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Non-Extensive Thermostatistical Approach of the Peculiar Velocity Function of Galaxy Clusters

by

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Abstract

We show that the observational data recently provided by Giovanelli et al. (1996 a, b) and discussed by Bahcall and Peng (1996) concerning the velocity distribution of clusters of galaxies can be naturally fitted by a statistical distribution which generalizes the Maxwell-Boltzmann one (herein recovered for the entropic index $q = 1$). Indeed a recent generalization of the Boltzmann-Gibbs thermostatistics suggests for this problem that the probability function is given, within a simple phenomenological model, by

$$P(>v) = \frac{\int_v^{v_{max}} dv\,[1-(1-q)(v/v_0)^2]^{\frac{q}{1-q}}}{\int_0^{v_{max}} dv\,[1-(1-q)(v/v_0)^2]^{\frac{q}{1-q}}},$$
$$v_{max} \equiv \begin{cases} v_0(1-q)^{-1/2} & \text{if } q < 1 \\ \infty & \text{if } q \geq 1. \end{cases}$$

A remarkably good fitting with the data is obtained for $q = 0.23^{+0.07}_{-0.05}$ and $v_0 = 490 \pm 5$ km s^{-1}.

Key-words: Non-extensive thermostatistics; Galaxy clusters; Cosmology; Velocity function.

The motions of clusters and group of galaxies have been studied by Bahcall et al. (1994). The sample of clusters of galaxies velocities recently observed by Giovanelli et al. (1996 a, b) based on Tully-Fisher distances of Sc galaxies has been examined and compared with model expectations by Bahcall and Peng (1996). In contrast with previous analysis of the data, the actual observed velocity function does not show a tail of high velocity clusters (clusters with velocities greater than ~ 600 km s^{-1} can be found with a probability less than five per cent). Bahcall and Peng (1996) determine the cluster velocity function using simulations based on four different cosmological models:
(a) CDM (cold dark matter), $\Omega \equiv$ *dimensionless matter density* $= 0.3$;
(b) PBI (primeval baryonic isocurvature), $\Omega = 0.3$;
(c) CDM, $\Omega = 1.0$;
(d) HDM (hot dark matter), $\Omega = 1.0$.
These authors conclude that model (a) and marginally model (b), in spite of the high energy tail, are consistent with the observed data (see Fig. 1), while models (c) and (d) show a too large high velocity tail to be in agreement with data. However, if we look in detail at the observations and simulations curves, we can see that not even the CDM, $\Omega = 0.3$ model is completely satisfactory, particularly in the region of velocities larger than 350 km s^{-1}. In fact, none of the four theoretical curves is fully satisfactory.

We want to show in this letter that to understand the Giovanelli et al. (1996 a, b) observational data is not simply a problem of which cosmological model is used, but it also depends on which statistical mechanics is used. In fact, if we adopt the recently introduced non-extensive statistics (Tsallis 1988, Curado and Tsallis 1991) to calculate the probability distribution function of cluster peculiar velocities, the agreement with experimental results is very satisfactory, particularly in foreseing that the number of galaxy clusters with velocities greater than ~ 600 km s^{-1} is practically zero. The non-extensive statistics, among several other applications, has been recently applied in calculating matter distribution of self-gravitating systems (Plastino and Plastino 1993, Aly 1993, Boghosian 1996), turbulence (Boghosian 1996), Levy-like anomalous diffusion (Zanette and Alemany 1995, Tsallis *et al.* 1995), solar neutrino fluxes (Kaniadakis *et al.* 1996), a cosmological model (Hamity and Barraco 1996), as well as linear response theory (Rajagopal 1996).

We firstly give a brief account of the non-extensive statistics. Then we calculate by means of it the probability of finding galaxy clusters of certain velocity v. Results are shown in the two Figures and discussed; conclusions are outlined.

