CBPF-NF-040/97

Bogomol'nyi equations in gauge theories

M.S. Cunha, H.R. Christiansen

*Centro Brasileiro de Pesquisas Físicas**

Fields and Particles Department, CBPF - DCP

Rua Dr. Xavier Sigaud 150, 22290-180

Rio de Janeiro, Brazil.

and

C.A.S. Almeida

Universidade Federal do Ceará †

Physics Department

C.P. 6030, 60455-760 Fortaleza-Ce, Brazil

By imposing self-duality conditions, we obtain the explicit form in which gauge theories spontaneously breakdown in the Bogomol'nyi limit. In this context, we reconsider the Abelian Higgs, Chern-Simons Higgs and Maxwell-Chern-Simons Higgs models. On the same footing, we find a topological Higgs potential for a Maxwell-Chern-Simons extended theory presenting both minimal and nonminimal coupling. Finally, we perform a numerical calculation in the asymmetric phase and show the solutions to the self-dual equations of motion of the topological theory. We argue about certain relations among the parameters of the model in order to obtain such vortex configurations.

Key-words: Self-dual vortices; Topological phase; Nonminimal coupling.

*Electronic address: marcony@cat.cbpf.br, hugo@cat.cbpf.br

†carlos@fisica.ufc.br

I. INTRODUCTION

The study of classical vortex solutions in gauge theories, extensively developed in the seventies, has motivated many works in the last few years. Actually, their existence has been claimed much earlier in connection to the Ginzburg-Landau model of superconductivity, in the pioneering work of Abrikosov [1]; later on, such topological configurations were experimentally observed in type-II superconductors [2].

In a relativistic framework, the issue of soliton solutions has been generalized to field theories such as the Abelian Higgs model in which vortices exhibit nonzero magnetic fluxes, but are electrically neutral [3]. By adding a Chern-Simons term, interesting features arise. In particular, vortex solutions gain an electric charge in the so-called topologically massive electrodynamics [4,5].

Self-duality, which allows the reduction of the second order equations of motion to a set of first order ones, is an interesting tool from the physical point of view since it is related to the minimization of the energy together with the action of the system [6].

Gauge fields whose dynamics exclusively depend on the Chern-Simons term and are minimally coupled to the scalar fields, have been thoroughly studied by Jackiw *et al.* in recent years [7,8]. Such models present topological and nontopological solitons and the Bogomol'nyi equations are obtained for specific sixth-order Higgs potentials.

On the other hand, when gauge field dynamics are also controlled by a Maxwell term, it seems necessary to introduce a neutral scalar field in order to ensure self-dual solutions to the model [10]. In fact, this can be avoided by appealing to an extra *nonminimal* contribution to the covariant derivative, which can be interpreted as an anomalous magnetic moment [11]. It is worth noting however that this analysis has been generally limited to nontopological potentials presenting just a symmetric phase [12,13].

The purpose of this paper is twofold. In a first part we review some well-known gauge theories presenting vortices and self-dual solutions. Though, in contrast to preceding authors, we work along the lines given in ref. [14] where self-duality conditions supply the well-known Higgs potentials for the Abelian Higgs and Chern-Simons Higgs models. Our aim at this point is to compare this procedure to the standard one and extract the Bogomol'nyi equations from the energy functional rather than imposing them together with the equations of motion as in [14].

In a second part, we go a step further and apply this idea to set the proper Bogomol'nyi conditions and obtain the resulting Higgs potential in a Maxwell-Chern-Simons Higgs model coupled nonminimally as above mentioned. This kind of coupling has been typically confined to a critical value that provides the model fractionary statistic solutions proper of pure CS theory. In this respect, let us stress that we work without any specific choice of the anomalous magnetic moment coupling, its value being only conditioned by the basic assumption of positive energy solutions. Neither we impose rotational symmetry before taking the Bogomol'nyi limit, which could hide true minimal energy solutions (see e.g. [13]). This freedom allows a tuning between Maxwell and Chern-Simons contributions, as will be shown later in the series expansion analysis of the potential. In this way we are able to find a *topological* potential for this model, without restricting us to a critical nonminimal coupling (*c.f.* [12] where only a nontopological sector is found in such a critical regime)). Finally, we perform a numerical calculation of the solutions to this theory for a convenient ansatz.

Our results put forward certain relations among the parameters of the model which, in particular, exclude the usual choice for the topological mass constants, see refs. [13,12].

