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(2,0)-Super-Yang-Mills Coupled to Non-Linear σ-Model

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Abstract

Considering a class of (2,0)-super Yang-Mills multiplets that accommodate a pair of independent gauge potentials in connection with a single symmetry group, we present here their coupling to ordinary matter and to non-linear σ-models in (2,0)-superspace. The dynamics and the coupling of the gauge potentials are discussed and the interesting feature that comes out is a sort of "chirality" for one of the gauge potentials once light-cone coordinates are chosen.

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The raise of interest on the investigation of geometrical aspects and quantum behaviour of two-dimensional systems, such as Yang-Mills theories and non-linear σ-models, especially if endowed with supersymmetry, has been broadly renewed in connection with the analysis of superstring background configurations [1, 2] and the study of conformal field theories and integrable models.

As for supersymmetries defined in two space-time dimensions, they may be generated by p left-handed and q right-handed independent Majorana charges: these are the so-called (p, q) supersymmetries [1, 3] and are of fundamental importance in the formulation of the heterotic superstrings [4].

Motivated by the understanding of a number of features related to the dynamics of world-sheet gauge fields [5, 6, 7, 8, 9, 10] and the possibility of finding new examples of conformal field theories, was considered the superspace formulation of a $(2,0)$ Yang-Mills model [11, 12] enlarged by the introduction of an extra gauge potential transforming under the same *simple* gauge group as the ordinary Yang-Mills field of the theory.

In the works of refs.[11, 12] one has discussed the role of the further gauge potential on the basis of discussing constraints on field-strength superfields in the algebra of gauge-covariant derivatives in $(2,0)$-superspace. The minimal coupling of this sort of less-constrained Yang-Mills model to matter superfields has been contemplated, and it has been ascertained that the additional gauge potential corresponds to non-interacting degrees of freedom in the Abelian case. For non-Abelian symmetries, the extra Yang-Mills field still decouples from matter, though it presents self-interactions with the gauge sector [11].

It is therefore our purpose in this letter to find out a possible dynamical role for the aditional gauge potential discussed in refs.[11, 12], by means of its coupling to matter superfields that describe the coordinates of the Kähler manifold adopted as the target space of a $(2,0)$ non-linear σ-model [13]. To pursue such an investigation, we shall gauge the isometry group of the σ-model under consideration while working in $(2,0)$-superspace; then, all we are left with is the task of coupling the $(2,0)$ Yang-Mills extended supermultiplets of ref.[11] to the superfields that define the $(2,0)$ σ-model that is gauged.

The coordinates we choose to parametrise the $(2,0)$-superspace are given by

$$z^A \equiv (x^{++}, x^{--}; \theta, \bar{\theta}), \tag{1}$$

where x^{++}, x^{--} denote the usual light-cone coordinates, whereas θ, $\bar{\theta}$ stand for complex right-handed Weyl spinors. The supersymmetry covariant derivatives are taken as:

$$D_+ \equiv \partial_\theta + i\bar{\theta}\partial_{++} \tag{2}$$

and

$$\bar{D}_+ \equiv \partial_{\bar{\theta}} + i\theta\partial_{++}, \tag{3}$$

where ∂_{++} (or ∂_{--}) represents the derivative with respect to the space-time coordinate x^{++} (or x^{--}). They fulfill the algebra:

$$D_+^2 = \bar{D}_+^2 = 0 \qquad\qquad \{D_+, \bar{D}_+\} = 2i\partial_{++}, \tag{4}$$

with this definition for D and $\bar{D}$, one can check that:

$$e^{i\theta\bar{\theta}\partial_+} D_+ e^{-i\theta\bar{\theta}\partial_+} = \partial_\theta \tag{5}$$

$$e^{-i\theta\bar{\theta}\partial_+} \bar{D}_+ e^{i\theta\bar{\theta}\partial_+} = \partial_{\bar{\theta}} \tag{6}$$

The fundamental matter superfields we shall deal with are "the chiral" scalar and left-handed spinor superfield, whose respective component-field expressions are given by:

$$\begin{aligned} \Phi(x;\theta,\bar{\theta}) &= e^{i\theta\bar{\theta}\partial_{++}}(\phi + \theta\lambda) \\ \Psi(x;\theta,\bar{\theta}) &= e^{i\theta\bar{\theta}\partial_{++}}(\psi + \theta\sigma) \end{aligned} \tag{7}$$

where ϕ and σ are scalars, whereas λ and ψ respectively right and left-handed Weyl spinors.

