

# Non-Abelian (2,0)-Super-Yang-Mills Coupled to Non-Linear $\sigma$ -Models

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Considering a class of (2,0)-super-Yang-Mills multiplets that accommodate a pair of independent gauge potentials in connection with a single symmetry group, we present here their non-Abelian coupling to ordinary matter and to non-linear  $\sigma$ -models in (2,0)-superspace. The dynamics and the couplings of the gauge potentials are discussed and the interesting feature that comes out is a sort of “chirality” for one of the gauge potentials whenever light-cone coordinates are chosen.

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In a previous paper [1], we discussed the dynamics and the couplings of the Abelian vector potentials of a class of (2,0)-gauge super multiplets([2]-[8]) in connection with a single U(1)-symmetry group. Since a number of interesting features came out, it was a natural question to ask how these fields would behave if the non-Abelian version of the theory was to be considered.

We can see that indeed some subtle changes occur. As we wish to make a full comparison between the two aspects(Abelian and non-Abelian) of the same sort of theory, all the characteristics of the original formulation were kept, namely, the coordinates we choose to parametrise the (2,0)-superspace are given by:

$$z^A \equiv (x^{++}, x^{--}; \theta, \bar{\theta}), \quad (1)$$

where  $x^{++}$ ,  $x^{--}$  denote the usual light-cone variables, whereas  $\theta$ ,  $\bar{\theta}$  stand for complex right-handed Weyl spinors. The supersymmetry covariant derivatives are taken as:

$$D_+ \equiv \partial_\theta + i\bar{\theta}\partial_{++} \quad (2)$$

and

$$\bar{D}_+ \equiv \partial_{\bar{\theta}} + i\theta\partial_{++}, \quad (3)$$

where  $\partial_{++}$  (or  $\partial_{--}$ ) represents the derivative with respect to the space-time coordinate  $x^{++}$  (or  $x^{--}$ ). They fulfill the algebra:

$$D_+^2 = \bar{D}_+^2 = 0 \quad \{D_+, \bar{D}_+\} = 2i\partial_{++}. \quad (4)$$

With these definitions for  $D$  and  $\bar{D}$ , one can check that:

$$e^{i\theta\bar{\theta}\partial_+} D_+ e^{-i\theta\bar{\theta}\partial_+} = \partial_\theta, \quad (5)$$

$$e^{-i\theta\bar{\theta}\partial_+} \bar{D}_+ e^{i\theta\bar{\theta}\partial_+} = \partial_{\bar{\theta}}. \quad (6)$$

The fundamental non-Abelian matter superfields are the “chiral” scalar and left-handed spinor superfields, whose respective component-field expressions are given by:

$$\begin{aligned} \Phi^i(x; \theta, \bar{\theta}) &= e^{i\theta\bar{\theta}\partial_+} (\phi^i + \theta\lambda^i), \\ \Psi^I(x; \theta, \bar{\theta}) &= e^{i\theta\bar{\theta}\partial_+} (\psi^I + \theta\sigma^I), \end{aligned} \quad (7)$$

the fields  $\phi^i$  and  $\sigma^I$  are scalars, whereas  $\lambda^i$  and  $\psi^I$  stand respectively for right- and left-handed Weyl spinors. The indices  $i$  and  $I$  label the representations where the corresponding matter fields are set.

We present below the gauge transformations of both  $\Phi$  and  $\Psi$ , assuming that we are dealing with a compact and simple gauge group,  $\mathcal{G}$ , with generators  $G_a$  that fulfill the algebra  $[G_a, G_b] = if_{abc}G_c$ . The transformations read:

$$\Phi'^i = R(\Lambda)_j^i \Phi^j, \quad \Psi'^I = S(\Lambda)_J^I \Psi^J, \quad (8)$$

where  $R$  and  $S$  are matrices that respectively represent a gauge group element in the representations under which  $\Phi$  and  $\Psi$  transform. Taking into account the chiral constraint on  $\Phi$  and  $\Psi$ , and bearing in mind the exponential representation for  $R$  and  $S$  in terms of the group generators, we find that the gauge parameter superfields,  $\Lambda^a$ , must satisfy the same sort of constraint. They can therefore be expanded as follows:

$$\Lambda^a(x; \theta, \bar{\theta}) = e^{i\theta\bar{\theta}\partial_+} (\alpha^a + \theta\beta^a), \quad (9)$$

where  $\alpha^a$  are scalars and  $\beta^a$  are right-handed spinors.