II. MINIMAL MODELS

Abelian Higgs Model

Let us start with a Higgs model Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}D^{\mu}\phi D_{\mu}\phi^{*} - U\left(\phi\right) \tag{1}$$

in $1+2$ dimensions with a Minkowski space-time signature $\eta_{\mu\nu}=(+,-,-)$. Here, ϕ is a complex scalar minimally coupled to an Abelian gauge field, U is an unknown potential and the covariant derivative is defined as

$$D_\mu\phi=(\partial_\mu-ieA_\mu)\phi \tag{2}$$

The equation of motion for A_μ is given by

$$\partial_\mu F^{\mu\nu}=J^\nu \tag{3}$$

where J^ν is the Nöether's (conserved) current

$$J_\mu=-\frac{ie}{2}(\phi^*D_\mu\phi-\phi D_\mu\phi) \tag{4}$$

The energy momentum tensor obtained from (1) is

$$T_{\mu\nu}=\frac{1}{2}(F_{\mu\alpha}F^{\alpha}_{\ \nu}+F_{\nu\alpha}F^{\alpha}_{\ \mu})+\frac{1}{2}(D_\mu\phi^*D_\nu\phi+D_v\phi^*D_\mu\phi)-\eta_{\mu\nu}\mathcal{L} \tag{5}$$

and integration of the T_{00} component yields

$$\mathcal{E}=\int d^2x\left\{\frac{1}{2}(B^2+\mathbf{E}^2)+\frac{1}{2}\left|D_0\phi\right|^2+\frac{1}{2}\left|D_i\phi\right|^2+U\right\} \tag{6}$$

In order to fix the self-duality conditions, we will focus on static topologically nontrivial solutions. Such classical configurations have been shown to exist in this theory and are known as the Nielsen-Olesen vortices [3]. By choosing the radiation gauge $A_0=0$ it can be easily seen that these are electrically neutral solutions. This implies a zero J_0 component and a vanishing electric field in the whole space.

Making use of the relation

$$\frac{1}{2}D_i\phi^*D_i\phi=\frac{1}{2}|(D_1\pm iD_2)\phi|^2\pm\frac{1}{2e}\varepsilon_{ij}\partial_iJ_j\mp\frac{e}{2}B|\phi|^2 \tag{7}$$

in eq.(6), we obtain after some algebra

$$\begin{aligned}\mathcal{E}=\frac{ev^2}{2}|\Phi_B|\pm\frac{1}{2e}\oint_{r\to\infty}\mathbf{J}.d\mathbf{l}\pm\int d^2x\left\{\frac{1}{2}\left(B\pm\sqrt{2U}\right)^2\right.\\ \left.\pm\left[\frac{e}{2}\left(v^2-|\phi|^2\right)-\sqrt{2U}\right]B+\frac{1}{2}|(D_1\pm iD_2)\phi|^2\right\}\end{aligned} \tag{8}$$

where the upper (lower) sign corresponds to positive (negative) values of the magnetic flux $\Phi_B \equiv -\int d^2xB$. Let us remind that Φ_B yields topological nontrivial classes for a potential allowing asymmetric vacuum phases.

Written in this form, the duality conditions to be imposed become apparent,

$$\begin{aligned} D_1\phi &= \mp i D_2\phi \\ B &= \pm\frac{e}{2}\left(|\phi|^2 - v^2\right) \end{aligned} \tag{9}$$

Note that since we are considering only finite energy configurations, the line integral of the vector current vanishes because the spatial components of covariant derivatives have to be zero at spatial infinity.

In the Bogomol'nyi limit, given by eq.(8), it can be seen that a lower bound for the energy exists; namely $\mathcal{E} \geq ev^2|\Phi_B|/2$, where the equal sign is saturated for

$$U(|\phi|^2) = \frac{e^2}{8}\left(|\phi|^2 - v^2\right)^2, \tag{10}$$

which is a well-known result.

Chern-Simons-Higgs Model

Bogomol'nyi-type vortex solutions are also encountered in the CSH model, whereas in this case vortices are electrically charged and the Higgs potential is of sixth-order [7–9]. On the other side, nontopological soliton solutions are also supported but in this case their magnetic fluxes are not quantized [8].

