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Einstein Quantum Gravity and Yang-Mills Quantum Fields from a Stringy Space-Time

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Abstract

We show that the dynamics of Einstein Gravitation Theory may be understood as an effective theory of a quantum theory for a fluctuating space-time implemented as a Bosonic string path-integral and interacting with a non-dynamics Einstein space-time metric field (the quantum gravitation vacuum). Additionally, we show how to deduce SU(N) Yang-Mills Gauge theories from a stringy SO(D) space-time with an intrinsic SU(N) fermionic structure at large N.

Introduction

At present times, the most important idea to understand the Physical world is the Einstein's view that all basic laws of physics comes from a single one. Presently, it has been proposed the use of Kaluza-Klein supersymmetric strings theories with imposed super conformal invariance [1], [3], [4] to describe the full spectrum of known elementary particles as string excitations. Although its "end of physics as we know" purpose, these super-string theories have not achieved yet an unambigous status of a complete and predictive physical theory.

In this letter, we propose a new and more modest "theory of everything" by considering the basic physical assumption that the space-time has a dynamical (fluctuating) string structure depending on the physical phenomena dynamics at large. In section of this letter, we deduce the Einstein quantum path integral from a space-time modelled as a Bosonic String. In section 2, we present our ideas to the case of Yang-Mills quantum fields.

1 Einstein Quantum Gravity from a Bosonic Stringy Space-Time

Let us start our analyze in this section 1 by considering as dynamical fields of our proposed theory, the non-dynamical Einstein gravitational metric $G_{\mu\nu}(X^\alpha)$ at large and the microscopic space-time vector position $X^\alpha(\xi)$, considered as the world-sheet of a closed Bosonic string. The quantum combined system path-integral will proposed to be given by the following σ-model covariant path-integral which may be regarded as the four-dimensional analogous of similar Path-Integral studied by Polyakov in ref. [1].

$$Z = \int_{(\bar{X},\mu,\nu)} \prod d[G_{\mu\nu}(\bar{X})] e^{-\mu^2 \int_M (\sqrt{G})(\bar{X}) d^D \bar{X}}$$
$$\left(\int_{\xi \in R^2} \prod (\sqrt{G} G^{\mu\nu}(X(\xi))^{1/2}) dX_\mu(\xi) \right)$$
$$exp\left(-\frac{1}{2\pi\alpha'}\right) \int_{R^2} d^2\xi [\sqrt{G} G_{\mu\nu}(X^\alpha(\xi))(\partial_a X^\mu \partial^a X^\nu]) \tag{1}$$

Let us show that at the large string scale $\alpha' \to 0$ (a (small) quantum piece of our classically observed space-time manifold M), the string path-integral in eq. (1) leads to an effective (dynamical) Path-Integral for metric fields $G_{\mu\nu}(\bar{X})$ weighted by the Einstein action.

In order to show this we take a (classical) fixed point $\bar{X}^\alpha$ and consider the string vector position geodesic expansion around it, but defined by the quantum string vector position until the $(\alpha')^2$ order, namelly:

$$X^\alpha(\xi) = \bar{X}^\alpha + \sqrt{\alpha'} Y^\alpha(\xi) \tag{2.a}$$
$$G_{\mu\nu}(X^\alpha(\xi)) = \eta_{\mu\nu}(\bar{X}^\alpha) + \frac{(\alpha')}{3} R_{\mu\alpha\nu\beta}(\bar{X}^\alpha) Y^\alpha(\xi) Y^\beta(\xi) + O(\alpha'^2) \tag{2.b}$$
$$D_{\sqrt{G}} X^\alpha(\xi) = \prod_{\xi \in R^2} dX^\alpha(\xi)[(\sqrt{G} G^{\mu\nu})^{1/2}(X^\beta(\xi))] \sim \prod dY^\alpha(\xi)$$
$$+O(\alpha') = D^F[Y^\alpha(\xi)] \tag{2.c}$$

As a consequence of the above made remarks we thus, should consider the following string path-integral at the space-time (quantum) chart of Hausdorf dimension four

$$V_{\bar{X}^\alpha} = \left\{ V \in R^4 | V^B = \bar{X}^\beta + \sqrt{\alpha'} Y^\beta(\xi) + O(\alpha') \right\}$$
$$Z_{V(\bar{X}^\alpha)}[G_{\mu\nu}(\bar{X}^\alpha)] = \int D^F[Y^\mu(\xi)] \; exp\left\{ -\frac{1}{2} \int_{R^2} d^2\xi \sqrt{G}(\bar{X}^\alpha) \right.$$
$$\left. [\eta_{\mu\nu}(\bar{X}^\alpha) + \frac{(\alpha')}{3} R_{\mu\alpha\nu\beta}(\bar{X}^\alpha) Y^\alpha(\xi) Y^\beta(\xi)](\partial_a Y^\mu(\xi) \partial^a Y^\nu(\xi) \right\} \tag{3}$$

