CBPF-NF-043/96

The tricritical phenomena in $(\lambda\varphi^4 + \sigma\varphi^6)_{D=3}$ model

Gino N.J.Añaños and *N.F. Svaiter**

Centro Brasileiro de Pesquisas Físicas - CBPF
Rua Dr. Xavier Sigaud, 150
22290-180 – Rio de Janeiro, RJ – Brazil

Abstract

The thermodynamics of the massive $\lambda\varphi^4 + \sigma\varphi^6$ model is analysed at finite temperature in the two-loop approximation. The behavior of the thermal mass and coupling constant is discussed. We demonstrate in the two-loops approximation the existence of a tricritical point.

Key-words: Finite temperature field theory.

Pacs numbers: 11.10.Ef, 11.10.Gh

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*E-MAIL:NFUXSVAI@LCA1.DRP.CBPF.BR

1 Introduction

The field theory with a $\lambda\varphi^4$ self-interaction has been extensively studied in the literature. General expressions for Feynman diagrams at zero temperature has been presented until the four-loops approximation [1]. In the same way, in the recent past the temperature dependence of the renormalized mass and coupling constant has been analysed by many authors [2]. More recently different methods has been used to study finite temperature quantum field theory [3] [4].

The purpose of this paper is to present a two-loop calculation of the $(\lambda\varphi^4 + \sigma\varphi^6)_{D=3}$ model. For simplicity we assume that the dimension of the order parameter is one. We obtained the thermal correction to the mass squared $m^2(\beta)$ and coupling constant $\lambda(\beta)$. Note that if the thermal coupling constant $\lambda(\beta)$ becomes negative (for positive mass squared $m^2(\beta)$) a first order phase transition could occur. For negative mass squared and positive $\lambda(\beta)$ we have a second order phase transition. The point $m^2(\beta) = \lambda(\beta) = 0$ defines the tricritical point [5]. Some systems as metamagnets (antifferomagnets in the presence of a strong external field) or in $He^3 - He^4$ mixture can present such kind of behavior. A tree level discussion of the tricritical phenomena can be found in Refs.[6] [7]. For a treatment using the Callan-Zymanzik equation see for example ref.[8]

In three recent papers, the thermal correction to the mass and coupling constants was analysed in the $\lambda\varphi^4$ and Efimov-Fradkin models. The possibility for vanishing the renormalized coupling constant in $(\lambda\varphi^4)_{D=4}$ model by thermal or topological effects was discussed in the first of these papers by Ford and Svaiter [9]. Assuming a trivial topology of the spacelike section Malbouisson and Svaiter [10] extended part of the discussion of Ford and Svaiter [9]. Assuming that the system is in thermal equilibrium with a reservoir, the thermal correction to the mass $\Delta m^2(\beta)$ and coupling constant $\Delta\lambda(\beta)$ was obtained in a generic D-dimensional spacetime using the one-loop approximation. Using a mix between dimensional and analytic regularization procedures and a modified minimal renormalization scheme the authors proved that in the one-loop approximation for $D \leq 4$, the renormalized coupling constant attains its maximum at zero temperature and decreases monotonically as the temperature of the thermal bath increases. As was discussed, for $D < 4$ the renormalized coupling constant may becomes negative above some temperature β_*^{-1}. As was discussed by Funakubo and Sakamoto and also Fendley [11], in the two-loop approximation the effect of the temperature over the physical parameters is opposite in the high temperature regime. Nevertheless we cannot disregards the posibility to vanish the thermal coupling constant at some intermediate temperature.

Part of the analysis of these above cited papers was extended by Malbouisson and Svaiter [12] assuming Lagrange densities with non-polynomial arguments. Still using the one-loop aproximation the behavior of the model at low and high temperatures was discussed. Using the notion of the critical dimension $D_c(n)$, these author proved that in the truncated model the renormalized coupling $\lambda_N(\beta^{-1})$ becomes negative above some temperature β_N^{-1} for $D < D_c$. In this situation the origin is a metastable vacuum.