This sort of chirality constraints yields the following component-field expansions for Φ^i and Ψ^i:

$$\begin{aligned} \Phi^i(x;\theta,\bar{\theta}) &= \phi^i(x) + \theta\lambda^i(x) + i\theta\bar{\theta}\partial_{++}\phi^i(x), \\ \Psi^i(x;\theta,\bar{\theta}) &= \psi^i(x) + \theta\sigma^i(x) + i\theta\bar{\theta}\partial_{++}\psi^i(x). \end{aligned} \tag{8}$$

The most general superspace action involving Φ and Ψ with interactions governed by dimensionless coupling parameters λ_1 and λ_2, reads

$$\begin{aligned} S &= \int d^2x d\theta d\bar{\theta}[i(\bar{\Phi}\partial_{--}\Phi - \Phi\partial_{--}\bar{\Phi}) + \bar{\Psi}\Psi + \\ &+ m(\Phi\Psi + \bar{\Phi}\Psi) + \\ &+ \lambda_1 P(\Phi,\bar{\Phi})(\bar{\Phi}\partial_{--}\Phi - \Phi\partial_{--}\bar{\Phi}) + \\ &+ \lambda_2 Q(\Phi,\bar{\Phi})\bar{\Psi}\Psi], \end{aligned} \tag{9}$$

where m is a mass parameter, and P and Q denotes real polynomials in Φ and $\bar{\Phi}$.

We now assume that Φ and Ψ transform under an arbitrary compact and simple gauge group, $\mathcal{G}$, according to

$$\Phi' = R(\Lambda)\Phi, \qquad \Psi' = S(\Lambda)\Psi \tag{10}$$

where R and S are matrices that respectively represent a gauge group element in the representation under which Φ and Ψ transform. Taking into account the constraint on Φ and Ψ, and bearing in mind the exponencial representation of R and S, we find that the gauge paramenter superfields Λ satisfy the same sort of constraint. It can therefore be expanded as follows:

$$\Lambda(x;\theta,\bar{\theta}) = e^{i\theta\bar{\theta}\partial_{++}}(\alpha + \theta\beta), \tag{11}$$

where α is a scalar and β is a right-handed spinor.

The kinetic part of the action (9) can be made invariant under the local transformations (10) by minimally coupling gauge potential superfields, $\Gamma_{--}(x;\theta,\bar{\theta})$ and $V(x;\theta,\bar{\theta})$ according to

$$S_{inv} = \int d^2x d\theta d\bar{\theta}\{i[\bar{\Phi}e^{hV}(\nabla_{--}\Phi) - \overline{(\nabla_{--}\Phi)e^{hV}\Phi}] + \bar{\Psi}e^{hV}\Psi\}. \tag{12}$$

The Yang-Mills supermultiplets are introduced by means of the gauge covariant derivatives which, according to the discussion of ref. [11], are defined as below:

$$\nabla_+ \equiv D_+ + \Gamma_+, \qquad \bar{\nabla}_+ \equiv \bar{D}_+, \tag{13}$$

$$\nabla_{++} \equiv \partial_{++} + ig\Gamma_{++} \qquad and \qquad \nabla_{--} \equiv \partial_{--} - ig\Gamma_{--}, \tag{14}$$

where the gauge superconnections Γ_+, Γ_{++} and Γ_{--} are all Lie-algebra-valued. The gauge coupling g and h could in principle be taken different. Γ_+ and Γ_{++} can be both expressed in terms of a real scalar superfield, $V(x;\theta,\bar{\theta})$, according to:

$$\Gamma_+ = e^{-gV}(D_+ e^{gV}) \tag{15}$$

and

$$\Gamma_{++} = -\frac{i}{2}\bar{D}_+[e^{-gV}(D_+ e^{gV})]. \tag{16}$$

Therefore, the gauging of the σ-model isometry group shall be achieved by minimally coupling the action of the $(2,0)$-supersymmetric σ-model to the gauge superfields V and Γ_{--}, as we shall see in the sequel.