In order to cover non-extensive systems (long-range microscopy memory, long range forces, fractal space time) the following generalized entropy has been proposed:

$$S_q = k\frac{1-\sum_i p_i^q}{q-1} \qquad \left(\sum_i p_i = 1; \qquad q\epsilon\Re\right), \tag{1}$$

where k is a positive constant. Optimization of S_q yields, for the canonical ensemble,

$$\rho_i = Z_q^{-1}\left[1-(1-q)\beta\epsilon_i\right]^{\frac{1}{1-q}}, \tag{2}$$

$$Z_q \equiv \sum_i \left[1-(1-q)\beta\epsilon_i\right]^{\frac{1}{1-q}} \tag{3}$$

and, when $q \to 1$, the Boltzmann-Gibbs result ($\rho_i \propto e^{-\beta\epsilon_i}$) is recovered. Let us stress that, for $q < 1$, ρ_i vanishes for $[1-(1-q)\beta\epsilon_i] \leq 0$. Within this formalism the observational data are to be identified with $\langle A\rangle_q = \sum_i \rho_i^q A_i$ where $\{A_i\}$ are the eigenvalues of an arbitrary observable A.

The application of this thermostatistics to stellar systems has shown that sensible distribution functions (stellar polytropes) can be derived if $q \in (-\infty, 7/9)$. Distributions with $q < 1$ give rise to the spatial cutoff of the mass distribution and the finite mass of the stellar polytrope (Plastino and Plastino 1993, Aly 1993 and Boghosian 1996). Observed distributions of pure-electron plasma with two dimensional turbulence are well described with $q = 0.5$ (Boghosian 1996). The logistic map at its threshold to chaos can be described using $q = 0.24$ (Tsallis *et al.* 1996). Solar neutrino fluxes can be understood using non-extensive statistics for the central core plasma and values of q slightly below one: $q = 0.997$ (ion plasma) and $q = 0.976$ (electron plasma) (Kaniadakis et al. 1996, Quarati et al. 1996).

We calculate now the probability $P(> v)$ of finding one-dimensional peculiar velocity function of clusters of galaxies (relative to a comoving cosmic frame) greater than v. It is given by

$$P(> v) = \frac{\int_v^{v_{max}} dv\,[1-(1-q)(v/v_0)^2]^{\frac{q}{1-q}}}{\int_0^{v_{max}} dv\,[1-(1-q)(v/v_0)^2]^{\frac{q}{1-q}}}, \tag{4}$$

$$v_{max} \equiv \begin{cases} v_0(1-q)^{-1/2} & \text{if } q < 1, \\ \infty & \text{if } q \geq 1, \end{cases} \tag{5}$$

where v_0 is a phenomenological characteristic velocity which incorporates all the averages over cluster configurations taking into account their gravitational interactions.

We find the best fit of the observed data and report it on Fig.1. The entropic parameter is $q = 0.23^{+0.07}_{-0.05}$ and $v_0 = 490 \pm 5$ km s^{-1}. This value of q, very different from one, reflects the fact that the long-range gravitational forces are, of course, in the case of galaxy clusters, very important. Moreover, by means of the relation $n \equiv$ *polytropic index* $= 3/2 + q/(1-q)$, (Boghosian 1996), we may derive that the system is characterized by $n \approx 7/4$. The fact that this value lies in the range of the typically observed (and theoretically allowed: Plastino and Plastino 1993, Aly 1993, Boghosian 1996) values constitutes a further, and independent, confirmation of the possible relevance of the present proposal. Peculiar of the non-extensive statistics is that our distribution is cut at $v \geq 520$ km/s, in perfect consistence with the observed data. In Fig. 2 we show $P(> v)$ of Eq.(4) for five different values of q: $q = 1$ (implicitely adopted by Bahcall and Peng 1996) is the Maxwell-Boltzmann distribution, $q = 0$ is a straight line, the $q < 0$ curves show clearly a pronounced cut tail.

