integrals are well defined analytic functions of the parameters in the region $Re(s) > s_0$ for some s_0. Performing an analytic extension of the expression for $Re(s) \le s_0$, poles will appear in the analytic extension and the final expression becomes finite after a renormalization procedure. To find the exact form of the counterterms let us use the renormalization conditions

$$\frac{\partial^2}{\partial\phi^2}V(\beta,\phi)|_{\phi=0} = m^2\mu^2 \tag{40}$$

and

$$\frac{\partial^4}{\partial\phi^4}V(\beta,\phi)|_{\phi=0} = \lambda\mu^4. \tag{41}$$

There is no need for wave function renormalization because the vacuum expectation value of the field has been chosen to be a constant. Substituting eq.(25), eq.(38) and eq.(39) in eq.(40) and eq.(41) it is possible to find the exact form of the countertems in such a way that they cancel the polar parts of the analytic extension of the terms $s = 1$ and $s = 2$. It is straighforward to show that both δm^2 and $\delta\lambda$ are temperature independent. The fact that if a model at zero temperature is renormalizable, with some counterterms it is also renormalizable at finite temperature with the same counterterms was proved in all orders of perturbation theory by Kislinger and Morley [23]. In general, only the divergent part of the counterterms are fixed by the renormalization conditions. In the neighbourhood of the poles $s = 1$ and $s = 2$ the regular part of the analytic extension of inhomogeneous Epstein zeta function has two contributions: one which is independent of the temperature and that can be absorbed by the counterterms and another that depends on it and cannot be absorbed by the counterterms. Let us identify the thermal contribution to the renormalized mass as the regular part of the second derivative of the effective potential with respect to the field at $\phi = 0$. (Although the renormalization point is arbitrary choosing $\phi = 0$ only the contribution of the pole at $s = 1$ will appear). It is clear that the temperature dependent mass is proportional to the regular part of the analytic extension of the inhomogeneous Epstein zeta function in the neighborhood of the pole $s = 1$. The same argument can be applied to the renormalized coupling constant. The thermal contribution to the renormalized coupling constant is proportional to the analytic extension of the inhomogeneous Epstein zeta function in the neighborhood of the pole $s = 2$. As we discussed above, the choice of the renormalization point $\phi = 0$ implies that only the regular part in the neighborhood of the pole $s = 1$ will appear in the renormalized mass. In the next section studying massless self-interacting fermion fields we will show that since it is necessary to choose the renormalization point at non-zero vacuum expectation value of the field all the terms of the series of the effective potential give a contribution to the renormalized mass and coupling constant. Before going back to the thermal mass and coupling constant let us discuss some questions related to spontaneous symmmetry breaking. Since $m^2 > 0$ the effective potential has its minimum at $\varphi_0 = 0$. If $m^2 < 0$ the true minima of the effective potential are no longer zero and using the condition $\frac{dV}{d\varphi_0} = 0$ for some non-zero value of φ_0 it is possible to find the value of φ_0 for which the minimum of the effective potential occurs. In this case the renormalization conditions must be changed in the sense that the renormalization point is the non-zero expectation value of the field which minimizes the effective potential. So, the critical temperature is defined exactly as the temperature at which the tadpole induced temperature contribution to the mass cancels the bare zero temperature mass. Note that the results are not modified by the fact that the effective potential has a mimimum for non-vanishing expectation value of the field. The only difference is the presence of all terms of the series in s in the thermal mass and thermal coupling constant and in this case the sign of the thermal coupling constant cannot be easily computed. From the above discussion we can write

$$m^2(\beta) = m^2 + \Delta m^2(\beta) \tag{42}$$

and

$$\lambda(\beta) = \lambda + \Delta\lambda(\beta). \tag{43}$$

where $m^2(\beta)$ and $\lambda(\beta)$ are respectivelly the temperature dependent renormalized mass square and coupling constant. It is straighforward to show that the thermal contribution to the renormalized mass square is given by:

$$\Delta m^2(\beta) - \Delta m^2(\infty) = \frac{1}{8\pi^2}\lambda \sum_{n=1}^{\infty} \frac{m}{\beta n} K_1(mn\beta). \tag{44}$$

Using the asymptotic representation of the Bessel function $K_n(z)$ for small arguments

$$K_n(z) \cong \frac{1}{2}\Gamma(n)(\frac{z}{2})^{-n} \quad , z \to 0 \quad n = 1, 2, ..,$$

we obtain that at high temperatures the temperature dependent mass square is proportional to $\lambda\beta^{-2}$. This result was obtained by many authors [24] and was discussed recently by Arnold and Carrington[25]. The result given by eq.(44) was also obtained by Braden [26] using the Schwinger's proper time method. This author also discussed the two-loop effective potential and the problem of overlapping divergences where the possibility of temperature dependent counterterms appear. Nevertheless these divergences must cancel as it was stressed by Kislinger and Morley [23].

