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$\pm J$ D-VECTOR SPIN GLASS PHASE DIAGRAM AND CRITICAL BEHAVIOUR

by

S. COUTINHO[+*] and M.L. LYRA[+]

Present address:

[+]Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

*Universidade Federal de Pernambuco
Departamento de Física
50739 - Recife, PE - Brasil

ABSTRACT

The phase diagram and the correlation length exponents of the $\pm J$ D-Vector Spin-Glass model are studied in the framework of the real space mean field renormalization group method. The boundary between the spin-glass (SG) and the ferromagnetic (F) phases is obtained from the renormalization flow equations and shows a re-entrant behaviour over the SG region. This re-entrance increases smoothly with the coordination number. Analytical expressions for the thermal and the correlation length exponents are calculated straightforwardly for all fixed points and figures are presented and compared with availables results from other methods and data.

Key-words: Spin glass; D-vector model; Mean field renormalization group; Critical exponents.

1 INTRODUCTION

The phase diagram and the critical behavior of the short range spin-glass model are nowadays receiving great attention. The random $\pm J$ Ising Spin-Glass model on the Bethe lattice has been studied by Kwon and Thouless[1], Carlson et al[2] and Thouless[3]. The phase diagram in an arbitrary lattice has been considered by Németh[4] and Nishimori[5]. Sophisticated numerical approaches like domain wall renormalization group (DWRG)[6], dynamic high - temperature series expansion[7] and Monte Carlo simulations[8,9] has been used to search for the transition in the three-dimensional $\pm J$ Ising spin-glass. Furthermore large-scale Monte Carlo simulations[10] has been applied to study the three-dimensional $\pm J$ Heisenberg spin-glass. The random Heisenberg model in two and three dimensions has been investigated within DWRG method[11] and the zero-temperature critical behaviour of the vector spin glasses has been studied numerically by the "defect energy" approach and by the Migdal-Kadanoff-like renormalization group method[12].

In this work we investigate the $\pm J$ D-Vector Spin-Glass model in the framework of the real space mean-field renormalization method (MFRG) introduced by Indekeu et al[13]. We focus our attention to the boundary transition from the spin-glass (SG) to the ferromagnetic (F) phase and to the critical exponent of the correlation lengths associated with the transitions. The boundaries lines from the paramagnetic (P) to the F and SG phases of this model with an asymmetric competing bond disorder has been considered by the authors within the same method[14].

Now we assume a $\pm J$ bond disorder, but the generalization to the J and $-\alpha J$ ($\alpha>0$) case can be done straighforwardly.

We consider the simplest choice of clusters with one and two-particles (see figure 1) which gives in the MFRG approach the exact ferromagnetic and spin-glass critical coupling of the present model on the Bethe lattice with the same coordination number. For instance we recover the results of Matsubara and Sakata[15] and the very recent one of Carlson et al[2], both for the Ising spin-glass model (D=1) in the Bethe lattice, the latter obtained after rigorous analysis. We believe that our results can be improved to some extend if large-size clusters are considered but with the cost of lengthy calculations. However, even for this choice of clusters the frustration effects introduced by the effective field symmetry boundary conditions can support the spin-glass phase. It has been shown that the present solution is the correct one for the Ising model on the Bethe lattice with uncorrelated boundary conditions[2].

2 THE MODEL HAMILTONIAN AND THE MFRG METHOD

The reduced model Hamiltonian is given by

$$-\beta\mathcal{H} \equiv H = \sum_{<i>} K_{ij}\vec{S}_i.\vec{S}_j + \sum_i \vec{B}.\vec{S}_i \qquad (1)$$

where $K_{ij}=\beta J_{ij}$ is the disordered reduced coupling constant for the nearest neighbour bonds in a hypercubic lattice, $\vec{S}$ is the vector spins variables with continuous symmetry in the D-dimensional space and $\vec{B}$ is the external uniform field in a particular direction of

the D-space. The D-vector spins variables are constrained by the normalizing condition $\sum_{r=1}^{D} (S_\nu)^2 = D$ which means the renormalization of the coupling constants relative to the spin dimensionality.