The stability of the $O(N)$ model in $D = 3$ was analysed by Appelquist and Heinz [13]. These authors studied the model with a Abelian gauge field coupled to a massless, charged N component field in the critical regime. They founded that the model contains a non-zero infrared stable fixed point in the scalar coupling and also in the gauge coupling. Bardeen, Moshe and Bander [16] studied the pure $O(N)$ $\lambda(\varphi_a\varphi_a)^2 + \eta(\varphi_a\varphi_a)^3$ model and founded an ultraviolet fixed point at finite coupling $\eta = \eta^*$. For $\eta > 16\pi^2$ the model has no stable ground state for large N.

As noted a long time ago by Dyson [18], in QED for negative coupling constant e^2 the Hamiltonian is unbounded from below and the vacuum is a metastable state with a mean life of the order $e^{(-\frac{1}{|g|})}$. For instance in the $(\lambda\varphi^4)_{D<4}$ model and $\lambda(\beta) < 0$ we have the same problems. In $D = 4$ the contribution from instantons to the massless model was analysed by Feinberg and Yofa [19]. These authors calculed high order correction to the instanton contribution to the Green's functions in the regime $\lambda < 0$. As was discussed by Parisi and others [20], asymptotic stimates in perturbation theory can be obtained using the semiclassical arguments computing perturbatively the imaginary part of the Green's functions for small negative coupling constant λ.

The purpose of this paper is to investigate the tricritical phenomena in the $(\lambda\varphi^4 + \sigma\varphi^6)_{D=3}$. We compute $\Gamma^{(2)}(0)$ and $\Gamma^{(4)}(0)$ up to second order in perturbation theory and proved that the two-loop approximation is enough to obtain the tricritical point where a line of second order phase transition merges smoothly at this point into a line of first order phase transition. This paper is organized as follows. In section II we will review some general formalism. In section III the thermal correction to the mass and coupling constant $\lambda(\beta)$ for the two-loop one particle irreducible diagrams $\Gamma^{(2)}(0)$ and $\Gamma^{(4)}(0)$ is presented. Conclusions are given in section IV. In this paper we use $\hbar = c = 1$.

2 General formalism

Let us consider the vacuum to vacuum persistence functional in the presence of an external scalar source $J(x)$.

$$Z(J) = \int \mathcal{D}\varphi \; exp\left(-\int d^D x \; (\mathcal{L}(\varphi) + J(x)\varphi(x))\right) \tag{1}$$

where

$$\mathcal{L}(\varphi) = \frac{1}{2}(\partial\varphi)^2 + \frac{1}{2}m^2\varphi^2 + \frac{1}{4!}\lambda\varphi^4 + \frac{1}{6!}\sigma\varphi^6. \tag{2}$$

The n-point correlation functions are defined as

$$\begin{aligned} G^{(n)}(x_1, x_2, .., x_n) =< \varphi(x_1)...\varphi(x_n) > &= \frac{1}{Z(J)}\frac{\delta^n Z(J)}{\delta J(x_1)\delta J(x_2)..\delta J(x_n)}|_{J=0} \\ &= \int \mathcal{D}\varphi\varphi(x_1)...\varphi(x_n) exp\left(-\int d^D x\ \mathcal{L}(\varphi)\right). \end{aligned} \tag{3}$$

We define the generating functional of the connected correlation functions of the fields by $W(J)$, where $W(J) = lnZ(J)$.

Thus

$$G_c^{(n)}(x_1, x_2, .., x_n) = \frac{\delta^n W(J)}{\delta J(x_1)\delta J(x_2)..\delta J(x_n)}|_{J=0} =< \varphi(x_1)...\varphi(x_n) >_c . \tag{4}$$

The generating functional of the connected one-particle irreducible correlation functions can be introduced, performing a Legendre transformation

$$\Gamma(\varphi_0) = -W(J) + \int d^D x\varphi(x)J(x) \tag{5}$$

and

$$\Gamma^{(n)}(x_1, x_2, .., x_n) = \frac{\delta^n \Gamma(\varphi_0)}{\delta\varphi_0(x_1)\delta\varphi_0(x_2)..\delta\varphi_0(x_n)}|_{\varphi_0=0}. \tag{6}$$

where

$$\varphi_0(x) = \frac{\delta W}{\delta J(x)} \tag{7}$$

If $\lambda = 0$ and $\sigma = 0$, the partition function $Z(J)$ can be calculed exactly i.e.