In order to explicitly obtain this potential let us define the Lagrangian density

$$\mathcal{L} = \frac{\kappa}{4}\varepsilon^{\mu\nu\rho}A_\mu F_{\nu\rho} + \frac{1}{2}D^\mu\phi D_\mu\phi^* - U(\phi). \tag{11}$$

Now, the equation of motion for the gauge field is a first order one, being given by

$$\kappa F^\mu = J^\mu \tag{12}$$

where $F^\mu \equiv \frac{1}{2}\,\varepsilon^{\mu\nu\rho}F_{\nu\rho}$ and J^μ is the same as in (4). The energy momentum tensor is derived as usual

$$T_{\mu\nu} = \frac{1}{2}(D_\mu\phi^* D_\nu\phi + D_v\phi^* D_\mu\phi) + \eta_{\mu\nu}(\frac{1}{2}|D_\alpha\phi|^2 + U) \tag{13}$$

so that the energy follows. Note however that the CS term is absent, thus, the energy functional simply reads

$$\mathcal{E} = \int d^2x \left\{\frac{1}{2}D_0\phi^* D_0\phi + U + \frac{1}{2}D_i\phi^* D_i\phi\right\} \tag{14}$$

Following similar steps as in the previous section, it can be written as

$$\begin{aligned}\mathcal{E} = \frac{ev^2}{2}|\Phi_B| \pm \int d^2x &\left\{\frac{1}{2\,|\phi|^2}\left|\phi^* D_0\phi \mp i\sqrt{2U\,|\phi|^2}\right|^2 \pm \frac{\sqrt{2U}}{e}\frac{J_0}{|\phi|}\right.\\ &\left.+\frac{1}{2}|(D_1 \pm iD_2)\phi|^2 \mp \frac{e}{2}B(|\phi|^2 - v^2)\right\}\end{aligned} \tag{15}$$

Notice that the second term in the equation above forces the magnetic field $B\ (= -J_0/\kappa)$ to vanish wherever ϕ does, so that the vortex becomes closed and the field lies within a toroidal region. This term, causing this kind of ring vortex configurations is physically relevant in order to stabilize nontopological soliton solutions. Compare it to $\kappa^2B^2/4e^2|\phi|^2$, obtained in ref. [8] for a particular sixth order Higgs potential (actually, this expression is reobtained when a lowest energy configuration is considered (see below)).

By imposing the following self-duality conditions

$$\begin{aligned}\phi^* D_0\phi &= \pm i\sqrt{2|\phi|^2 U}\\ D_1\phi &= \mp iD_2\phi\end{aligned} \tag{16}$$

and using the time component of eq.(12) one has

$$\mathcal{E} = \frac{e}{2}v^2|\Phi_B| \pm \int d^2x \left(\frac{e}{2}(v^2 - |\phi|^2) - \sqrt{\frac{2U}{|\phi|^2}}\kappa\right) B. \tag{17}$$

Again, for

$$U(|\phi|^2) = \frac{e^4}{8\kappa^2}|\phi|^2\left(|\phi|^2 - v^2\right)^2. \tag{18}$$

the lower bound is reached. Now, equation (16) can be manipulated so as to produce a self-duality condition in terms of J_0

$$J_0 = \pm e\sqrt{2|\phi|^2 U} \tag{19}$$

Hence, using once more eq.(12), eq.(18) implies

$$B = \pm\frac{e^3}{2\kappa^2}|\phi|^2\left(|\phi|^2 - v^2\right) \tag{20}$$

in agreement with the literature [7,8]. Notice that in this (minimal) model a constraint on the gauge field arise; namely, A_0 is related to the magnetic field by means of the time component of eq. (12), *i.e.*,

$$A_0 = \frac{e}{\kappa^2}\frac{B}{|\phi|^2}. \tag{21}$$

This implies of course that the radiation gauge choice is prohibited in this case.

Maxwell-Chern-Simons Higgs Model

In the present section we would like to recover the well known Higgs potential for the spontaneously broken Maxwell-Chern-Simons model [10]. The (minimally coupled) Lagrangian density reads

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{\kappa}{4}\varepsilon^{\mu\nu\rho}A_\mu F_{\nu\rho} + \frac{1}{2}D^\mu\phi D_\mu\phi^* \\ &+\frac{1}{2}\partial_\mu N\partial^\mu N - V - |\phi|^2 W\end{aligned} \tag{22}$$

where for convenience, the potential has been written as $U = V + |\phi|^2 W$ and N is a neutral scalar field.