Following closelly ref.[2], let us introduce composite fields to write eq. (4) as a gaussian path-integral over the string vector position $Y^{\beta}(\xi)$ describing the stringy space-time fluctuations

$$
\begin{aligned}
& Z_{V\{\bar{x}\}}[G_{\mu\nu}(\bar{x}^{\alpha})] = \int D^F[Y^{\mu}(\xi)]D^F[\lambda(\xi)] \\
& exp\left\{-\frac{1}{2}\int d^2\xi\sqrt{G(\bar{X}^{\alpha})}\left[(Y^{\mu}(\xi))(-\partial_{\xi}^2)\eta_{\mu\nu}(\bar{X}^{\alpha})Y^{\nu}(\xi))+\right.\right. \\
& \left.\left.+\frac{1}{3}(\alpha')R_{\mu\alpha\nu\beta}(\bar{X}^{\alpha})\sigma^{\alpha\beta}(\xi)(\partial_a Y^{\mu}\partial^a Y^{\nu})(\xi)\right]\right\} \\
& exp\left\{i\int_{R^2} d^2\xi\sqrt{G(\bar{X}^{\alpha})}\lambda(\xi)\eta_{\alpha\beta}(\bar{X}^{\alpha})(\sigma^{\alpha\beta}(\xi)-Y^{\alpha}(\xi)Y^{\beta}(\xi))\right\}
\end{aligned} \tag{4}
$$

At this point, we propose to replace the functional integral constraint over the $\lambda(\xi)$ scalar field variable by the usual (non-functional) saddle-point limit

$$
\begin{aligned}
& exp\left\{i\int_{R^2} d^2\xi\ \sqrt{G(\bar{X})^{\alpha}}\lambda(\xi)\eta_{ab}(\bar{X}^{\alpha})(\sigma^{\alpha\beta}(\xi)-Y^{\alpha}(\xi)Y^{\beta}(\xi)\right\} \\
& \lim_{\lambda\to\infty}\ exp\left\{i\lambda\int_{R^2} d^2\xi\sqrt{G(\bar{X}^{\alpha})}\eta_{\alpha\beta}(\bar{X})^{\alpha}(\sigma^{\alpha\beta}(\xi)-Y^{\alpha}(\xi)Y^{\beta}(\xi))\right\}
\end{aligned} \tag{5}
$$

Additionally, we consider the condensate approximation for the Lagrange multiplier field $\sigma^{\alpha\beta}(\xi)$ (see ref. [2] for details)

$$
\sigma^{\alpha\beta}(\xi) = \langle\sigma\rangle\eta^{\alpha\beta}(\bar{X}^{\alpha}) + O(\alpha') \tag{6}
$$

Substituting eq. (5)-eq.(6) into eq.(4) we get a $Y_{\mu}(\xi)$ gaussian functional integral with the following result

$$
\begin{aligned}
& Z_{V\{\bar{X}\}}[G_{\mu\nu}(\bar{X}^{\alpha})] = \lim_{\substack{\alpha'\to 0\\ <\lambda>\to\infty}} \\
& \left\{det_F^{-1/2}\left[(\eta_{\mu\nu}(\bar{X})+\frac{<\sigma>}{3}(\alpha')R_{\mu\alpha\nu\beta}(\bar{X})G^{\alpha\beta}(\bar{X})(-\partial^2)_{\xi}+<\lambda>G_{\mu\nu}(\bar{X})\right]\right\}
\end{aligned} \tag{7}
$$

or equivalently (since $\alpha'\to 0$):

$$
\begin{aligned}
& Z_{V\{\bar{x}\}}[G_{\mu\nu}(\bar{x}^{\alpha})] = \lim_{<\lambda>\to\infty}\ \lim_{\alpha'\to 0}\left\{det_F^{-1/2}[(-\partial^2)_{\xi}\ \ \eta_{\mu\nu}(\bar{X})+\right. \\
& \left.+<\lambda>G_{\mu\rho}(\bar{X})(\eta_{\rho\nu}(\bar{X})-\frac{<\sigma>}{3}(\alpha')R_{\rho\alpha\nu\beta}(\bar{X})G^{\alpha\beta}(\bar{X})]\right\}
\end{aligned} \tag{8}
$$