$$Z_0(J) = exp\left(-\frac{1}{2}\int d^D x J(x) D(x-y,m^2) J(y)\right), \tag{8}$$

where

$$(-\Delta_x + m^2) D(x-y,m^2) = \delta^D(x-y). \tag{9}$$

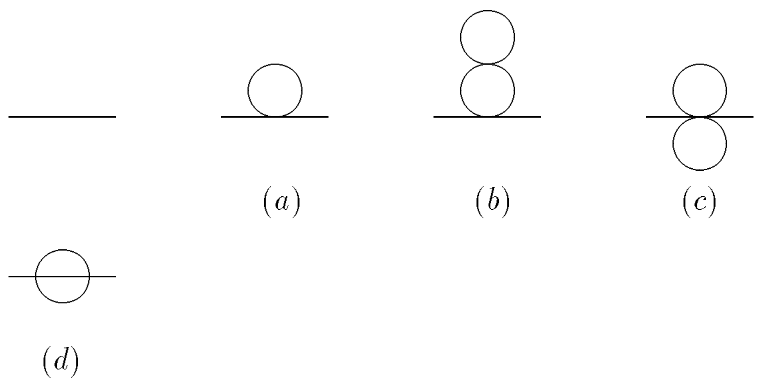
For $\lambda \neq 0$ and $\sigma \neq 0$ it is not possible to find exactly $Z(J)$ and perturbation theory is mandatory, so we have

$$Z(J) = exp\left(-\frac{\lambda}{4!}\int d^D x \left(\frac{\delta}{\delta J(x)}\right)^4\right) exp\left(-\frac{\sigma}{6!}\int d^D y \left(\frac{\delta}{\delta J(y)}\right)^6\right) Z_0(J). \tag{10}$$

and

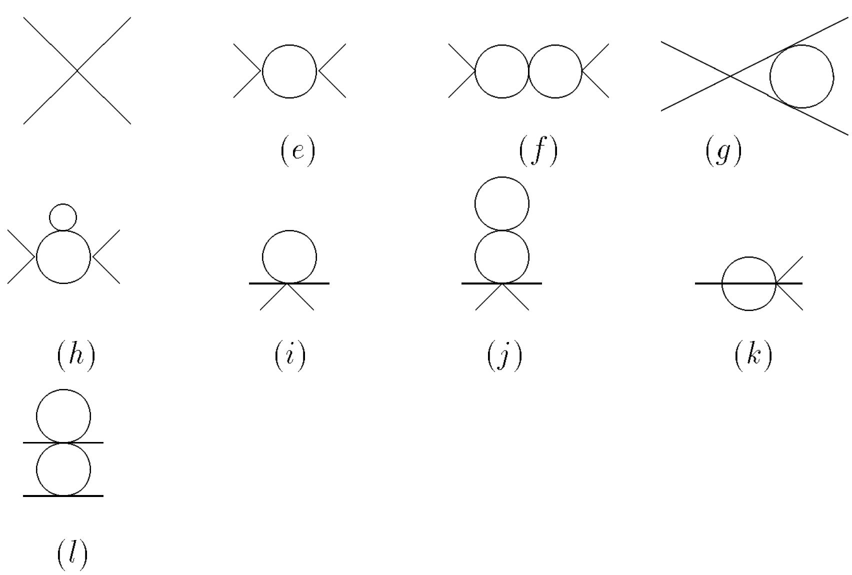
$$\Gamma^{(n)}(p_1, p_2, ...p_{n-1}, p_k) =$$
$$\int d^D x_1 d^D x_2 ... d^D x_{n-1} d^D y \; exp\left(i(p_1 x_1 + p_2 x_2 + ...(p_{n-1} x_{n-1} + ky)\right)\Gamma(x_1, ...x_{n-1}, 0). \tag{11}$$