The equation of motion for the gauge field now reads

$$\partial_\mu F^{\mu\nu} + \kappa F^\nu = J^\nu \tag{23}$$

implying the following “Gauss Law”

$$\partial_i E_i + \kappa B + J_0 = 0 \tag{24}$$

On the other side, for a static N field the energy functional of the model is given by

$$\begin{aligned}\mathcal{E} = \int d^2x \left\{ \frac{1}{2}(B^2 + \mathbf{E}^2) + \frac{1}{2} D_0\phi^* D_0\phi + \frac{1}{2} D_i\phi^* D_i\phi \right.\\ \left. + \frac{1}{2}\partial_i N \partial_i N + V(|\phi|^2) + |\phi|^2 W \right\}\end{aligned} \tag{25}$$

which after some algebra reads

$$\begin{aligned}\mathcal{E} = \int d^2x \left\{ \frac{1}{2}(B \pm \sqrt{2V})^2 \mp B\sqrt{2V} \pm \frac{\sqrt{2W}}{e} J_0 \right.\\ + \frac{1}{2}\left| D_0\phi \mp i\sqrt{2W}\phi \right|^2 + \frac{1}{2}|(D_1 \pm iD_2)\phi|^2 \\ \left. \mp \frac{e}{2} B|\phi|^2 + \frac{1}{2}(E_i \pm \partial_i N)^2 \mp E_i \partial_i N \right\}\end{aligned} \tag{26}$$

By imposing the following self-duality conditions

$$\begin{aligned} B &= \mp\sqrt{2V} \\ D_0\phi &= \pm i\sqrt{2W}\phi \\ D_1\phi &= \mp i D_2\phi \\ E_i &= \mp\partial_i N \ , \end{aligned} \tag{27}$$

eliminating J_0 (eq. 24), and integrating $E_i\partial_i N$ by parts we have

$$\begin{aligned}\mathcal{E} = \frac{ev^2}{2}|\Phi_B| + \int d^2x \left\{ \pm B \left[\frac{e}{2}(v^2 - |\phi|^2) - \sqrt{2V} - \frac{\sqrt{2W}}{e}\kappa \right] \right.\\ \left. \pm \left(N - \frac{\sqrt{2W}}{e} \right) \partial_i E_i \right\}\end{aligned} \tag{28}$$

It is clear that the system will lie on its lower bound limit provided that

$$\begin{aligned}\sqrt{2V} &= \frac{e}{2}(v^2 - |\phi|^2) - \frac{\sqrt{2W}}{e}\kappa \\ \frac{\sqrt{2W}}{e} &= N\end{aligned} \tag{29}$$

From eqs.(27) and (29) above, one can read out the relation between the magnetic and the scalar fields

$$B = \pm\frac{e}{2}(|\phi|^2 - v^2 + \frac{2\kappa}{e}N) \quad (30)$$

bringing about the explicit form of the Higgs potential

$$U(N, |\phi|^2) = \frac{e^2}{2}N^2|\phi|^2 + \frac{e^2}{8}(|\phi|^2 - v^2 + \frac{2\kappa}{e}N)^2 \quad (31)$$

which coincides with the potential used in [10].

III. NONMINIMAL MODELS

In three space-time dimensions Pauli type coupling is known to give a meaningful contribution to the magnetic moment of fields, without any reference to their spin statistics. Namely, even scalar fields can present a nonzero magnetic moment. The interest on such 'nonminimal' coupling was recently renewed [11] since, using some critical value for this kind of coupling in a MCSH model, one recovers the ideal anyon behavior proper of pure CS theory. Later on further investigation was performed including a nontopological Higgs potential [12,13,15] where axially symmetric self-dual solutions were encountered for a critical value of the anomalous coupling.

Here we analyze the MCSH model in the topological sector and we relax the condition on the nonminimal coupling out of its critical value. Further, we do not impose rotational symmetry before setting the Bogomolnyi limit in order to ensure minimal energy solutions.

Maxwell-Chern-Simons-Higgs model with anomalous magnetic moment

Let us consider a MCSH Lagrangian with the anomalous magnetic moment term characterized by the coupling constant g

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{\kappa}{4}\varepsilon^{\mu\nu\rho}A_\mu F_{\nu\rho} + \frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi^* - U(\phi) \quad (32)$$

with ∇_μ defined as

$$\nabla_\mu\phi \equiv (\partial_\mu - ieA_\mu - igF_\mu)\phi$$

The equation of motion for A_μ is

$$\partial_\mu F^{\mu\rho} + \kappa F^\rho = \mathcal{J}^\rho + \frac{g}{e}\varepsilon^{\mu\nu\rho}\partial_\mu \mathcal{J}_\nu \tag{33}$$

where $\mathcal{J}_\mu$ is defined by

$$\mathcal{J}_\mu \equiv -\frac{ie}{2}(\phi^* \nabla_\mu \phi - \phi \nabla_\mu \phi)$$