To stablish contact with a component-field formulation and to actually identify the presence of an aditional gauge potential, we write down the θ-expansions for V and Γ_{--}:

$$V(x;\theta,\bar{\theta}) = C + \theta\xi - \bar{\theta}\bar{\xi} + \theta\bar{\theta}v_{++} \tag{17}$$

and

$$\Gamma_{--}(x;\theta,\bar{\theta}) = \frac{1}{2}(A_{--} + iB_{--}) + i\theta(\rho + i\eta) + i\bar{\theta}(\chi + i\omega) + \frac{1}{2}\theta\bar{\theta}(M + iN). \tag{18}$$

A_{--}, B_{--} and v_{++} are the light-cone components of the gauge potential fields; ρ, η, χ and ω are left-handed Majorana spinors; M, N and C are real scalars and ξ is a complex right-handed spinor.

The gauge transformations of the component fields above read

$$\begin{aligned}
\delta C &= \frac{2}{h}\Im m\alpha \\
\delta\xi &= -\frac{i}{h}\beta \\
\delta v_{++} &= \frac{2}{h}\partial_{++}\Re e\alpha \\
\delta A_{--} &= \frac{2}{g}\partial_{--}\Re e\alpha \\
\delta B_{--} &= \frac{2}{g}\partial_{--}\Im m\alpha
\end{aligned}$$

$$\begin{aligned}
\delta\eta &= -\frac{1}{g}\partial_{--}\Re e\beta \\
\delta\rho &= \frac{1}{g}\partial_{--}\Im m\beta, \\
\delta M &= -\frac{2}{g}\partial_{--}\partial_{++}\Im m\alpha, \\
\delta N &= \frac{2}{g}\partial_{--}\partial_{++}\Re e\alpha, \\
\delta\chi &= 0, \\
\delta\omega &= 0
\end{aligned} \tag{19}$$

and they suggest that we can take $h = g$, so the component v_{++} should be identified as the light-cone partner of A_{--},

$$v_{++} \equiv A_{++}, \tag{20}$$

and we end up with two component-field gauge potentials: $A^\mu \equiv (A^0, A^i)$ and $B_{--}(x)$. At this point, we should set a non-trivial remark: the $\theta\bar{\theta}$-component of Γ_{--} should be identified as bellow:

$$M + iN = i\partial_{++}(A_{--} + iB_{--}), \tag{21}$$

so as to ensure that $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ appears as a component accomodated in the field-strength superfield defined by:

$$[\nabla_+, \nabla_{--}] \equiv X = -igD_+\Gamma_{--} - \partial_{--}\Gamma_+. \tag{22}$$

This does not break supersymmetry, for χ and ω are non-dynamical degrees of freedom and drop out from the field-strength superfield X. In practice, after the identification given in equation (21), Γ_{--} carries two bosonic and two fermionic degrees of freedom.

Using the field-strength defined in (22), we can build up the gauge invariant kinetic Lagrangian:

$$S_{kin} = -\frac{1}{8g^2}\int d^2x d\theta d\bar{\theta}\bar{X}X. \tag{23}$$

This action leads to the component Lagrangian as below:

$$\mathcal{L}_{kin} = \mathcal{L}_{kin}(\rho, \eta, \xi) + \mathcal{L}_{kin}(A) + \mathcal{L}_{kin}(B_{--}, C) \tag{24}$$

where

$$\mathcal{L}_{kin}(\rho,\eta,\xi) = \frac{i}{8}(\bar{\rho} - i\bar{\eta} - \partial_{--}\bar{\xi}) \leftrightarrow \partial_{++}(\rho + i\eta - \partial_{--}\xi), \tag{25}$$

where $A \leftrightarrow \partial B = (\partial A)B - A(\partial B)$.

$$\mathcal{L}_{kin}(A) = \frac{1}{2}A^{\nu}(\Box\eta_{\mu\nu} - \partial_{\mu}\partial_{\nu})A^{\mu} = \frac{1}{2}A^{\nu}R_{\mu\nu}A^{\mu}, \tag{26}$$

and

$$\mathcal{L}_{kin}(B_{--},C) = \frac{1}{8}(\partial_{++}B_{--} - \partial_{++}\partial_{--}C)^2 = \frac{1}{8}(B_{--}\ \ C)K(B_{--}\ \ C)^t. \tag{27}$$

Notice that, as already mentioned above, χ and ω are <u>not</u> present in the kinetic Lagrangian (25).