The kinetic action for  $\Phi^i$  and  $\Psi^I$  can be made invariant under the local transformations (8) by minimally coupling gauge potential superfields,  $\Gamma_{-}^a(x; \theta, \bar{\theta})$  and  $V^a(x; \theta, \bar{\theta})$ , according to the minimal coupling prescriptions:

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$$S_{inv} = \int d^2x d\theta d\bar{\theta} \{ i[\bar{\Phi} e^{hV} (\nabla_{--} \Phi) - (\bar{\nabla}_{--} \bar{\Phi}) e^{hV} \Phi] + \bar{\Psi} e^{hV} \Psi \}, \quad (10)$$

where the gauge-covariant derivatives are defined in the sequel.

The Yang-Mills supermultiplets are introduced by

$$\begin{aligned} \nabla_+ &\equiv D_+ + \Gamma_+, & \bar{\nabla}_+ &\equiv \bar{D}_+, & (11) \\ \nabla_{++} &\equiv \partial_{++} + \Gamma_{++} & \text{and} & & (12) \\ \nabla_{--} &\equiv \partial_{--} - ig\Gamma_{--}, \end{aligned}$$

with the gauge superconnections  $\Gamma_+$ ,  $\Gamma_{++}$  and  $\Gamma_{--}$  being all Lie-algebra-valued. Note that  $\Gamma_{++}$  does not enter the Lagrangian density of eq.(10). The gauge couplings,  $g$  and  $h$ , can in principle be taken different; nevertheless, this would not mean that we are gauging two independent symmetries. There is a single simple gauge group,  $\mathcal{G}$ , with just one gauge-superfield parameter,  $\Lambda$ . It is the particular form of the  $(2,0)$ -minimal coupling (realised by the exponentiation of  $V$  and the connection present in  $\nabla_{--}$ ) that opens up the freedom to associate, in principle, different coupling parameters to the gauge superfields  $V$  and  $\Gamma_{--}$ .  $\Gamma_+$  and  $\Gamma_{++}$  can be both expressed in terms of the real scalar superfield,  $V(x; \theta, \bar{\theta})$ , according to [1]:

$$\Gamma_+ = e^{-gV} (D_+ e^{gV}) \quad (13)$$

and

$$\Gamma_{++} = -\frac{i}{2} \bar{D}_+ [e^{-gV} (D_+ e^{gV})]. \quad (14)$$

To establish contact with a component-field formulation and to actually identify the presence of an additional gauge potential, we write down the  $\theta$ -expansions for  $V^a$  and  $\Gamma_{--}^a$ :

$$V^a(x; \theta, \bar{\theta}) = C^a + \theta \xi^a - \bar{\theta} \bar{\xi}^a + \theta \bar{\theta} v_{++}^a \quad (15)$$

and

$$\begin{aligned} \Gamma_{--}^a(x; \theta, \bar{\theta}) &= \frac{1}{2} (A_{--}^a + iB_{--}^a) + i\theta(\rho^a + i\eta^a) \\ &+ i\bar{\theta}(\chi^a + i\omega^a) + \frac{1}{2} \theta \bar{\theta} (M^a + iN^a) \end{aligned} \quad (16)$$

$A_{--}^a$ ,  $B_{--}^a$  and  $v_{++}^a$  are the light-cone components of the gauge potential fields;  $\rho^a$ ,  $\eta^a$ ,  $\chi^a$  and  $\omega^a$  are left-handed Majorana spinors;  $M^a$ ,  $N^a$  and  $C^a$  are real scalars and  $\xi^a$  is a complex right-handed spinor.

means of the gauge-covariant derivatives which, according to the discussion of ref. [7], are defined as below:

The infinitesimal gauge transformations for  $V^a$  and  $\Gamma^a$  are given by

$$\delta V^a = \frac{i}{h} (\bar{\Lambda} - \Lambda)^a - \frac{1}{2} f^{abc} (\bar{\Lambda} + \Lambda)_b V_c \quad (17)$$

and

$$\delta \Gamma_{--}^a = -f^{abc} \Lambda_b \Gamma_{c--} + \frac{1}{g} \partial_{--} \Lambda_a. \quad (18)$$

No derivative acts on the  $\Lambda^a$ 's in eq.(17), which suggests the possibility of choosing a Wess-Zumino gauge for  $V^a$ . If such a choice is adopted, it can be shown that the gauge transformations of the  $\theta$ -component fields above read as follows:

$$\begin{aligned} &= \frac{2}{h} \Im m \alpha, \\ \delta v_{++}^a &= \frac{2}{h} \partial_{++} \alpha^a - f_{abc} \alpha^b v_{++}^c, \\ \delta A_{--}^a &= \frac{2}{g} \partial_{--} \alpha^a - f_{abc} \alpha^b A_{--}^c, \\ \delta B_{--}^a &= -f_{abc} \alpha^b B_{--}^c, \\ \delta \eta^a &= -f_{abc} \alpha^b \eta^c, \\ \delta \rho^a &= -f_{abc} \alpha^b \rho^c, \\ \delta M^a &= -f_{abc} \alpha^b M^c + f_{abc} \partial_{++} \alpha^b B_{--}^c, \\ \delta N^a &= \frac{2}{g} \partial_{++} \partial_{--} \alpha^a - f_{abc} \alpha^b N^c - f_{abc} \partial_{++} \alpha^b A_{--}^c, \\ \delta \chi^a &= -f_{abc} \alpha^b \chi^c, \\ \delta \omega^a &= -f_{abc} \alpha^b \omega^c, \end{aligned} \quad (19)$$

and they suggest that we should take  $h = g$ , so that the  $v_{++}^a$ -component could be identified as the light-cone partner of  $A_{--}^a$ ,

$$v_{++}^a \equiv A_{++}^a; \quad (20)$$

this procedure yields two component-field gauge potentials:  $A^\mu \equiv (A^0, A^1) = (A^{++}; A^{--})$  and  $B_{--}$ .

It is interesting to point out here that the first difference between the Abelian and the non-Abelian version of the theory arises. In the Abelian version [1], it was shown that both fields  $\chi$  and  $\omega$  were gauge invariant and the fields  $M$  and  $N$  could be identified with a combination of  $A_{--}$  and  $B_{--}$ . This combination, which was naturally dictated by the form of the gauge trans-

formations, ensured the symmetry of the Lagrangian. In the present situation, the gauge transformations do not undertake that we express  $M$  and  $N$  in terms of  $A_{--}$  and  $B_{--}$ , as it was done before but; on the other hand, the  $\chi$ - and  $\omega$ -fields are no longer auxiliary fields as they were in the Abelian version.

To discuss the field-strength superfields, we start analysing the algebra of the gauge covariant derivatives. So, the field strengths are defined such that:

$$\begin{aligned}
\{\nabla_+, \nabla_+\} &\equiv \mathcal{F} = 2D_+\Gamma_+, \\
\{\nabla_+, \bar{\nabla}_+\} &\equiv 2i\nabla_{++}, \\
[\nabla_+, \nabla_{++}] &\equiv W_- = D_+\Gamma_{++} - \partial_{++}\Gamma_+, \\
[\nabla_+, \nabla_{--}] &\equiv W_+ = -igD_+\Gamma_{--} - \partial_{++}\Gamma_+ - ig[\Gamma_+, \Gamma_{--}], \\
[\bar{\nabla}_+, \nabla_{++}] &\equiv U_+, \\
[\bar{\nabla}_+, \nabla_{--}] &\equiv U_- = -ig\bar{D}_+\Gamma_{--}, \\
[\nabla_{++}, \nabla_{--}] &\equiv \mathcal{Z}_{+-} = -ig\partial_{++}\Gamma_{--} - \partial_{--}\Gamma_{++} - ig[\Gamma_+, \Gamma_{--}].
\end{aligned} \tag{21}$$