Our results indicate that in order to distinguish which, among the four models judiciously used by Bahcall and Peng (1996) (each of them using several fitting parameters), is the most appropriate to describe the velocity distribution, it would be necessary to repeat the simulations within the non-extensive statistics with $q = 0.23^{+0.07}_{-0.05}$. The best value of v_0 can be selected and, possibly, one could obtain information on the quantity Ω. Finally, we note that the efficiency coming from the modification of the statistics is greater and more important than the efficiency obtained from the modification of the model (precisely the same occured for experiments done in pure-electron plasma turbulence: see Huang and Driscoll 1994 and Boghosian 1996). In fact, our fit of the experimental results, based on the non-extensive statistics, has been derived within a classical ideal gas picture. Incidentally, we notice that the value $q = 0.23^{+0.07}_{-0.05}$ is very close to the value $q = 0.24$, which is appropriate to chaotic dynamics (Tsallis et al. 1996). It would be interesting to investigate if this approximate equality between the two above figures is just a numerical coincidence or is due to some fundamental reasons.

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Figure Captions

Fig. 1. – The observed data of the clusters velocity function are reported together with the four curves corresponding to the four different models of $P(>v)$ elaborated by Bahcall and Peng (see their Fig. 1b). Our fit of the observed CVF, based on non-extensive statistics ($q = 0.23$ and $v_0 = 490$ km s^{-1}), is also shown.

Fig. 2. – The function $P(>x)$ versus $x = v/v_0$ is reported for five different values of the entropic parameter q $(-5, -1, 0, 0.5, 1)$. The case $q = 1$ is the well known Maxwell-Boltzmann distribution. We note that the curve $q = 0$ is a straight line. Curves with negative values of q have an abruptly cutted tail.

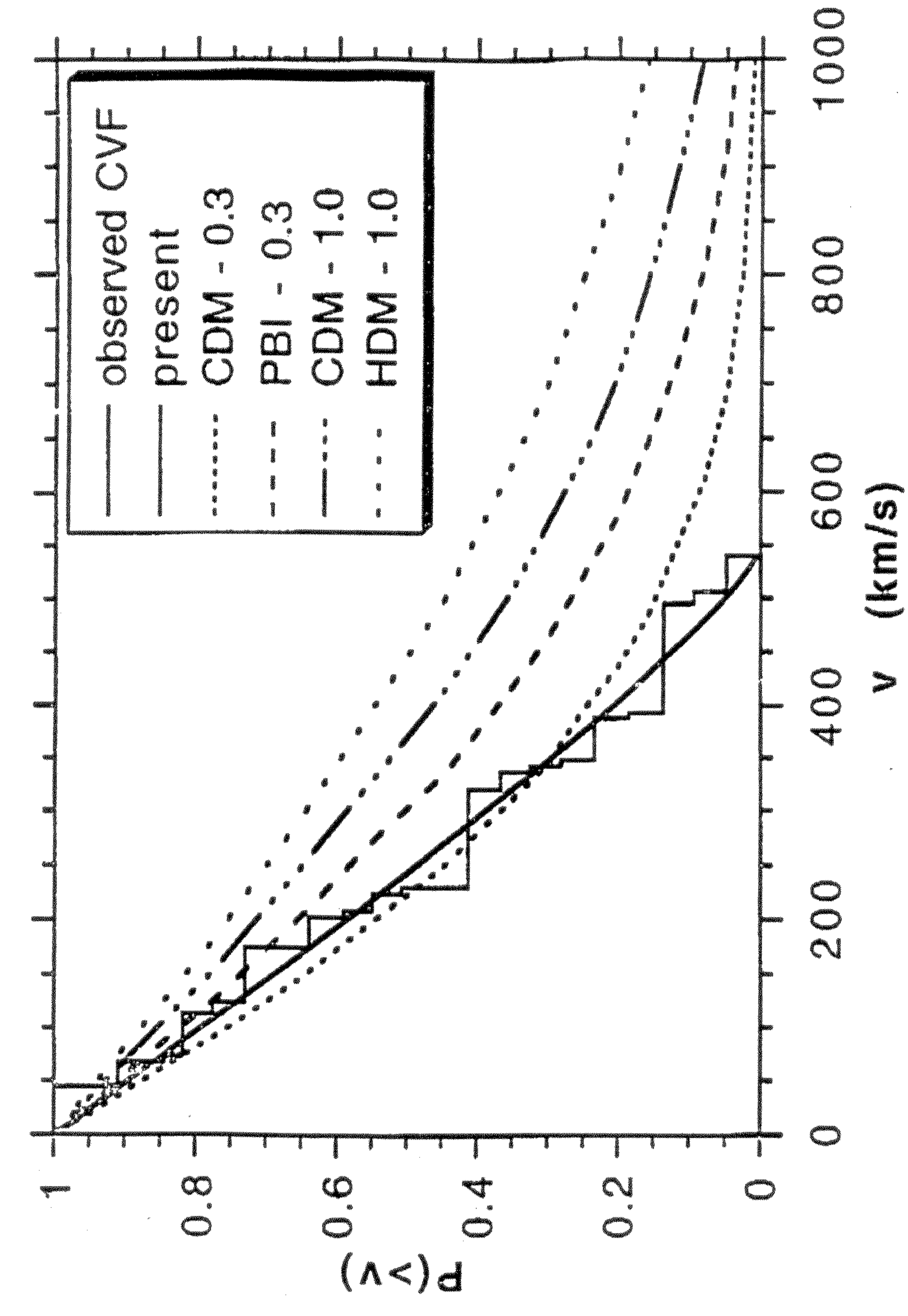
observed CVF
present
CDM - 0.3
PBI - 0.3
CDM - 1.0
HDM - 1.0
P(>v)
v (km/s)
0
200
400
600
800
1000
0.2
0.4
0.6
0.8
1