We can see that $R_{\mu\nu}$ and K are non-invertible matrices, so it is necessary to write down the gauge fixing Lagrangian which is given by:

$$\begin{aligned} S_{gf} &= k\int d^2x d\theta d\bar{\theta}\bar{G}G \\ &= -\frac{1}{2\alpha}(\partial_{\mu}A^{\mu})^2 - \frac{i}{4\alpha}(\bar{\rho} - i\bar{\eta} - \partial_{--}\bar{\xi})\partial_{++}(\rho + i\eta - \partial_{--}\xi) + \\ &- \frac{1}{8\alpha}(\partial_{++}B_{--} + \partial_{--}\partial_{++}C) \end{aligned} \tag{28}$$

where $G = D_+\partial_{--}V - iD_+\Gamma_{--}$. Using the gauge fixing, eq.(28), together with equations (26) and (27), we can write down the propagators for A, B_{--} and C:

$$\begin{aligned} \langle AA\rangle &= -\frac{2i}{\Box}(\theta^{\mu\nu} + \alpha\omega^{\mu\nu}), \\ \langle BB\rangle &= -\frac{8i}{\Box^2}(\alpha - 1)\partial_{--}^2, \\ \langle BC\rangle &= -\langle CB\rangle = \frac{8i}{\Box^2}(\alpha + 1)\partial_{--}, \\ \langle CC\rangle &= \frac{8i}{\Box^2}(\alpha - 1) \end{aligned} \tag{29}$$

Expressing the action of equation 12 in terms of component-fields, and adopting the Wess-Zumino gauge, the Lagrangian reads as bellow:

$$\begin{aligned} \mathcal{L}_{comp} &= 2\phi\Box\phi^* - igA_{--}[\phi\partial_{++}\phi^* - c.c] + igA_{++}[\phi\partial_{--}\phi^* - c.c] + \\ &+ g\phi\phi^*[\partial_{++}A_{--} + gA_{++}A_{--}] + 2i\bar{\lambda}\partial_{--}\lambda - gA_{--}\bar{\lambda}\lambda + \\ &+ ig[\phi^*(\chi + \bar{\rho} + i\omega - i\bar{\eta})\lambda - c.c] \end{aligned} \tag{30}$$

one immediately checks that the extra gauge field B_{--} does not decouple from the matter vector. Our point of view of leaving the superconnection Γ_{--} as a complex superfield naturally introduced an extra gauge potential, B_{--} in addition to the usual gauge field A_μ. B_{--} behaves as second gauge field. The fact that its massless pole is of order two may harm the unitarity. However, the mixing with the C-component of V, which is a compensating field, indicates us that we should couple them to external currents and analyse the imaginary part of the current-current amplitude at the pole. In so doing, this imaginary part turns out to be positive-definite, and so no ghosts are present. This insures us to state that B_{--} behaves as a physical gauge field. It has dynamics and couples to matter. The only peculiarity regards the presence of a single component in the light-cone coordinates. The B-field plays rather the role of a "chiral gauge potential". Despite the presence of the pair of gauge fields, a gauge-invariant mass term cannot be introduced since B does not present the B_{++}-component, contrary to what happens with A^μ. Let us now turn to the coupling of the two gauge potentials, A_μ and B_{--}, to the non-linear σ-model.

It is our main purpose henceforth to carry out the coupling of a $(2,0)$ σ-model to the relaxed gauge superfields of the ref. [11], and show that the extra degrees of freedom do not decouple now from the matter fields (that is the target space coordinates). The extra gauge potential obtained upon relaxing constraints can therefore acquire a dynamical significance by means of the coupling between the σ-model and the Yang-Mills fields of ref.[11]. Moreover, this system might provide another example of a gauge invariant conformal field theory.

The $(2,0)$-supersymmetric σ-model action written in $(2,0)$-superspace reads [13]:

$$S = -\frac{i}{2}\int d^2x d\theta d\bar{\theta}\Big[K_i(\Phi,\bar{\Phi})\partial_{--}\Phi^i - c.c.\Big], \tag{31}$$

where the target space vector $K_i(\Phi,\bar{\Phi})$ can be expressed as the gradient of a real scalar (Kähler) potential, $K(\Phi,\bar{\Phi})$, whenever the Wess-Zumino term is absent (*i.e.*, torsion-free case) [1]:

$$K_i(\Phi,\bar{\Phi}) = \partial_i K(\Phi,\bar{\Phi}) \equiv \frac{\partial}{\partial\Phi^i}K(\Phi,\bar{\Phi}). \tag{32}$$

We shall draw our attention to Kählerian target manifolds of the coset type, G/H.