By the same reasoning, the thermal contribution to the renormalized coupling constant is given by:

$$\Delta\lambda(\beta) - \Delta\lambda(\infty) = -\frac{3}{16\pi^2}\lambda^2 \sum_{n=1}^{\infty} K_0(mn\beta). \tag{45}$$

The function $K_0(z)$ is positive and decreasing for $z > 0$. Therefore let us present an interesting result: the renormalized coupling constant attains its maximum at zero temperature ($\beta^{-1} = \infty$) and decrease monotonically as the temperature increases. In other words, the thermal contribution to the renormalized coupling constant $\Delta\lambda(\beta) - \Delta\lambda(\infty)$ is negative, and increase in modulus with the temperature. We are discussing thermal effects, then in the limit of zero temperature the thermal contribution to the thermal mass and the thermal contribution to the thermal coupling constant must vanish as we can see easily from eq.(42) and eq.(43). Since the thermal contribution to the renormalized coupling constant is negative someone can enquire: it is possible to vanish the renormalized coupling constant ? In other words, there is a temperature of triviality? Since $\Delta\lambda(\beta)$ is $O(\lambda^2)$ if we assume $D = 4$, it is not possible to implement such mechanism. But, if $D < 4$ the renormalized coupling constant is not necessarilly a small quantity. For spacetime dimensions D smaller than four, this is rather a large quantity, due to its positive dimension $4 - D$ in terms of the mass parameter μ (or using the language of critical phenomena, due to its positive dimension $4 - D$ in terms of the scale $\frac{1}{a}$ where a is the lattice spacing). Following this argument, for $D < 4$ it may have a temperature β_T^{-1} such that the renormalized coupling constant vanishes (the temperature of triviality). By completness we will consider the two cases: $D < 4$ and also the case $D > 4$. Of course if we are interested in critical phenomena, the study of the model in dimensions below $D = 4$ is obligatory. In this case the model becomes superrenormalizable and there is only a finite number of divergent graphs. Note that in the one-loop aproximation for $D = 4$ the are only two divergent graphs and for $D < 4$ only one graph is divergent. This result can be easilly obtained investigating eq.(38). In this equation the divergent part of the effective potential is given by $\Gamma(s-\frac{D}{2})$ and for $D<4$ only the $s=1$ pole will appear. In other words, for $D<4$ there is only finite coupling constant renormalization at the one-loop aproximation. The graph $s=2$ gives a finite and negative contribution to the coupling constant and triviality can be achieved. Since in superrenormalizable models the renormalizable coupling constant may not be a smalll quantity it could be argued that our calculations are meaningless since we have a breakdown of the perturbative expansion. This is not a definitive argument. Even if the renormalized coupling constant is a small quantity, the perturbative series may be divergent; there are strong evidences that perturbative series are divergent in QED, which is the most sucessful field theory. In some cases, if the (divergent) perturbative expansion satisfies the conditions of the Watson-Nevanlinna-Sokal theorem [27], it may be Borel summable, in which case the model is meaningfull even if it is not mathematically defined in perturbation theory [28]. For $D>4$ the renormalization of the coupling constant is obligatory (note the presence of the pole in $s=2$). Going back to the D dimensional case the renormalization conditions also are given by eq.(40) and eq.(41) Using the renormalization conditions in eq.(38), it is not difficult to find the regular part of the analytic extension which gives a finite contribution to the renormalized mass square $\Delta m^2(D,m,\lambda,\beta)$ and coupling constant $\Delta\lambda(D,m,\lambda,\beta)$ in a D dimensional flat spacetime (note that by simplicity we will write $\Delta m^2(\beta)$ and $\Delta\lambda(\beta)$). For D even are given respectively by:

$$\Delta m^2(\beta)=\frac{\mu^{D-2}\lambda}{2(2\pi)^{D/2}}\Big(\frac{(-1)^{\frac{D}{2}-1}}{(\frac{D}{2}-1)!}\psi(\frac{D}{2})(\frac{m}{\mu})^{D-2}+\sum_{n=1}^{\infty}\Big(\frac{m}{\mu^2\beta n}\Big)^{\frac{D}{2}-1}K_{\frac{D}{2}-1}(mn\beta)\Big) \tag{46}$$

and

$$\Delta\lambda(\beta)=-\frac{3}{4}\frac{\mu^{D-4}\lambda^2}{(2\pi)^{D/2}}\Big(\frac{(-1)^{\frac{D}{2}-2}}{(\frac{D}{2}-2)!}\psi(\frac{D}{2}-1)(\frac{m}{\mu})^{D-4}+\sum_{n=1}^{\infty}\Big(\frac{m}{\mu^2\beta n}\Big)^{\frac{D}{2}-2}K_{\frac{D}{2}-2}(mn\beta)\Big) \tag{47}$$

where $\psi(s)=\frac{d}{ds}ln\Gamma(s)$. For D odd, the first term between the parentesis in eq.(46) and eq.(47) must be changed by $\Gamma(1-\frac{D}{2})(\frac{m}{\mu})^{D-2}$ and $\Gamma(2-\frac{D}{2})(\frac{m}{\mu})^{D-4}$ respectivelly. Since the first terms between the parentesis of eq.(46) and eq.(47) are temperature independent it is possible to isolate the thermal contribution to the renormalized mass and coupling constant in a generic D dimensional spacetime in the one-loop aproximation. Using eq.(46) and eq.(47) we obtain the following contribution to the thermal mass and coupling constant respectivelly

$$\Delta m^2(\beta)-\Delta m^2(\infty)=\frac{\mu^{D-2}\lambda}{2(2\pi)^{D/2}}\sum_{n=1}^{\infty}\left(\frac{m}{\mu^2\beta n}\right)^{\frac{D}{2}-1}K_{\frac{D}{2}-1}(mn\beta) \tag{48}$$

and

$$\Delta\lambda(\beta)-\Delta\lambda(\infty)=-\frac{3}{4}\frac{\mu^{D-4}\lambda^2}{(2\pi)^{D/2}}\sum_{n=1}^{\infty}\left(\frac{m}{\mu^2\beta n}\right)^{\frac{D}{2}-2}K_{\frac{D}{2}-2}(mn\beta). \tag{49}$$