The time component of eq.(33) defines the modified "Gauss Law"

$$\partial_i E_i + \kappa B + \frac{g}{e}\varepsilon_{ij}\partial_i \mathcal{J}_j + \mathcal{J}_0 = 0 \tag{34}$$

The gauge invariant modes are now short-range due the mass term resulting from the modified equation of motion. Hence, the first term in eq.(34) has a vanishing integral. On the other hand, the third term results in a line integral taken at infinity which also vanishes for finite energy configurations. Therefore, it can be seen from the remaining piece that one has the charge of the vortex solutions related to nonzero magnetic fluxes as follows

$$Q = \kappa \Phi_B \tag{35}$$

Unlike the preceding models, the nonminimal coupling allows a temporal gauge choice $A_0 = 0$, which simplifies the handling of the equations. For example, the electric charge reads

$$Q = \int d^2x \; J_0 = e\,g \int d^2x \; |\phi|^2 B \tag{36}$$

Then, from eqs.(35,36) (assuming that $-ge/\kappa > 0$) it follows that

$$-gev^2/\kappa > 1 \tag{37}$$

where v gives the minimum value of the symmetry breaking potential. (Of course it is true provided that v is the maximum value of the field ϕ; it will be shown below.)

On the other hand, the energy functional is given by

$$\mathcal{E} = \int d^2x \left\{ \frac{1}{2} G(\mathbf{E}^2 + B^2) + \frac{1}{2} D_0\phi^* D_0\phi + \frac{1}{2} D_i\phi^* D_i\phi + U \right\} \tag{38}$$

with

$$G = 1 - g^2|\phi|^2$$

In order to ensure a positive definite energy one then needs

$$|g| < 1/v \tag{39}$$

which together with eq.(37) implies

$$\kappa < ev \tag{40}$$

and therefore

$$|g| < e/\kappa \tag{41}$$

Actually, this relation excludes the constraint usually imposed on g in order to obtain a set of equations of motion of the first order without imposing self-duality conditions [12,13].

In the $A_0 = 0$ gauge, and after some calculation, the energy functional can be written as

$$\begin{aligned} \mathcal{E} = \frac{ev^2}{2}|\Phi_B| + \int d^2x \left\{ \pm \left[\frac{e}{2}(v^2 - |\phi|^2) - \sqrt{2GU} \right] B \right. \\ \left. \frac{1}{2} G \left(B \pm \sqrt{2UG} \right)^2 + \frac{1}{2} |(D_1 \pm iD_2)\phi|^2 \right\} \end{aligned} \tag{42}$$

By imposing the self-dual equations,

$$\begin{aligned} D_1\phi &= \mp i D_2\phi \\ B &= \pm G^{-1}\frac{e}{2}(|\phi|^2 - v^2) \end{aligned} \tag{43}$$

the *topological* potential can be then determined in order to achieve the lower bound limit. So we have,

$$U(|\phi|^2) = G^{-1}\frac{e^2}{8}(|\phi|^2 - v^2)^2. \tag{44}$$

As already pointed out in the introduction, this potential leads to topologically stable vortex solutions for the Maxwell-Chern-Simons Higgs model with no reference to scalar field introduced *ad-hoc* [10].

Now, for small values of g we can perform a series expansion of the topological Higgs potential just found, in order to obtain the non anomalous phase limit

$$U(|\phi|^2) = \frac{e^2}{8}(|\phi|^2 - v^2)^2 + g^2\frac{e^2}{8}|\phi|^2(|\phi|^2 - v^2)^2 + \dots \tag{45}$$

Notice that in the $g = 0$ limit the sixth order term, characteristic of a CS contribution, is absent. This is consistent with eqs.(35-37) which impose a vanishing κ whenever $g \to 0$.

Let us write eq.(45) in the following way

$$U(|\phi|^2) = \frac{e^2}{8}(|\phi|^2 - v^2)^2 + m^2\frac{e^4}{8\kappa^2}|\phi|^2(|\phi|^2 - v^2)^2 + \dots \tag{46}$$

with $m < 1$, defined by $|g| = me/\kappa$ [see eq.(41)] (notice that while eq.(37) imposes that $\kappa/ev^2 < |g|$, m is assumed to be small enough to make the CS term above a second order correction). Thus, for a small a.m.m one can see in eq.(46) the remaining of the more elementary theories contained in the present model, namely, both a Maxwell (Nielsen-Olesen) topological potential term and a typical sixth order CS one.