The MFRG method is based on phenomenological scaling relations between the order parameters and between the associated effective symmetry breaking boundary fields of the clusters to be renormalized. The renormalization recursion equations are formally obtained by considering two finite clusters with N' and N spins (N'<N) and by imposing a finite size scaling relation between the order parameters of both clusters. The order parameters are calculated by assuming a symmetry breaking effective field at the edges in the spirit of the mean field approximation. The same scaling relation is self consistently imposed between the fields. For second order transitions one lead to a recursion relation for each order parameter given by[13]

$$\left.\frac{\partial f_{N'}}{\partial b'}\right|_{b'=0} = \left.\frac{\partial f_N}{\partial b}\right|_{b=0} \qquad (2)$$

where $f_N(f_{N'})$ and $b(b')$ are the order parameter and the effective field of the N(N') cluster. Note that eq.(2) is independent of the scaling factor. The critical coupling K_c for each transition is given by the fixed point solutions of eq.(2).

For the present model we consider the quenched average magnetization as the order parameter for the ferromagnetic transition and the Edwards-Anderson order parameter[16] for the spin-glass transition.

3 RENORMALIZATION FLOW EQUATIONS AND PHASE DIAGRAM

The renormalization recursion equations for the ferromagnetic and the spin glass order parameters for the clusters with $N'=1$ and $N=2$ spins shown in figure 1 are given respectively by[17]

$$\left.\frac{\partial\overline{\langle S'^1\rangle}}{\partial b'}\right|_{b'=0} = \left.\frac{\partial\overline{\langle S^1\rangle}}{\partial b}\right|_{b=0} \tag{3}$$

$$\left.\frac{\partial\overline{\langle S'^1\rangle^2}}{\partial h'}\right|_{h'=0} = \left.\frac{\partial\overline{\langle S^1\rangle^2}}{\partial h}\right|_{h=0} \tag{4}$$

where

$$\langle S'^1\rangle = \sum_{i=1}^{z} K'_i\, b'^1_i \tag{5}$$

$$\langle S^1\rangle = \sum_{j=1}^{q} K_{1j}\, b^1_{1j} + th_{D/2}(DK_{12}) \sum_{j=1}^{q} K_{2j}\, b^1_{2j} \tag{6}$$

where $K'_i(K_{\ell j})$ and $b'^{\nu}_i(b^{\nu}_{\ell j})$ $(\ell=1,2)$ are the temperature reduced coupling constant and the ν-component of the boundary effective field of the $N'=1$ $(N=2)$ cell respectively, $q=(z-1)$, $z=2d$ being the coordination number of the d-dimensional hypercubic lattice. $th_{D/2}(DK_{12})$ is the generalized hyperbolic tangent given by $I_n(y)/I_{n-1}(y)$, $I_n(y)$ being the modified Bessel function of first kind of order $n=D/2$ and $y=DK_{12}$. In eqs.(3) and (4) $\langle\cdots\rangle$ means the thermal average while the bar represents the configurational quenched average,

$$b' = \overline{b'^1_i}\ ,\quad b = \overline{b^1_{\ell j}}\ ,\quad h' = \overline{(b'^1_i)^2} \quad \text{and} \quad h = \overline{(b^1_{\ell j})^2}\ .$$

To get (5) and (6) we have assumed second order ferromagnetic and

spin-glass phase transitions. Furthermore, for each transition the effective fields are subjected to appropriated boundary conditions to simulate the effect of the surrounding spins of the infinite system with the symmetry of the corresponding order parameter.

To perform the configurational quenched average we assume that the k's and b's are independent random variables according to a probability distribution. Moreover we force the bond distribution to be the same for both clusters leaving a disorder parameter (the concentration of positive bonds, for instance) to be renormalized. From eqs.(3-6) we get

$$\text{P-F:} \qquad z\,\overline{K'} = q\,\overline{K}(1+\overline{t_n}) \tag{7}$$

$$\text{P-SG:} \qquad z\,\overline{K'^2} = q\,\overline{K^2}(1+\overline{t_n^2}) \tag{8}$$

for the ferromagnetic-paramagnetic (P-F) and spin-glass-paramagnetic (P-SG) transitions, where $t_n=th_n(DK_{12})$, $(n=D/2)$.

For simplicity we assume for the bond disorder the binary independent distribution with probability p for the +J and (1-p) for the -J bonds. Therefore from (7) and (8) we get

$$\text{P-F:} \qquad z\,x'K' = qxK(1+xt_n) \tag{9}$$

$$\text{P-SG:} \qquad z\,K'^2 = qK^2(1+t_n^2) \tag{10}$$

where $x=(2p-1)$ and $K=\beta J$. Clearly the paramagnetic trivial solution $K=0$ is a fixed point of both recursion relations (9) and (10) for all values of x (or p). As the temperature is lowered, the paramagnetic solution becomes unstable and a phase transition occurs. The phase boundaries from the paramagnetic to the ferromagnetic and to