Since $\Delta\lambda(\beta)-\Delta\lambda(\infty)<0$ we may have a temperature of triviality β_T^{-1} in the model for $D<4$. This is one of the main results of the paper. We cannot overlook the Frohlich result [29] who has obtained that all the Green's functions of the theory for $D>4$ correspond to a free field i.e. the model is gaussian at zero temperature above four spacetime dimensions. One could related the Frohlich and our result? Note that only if the dimensionality of the order parameter is one, for $D<4$ there is a temperature where the model becomes free i.e. Gaussian. Defining a dimensionless effective potential $v=\frac{V}{\mu^D}$ we have:

$$\begin{aligned} v(\beta,\phi) &= \frac{1}{2}m^2\mu^{2-D}\phi^2+\frac{\lambda}{4(2\pi)^{D/2}}\sum_{n=1}^{\infty}\left(\frac{m}{\mu^2\beta n}\right)^{\frac{D}{2}-1}K_{\frac{D}{2}-1}(mn\beta)\phi^2 \\ &+ \frac{\lambda}{4!}\mu^{4-D}\phi^4-\frac{1}{32}\frac{\lambda^2}{(2\pi)^{D/2}}\sum_{n=1}^{\infty}\left(\frac{m}{\mu^2\beta n}\right)^{\frac{D}{2}-2}K_{\frac{D}{2}-2}(mn\beta)\phi^4, \end{aligned} \tag{50}$$

Where higher order terms in s are neglected. For low temperatures the dimensionless effective potential has only one global minimum. If we wish to heat the system above the temperature of triviality the renormalized coupling constant would become negative and the effective potential would be unbounded below. In this case the perturbative vacuum is a metastable state. Negative renormalized coupling constant means that instead of a repulsive force between the quanta of the field it appear an atractive force. This is related with the stability of the system and the mean life of the vacuum state. It is possible to draw a graph of the dimensionless effective potential as a function of the temperature and ϕ. We take as an example the case $D = 4$, $\lambda = m = \mu = 1$. See fig.(1). The temperature is the parameter that allows to interpolate between the two configurations: a stable vacuum at low temperatures and an metastable state in the high temperature limit. Before studying a model with asymptotic freedom we will present the $\mathcal{B}$ coeficient of the Callan-Zymanzik equation for the case $D = 4$, $D > 4$ and $D < 4$. Since we used a mix between dimensional regularization and the zeta function analytic regularization, the renormalized coupling constant acquire mass dimension for $D \neq 4$. To isolate these mass dimension explicity let us define:

$$g_r = g_R \mu^{\epsilon} \tag{51}$$

where g_R is the dimensionless renormalized coupling constant and $\epsilon = 4 - D$. The $\mathcal{B}$ coefficient is given by,

$$\mu \frac{d}{d\mu} g_R = \mathcal{B}. \tag{52}$$

It is not dificult to find the expressions for the $\mathcal{B}(\lambda)$ coefficient of the Callan-Zymanzik equation in both cases $D = 4$ and $D \neq 4$. For $D = 4$ we have

$$\mathcal{B}(\lambda) = \lambda^2(\alpha + f(\beta)) \tag{53}$$

where α is a constant proportional to the first term in eq.(47) and $f(\beta)$ is proportional to the second one (temperature dependent) of the same equation. It is clear that $f(\beta) \to 0$ if $\beta \to \infty$ and $f(\beta) \to \infty$ if $\beta \to 0$. For any temperature the model has only the trivial infrared fixed point. For $D \neq 4$ we have:

$$\beta(\lambda) = (4 - D)(-\lambda + \lambda^2(\alpha' + f(\beta)), \tag{54}$$

where α' is also a constant. In the case $D < 4$ we get the well known result for low temperatures: we have a trivial ultraviolet stable fixed point and a non-trivial infrared stable fixed point. If we increase the temperature the infrared fixed point approaches the origin and at high temperatures the model has only a trivial infrared stable fixed point (infrared asymptotically free). For $D > 4$ we have an opposite behavior. For low temperatures we have a trivial infrared stable fixed point and a non-trivial ultraviolet stable fixed point. As in the preceding case, if we start to heat the system the ultraviolet fixed point goes to the origen. For high temperatures the model becomes ultraviolet asymptotically free. In the same way as it is possible to draw the surface of the equation of state for a ferromagnet in the space of the magnetization, external magnetic field and temperature, we will draw a surface in the space of the $\mathcal{B}$ coefficient, the inverse of the temperature β and λ. See figs (2) and (3). Note that for low temperatures the standard results and ours coincide, since $f(\beta)$ goes to zero at low temperatures. The effect of rising the temperature is to approach the non-trivial fixed points (infrared and ultraviolet) to the origin. The renormalization group of the $\lambda\varphi^4$ model in $D = 4$ has been recently studied by Kerman and Martin[30]. These authors using the Gaussian aproximation have shown that the $\mathcal{B}$ coefficient of $\lambda\varphi^4$ is negative in the broken phase of the model i.e. the model becomes asymptoticaly free in this phase. We would like to stress that our work has been done at one-loop aproximation. To improve these results it would be interesting to go beyond one-loop aproximation to analise the sign of the thermal contribution to the coupling constant. A proof that the sign of the thermal correction to the coupling constant is also negative if we take in account higher order loops is still under investigation.