The generators of the isometry group, G, are denoted by $Q_\alpha(\alpha = 1, ..., dimG)$, whereas the isotropy group, H, has its generators denoted by $Q_{\bar{\alpha}}(\bar{\alpha} = 1, ..., dimH)$. The transformations of the isotropy group are linearly realised on the superfields Φ and $\bar{\Phi}$, and act as matrix multiplication, just like for flat target manifolds. The isometry transformations instead are non-linear, and their infinitesimal action on the points of G/H can be written as:

$$\delta\Phi^i = \lambda^\alpha k^i_\alpha(\Phi) \tag{33}$$

and

$$\delta\bar{\Phi}_i = \lambda^\alpha \bar{k}_{\alpha i}(\Phi), \tag{34}$$

where $k_{\alpha i}$ and $\bar{k}_{\alpha i}$ are respectively holomorphic and anti-holomorphic Killing vectors of the target manifold. The finite versions of the isometry transformations above read:

$$\Phi'^i = exp(\mathbf{L}_{\lambda.k})\Phi^i \tag{35}$$

and

$$\Phi'^i = exp(\mathbf{L}_{\lambda.\bar{k}})\bar{\Phi}^i \tag{36}$$

with

$$\mathbf{L}_{\lambda.k}\Phi^i \equiv \left[\lambda^\alpha k^i_\alpha \frac{\partial}{\partial\Phi^j}, \Phi^i\right] = \delta\Phi^i. \tag{37}$$

Though the Kähler scalar potential can always be taken H-invariant, isometry transformations induce on K a variation given by:

$$\delta K = \lambda^\alpha[(\partial_i K)k_{\alpha i} + (\bar{\partial}^i K)\bar{k}_{\alpha i}] = \lambda^\alpha[\eta_\alpha(\Phi) + \bar{\eta}_\alpha(\bar{\Phi})], \tag{38}$$

where the holomorphic and anti-holomorphic functions η_α and $\bar{\eta}_\alpha$ can be determined up to a purely imaginary quantity as below:

$$(\partial_i K)k^i_\alpha \equiv \eta_\alpha + iM_\alpha(\Phi, \bar{\Phi}) \tag{39}$$

and

$$(\bar{\partial}^i K)\bar{k}_{\alpha i} \equiv \bar{\eta} - iM_\alpha(\Phi, \bar{\Phi}). \tag{40}$$

The real functions M_α, along with the holomorphic and anti-holomorphic functions η_α and $\bar{\eta}_\alpha$, play a crucial role in discussing the gauging of the isometry group of the target manifold [14, 15]. Therefore, by virtue of the transformation (38) and the constraints imposed on Φ and $\bar{\Phi}$, it can be readily checked that the superspace action (31) is *invariant* under global isometry transformations.

Proceeding further with the study of the isometries, a relevant issue in the framework of $(2,0)$-supersymmetric σ-models is the gauging of the isometry group G of the Kählerian target manifold. This in turn means that one should contemplate the minimal coupling of the $(2,0)$-σ-model to the Yang-Mills supermultiplets of $(2,0)$-supersymmetry [16]. An eventual motivation for pursuing such an analysis is related to the 2-dimensional conformal field theories. It is known that 2-dimensional σ-models define conformal field theories provided that suitable constraints are imposed upon the target space geometry [1, 2]. Now, the coupling of these models to the Yang-Mills sector might hopefully yield new conformal field theories of interest.

The study of $(2,0)$-supersymmetric Yang-Mills theories has been carried out in ref.[16] and the gauging of σ-model isometries in $(2,0)$-superspace has been considered in ref.[17]. On the other hand an alternative less constrained version of $(2,0)$ gauge multiplets has been proposed and discussed in refs.[11, 12]. It has been shown that the elimination of some constraints on the gauge superconnections and on field-strength superfields leads to the appearence of an extra gauge potential that shares a common gauge field. Nevertheless, this extra potential is shown to decouple from the $(2,0)$-matter superfields whenever they are minimally coupled to the Yang-Mills sector.