The problem we will study is to find a tricritical temperature $\beta^{-1}(m,\lambda,\sigma)$ for a set of values of m, λ and σ where the 1PI diagrams $\Gamma^{(2)}(0)$ and $\Gamma^{(4)}(0)$ vanishes. This point defines the tricritical point. Consequently let us examinate thermal effects over $\Gamma^{(2)}(0)$ and $\Gamma^{(4)}(0,0,0,0) \equiv \Gamma^{(4)}(0)$. We will calculate explicity the two-loop contribution to the renormalized thermal mass and coupling constant. The diagrams contributing to the two-point function are:



$fig.(1)$

The diagrams that contributes to the four-point functions are:



$fig.(2)$

It is possible to obtain the expressions of $\Gamma^{(2)}(0)$ and $\Gamma^{(4)}(0)$ given by:

$$\Gamma^{(2)}(0) = m^2 + \frac{1}{2}\lambda(a) - \frac{1}{4}\lambda^2(b) - \frac{1}{6}\lambda^2(c) - \frac{1}{8}\sigma(d) \tag{12}$$

and

$$\begin{aligned}\Gamma^{(4)}(0) &= \lambda - \frac{3}{2}\lambda^2(e) + \frac{3}{4}\lambda^3(f) + 3\lambda^3(g) + \frac{3}{2}\lambda^3(h) \\ &- \frac{1}{2}\sigma(i) + \frac{1}{4}\lambda\sigma(j) + \frac{2}{3}\lambda\sigma(k) + \frac{1}{2}\lambda\sigma(l),\end{aligned} \tag{13}$$

where:

$$(a) = \frac{1}{(2\pi)^D}\int \frac{d^Dq}{(q^2+m^2)} \tag{14}$$

$$(b) = \frac{1}{(2\pi)^{2D}}\int \frac{d^Dq_1}{(q_1^2+m^2)}\int \frac{d^Dq_2}{(q_2^2+m^2)^2}. \tag{15}$$

$$(c) = \frac{1}{(2\pi)^{2D}}\int \frac{d^Dq_1 d^Dq_2}{(q_1^2+m^2)(q_2^2+m^2)((q_1+q_2)^2+m^2)} \tag{16}$$

$$(d) = \left(\frac{1}{(2\pi)^D}\int \frac{d^Dq}{(q^2+m^2)}\right)^2 \tag{17}$$

$$(e) = \frac{1}{(2\pi)^D}\int \frac{d^Dq}{(q^2+m^2)^2} \tag{18}$$

$$(f) = \left(\frac{1}{(2\pi)^D}\int \frac{d^Dq}{(q^2+m^2)^2}\right)^2 \tag{19}$$

$$(g) = \frac{1}{(2\pi)^{2D}} \int \frac{d^D q_1 d^D q_2}{(q_1^2 + m^2)(q_2^2 + m^2)((q_1 + q_2)^2 + m^2)^2} \tag{20}$$

$$(h) = \frac{1}{(2\pi)^{2D}} \int \frac{d^D q_1}{(q_1^2 + m^2)} \int \frac{d^D q_2}{(q_2^2 + m^2)^3} \tag{21}$$

$$(i) = \frac{1}{(2\pi)^{D}} \int \frac{d^D q}{(q^2 + m^2)} \tag{22}$$

$$(j) = \frac{1}{(2\pi)^{2D}} \int \frac{d^D q_1}{(q_1^2 + m^2)} \int \frac{d^D q_2}{(q_2^2 + m^2)^2} \tag{23}$$

$$(k) = \frac{1}{(2\pi)^{2D}} \int \frac{d^D q_1 d^D q_2}{(q_1^2 + m^2)(q_2^2 + m^2)((q_1 + q_2)^2 + m^2)} \tag{24}$$

$$(l) = \frac{1}{(2\pi)^{2D}} \int \frac{d^D q_1}{(q_1^2 + m^2)} \int \frac{d^D q_2}{(q_2^2 + m^2)^2} \tag{25}$$

Before go to the next section to calculate the integrals at finite temperature and show how the tricritical behavior appear, we would like to make some remarks. Although we use the terminology "thermal mass" to $\Gamma^2(0)$, interacting thermal field theory does not admit the notion of mass in the usual sense of a real axis pole in the full propagator [14]. Nevertheless we still use the term physical mass since the physical mass and this quantity are related by a finite quantity (using the renormalization group arguments).