In order to compute the above written functional determinant at the limit of large $<\lambda>$, we regulate this determinant by the proper-time method and evaluate the $<\lambda>\to\infty$ limit as in ref. [6], namelly:

$$\lim_{\langle\lambda\rangle\to\infty} log\ det_F\left[(-\partial^2)_\xi\eta_{\mu\nu}(\bar{X})+\langle\lambda\rangle G_{\mu\nu}(\bar{X})-\frac{\alpha'\langle\sigma\rangle}{3}\lambda R_{\mu\alpha\nu\beta}(\bar{X})G^{\alpha\beta}(\bar{X})\right]$$
$$\lim_{\langle\lambda\rangle\to\infty} 4\int_0^\infty \frac{dt}{t}\ e^{-t\lambda}$$
$$\lim_{t\to 0^+}\left\{exp-t\left((-\partial^2)_\xi\eta_{\mu\nu}(\bar{X})+\langle\lambda\rangle G_{\mu\nu}(\bar{X})-\frac{\alpha'\langle\sigma\rangle}{3}\lambda R_{\mu\alpha\nu\beta}(\bar{X})G^{\alpha\beta}(\bar{X})\right)\right\} \quad (9)$$

After using the usual Secley-De Witt expansion for the 2D differential operator inside eq. (10) we, thus, obtain the Einstein-Kilbert action with a cosmological constant in the (Euclidean) space-time quantum chart $V(\bar{X})$, namelly

$$Z_{V(\bar{X})}[G_{\mu\nu}(\bar{X}^\alpha)]=$$
$$\lim_{\substack{\langle\lambda\rangle\to\infty\\ \alpha'\to 0}} exp\left\{-\sqrt{G(\bar{X}^\alpha)}\left\{4A\int_0^\infty\frac{dt}{t^2}\ e^{-t\langle\lambda\rangle}\right\}\right.$$
$$\left.-\sqrt{G(\bar{X}^\alpha)}R(\bar{X}^\alpha)\left\{\frac{4A\langle\lambda\rangle\langle\sigma\rangle\alpha'}{3}\int_0^\infty\frac{dt}{t}e^{-t\langle\lambda\rangle}\right\}\right.$$
$$\sim exp\left\{-\frac{1}{G_{Newton}}(R(G^{\mu\nu}(\bar{X}^\beta))\sqrt{G(\bar{X}^\beta)})-\mu_{cm}\sqrt{G(\bar{X}^\beta)}\right\} \quad (10.a)$$

It is worth point out that the Newton gravitational and the cosmological constants in our approach are not fundamental in our theory and are defined in terms of the microscopic string constants as follows from eq. (11)

$$\frac{1}{G_{Newton}}=\frac{4A\langle\lambda\rangle\langle\sigma\rangle\alpha'}{3}\int_0^\infty\frac{dt}{t}e^{-t\langle\lambda\rangle} \quad (10.b)$$
$$\mu_{cm}=4A\int_0^\infty\frac{dt}{t}e^{-t\langle\lambda\rangle} \quad (10.c)$$

where $A=\int d^2\xi$ denotes the internal string area .

The complete path-integral eq. (1) at large is finally given by sum of eq. (10) over all space-time charts $V_{\{\bar{X}\}}$ with $\bar{X} \in R^4$

$$Z = \int \prod_{\bar{X},\mu\nu} d[G_{\mu\nu}(\bar{X})^\alpha] exp\{-\mu \int_M (\sqrt{G})(\bar{X}) d^4 \bar{X}^\alpha\} \left(\sum_{\{\bar{X}\}\in R^4} \left\{ Z_{V\{\bar{X}_\alpha\}}[G_{\mu\nu}(\bar{X}^\alpha)] \right) \right\}$$
$$= \int \prod_{(\bar{X},\mu,\nu)} d[G_{\mu\nu}(\bar{X}^\alpha)] exp\{-\mu_{cm} \int_{R^4} (\sqrt{G})(\bar{X}^\alpha) d^D \bar{X}^\alpha\}$$
$$exp\left\{ -\frac{1}{G_{Newton}} \int_{R^4} d^D \bar{X}^\alpha (\sqrt{G} R(G^{\alpha\nu}))(\bar{X}^\alpha) \right\} \tag{11}$$

At this point we remark that the Quantum Field Theory Path Integral eq. (11) must be well-defined (quantized) independent of possessing an effective nature ([7]).

2 Yang-Mills from a Fermionic Stringy Space-Time

In this section we present the same procedure exposed in previous section to deduce the Yang-Mills quantum field path integral from a string theory, modelling space-time fluctuations.