the spin-glass phases are given by the non-trivial solutions of eqs.(9) and (10) respectively, that is

$$\text{P-F:} \qquad 1 = q\,x\,t_n \tag{11}$$

$$\text{P-SG:} \qquad 1 = q\,t_n^2 \tag{12}$$

Both eqs.(11) and (12) are lines of fixed points. They also give the exact boundaries of the paramagnetic phase for the model hamiltonian on the Bethe lattice of coordination number $z=(q+1)$ as mentioned before. Furthermore the renormalization flow in the (t_n,x)-space, obtained by eqs.(9) and (10), that is

$$x' = \frac{q\,x(1+x\,t_n)}{[qz(1+t_n^2)]^{1/2}} \tag{13}$$

$$t_n' = th_n\left[[q(1+t_n^2)/z]^{1/2}\, th_n^{-1}(t_n)\right] \tag{14}$$

gives the critical line of the phase boundary between the ferromagnetic and spin-glass phases in the vinicity of the multicritical point $x_c=t_c=1/\sqrt{q}$ which is the common solution of eqs.(11) and (12) as expected. In the neighborhood of (x_c,t_c) the F-SG boundary is given by flux line which can be well aproximated by the non-trivial solution of eq.(8) that is

$$x = \frac{1}{t_n}\left[[z(1+t_n^2)/q]^{1/2} - 1\right] \tag{15}$$

The renormalization flow eqs.(13) and (14) have a zero temperature unstable fixed point given by $x_o=2[z/2q]^{1/2}-1$, $t_o=1$ which is obviously a solution of eq.(15).

In figure (2) we show the squematic phase diagram $(1-t_n,x)$

which is similar to the one obtained by Carlson et al 1987 for the Ising case (D=1). At high temperatures the system is paramagnetic (P). At low temperatures and high concentrations of +J bonds ($x=1$) we find the ferromagnetic (F) phase, but for concentrations lower than $x_c=1/\sqrt{q}$ (or $p_c=\frac{1}{2}(1+1/\sqrt{q})$ the SG-phase is stable. Note that the antiferromagnetic phase should appears for low concentrations $p\simeq 0 (x\simeq -1)$. The phase diagram for $x<0$ can be obtained by reflection of figure (1) across the line $x=0$. Therefore the SG-phase is stable between $\frac{1}{2}(1-1/\sqrt{q}) < \phi < \frac{1}{2}(1+1/\sqrt{q})$ and $t_n > t_c = 1/\sqrt{q}$. However, at zero temperature ($t_n=1$) the transition between the SG and the F phases occurs, for the approximation considered here, at the concentration $p_o=(z/2q)^{1/2}$. Since $p_o > p_c$ the F-phase is reentrant in the SG-phase. The width of this reentrance is given by $\Delta_q=(p_o-p_c)/p_c$ which increases with q from $\Delta_2=0.0146$ to $\Delta_\infty=0.4142$.

The SG-F boundary given by eq.(15) should be the same line found by Carlson et al.[2] for the boundary between the SG phase and a MSG-phase (magnetized spin glass phase). For instance, eq. (15) has the same behavior near the multicritical point $t_c=x_c=1/\sqrt{q}$, that is

$$\sqrt{t_c}\,|t-t_c| = \frac{2\sqrt{q+1}}{q}\sqrt{|p-p_c|} \qquad (16)$$

and end at the point $x_o=0.732$, $t_o=1$ which is very close to the accurate value 0.739 obtained by Kwon and Thouless[1]. We did not succeed to find in the framework of the MFRG method the MSG-phase, a mixed phase between the spin-glass and the ferromagnetic phases[1,2]. Moreover we believe that the exact phase boundary between the spin-glass and the magnetized phases (F or MSG) should shows the re-entrant behaviour in contrast with the conclusions of Németh[4] and

Nishimori[5] for the $\pm J$ Ising model who argue that a vertical phase boundary should occurs as "a manifestation of a singularity in the entropy of distribution of frustration".