To write down the local version of the isometry transformations (33) and (34), we have to replace the global parameter λ^α by a pair of chiral and antichiral superfields, $\Lambda^\alpha(x;\theta,\bar{\theta})$ and $\bar{\Lambda}^\alpha(x;\theta,\bar{\theta})$, by virtues of the constraints satisfied by Φ and $\bar{\Phi}$. This can be realised according to:

$$\Phi'^i = exp(\mathbf{L}_{\Lambda.k})\Phi^i \tag{41}$$

and

$$\bar{\Phi}'^i = exp(\mathbf{L}_{\bar{\Lambda}.\bar{k}})\bar{\Phi}^i. \tag{42}$$

In order to get closer to the case of global transformations, and to express all gauge variations exclusively in terms of the superfield parameters Λ^α, we propose a field redefinition that consist in replacing $\bar{\Phi}$ by a new superfield, $\tilde{\Phi}$, as it follows:

$$\tilde{\Phi}_i \equiv exp(i\mathbf{L}_{v.\bar{k}})\bar{\Phi}_i. \tag{43}$$

¿From the expression for the gauge transformation of the prepotential V, it can be shown that:

$$exp(i\mathbf{L}_{V'.\bar{k}}) = exp(\mathbf{L}_{\Lambda.\bar{k}})exp(i\mathbf{L}_{v.\bar{k}})exp(-\mathbf{L}_{\bar{\Lambda}\bar{k}}), \tag{44}$$

and $\tilde{\Phi}_i$ consequently transform with the gauge parameter Λ^α:

$$\tilde{\Phi}'_i = exp(\mathbf{L}_{\Lambda\bar{k}})\tilde{\Phi}_i, \tag{45}$$

which infinitesimally reads:

$$\delta\tilde{\Phi}_i = \Lambda^\alpha(x;\theta\bar{\theta})\bar{k}_{\alpha i}(\bar{\Phi}). \tag{46}$$

Now, an infinitesimal isometry transformation induces on the modified Kähler potential, $K(\Phi,\tilde{\Phi})$, a variation given by:

$$\delta K(\Phi,\tilde{\Phi}) = \Lambda^\alpha(\eta_\alpha + \tilde{\eta}_\alpha), \tag{47}$$

where

$$\tilde{\eta}_\alpha = (\tilde{\partial}^i K)\bar{k}_{\alpha i}(\tilde{\Phi}) + iM_\alpha(\Phi,\tilde{\Phi}), \tag{48}$$

with $\tilde{\partial}$ denoting a partial derivative with respect to $\tilde{\Phi}$. The isometry variation δK computed above reads just like a Kähler transformation and is a direct consequence of the existence of the real scalar $M_\alpha(\Phi,\tilde{\Phi})$, as discussed in refs [14, 15].

The form of the isometry variation of $K(\Phi,\tilde{\Phi})$ suggest the introduction of a pair of *chiral* and *antichiral* superfields, $\xi(\Phi)$ and $\bar{\xi}(\bar{\Phi})$, whose respective gauge transformations are fixed in such a way that they compensate the change of K under isometries. This can be achieved by means of the Lagrangian defined as:

$$\begin{aligned}\mathcal{L}_\xi &= \partial_i[K(\Phi,\tilde{\Phi}) - \xi(\Phi) - \tilde{\xi}(\tilde{\Phi})]\nabla_{--}\Phi^i + \\ &- \tilde{\partial}_i[K(\Phi,\tilde{\Phi}) - \xi(\Phi) - \tilde{\xi}(\tilde{\Phi})]\nabla_{--}\tilde{\Phi}^i,\end{aligned} \tag{49}$$

where the covariant derivatives $\nabla_{--}\Phi^i$ and $\nabla_{--}\tilde{\Phi}^i$ are defined in perfect analogy to what is done in the case of the bosonic σ-model:

$$\nabla_{--}\Phi_i \equiv \partial_{--}\Phi_i - g\Gamma^{\alpha}_{--}k^i_{\alpha} \tag{50}$$

and

$$\nabla_{--}\tilde{\Phi}_i \equiv \partial_{--}\tilde{\Phi}_i - g\Gamma^{\alpha}_{--}\bar{k}_{\alpha i}(\tilde{\Phi}). \tag{51}$$