4 The one-loop effective potential in the massless Gross-Neveu model at finite temperature

The idea of this section is to examinate the mechanism of triviality in a model involving fermions with a quartic interaction with a coupling constant g^2. It is well known that in two-dimensional spacetime ($D = 2$) the model is renormalizable and asymptotically free. We will consider an N-component fermion field and the limit of large N will be investigated. As was stressed in ref.(4) due to the quartic nature of the interaction it is possible to introduce an ultralocal auxiliar scalar field φ which is formally equal to $g\overline{\psi}\psi$ where $\psi(x)$ is the fermionic field, in order to present the effective potential of the model. Exactly as we did in the section II we suppose that the quantum field is in thermal equilibrium with a reservoir at temperature β^{-1}. We will show that for $D = 2$ and $D = 4$ the sign of the thermal correction to the renormalized coupling constant cannot be easily calculed, then the mechanism of triviality cannot be proved. Nevertheless for $D = 3$ the thermal correction to the renormalized coupling constant is zero and triviality cannot be achieved at one-loop aproximation.

The Lagrange density of the massless model is given by:

$$\mathcal{L}(\overline{\psi}, \psi, \varphi) = i\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - \frac{1}{2}\varphi^2 - g\varphi\overline{\psi}\psi. \tag{55}$$

Defining φ_0 as the vacuum expectation value of φ, i.e. $\varphi_0 =< 0|\varphi|0 >=< 0|g\overline{\psi}\psi|0 >$, the leading terms in the effective potential for large N are given by the tree-level graphs plus all one-loop graphs,

$$V(\varphi) = \frac{1}{2}\varphi_0^2 - iN\sum_{s=1}^{\infty}\frac{1}{2s}(g\varphi_0)^{2s}\int\frac{d^Dq}{(2\pi)^D}\frac{1}{k^{2s}}. \tag{56}$$

After a Wick rotation we identify the effective potential as the free energy of the system. To introduce finite temperature effects we assume that the Grasmannian integration in the path integral goes over anti-periodic configurations in Euclidean time. In the effective potential this is equivalent to the replacement given by eq.(21) and

$$\omega \rightarrow \frac{2\pi}{\beta}(n+\frac{1}{2}). \tag{57}$$

Using eq.(33) and defining $f(D,s)$ by:

$$p(D,s) = \frac{1}{2^{2s+1}} \frac{1}{\pi^{2s-\frac{d}{2}}} \frac{(-1)^s}{s} \frac{\Gamma(s-\frac{d}{2})}{\Gamma(s)}, \tag{58}$$

it is not difficult to show that $V(\beta,\varphi_0)$ is given by:

$$V(\beta,\varphi_0) = \frac{1}{2}\varphi_0^2 + N\sum_{s=1}^{\infty} p(D,s)(g\varphi_0)^{2s}\beta^{2s-D} \sum_{n=-\infty}^{\infty} \frac{1}{(n+\frac{1}{2})^{2s-d}}. \tag{59}$$

Before going on, we would like to make some coments. Note that we are using dimensional regularization in eq.(56) and it is well known that for massless fields this technique requires modification in order to deal with infrared divergences [31]. Since we are regularizing only a $d = D - 1$ dimensional integral, this procedure is equivalent of inserting a mass into the d dimensional integral. In other words, the Matsubara frequency play the role of a "mass" in the integral, provided we exclude the limit $\beta \rightarrow \infty$ which means that we must be restricted to non-zero temperature.

Again as in eq.(30) we can define a new field $\phi = \frac{\varphi_0}{\mu}$ (no confusion must be done between the present auxiliar scalar field and the previous scalar field). Using eq.(24) we obtain

$$V(\beta,\phi) = \frac{1}{2}\mu^2\phi^2 + N\mu^D\sum_{s=1}^{\infty} p(D,s)a^{\frac{D}{2}-s}(g\phi)^{2s} \sum_{n=-\infty}^{\infty} \frac{1}{(n+\frac{1}{2})^{2s-d}}. \tag{60}$$

Let us define the Hurwitz zeta function

$$\zeta(z,q) = \sum_{n=0}^{\infty} \frac{1}{(n+q)^z} \tag{61}$$

for $Re(z) > 1$ and $q \neq 0, -1, \ldots$. It is clear that for $q = 1$ we recover the usual Riemann zeta function. Defining:

$$r(D,s) = p(D,s)\Big(\zeta(2s-d,\frac{1}{2}) + (-1)^{2s-d}\zeta(2s-d,-\frac{1}{2}) - \frac{1}{2^{d-2s}}\Big) \tag{62}$$

we have:

$$V(\beta,\phi) = \frac{1}{2}\mu^2\phi^2 + N\mu^D\sum_{s=1}^{\infty} r(D,s)a^{\frac{D}{2}-s}(g\phi)^{2s}. \tag{63}$$

The effective potential is still ill defined and it will be regularized by the Principle of Analytic Extension. The function $r(D,s)$ valid in the begining in an open connected set of points, i.e. for $Re(z) > 1$. Since we are considering even nonrenormalizable models let us study the cases $D = 2, 3$ and 4. We would like to stress that even in this nonrenormalizable models it is possible to make qualitative predictions and we will regularize and renormalize the model in the standard way. A strong argument in favor of the study of the Gross-Neveu model even for $D > 2$ is that the nonrenormalizability does not appear in the leading $\frac{1}{N}$ approximation.