The theory possess two massive gauge propagating modes. The masses of these gauge excitations read

$$m_{A\pm} = \frac{\kappa I}{2G_v} \pm \sqrt{(\frac{\kappa I}{2G_v})^2 + \frac{e^2v^2}{G_v}} \tag{47}$$

where $I = 1 + 4egv^2/\kappa$ and $G_v = 1 - g^2v^2$, and for $g \to 0$ one obtains the Nielsen-Olesen model mass $m_{NO} = ev$. On the other hand, the Higgs mass is easily seen to be

$$m_H = ev/\sqrt{G_v} \tag{48}$$

which of course approaches also m_{NO} for a vanishing g.

Solutions and numerical analysis

By means of proper gauge transformations, static rotationally symmetric configurations can be written in polar coordinates as

$$\phi(r) = vR(r)e^{in\theta} \tag{49}$$

$$e\mathbf{A}(r) = -\frac{\hat{\theta}}{r}\left[a(r) - n\right] \tag{50}$$

where R and a are real functions of r, and n an integer indicating the topological charge of the vortex. Now, under transformation $r \to (\sqrt{2}/ev)\, r$ the self-duality equations (43) become

$$R' = \pm\frac{a}{r}\ R \tag{51}$$

$$\frac{a'}{r} = \pm\frac{1}{1-\gamma^2R^2}(R^2-1) \tag{52}$$

where $\gamma < 1$ is defined by $g = \gamma/v$.

The natural boundary conditions at spatial infinity result from the requirement of finite energy, namely, $R(\infty) = 1$ and $a(\infty) = 0$ for any nontrivial vorticity n. On the other hand, at the origin one must expect nonsingular fields, implying $R(0) = 0$ and $a(0) = n$. Hence, the magnetic field reads

$$B = -\frac{ev^2}{2}\ \frac{a'}{r} \tag{53}$$

and its flux is, as expected

$$\Phi_B = \frac{2\pi}{e}[a(0) - a(\infty)] = \frac{2\pi}{e}n. \tag{54}$$

At large values of r it is easy to see that the $n > 0$ solutions behave like

$$R(r) \to 1 - cK_0(r) \tag{55}$$

and

$$a(r) \to d\, rK_1(r) \tag{56}$$

where c and d are constants. The $n < 0$ configurations are related to these ones by the transformation $a \to -a$ and $R \to R$. The behavior of the solutions at small values of r is power like, and without any loss of generality we can simply assume

$$R(r) \sim c_n r,$$
$$a(r) \sim n \qquad (57)$$

Notice that c_n is determined by the shape of the fields at infinity rather than their behavior at the origin. Indeed, we have numerically solved the self-duality equations of motion by means of an iterative procedure which involves a tentative value for c_n which is corrected each time by imposing that both $R \to 1$ and $a \to 0$ hold together at infinity.

For $n = 1, 2$ and 3 we have found $c_1 = 8.891 \times 10^{-1}$, $c_2 = 4.796 \times 10^{-6}$ and $c_3 = 6.877 \times 10^{-9}$ (see fig.1 for higher precision). In fig.1 we show the topologically nontrivial solutions $R(r)$ and $a(r)$ and in fig.2 we plot the corresponding magnetic fields. In fig.3 we plot the magnetic field for $n = 1$ vorticity at different values of γ.