3 The tricritical phenomena

The aim of this section is to show that there is a temperature where the tricritical phenomena appear. To regularize the model we can choose between a plethora of regularization procedures:

Pauli-Villars, dimensional regularization, momentum cutoff etc. We prefer to use a mix between dimensional and zeta function analytic regularization. Therefore let us define:

$$I_\beta(D,s) = 1/\beta \sum_{n=-\infty}^{\infty} \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \frac{1}{(\omega_n^2 + k^2 + m^2)^s} \tag{26}$$

Writing $\Gamma^{(2)}(0)$ and $\Gamma^{(4)}(0)$ as a function of $I_\beta(D,s)$ we have:

$$\begin{aligned}\Gamma^{(2)}(0) &= m^2 + \frac{1}{2}\lambda I_\beta(D,1) - \frac{1}{4}\lambda^2 I_\beta(D,1) I_\beta(D,2) \\ &\quad -\frac{1}{6}\lambda^2(c) - \frac{1}{8}\sigma\left(I_\beta(D,1)\right)^2 .\end{aligned} \tag{27}$$

and

$$\begin{aligned}\Gamma^{(4)}(0) &= \lambda - \frac{3}{2}\lambda^2 I_\beta(D,1) + \frac{3}{4}\lambda^3 (I_\beta(D,2))^2 + 3\lambda^3(g) + \frac{3}{2}\lambda^3 I_\beta(D,1) I_\beta(D,3) \\ &\quad -\frac{1}{2}\sigma I_\beta(D,1) + \frac{1}{4}\lambda\sigma I_\beta(D,1) I_\beta(D,2) + \frac{2}{3}\lambda\sigma(k) \\ &\quad +\frac{1}{2}\lambda\sigma I_\beta(D,1) I_\beta(D,2).\end{aligned} \tag{28}$$

Using the analytic extension of the inhomogeneous Epstein zeta function it is possible to obtain $I_\beta(D,s)$;

$$I_\beta(D,s) = \frac{m^{D-2s}}{(2\pi^{\frac{1}{2}})^D \Gamma(s)} \left(\Gamma(s - \frac{D}{2}) + 4 \sum_{n=1}^{\infty} (\frac{2}{mn\beta})^{\frac{D}{2}-s} K_{\frac{D}{2}-s}(mn\beta) \right), \tag{29}$$

where $K_\nu(z)$ is the modified Bessel function of thirth kind [15]

We would like to stress that for $D > 3$ the φ^6 interaction is irrelevant and cannot affect the infrared structure of the model. Nevertheless for $D = 3$ the φ^6 interaction is marginal and the above comment does not follow. We will have that there is a set of values of the parameters m^2, σ and λ for each temperature which lead to the vanishing of the thermal physical mass $m^2(\beta)$ and coupling constant $\lambda(\beta)$ i.e., the critical line in the parameter space. Note that the basis of all considerations above assume that the sunset and related diagrams can not modify the tricritical behavior.

A straighforward calculation gives for $I_\beta(3,1)$, $I_\beta(3,2)$ and $I_\beta(3,3)$

$$I_\beta(3,1) = \frac{m}{2\pi}\left(-\frac{1}{2} + \sum_{n=1}^{\infty}(\frac{1}{mn\beta})e^{-mn\beta}\right) \tag{30}$$

$$I_\beta(3,2) = \frac{1}{16\pi m}\left(\frac{1}{2} + \frac{1}{e^{m\beta}-1}\right) \tag{31}$$

and finally

$$I_\beta(3,3) = \frac{1}{16m^3}\left(\frac{1}{2} + \frac{1}{e^{m\beta}-1} + \sum_{n=1}^{\infty}(mn\beta)e^{-mn\beta}\right) \tag{32}$$

To evaluate the sum in $I_\beta(3,1)$ we use the following trick

$$\sum_{n=1}^{\infty}(\frac{1}{mn\beta})e^{-mn\beta} = \frac{1}{m\beta}\sum_{n=1}^{\infty}\frac{(e^{-m\beta})^n}{n} = -\frac{1}{m\beta}ln(1-e^{-m\beta}) \tag{33}$$