After the analytic continuation, the effective potential requires a renormalization procedure in the points $s = 1, 2..$ The renormalization condition which will fix the form of the counterterm of the pole $s = 1$ is:

$$\frac{\partial^2 V}{\partial\phi^2}|_{\phi=cte} = \mu^2 \tag{64}$$

Due to infrared divergences, we must follow Coleman and Weinberg [18] and choose the renormalization point at non-zero ϕ. To evaluate the renormalized effective potential it is necessary to use the Hermite formula of the analytic extension for the Hurwitz zeta function given by [32]

$$\zeta(z,q) = \frac{1}{2q^z} + \frac{q^{1-z}}{z-1} + 2\int_0^{\infty}(q^2+y^2)^{\frac{-z}{2}}\sin(z\arctan\frac{y}{q})\frac{1}{e^{2\pi y}-1}dy. \tag{65}$$

It is not difficult to show that the thermal contribution to the renormalized coupling constant is,

$$\Delta g(\beta) = N\mu^{D-2}\sum_{s=1}^{\infty} r(D,s)(2s)(2s-1)g^{2s}(\beta\mu)^{2s-D}. \tag{66}$$

The situation is different from the massive $\lambda\varphi^4$ model, since we have contribution of all terms of the series in s and the sign of the thermal contribution cannot be easily obtained. Nevertheless, for g sufficiently small the leading term is $O(g^2)$. In this case, for $D = 3$ using the fact that $\zeta(0,q) = \frac{1}{2} - q$ we obtain that $\Delta g = 0$. We found here that there is no thermal correction to the coupling constant at least in the one-loop approximation. For $D = 2$ and $D = 4$ although only one term in eq.(66) contributes, the sign of the function $r(D,s)$ cannot be easilly obtained.

Note that $\Delta g(\beta)$ is still ill-behaved. The terms $s > \frac{D}{2}$ are divergent in the low temperature limit (this situation is remanescent from the use of dimensional regularization in the begining of the calculations). For $s < \frac{D}{2}$ the high temperature limit of the model is problematic due to the well known fact that ultraviolet divergences are worst as the spacetime dimension increases. The one-loop renormalization of the Gross-Neveu model was recently studied by Kang and Kim [33], where the real time formulation was developed. These authors obtained the renormalized two and four point functions, and in the latter the thermal dependence of the coupling constant appears.

5 Conclusions

In this paper we studied the renormalization program assuming that scalar or fermionic fields are in equilibrium with a thermal reservoir at temperature β^{-1}. We have attempted to analise the consequences of the fact that not only the renormalized mass but also the renormalized coupling constant acquire thermal corrections. It is well known that if we have a system at finite temperature and one spatial dimension compactified, if we have a spontaneous symmetry breaking there are two diferent ways to restorate the symmetry. Since the compactification of one spatial dimension gives us the well known mechanism of topological generation of mass, it is possible to restorate the symmetry by thermal or topological effect. There is a very simple way to interpret the origin of the thermal and topological mass. The effective potential is ill-defined. Using the Principle of the Analytic Extension, we regularize the model and the introduction of counterterms remove the principal part of the analytic extension, and the model becomes finite. Meanwhile, in the neighbourhood of the poles the regular part of the analytic extension does not vanish. Since these contributions cannot be absorbed by the counterterms, we are forced to admit that the renormalized mass has a thermal contribution. This thermal mass is a well known result in the literature. Since in the model we have two physical parameters, mass and coupling constant, we cannot disregard the possibility that not only the renormalized mass, but also the coupling constant acquire a temperature dependent correction that also cannot be absorbed by the counterterms. It was proved that in the $\lambda\varphi^4$, in the one-loop aproximation, the thermal correction to the renormalized mass is positive and the thermal correction to the renormalized coupling constant is negative. Then the renormalized coupling constant attains its maximum at zero temperature and decreases monotonically as the temperature increases. Since in $D = 4$, $\Delta\lambda(\beta)$ is $O(\lambda^2)$ it is not possible to change the model to a trivial case, only increasing the temperature of the thermal bath. Nevertheless, for $D < 4$ triviality can be achieved. In the same way as the critical temperature is defined as the temperature at which the "tadpole" induced temperature dependent contribution to the mass cancels the bare zero temperature mass, the temperature of triviality is defined as the temperature at which the one-loop four point function induced temperature dependent contribution to the coupling constant cancels the bare zero temperature coupling constant. Although the assumption of thermal dependence of the renormalized coupling constant gives more problems that answers, since the extension to the the region above the temperature of triviality is still to be constructed, we would like to stress that m_0 and λ_0 are only parameters in the bare Lagrange density and both (after the regularization and renormalization procedure) must depend on the temperature of the thermal bath.