Finally, all we have to do in order that the Lagrangian $\mathcal{L}_\xi$ given above be invariant under local isometries is to fix the gauge variations of the auxiliary scalar superfields ξ and $\tilde{\xi}$. If the latter are so chosen that:

$$(\partial_i\xi)k^i_{\alpha}(\Phi) = \eta_{\alpha}(\Phi) \tag{52}$$

and

$$(\tilde{\partial}^i\tilde{\xi})\bar{k}_{\alpha i}(\tilde{\Phi}) = \tilde{\eta}_{\alpha}(\tilde{\Phi}), \tag{53}$$

then it can be readily verified that the Kähler-transformed potential

$$[K(\Phi,\bar{\Phi}) - \xi(\Phi) - \tilde{\xi}(\tilde{\Phi})] \tag{54}$$

is isometry-invariant, and the Lagrangian $\mathcal{L}_\xi$ of eq. (49) is indeed symmetric under the gauged isometry group.

The interesting point we would like to stress is that the extra gauge degrees of freedom accommodated in the component-field $B_{--}(x)$ of the superconnection Γ_{--} behave as a genuine gauge field that shares with A^μ the coupling to matter and to σ-model [11]. This result can be explicitly read off from the component-field Lagrangian projected out of the superfield Lagrangian $\mathcal{L}_\xi$. We therefore conclude that our less constrained $(2,0)$-gauge theory yields a pair of gauge potentials that naturally transform under the action of a single Abelian gauge group.

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References

[1] C. M. Hull and E. Witten, *Phys. Lett.***160B**(1985) 398.

[2] P. Candelas, G. Horowitz, A. Strominger and E. Witten, *Nucl. Phys.***B258**(1985) 46; C. M. Hull, *Nucl. Phys.***B260**(1985) 182 and *Nucl. Phys.***B267**(1986) 266; A. Sen, *Phys. Rev.***D32**(1985) and *Phys. Rev. Lett.***55**(1985) 1846.

[3] M. Sakamoto, *Phys. Lett.***B151**(1985) 115.

[4] D. J. Gross, J. A. Harvey, E. J. Martinec and R. Rohm, *Phys. Rev. Lett.***54**(1985) 502.

[5] M. Porrati and E. T. Tomboulis, *Nucl. Phys.***B 315**(1989) 615; J. Quackenbush, *Phys. Lett.***B234**(1990) 285.

[6] H. Oguri and C. Vafa, *Mod. Phys. Lett.* **A5** (1990) 1389; *Nucl. Phys.* **B361** (1991) 469; *Nucl. Phys.* **B367** (1991) 83.

[7] S. J. Gates Jr. and H. Nishino, *Mod. Phys. Lett.* **A7** (1992) 2543.

[8] S. J. Gates Jr., S. V. Ketov and H. Nishino, *Phys. Lett.* **B307** (1993) 323.

[9] S. J. Gates Jr., S. V. Ketov and H. Nishino, *Phys. Lett.* **B297** (1992) 99; *Nucl. Phys.* **B393** (1993) 149.

[10] M. A. De Andrade, O. M. Del Cima and L. P. Colatto, *Phys. Lett.* **B370** (1996) 59.

[11] N. Chair, J. A. Helayël-Neto and A. William Smith, *Phys. Lett.***B 233**(1989) 173.

[12] C. A. S. Almeida and R. M. Doria, **CBPF-NF-032-90**

[13] M. Dine and N. Seiberg, , *Phys. Lett.***180B**(1986) 364.

[14] J. Bagger and E. Witten, *Phys. Lett.***118B**(1982) 103.

[15] C. M. Hull, A. Karlhede, U. Lindström and M. Roček, *Nucl. Phys.***B266**(1986) 1; M. Carvalho, M. W. de Oliveira and J. A. Helayël-Neto, " The Geometry of N=1

and N=2 Real Supersymmetric Non-Linear Sigma-Models in the Atiyah-Ward Space-Time", **hep-th/9811103**.

[16] R. Brooks. F. Muhammad and S. J. Gates Jr., *Nucl. Phys.***B268**(1986) 599.

[17] C. A. S. Almeida, J. A. Helayël-Neto and A. William Smith, *Mod. Phys. Lett.***A6**(1991) 1397 and *Phys. Lett.***279B**(1992) 75.