Let us, thus, consider the following combined path integral of a Yang-Mills field and a Bosonic string moving on a sphere of radius $R = 1$ in the Euclidean space-time, R^D with a fermionic SU(N) structure ([7]).

$$Z = \int_{(\bar{X},\mu,a)} (\Pi dA_\mu^a(\bar{X}))^{Haar}$$
$$\int D^F[X^\mu(\xi)] exp\left\{ -\frac{1}{2} \int_{R^2} d^2\xi (\partial_A X^\mu \partial^A X_\mu)(\xi) \right\} \delta^{(F)}((X_\mu X^\mu(\xi) - 1)$$
$$\int D^F[\psi_a, \bar{\psi}_a] exp\left\{ -\frac{1}{2} \int_{R^2} d^2\xi (\bar{\psi}(i\gamma^A \partial_A)\psi)(\xi) \right\}$$
$$exp\left\{ ie \int_{R^2} d^2\xi [A_\mu^i(X^\beta(\xi))(\bar{\psi}_a \gamma^A (\lambda_i)_{ab} \psi)(\xi)(\partial_A X^\nu)(\xi)] \right\} \tag{11}$$

In order to implement our previous studies of section 1 to this case, we consider the "Harmonic gauge" fixing in the Haar-Yang-Mills path integral in eq. (13), namelly $\bar{X}_\mu \cdot A_\mu(\bar{X}) = 0$ which allow us by its turn to rewrite the interaction term in eq. (13) in terms of the Yang-Mills strenght field in the chart $V(\bar{X})$ at large

$$I_{V(\bar{X})}[A^\mu_a(\bar{X})] = \int D^F[X^\mu(\xi)] exp\left\{-\frac{1}{2}\int_{R^2} d^2\xi(\partial_A X^\mu \partial^A X_\mu)(\xi)\right\}$$
$$\left(\lim_{\lambda\to\infty} exp\left\{-\lambda\int_{R^2} d^2\xi[(X^\mu X_\mu)(\xi)-1]\right\}\right)$$
$$\int D^F[\psi_a,\bar{\psi}_a] exp\left\{-\frac{1}{2}\int_{R^2} d^2\xi(\bar{\psi}(i\gamma^A\partial_A)\psi)(\xi)\right\}$$
$$exp\left\{ie\alpha'\int_{R^2} d^2\xi\ \frac{1}{2}Y^\rho(\xi)F^i_{\rho\mu}(\bar{X}^\alpha)(\bar{\psi}_a\gamma^A(\lambda_i)_{ab}\psi)(\xi)(\partial_A Y^\mu)(\xi)\right\} \tag{12}$$

At this point we evaluate the $X_\mu(\xi)$-Gaussian functional integral with the exact result

$$I_{V(\bar{X})}[A^\mu_a(\bar{X})] = \lim_{\langle\lambda\rangle\to\infty}\left\langle det^{-\frac{1}{2}}\left[(-\partial^2)_\xi\eta^{\mu\nu}(\bar{X}) + \frac{1}{2}\left(F^{\mu\nu}_i(\bar{X})j^i_a(\xi)\right)\partial^a_\xi - \langle\lambda\rangle\right]\right\rangle_{\psi,\bar{\psi}} \tag{13}$$

where $\langle\ ,\ \rangle_{\psi,\bar{\psi}}$ denotes the functional integral over the SU(N) string intrinsic Dirac fields and $j^i_a(\xi)$ is the conserved fermion SU(N) current on the string world-sheet.

At $\langle\lambda\rangle\to\infty$, we obtain the following result for eq. (15)

$$I_{V(\bar{X})}[A^a_\mu(\bar{X})] \sim \left\langle exp\left\{-\frac{1}{16\pi}\ F^i_{\mu\nu}(\bar{X})F^p_{\mu\nu}(\bar{X})(j^a_i(\xi)j^a_p(\xi))\right\}\right\rangle_{\psi,\bar{\psi}} \tag{14}$$

which at large N, give us the final result

$$I_{V(\bar{X})}[A^a_\mu(\bar{X})]_{(N\to\infty)} = exp\left\{-\left(\frac{\langle\int d^2\xi j^a_i(\xi)j^a_i(\xi)\rangle^{(N\to\infty)}_{\psi,\bar{\psi}}}{16\pi}\right)F^i_{\mu\nu}(\bar{X})F^i_{\mu\nu}(\bar{X})\right\} \tag{15}$$

The complete path-integral eq. (13) is, thus, exactly the SU(∞) Yang-Mills quantum field path integral for the space-time at large (after integrating out the space-time microscopic stringy fluctuations $Y^\mu(\xi)$)

$$Z = \int D^F[A^a_\mu(X)] exp\left\{-\frac{1}{16g^2_{Q.C.D}}\int d^D\bar{X}(F^2_{\mu\nu}(\bar{X})\right\} \tag{16}$$

Note that the .Q.C.D$_{N_c=+\infty}$ coupling constant is expressed in terms of the microscopic string parameters

$$g^2_{Q.C.D} = \frac{1}{\pi}\left\langle\int d^2\xi j^a_i(\xi)j^a_i(\xi)\right\rangle^{(N\to\infty)}_{\psi,\bar{\psi}} \tag{17}$$

Finally we remark that the supersymmetric versions of our proposed string structure for the space-time leads to the super-gravity and super Yang-Mills theories.

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