In section IV we study the Gross-Neveu model and we showed that at the leading order in the coupling constant, for $D = 3$ the thermal correction to the coupling constant is zero at least in the one-loop aproximation. Unfortunately, for $D = 2$ or $D = 4$ even at the leading order in the coupling constant, it is very difficult to evaluate the integral in eq.(65), and the sign of $\Delta g(\beta)$ cannot be obtained. Finally we would like to discuss some open questions. Since we proved that if we start with a $\lambda\varphi^4$ model, with $m^2 > 0$ in equilibrium with a thermal reservoir at finite temperature, in the process of regularization and renormalization a thermal mass and coupling constant arises, someone can argue that the "thermal photon" resulting from the quantization of QED at finite temperature could have a thermal mass. Indeed, as shown recently by Loewe and Rojas [15], this seems to be the case. Of course, some symmetries prevent the generation of mass at zero temperature; as for example in the case of gauge fields the gauge symmetry prevents the fields to acquire mass. Is the gauge invariance maintained at finite temperature? Or if we go further, is it possible that temperature effect brake a symmetry valid at zero temperature? This is related with the regularization procedure? We would like to stress that some regularization procedures are superior to others in that they allow to preserve the initial symmetries of the Lagrange density at every stage of perturbation calculation. A trivial example is the Schwinger regularization or the Speer analytic regularization in QED. Both cases in $D = 4$ does not give a transverse vacuum polarization tensor, then violating the gauge invariance of the model. Note that the analytic regularization method in $D = 3$ is perfectly suitable. The $D = 3$ QED is a good example of the ambiguity of the regularization procedures [34]. Starting with a Lagrange density with a parity-violating fermion mass and withouth a topological mass term to the gauge field at the tree level, and using a gauge invariant regularization procedure as dimensional regularization, it is easy to show that a topological mass to the gauge fields is induced at one-loop level. Note that using the Pauli-Villars regularization this fact does not occur[35]. It is not clear to us that the mixed regularization method, first dimensional regularization and after the zeta function analytic regularization (which deal with the divergent contribution of the Matsubara frequencies) violates the Ward-Takahashi identities, or in other words, does not give a transverse vacuum polarization tensor. To study such kind of problem the strategy must be the following. First let us write the polarizarion tensor in the form

$$\Pi_{\mu\nu} = \frac{1}{\epsilon}\Pi_{\mu\nu}(pole) + \Pi_{\mu\nu}(finite), \tag{67}$$

where ϵ is the deviation from the physical spacetime dimension one is interested in. It must be verified that not only the singular part obey the Ward identity but also the finite term also obey this identity. This possibility has been investigated by Ford, studying the vacuum polarization tensor of QED in a nonsimply connected spacetime. Both, the untwisted and twisted spinors coupled to the gauge field was investigated. If we accept that the two situations: QED at finite temperature in a spacetime with trivial topology of the spacelike sections and QED at zero temperature with topology $R^2 \times S$ of the spacelike sections are totally equivalent (for untwisted spinors), the conclusions obtained in the second situation must be applied to the first one. Ford proved that although the renormalized polarization tensor has two contributions

$$\Pi^R_{\mu\nu}(q) = \Pi^0_{\mu\nu}(q) + \Pi^1_{\mu\nu}(q) \tag{68}$$

where $\Pi^1_{\mu\nu}$ contain the correction due to the finiteness L (the radius of the compact spatial dimension) it can be shown that

$$q^{\mu}\Pi^{0}_{\mu\nu}(q) = 0 \tag{69}$$

and

$$q^{\mu}\Pi^{1}_{\mu\nu}(q) = 0. \tag{70}$$

Thus the gauge invariance of the model is mantained [36]. Of course, the above argument cannot be considered definitive since the proof of the equivalence between both situations has been not proved. To sheds new light on this problem, a natural extension of this paper is to investigate if the Ward-Takahashi identities are mantained if the gauge and fermion fields are in equilibrium with a thermal reservoir at temperature β^{-1}. Another possibility is a sistematic study of triviality in the $\lambda\varphi^4$ at two-loops level. Since triviality means that no ground state exist and the vacuum is a metastable state (a resonance) it is difficult to escape to the conclusion that the theory is not defined for negative λ. A possibility to remove the problem of the unboundedness of the effective potential at temperatures above β_T^{-1} is to include high order terms of the field in the Lagrange density. This subject will be presented in a forthcomming paper[37].