The results obtained for the field-strengths are con-

sistent with the Bianchi identities. The identity for  $U_+$ ,

$$[\bar{\nabla}_+, \{\nabla_+, \bar{\nabla}_+\}] + [\nabla_+, \{\bar{\nabla}_+, \bar{\nabla}_+\}] + [\bar{\nabla}_+, \{\bar{\nabla}_+, \nabla_+\}] = 0 \tag{22}$$

gives immediately that  $U_+ = 0$ . The Bianchi identity

for  $Z_{+-}$ ,

$$[\nabla_{--}, \{\nabla_+, \bar{\nabla}_+\}] + \{\nabla_+, [\bar{\nabla}_+, \nabla_{--}]\} - \{\bar{\nabla}_+, [\nabla_{--}, \bar{\nabla}_+]\} = 0, \tag{23}$$

allows us to express  $Z_{+-}$  as

$$Z_{+-} = -\frac{i}{2}\nabla_+U_- - \frac{i}{2}\bar{\nabla}_+W_-; \tag{24}$$

and, finally, the Bianchi identity

$$[\bar{\nabla}_+, \{\nabla_+, \nabla_+\}] + [\nabla_+, \{\nabla_+, \bar{\nabla}_+\}] + [\nabla_+, \{\bar{\nabla}_+, \nabla_+\}] = 0 \tag{25}$$

leads to

$$W_+ = \frac{i}{4} \bar{D}_+ \mathcal{F}. \quad (26)$$

These are the relevant results yielded by pursuing an

investigation of the Bianchi identities.

The gauge field,  $A_\mu$ , has its field strength,  $F_{\mu\nu}$ , located at the  $\theta$ -component of the combination  $\Omega \equiv W_- + \bar{U}_-$ . This suggests the following kinetic action for the Yang-Mills sector:

$$\begin{aligned} S_{YM} &= \frac{1}{8g^2} \int d^2x d\theta d\bar{\theta} Tr \Omega \bar{\Omega} \\ &= \int d^2x Tr \left[ \frac{-1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{4} \Sigma \bar{\partial}_{++} \bar{\Sigma} + \frac{1}{4} M^2 \right], \end{aligned} \quad (27)$$

where  $\Sigma = \rho + i\eta + \bar{\chi} - i\bar{\omega}$  and  $A \bar{\partial} B \equiv (\partial A)B - A(\partial B)$ .

Choosing now a supersymmetry-covariant gauge-fixing, instead of the Wess-Zumino, we propose the following gauge-fixing term in superspace:

$$S_{gf} = -\frac{1}{2\alpha} \int d^2x d\theta d\bar{\theta} Tr [\Pi \bar{\Pi}]$$

$$\begin{aligned} &= -\frac{1}{2\alpha} \int d^2x \{ [(\partial_\mu A^\mu)^2 + (\partial_\mu A^\mu)N + \frac{1}{4}N^2] \\ &+ \frac{1}{4}[M^2 - 2M\partial_{++}B_{--} + (\partial_{++}B_{--})^2] \\ &- i(\rho + i\eta) \bar{\partial}_{++} (\bar{\rho} - i\bar{\eta}) \}, \end{aligned} \quad (28)$$

where  $\Pi = -iD_+\Gamma_{--} + \frac{1}{2}D_+\partial_{--}V$ .

So, the total action reads as follows:

$$\begin{aligned} S &= \int d^2x Tr \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\alpha} (\partial_\mu A^\mu)^2 - \frac{1}{2\alpha} (\partial_\mu A^\mu)N - \frac{1}{8\alpha} N^2 \right. \\ &+ \frac{1}{4} \left(1 - \frac{1}{2\alpha}\right) M^2 + \frac{1}{4\alpha} M(\partial_{++}B_{--}) - \frac{1}{8\alpha} (\partial_{++}B_{--})^2 \\ &\left. + \frac{i}{2\alpha} (\rho + i\eta) \bar{\partial}_{++} (\bar{\rho} - i\bar{\eta}) + \frac{i}{4} \Sigma \bar{\partial}_{++} \bar{\Sigma} \right\}. \end{aligned} \quad (29)$$