The eq.(32) can be written as

$$I_\beta(3,3) = \frac{1}{16m^3}\left(\frac{1}{2} + \frac{1}{e^{m\beta}-1} + m\beta\frac{e^{-m\beta}}{(e^{m\beta}-1)^2}\right) \tag{34}$$

The idea is to define the quantities $x = m\beta$, $y = \frac{\lambda}{m}$ and $z = \sigma$. In the space (x,y,z) the condition $\Gamma^2(0) = 0$ define a surface. The same happens for $\Gamma^4(0)$. The intersection of both surfaces define a tricritical line. See fig.(3). The effective potential as a function of the vacuum expectation value of the field and $m\beta$ can be ploted. The temperature is the parameter that allows us to interpolate between the two configurations: a metastable state at $<\varphi>=0$ in the low temperature regime with first order phase transition and a second order phase transition in the high temperature regime. See fig.(4). At some intermediate temperature the tricritical point appears.

In the high temperature regime it is possible to write

$$I_\beta(3,1)=\frac{m}{2\pi}\left(-\frac{1}{2}+\sum_{k=1}^{\infty}\frac{(m\beta)^{k-1}}{k!}\right). \tag{35}$$

and

$$I_\beta(3,2)=\frac{1}{16m\pi}\sum_{k=0}^{\infty}\frac{B_{2k}}{(2k)!}(m\beta)^{2k-1}. \tag{36}$$

A possivel method to deal with the system in the high temperature regime is dimensional reduction (DM). This approach has been used by many authors [22]. The basic idea is that in the imaginary time formalism the free propagator has a form $(\omega_n+p^2+m^2)^{-1}$. The Matsburara frequency act like a mass so in the high temperature regime the non-static modes ($n\neq 0$) decouple and we have a three dimensional theory after the integration of the non-zero modes. Of course this efective model will describe the original model only for distances $R>>\beta^{-1}$.

4 Conclusion

Studying the $(\lambda\varphi^4+\sigma\varphi^6)_{D=3}$ model at finite temperature we obtained a well known result. We proved that for each set of values of m, λ and σ there is a temperature $\beta^{-1}(m,\lambda,\sigma)$ where the physical thermal mass $m^2(\beta)$ and coupling constant $\lambda(\beta)$ vanishes. Two remarks should be made.

First is that the existence of the tricritical point can not be modified with the inclusion of the sunset and related graphs. The inclusion of these graphs will only change the temperature of the tricritical point. The second is the existence of a pole in $\Gamma^4(0)$ at zero temperature as was noted by Bardeen, Moshe and Bander[16]. In the pure $\lambda\varphi^4$ it is possible to sum a infinite series of diagrams (ring diagrams) to circumvent the problem, since the field acquire a mass proportional to $(\lambda)^{\frac{1}{2}}\beta^{-1}$. We conjectured that the same can be done in the $\lambda\varphi^4 + \sigma\varphi^6$ model in such a way that the dynamically generated mass throws away the infrared divergence. A more elaborated argument was given by Parisi [21], where the introduction of multi-local operators as counterterms can eliminate the infrared divergences.

A natural extension of this paper is to calculate the decay rate of the metastable state $< \varphi >= 0$ with nucleation of bubbles in the low temperature regime [23]. For $m\beta > m\beta_*$ the solution $< \varphi >= 0$ is a unstable minimum of the potential (the false vacuum), and it is posssible to evaluate the probability per unit time and volume for the falso vacuum to decay into the true vacuum of the model. To calculate then decay rate it is necessary to evaluate the instanton solution and a gaussian integral around the instanton. This subject is under investigation in this model.

Acknowledgment

We would like to thanks Prof.A.P.C.Malbouisson for several helpful discussions. This work was supported by Conselho Nacional de desenvolvimento Cientifico e Tecnologico do Brasil-CNPq.