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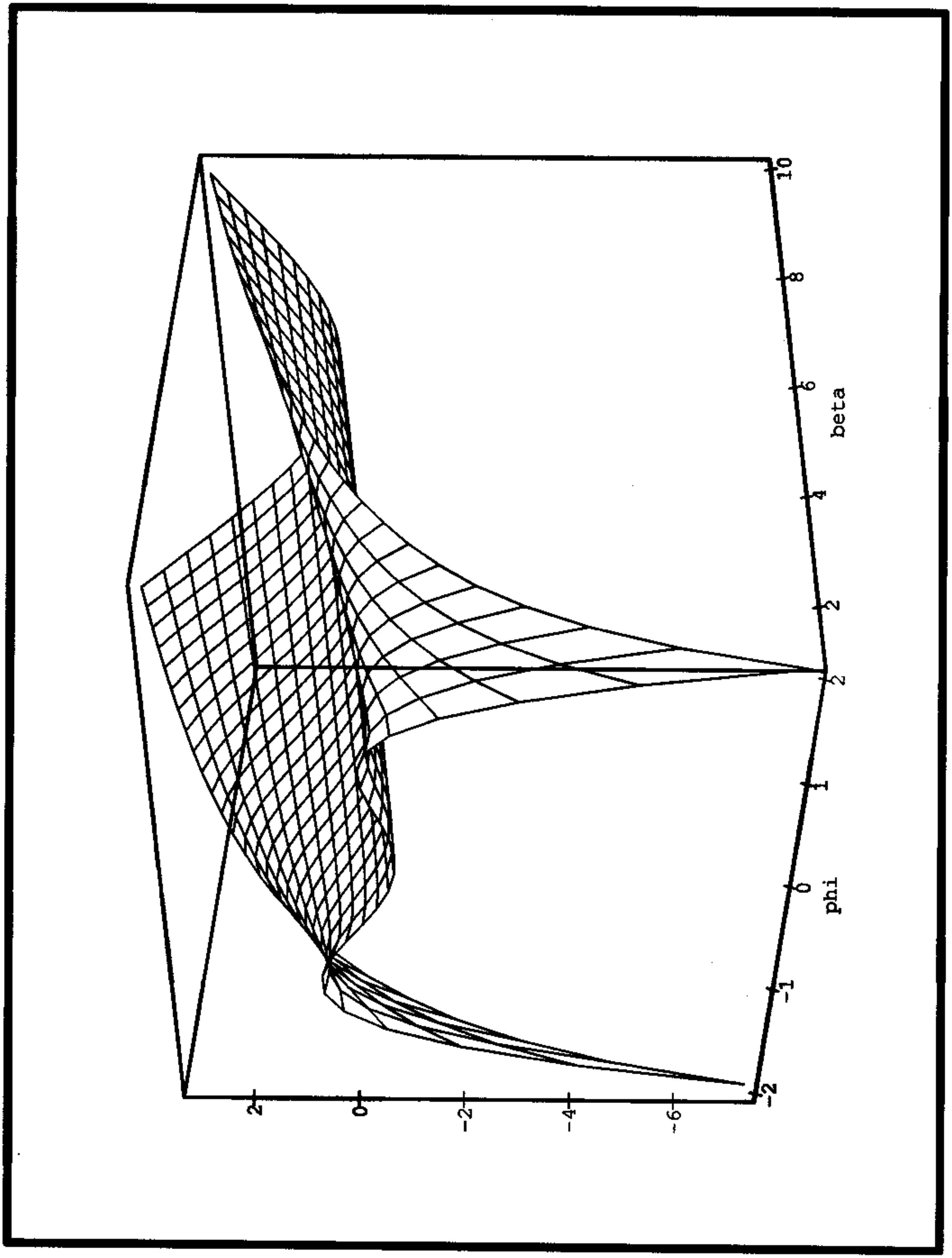
Fig(1) - The dimensionless effective potential as a function of the vacuum expectation value of the field and the inverse of the temperature. For low temperatures it has a global minimum and for high temperatures the potential becomes unbounded bellow. At high temperatures the vacuum is metastable.

Fig(2) - The $\mathcal{B}$ coefficient as a function of the coupling constant and the inverse of the temperature in the case $D < 4$.