Using eq.(29), we are ready to write down the propagators for  $A^a$ ,  $B_{--}^a$ ,  $N^a$ ,  $M^a$ ,  $\rho^a$ ,  $\eta^a$ ,  $\chi^a$  and  $\omega^a$ :

$$\begin{aligned} \langle AA \rangle &= -\frac{i}{2\alpha(1-\alpha)} \omega^{\mu\nu}, \\ \langle BB \rangle &= -\frac{i(2\alpha-1)}{4\alpha(1-\alpha)} \frac{\partial_{--}^2}{\square^2}, \\ \langle AN \rangle &= \frac{i\alpha}{\alpha(1-\alpha)} (1-\alpha+\alpha)\partial^\mu, \\ \langle NA \rangle &= \frac{i}{(1-\alpha)\alpha} \partial^\nu \\ \langle NN \rangle &= -\frac{2i\alpha^2}{(1-\alpha)} \\ \langle MM \rangle &= -\frac{i}{16\alpha} \frac{1}{(1-\alpha)} \end{aligned}$$

$$\langle MB \rangle = -\langle BM \rangle = \frac{i}{8\alpha(1-\alpha)} \frac{\partial_{--}}{\square}$$

$$\langle (\rho + i\eta)(\bar{\rho} - i\bar{\eta}) \rangle = -\frac{2\alpha}{(\alpha-1)} \frac{\bar{\partial}_{--}}{4\square}$$

$$\langle (\rho + i\eta)(\chi + i\omega) \rangle = -\frac{\alpha}{4} \frac{\bar{\partial}_{--}}{\square}$$

$$\langle (\bar{\chi} - i\bar{\omega})(\bar{\rho} - i\bar{\eta}) \rangle = +\frac{\alpha}{4(\alpha-1)} \frac{\bar{\partial}_{--}}{\square}$$

$$\langle (\bar{\chi} - i\bar{\omega})(\chi + i\omega) \rangle = +\frac{(\alpha+2)}{4} \frac{\bar{\partial}_{--}}{\square}. \quad (30)$$

Expressing the action of equation (10) in terms of component fields, and coming back to the (2, 0)-version of the Wess-Zumino gauge, the matter-gauge sector Lagrangian reads:

$$\begin{aligned}
\mathcal{L}_{matter-gauge} = & 2\phi^{*i}\square\phi_i - ig[\phi^{*i}A_{--}^a(G_a)_i^j\partial_{++}\phi_j - c.c] + \bar{\sigma}^i\sigma_i + \\
& - ig[\phi^{*i}A_{++}^a(G_a)_i^j\partial_{--}\phi_j - c.c] - g\phi^{*i}M^a(G_a)_i^j\phi_j + \\
& - \frac{i}{2}g^2\phi^{*i}A_{++}^aA_{--}^b\phi_id_{abc}G_c - g\bar{\lambda}^iA_{--}^a(G_a)_i^j\lambda_j + \\
& - \frac{1}{2}\phi^{*i}A_{++}^aB_{--}^b\phi_if_{abc}G_c + 2i\bar{\lambda}^i\partial_{--}\lambda_i + \\
& - ig\phi^{*i}[(\chi^a + \bar{\rho}^a + i\omega^a - i\bar{\eta}^a)(G_a)_i^j\lambda_j - c.c] + \\
& - 2i\bar{\psi}^i\partial_{++}\psi_i - g\bar{\psi}^iA_{++}^a(G_a)_i^j\psi_j, \tag{31}
\end{aligned}$$

where  $d_{abc}$  are the (representation-dependent) symmetric coefficients associated to  $\{G_a, G_b\}$ .