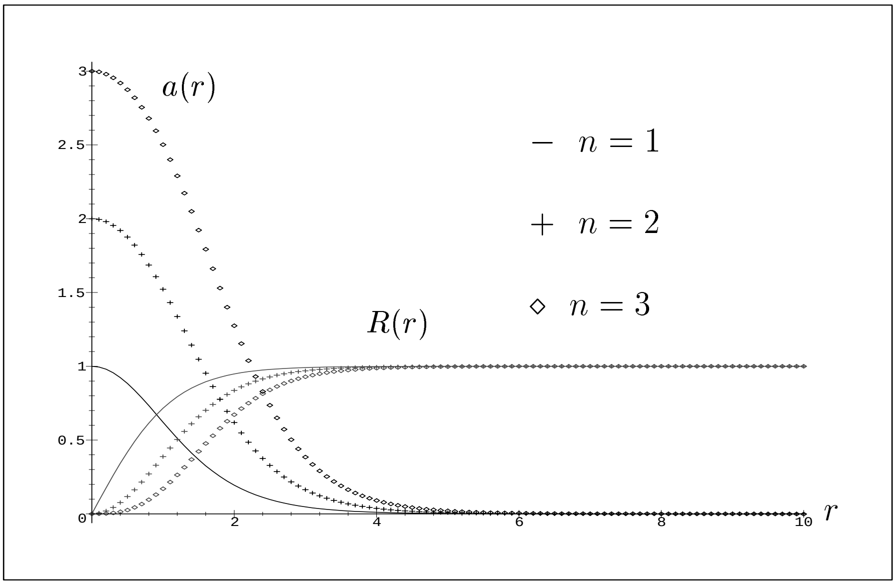

FIG. 1. The scalar R and the gauge field a as a function of r. The values of the c_n constants are fixed by the shape of the fields at infinity: $(-)$ $c_1 = 8.891308075 \times 10^{-1}$, $(+)$ $c_2 = 4.796825890 \times 10^{-6}$, $(\diamond)$ $c_3 = 6.877604870 \times 10^{-9}$.

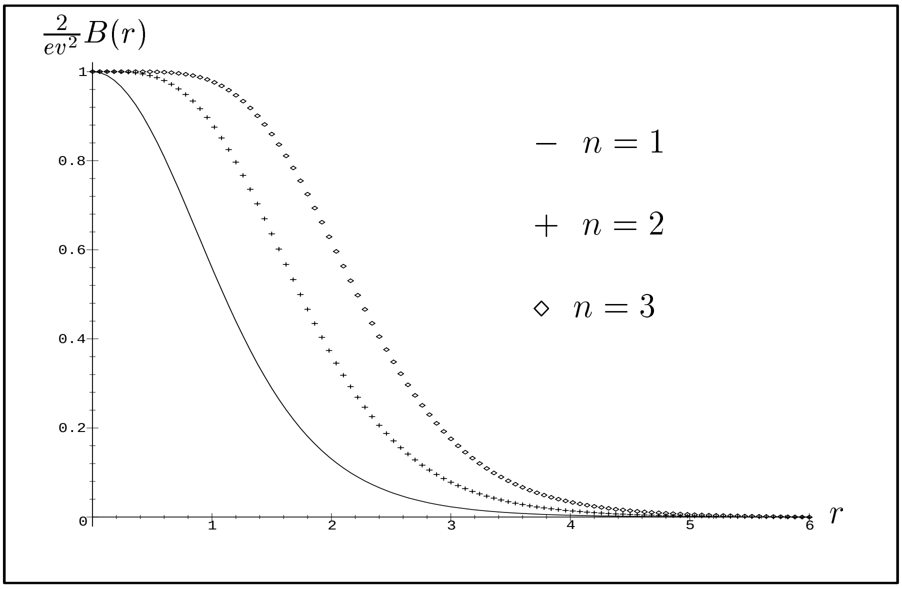

FIG. 2. The magnetic field B as function of r for n = 1, 2, and 3 vorticities. Notice that the field distributions are concentrated at the origin – like NO vortices.

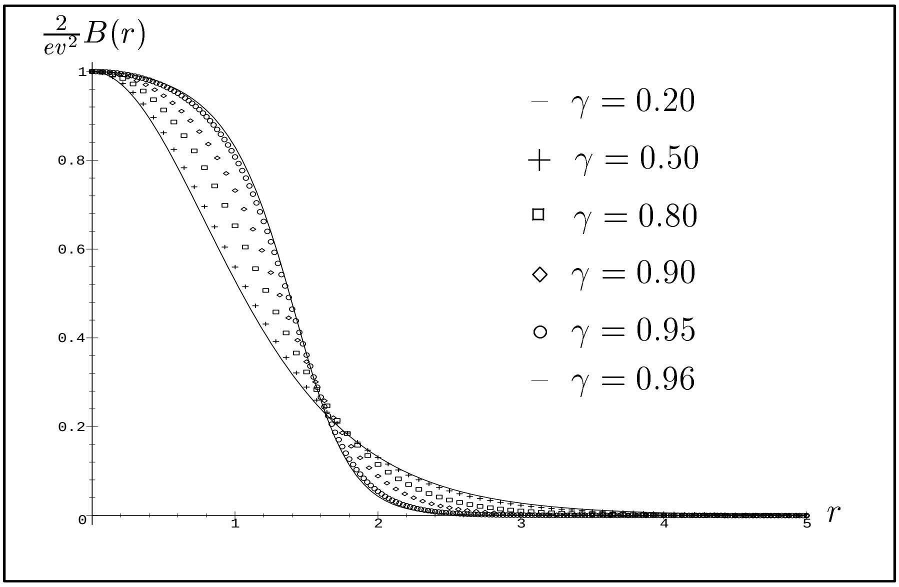

FIG. 3. The magnetic field B as a function of r for various values of the anomalous magnetic moment parameter $\gamma\,(=vg)$.