Fig. 1

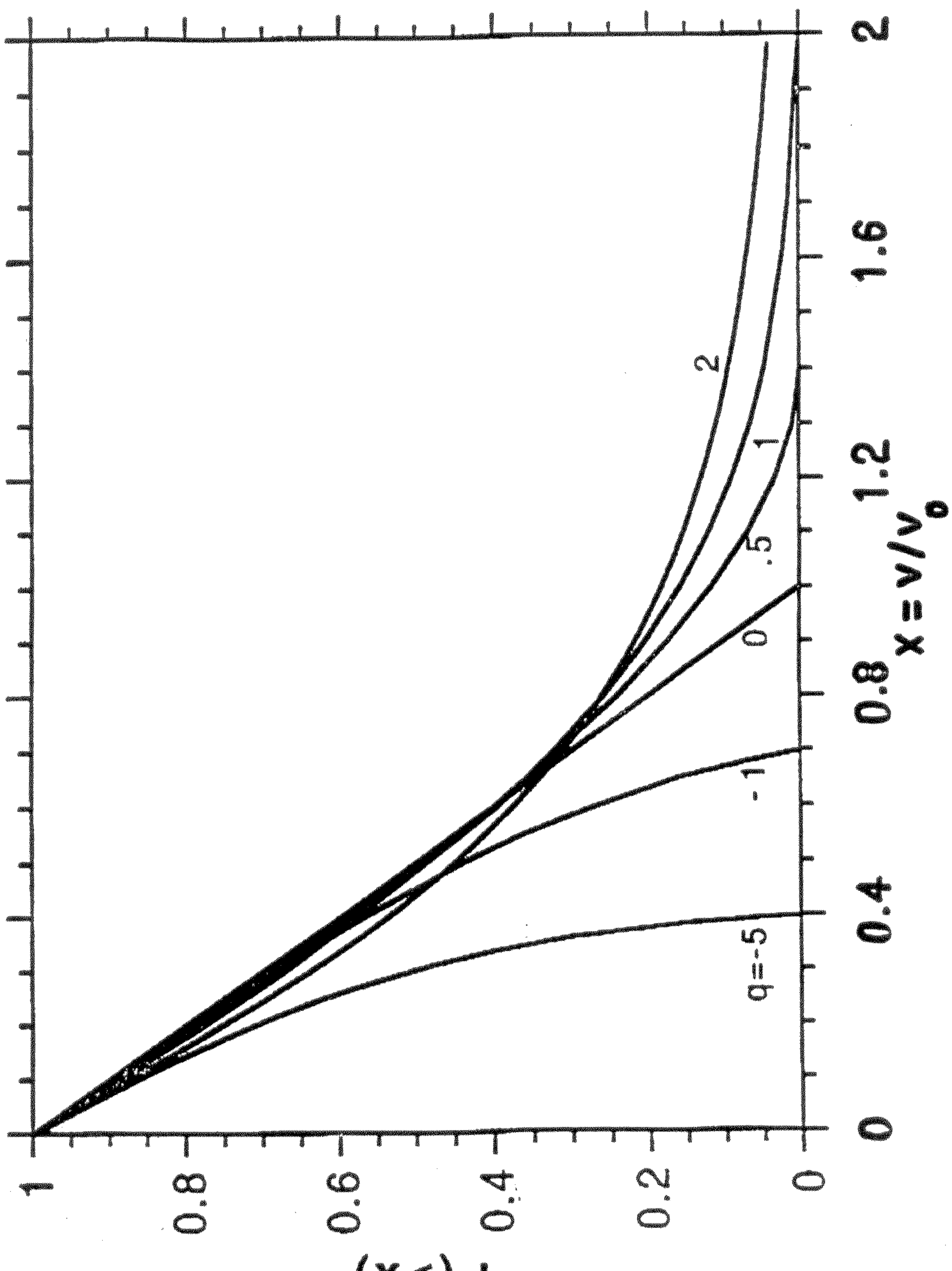

Fig. 2

References

[1] Aly, J. 1993, in *N-body problems and gravitational dynamics*, ed. F. Combes and E. Athanassoula (Paris: Publications de l'Observatoire de Paris), 19.

[2] Bahcall, N. A., Gramann, Cen, R. 1994, ApJ, 436, 23.

[3] Bahcall, N. A., Peng, S. O. 1996, ApJ, 462, L49.

[4] Boghosian, B. 1996, Phys. Rev. E, 53, 4754.

[5] Curado, E., Tsallis, C. 1991, J. Phys. A: Math. Gen., 24, L69; Corrigenda: 24, 3187 (1991) and 25, 1019 (1992).

[6] Giovanelli, R. et al. 1996a, in IAU Symp. 168, *Examining the Big Bang and Diffuse Background Radiation*, ed. M. Kafates and Y. Kondo (Dordrecht: Kluver).

[7] ———— 1996b, in preparation.

[8] Huang, X.-P., Driscoll, C.F. 1994, Phys. Rev. Lett. 72, 2187.

[9] Hamity, V. H., Barraco D.E. 1996, Phys. Rev. Lett. 76, 4664.

[10] Kaniadakis, G., Lavagno, A., Quarati, P. 1996, Phys. Lett. B, 369, 308.

[11] Plastino, A. R., Plastino, A. 1993, Phys. Lett. A, 174, 384.

[12] Quarati, P., Carbone, A., Gervino, G., Kaniadakis, G., Lavagno, A. and Miraldi, E. 1996, contribution to Nuclei in the Cosmos IV (Bloomington).

[13] Rajagopal, A.K. 1996, Phys. Rev. Lett. 76, 3469.

[14] Tsallis, C., 1988, J. Stat. Phys., 52, 479.

[15] Tsallis, C., Levy, S., de Souza, A. and Maynard, R. 1995, Phys. Rev. Lett., 75, 3589.

[16] Tsallis, C., Plastino, A. R. and Zheng, W.-M. 1996, *Power law sensitivity to the initial conditions-New entropic representation*, preprint CBPF (Rio de Janeiro).

[17] Zanette, D.H., Alemany, P.A. 1995, Phys. Rev. Lett. 75, 366.