Fig(3) - The $\mathcal{B}$ coefficient as a function of the coupling constant and the inverse of the temperature in the case $D > 4$.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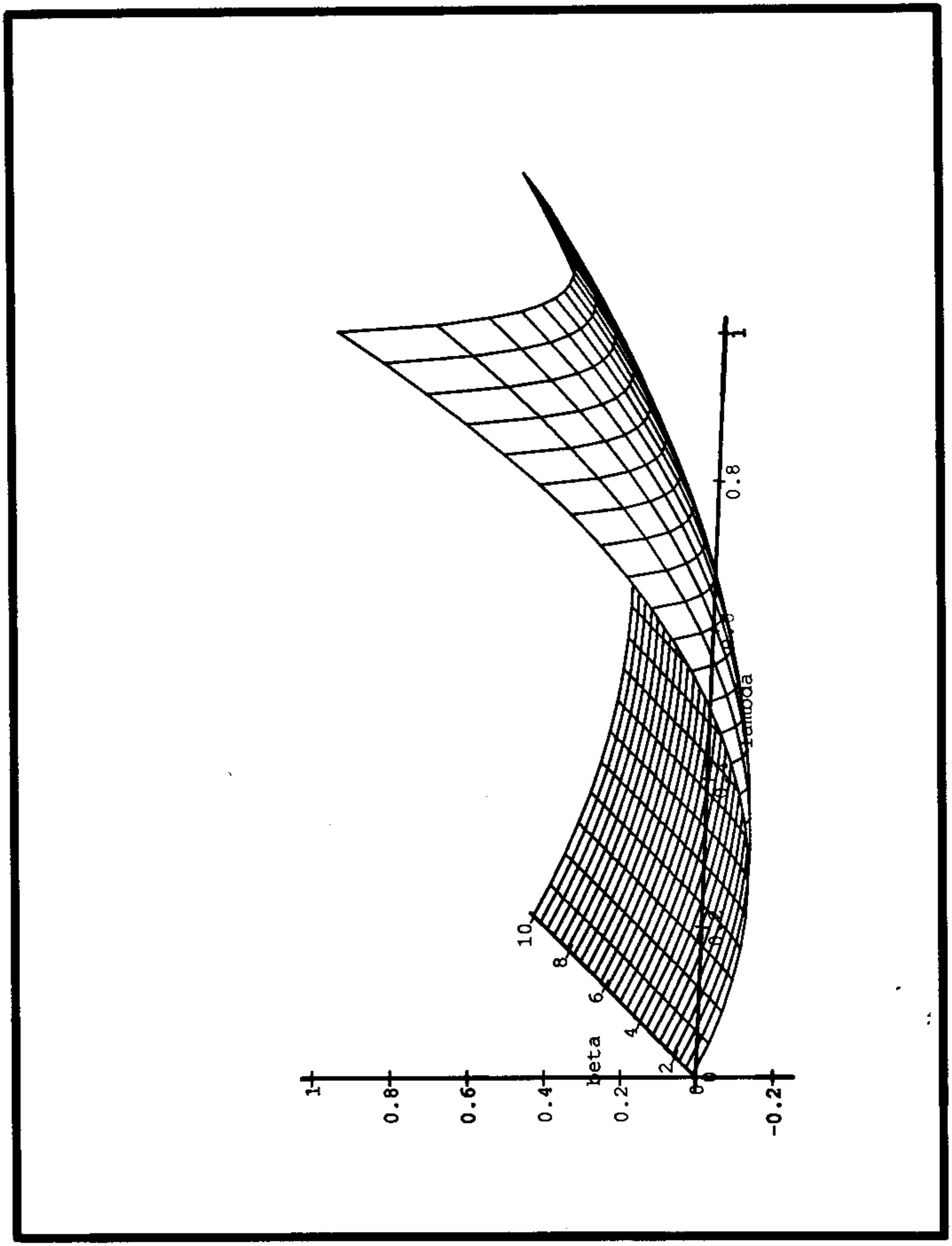
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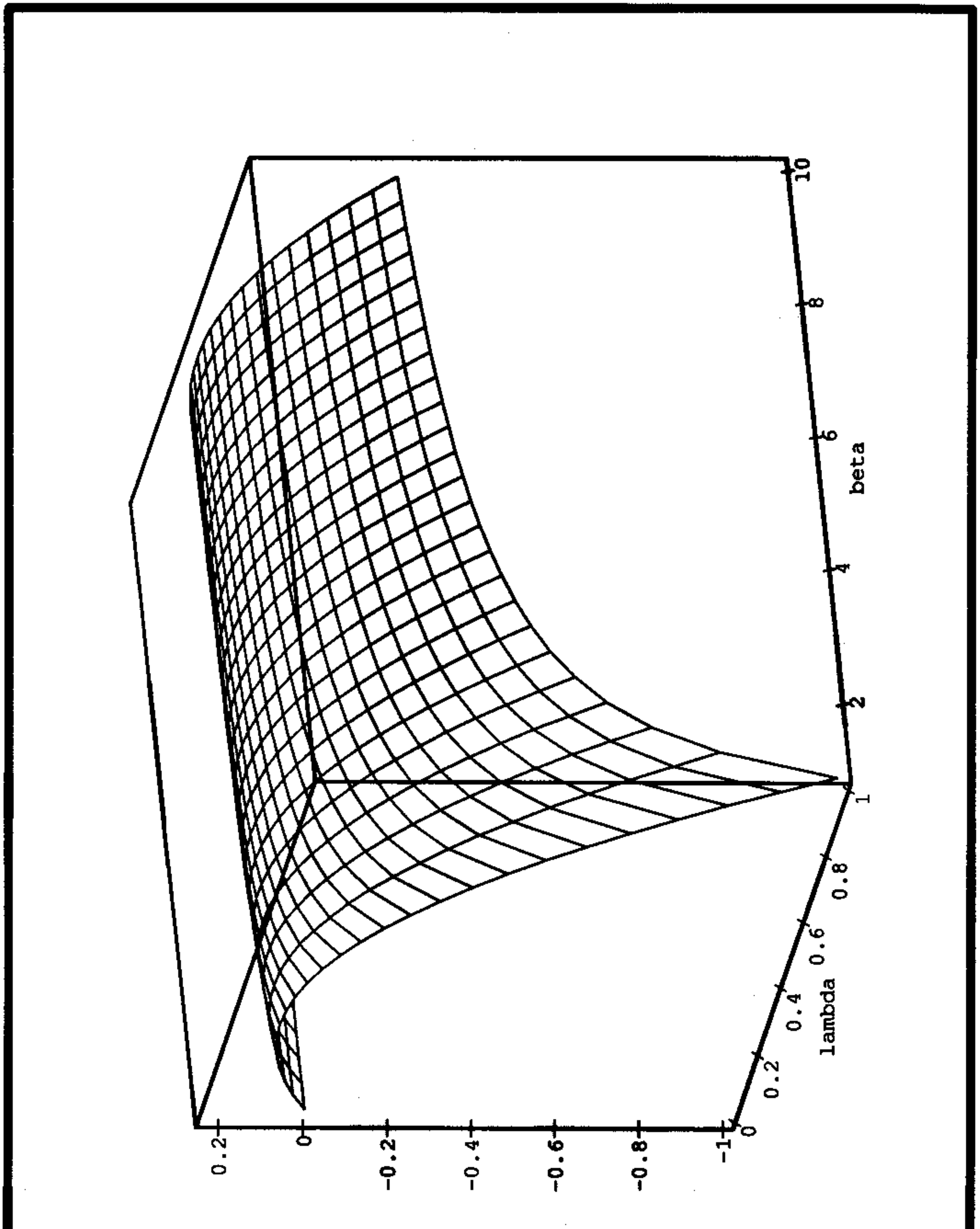
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