IV. SUMMARY AND CONCLUSIONS

We have shown that a Higgs potential, necessary to get vortex solutions of minimal energy in the Bogomol'nyi limit, can be obtained by imposing self-dual equations from the very beginning, in contrast to the regular procedure found in the literature [3]- [12]. From this point of view, we have revisited the Abelian Higgs, Chern-Simons Higgs and Maxwell-Chern-Simons Higgs models, and we have regained the standard outcomes.

Thereafter, guided by this outlook, we obtained a topological Higgs potential for a generalized MCSH theory, modified by the inclusion of a nonminimal coupling controlled by a parameter g which is introduced in the covariant derivative. By means of a series expansion in g, this potential has shown to contain traces of more elementary models, namely, Abelian Higgs and Chern-Simons Higgs contributions, precisely in their usual topological phase.

In order to get the topological potential, we worked in a temporal gauge which clarifies the relations among the constants of the model e, κ, v and g. This gauge choice also prevents the choice of the critical value $g = -e/\kappa$ commonly adopted; this relation, in fact, would lead to the already known *nontopological* solutions to the model [12].

The generalized MCSH model that we have analyzed, possess two massive propagating modes. The masses are different from the Higgs mass, even in the Bogomol'nyi limit, a result which is produced by the anomalous magnetic moment. On the other hand, it has been recently argued that bosonic theories in the Bogomol'nyi limit could be closely connected to their $N = 2$ supersymmetric extension [16]. Thus, it is worth enquiring at this point if the supersymmetric extension of this specific model requires different conditions on the coupling constants (in this direction, see [17]). We hope to report on these issues in a future work.

ACKNOWLEDGMENTS

The authors would like to thank Professor J.A. Helayel-Neto for stimulating and useful discussions. M.S.C. would like to thank the Physics Department of the Universidade Federal do Ceará for warm hospitality. H.R.C. is grateful to Centro Latinoamericano de Física (CLAF-CNPq) and CBPF-CNPq for warm hospitality and financial support. M.S.C. and C.A.S.A are financially supported by CNPq (Brazilian Research Agency).

---

[1] A.A. Abrikosov, Sov. Phys. JETP **5** (1957) 1174.

[2] G. Rickayzen, *Theory of Superconductivity* (John Wiley & Sons, 1965);
E. Fradkin, *Field Theories of Condensed Matter Systems* (Addison-Wesley Publishing Company, 1991).

[3] H. B. Nielsen and P. Olesen, Nucl. Phys. **B61** (1973) 45.

[4] S. Paul and A. Khare, Phys. Lett. **B174** (1986) 420.

[5] H. de Vega and F.A. Schaposnik, Phys. Rev. Lett. **56** (1986) 2564.

[6] G. Dunne, *Self-Dual Chern-Simons Theories,* Lectures on Physics Vol. **36** (Springer, Berlin, 1995).

[7] R. Jackiw and E.J. Weinberg, Phys. Rev. Lett. **64** (1990) 2234.

[8] R. Jackiw, K. Lee, and E. J. Weinberg, Phys. Rev. **D42** (1990) 3488.

[9] J. Hong, Y. Kim, and P.Y. Pac, Phys. Rev. Lett. **64** (1990) 2230.

[10] C. Lee, K. Lee, and H. Min, Phys. Lett. **B252** (1990) 79.

[11] J. Stern, Phys. Rev. **D46** (1991) 119.

[12] M. Torres, Phys. Rev. **D46** (1992) R2295;

J. Escalona, M. Torres, and A. Antillon, Mod. Phys. Lett. **A8** (1993).

[13] T. Lee and H. Min, Phys. Rev. **D50** (1994) 7738.

[14] P.P. Srivastava and P. Tanaka, Phys. Lett. **B256** (1991) 427.

[15] P.K. Ghosh, Phys. Rev. **D49** (1994) 5458.

[16] J. Edelstein, C. Núñez, F. Schaposnik, Phys. Lett. **B329** (1994) 39.

[17] P. Navrátil, Phys. Lett. **B365** (1996) 119.