One immediately checks that the extra gauge field,  $B_{--}$ , does not decouple from the matter sector. Our point of view of leaving the superconnection  $\Gamma_{--}$  as a complex superfield naturally introduced this extra gauge potential in addition to the usual gauge field,  $A_\mu$ :  $B_{--}$  behaves as a second gauge field. The fact that it yields a massless pole of order two in the spectrum may harm unitarity. However, the mixing with the  $M$ -component of  $\Gamma_{--}$ , which is a compensating field, indicates that we should couple them to external currents and analyse the imaginary part of the current-current amplitude at the pole. In so doing, this imaginary part turns out to be positive-definite, and so no ghosts are present. It is very interesting to point out that, in the Abelian case,  $B_{--}$  showed the same behaviour [1]. It coupled to  $C$  instead of  $M$ , but these two fields show the same kind of behaviour:  $C$  (in the Abelian case) and  $M$  (in the non-Abelian case) are both compensating fields. This ensures us to state that  $B_{--}$  behaves as a physical gauge field: it has dynamics and couples both to matter and the gauge field  $A^\mu$ . Its only peculiarity regards the presence of a single component in the light-cone coordinates. The  $B$ -field plays rather the rôle of a ‘‘chiral gauge potential’’. Despite the presence of the pair of gauge fields, a gauge-invariant mass term cannot be introduced, since  $B$  does not carry the  $B_{++}$ -component, contrary to what happens with  $A^\mu$ .

Let us now turn to the coupling of the two gauge potentials,  $A_\mu$  and  $B_{--}$ , to a non-linear  $\sigma$ -model always keeping a supersymmetric scenario. It is our main purpose henceforth to carry out the coupling of a  $(2,0)$   $\sigma$ -model to the relaxed gauge superfields of the ref. [7], and show that the extra vector degrees of freedom do not decouple from the matter fields (that is, the target space coordinates)[9][10][11][12]. The extra gauge potential,  $B_{--}$ , obtained upon relaxing constraints can therefore acquire a dynamical significance by means of the coupling between the  $\sigma$ -model and the Yang-Mills fields of ref.[7]. To perform the coupling of the  $\sigma$ -model to the Yang-Mills fields we reason along the same considerations as in ref.[1] and find out that:

$$\mathcal{L}_\xi = \partial_i[K(\Phi, \tilde{\Phi}) - \xi(\Phi) - \tilde{\xi}(\tilde{\Phi})]\nabla_{--}\tilde{\Phi}^i +$$

$$- \tilde{\partial}_i[K(\Phi, \tilde{\Phi}) - \xi(\Phi) - \tilde{\xi}(\tilde{\Phi})]\nabla_{--}\tilde{\Phi}^i, \tag{32}$$

where  $\xi(\Phi)$  and  $\tilde{\xi}(\tilde{\Phi})$  are a pair of *chiral* and *antichiral* superfields,  $\tilde{\Phi}_i \equiv \exp(i\mathbf{L}_{V,\bar{k}})\tilde{\Phi}_i$  and  $\nabla_{--}\tilde{\Phi}^i$  and  $\nabla_{--}\tilde{\Phi}^i$  are defined in perfect analogy to what is done in the case of the bosonic  $\sigma$ -model:

$$\nabla_{--}\tilde{\Phi}_i \equiv \partial_{--}\tilde{\Phi}_i - g\Gamma_{--}^\alpha \bar{k}_\alpha^i(\Phi) \tag{33}$$

and

$$\nabla_{--}\tilde{\Phi}^i \equiv \partial_{--}\tilde{\Phi}^i - g\Gamma_{--}^\alpha \bar{k}_{\alpha i}(\tilde{\Phi}). \tag{34}$$

The interesting point we would like to stress is that the extra gauge degrees of freedom accommodated in the component-field  $B_{--}(x)$  of the superconnection  $\Gamma_{--}$  behave as a genuine gauge field that shares with  $A^\mu$  the feature of coupling to matter and to  $\sigma$ -model [7]. This result can be explicitly read off from the component-field Lagrangian projected out from the superfield Lagrangian  $\mathcal{L}_\xi$ . We therefore conclude that our less constrained  $(2,0)$ -gauge theory yields a pair of gauge potentials that naturally transform under the action of a single compact and simple gauge group and may be consistently coupled to matter fields as well as to the  $(2,0)$  non-linear  $\sigma$ -models by means of the gauging of their isotropy and isometry groups. Relaxing constraints in the  $N = 1$ - and  $N = 2 - D = 3$  supersymmetric algebra of covariant derivatives may lead to a number of peculiar features of the gauged  $O(3)$ - $\sigma$ -model[13] in the presence of Born-Infeld terms for the pair of gauge potentials; of special interest are the self-dual equations[14].

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