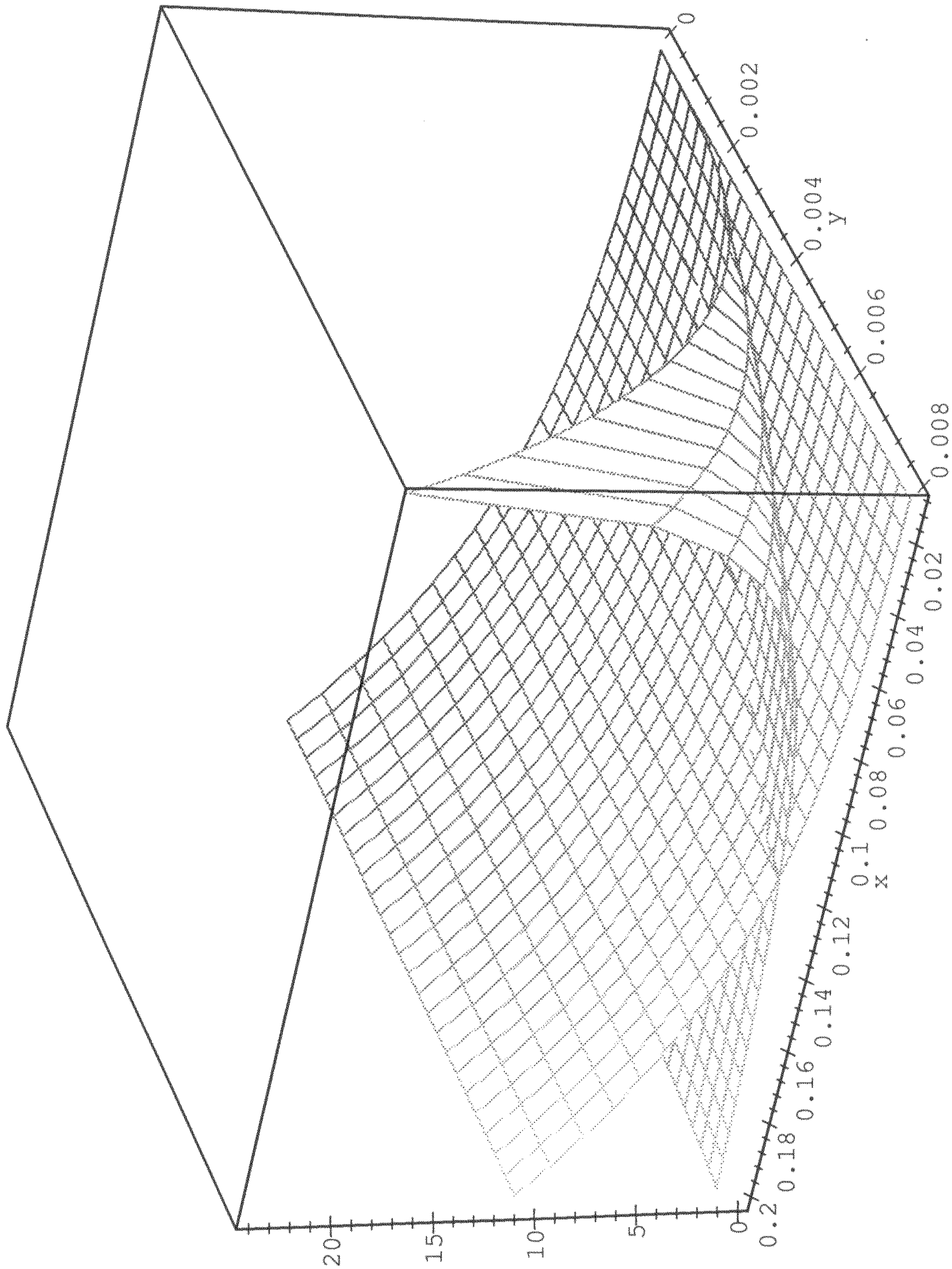
Figure Caption

fig.(1) - The diagrams that contributes to the 1PI two-point functions.

fig.(2) - The diagrams that contributes to the 1PI four-point functions.

fig.(3) - The two surfaces $\Gamma^{(2)}(0) = 0$ and $\Gamma^{(4)}(0) = 0$ in the space $x = m\beta$, $y = \frac{\lambda}{m}$ and $z = \sigma$.

fig.(4) - The effective potential as a function of the vacuum expectation value of the field and $m\beta$. In the low temperature regime, there is a metastable minimum at $<\varphi>= 0$ (there is a true minimum outside the origin that does not appears in the figure). Increasing the temperature appears the tricritical temperature β_*^{-1}. In the high temperature regime there is only a second order phase transition.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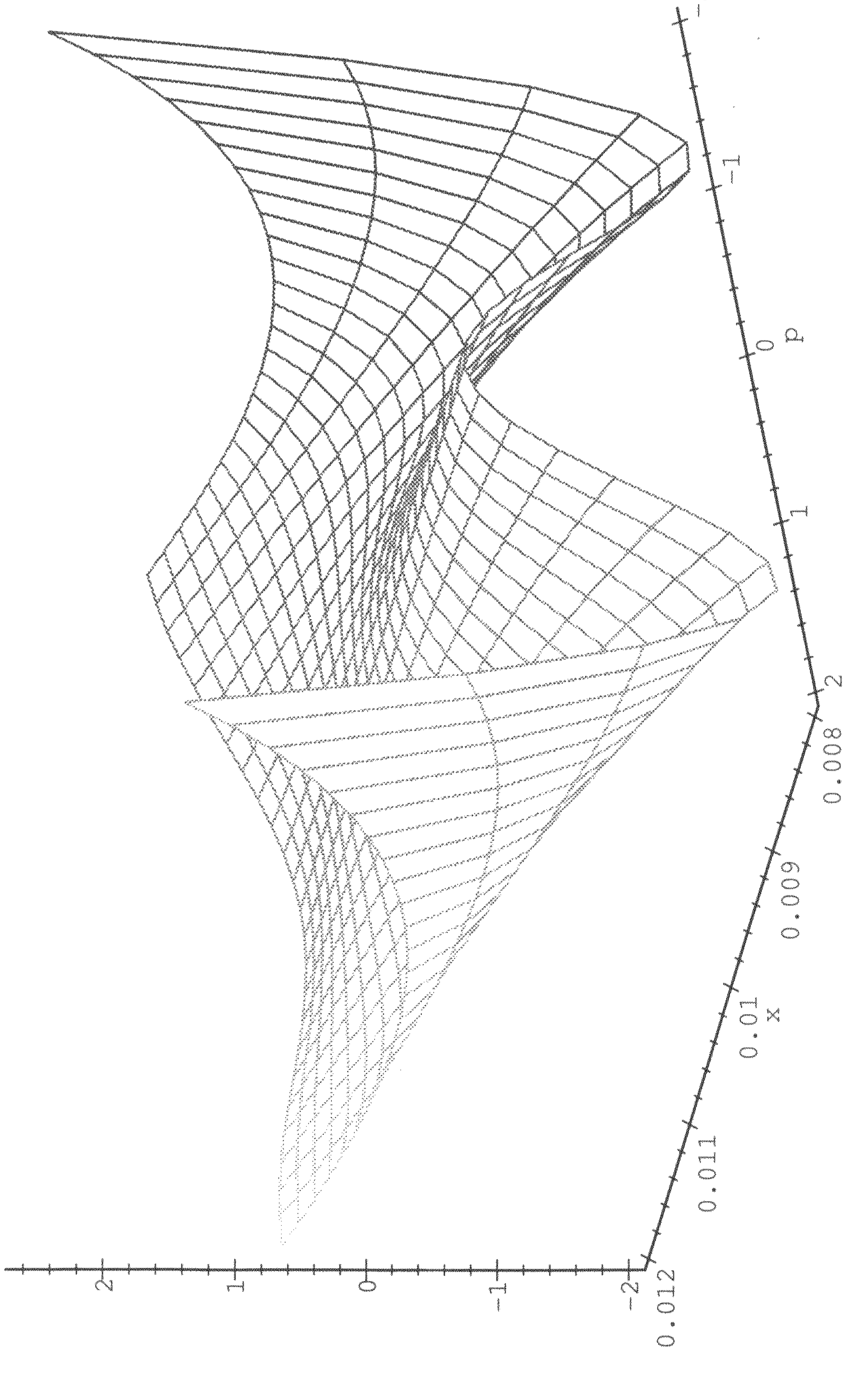
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