4 CORRELATION LENGTH CRITICAL EXPONENTS

The critical exponent of the thermal correlation length ν_T can be also extracted from eqs.(9) and (10) as usual, that is

$$\ell^{1/\nu_T} = \left.\frac{dK'}{dK}\right|_{K_c} \tag{17}$$

where ℓ is the scaling factor. The critical exponent ν_T at the fixed points A,C and O of figure (1), can be evaluated analitically as a function of the coordination number and the spin dimensionality. Actually from eqs.(17), (9) and (10) we get

$$\nu_{TA}^{-1} = \frac{1}{\ell n\, \ell}\, \ell n\left[1 - \frac{D-1}{q+1} + \frac{q-1}{q}\, th_{D/2}^{-1}\left(\frac{1}{q}\right)\right] \tag{18}$$

$$\nu_{TC}^{-1} = \frac{1}{\ell n\, \ell}\, \ell n\left[1 - \frac{D-1}{q+1} + \frac{q-1}{q+1}\, \frac{1}{\sqrt{q}}\, th_{D/2}^{-1}\left(\frac{1}{\sqrt{q}}\right)\right] \tag{19}$$

$$\nu_{TO}^{-1} = \frac{1}{2\ell n\, \ell}\, \ell n\left[\frac{2q}{q+1}\right] \tag{20}$$

for finite D, where $th_{D/2}^{-1}(...)$ is the inverse of the generalized hyperbolic tangent function, and $\ell = [9(q+1)/(9q-1)]^{1/2}$ is the new definition of the scaling factor for the MFRG proposed by Slotle[18] for the one and two-particles clusters in a d-dimensional hypercubic lattice of coordination number q+1. Note that ν_{TO}^{-1} is D independent.

The $D \to \infty$ limit of ν_{TA}^{-1} and ν_{TC}^{-1} are given respectively by

$$\nu_{TA}^{-1}(\infty) = \frac{q+1}{2\ell n 2}\, \ell n \left[\frac{q(q+1)}{(q^2+1)}\right] \tag{21}$$

$$\nu_{TA}^{-1}(\infty) = \frac{q+1}{2\ell n 2}\, \ell n \left[\frac{q(q+3)}{(q+1)^2}\right] \tag{22}$$

The concentration correlation length exponent ν_p can be defined by analogy with the thermal one by

$$\ell^{1/\nu_p} = \left.\frac{dx'}{dx}\right|_{x^*} \tag{23}$$

which can be calculated with help of eq.(13). For the fixed points C and O we get

$$\nu_{cc}^{-1} = \frac{1}{\ell n\, \ell}\, \ell n \left[\frac{q+2}{q+1}\right] \tag{24}$$

$$\nu_{co}^{-1} = \frac{1}{\ell n\, \ell}\, \ell n \left[2 - \sqrt{\frac{q}{2(q+1)}}\right] \tag{25}$$

which are D independent.

In table I we show the pure ferromagnetic thermal correlation length exponent ν_{TA} given by eqs.(18) and (21) as function of the hypercubic lattice and spin dimensionalities in comparison with results obtained from several methods and experimental data. The MFRG correlation length exponents are very sensitive to the scaling factor which is somewhat arbitrary in the MFRG method for anisotropic clusters. Slotle[18] has proposed a reasonable definition for the scaling factor that has been adopted in the present work. In table II we show the spin-glass thermal correlation length exponent ν_{TC} for several spin dimensionalities and hypercubic lattice dimensions. The figures in Tables I and II should not be compared in accuracy

with the ones obtained by sophisticated approaches and data but viewed as a qualitative description of the dependence of the correlation lenght exponents with the hypercubic lattice and spin dimensionalities. Even though the results for the pure ferromagnetic exponent ν_{TA} (Table I) are in good position comparing with others real space renormalization group methods. We note that ν_{TA} increases smoothly with the increasing of the spin dimensionality and decreases smoothly with the increasing of the lattice dimension. This behavior is also followed by the availables figures obtained by others methods and data. Moreover in the infinite coordination number limit ($d \to \infty$) $\nu_{TA} \to 5/6 = 0.555...$ for all spins dimensionalities which is close to the $\frac{1}{2}$ mean field value. For the spin glass transition (Table II) ν_{TC} decreases with the lattice dimensionality as expected. For $d \to \infty$ $\nu_{TC}=0.555...$ which is close to the $\frac{1}{2}$ mean field value.We also point out that the present results show that ν_{TC} increases with the spin dimensionality. However figures given by Morris et al.[12] (defect energy and real space renormalization group methods) for the XY model (D=2) and the ones given by McMillan[11] (domain wall renormalization group) for the Heisenberg model (D=3) in d=2 and 3 dimensions show an opposite behavior. However regarding to other results listed in Table II the picture is not conclusive since those sophisticated methods do not covers systematically the whole table. It is worth to mention that for the present choice of clusters one is not able to predict a lower critical dimension. As we mentioned above our critical temperatures follow the Bethe-Peierls approximation. We believe that these difficulties can be overcome if large clusters preserving the lattice symmetry are chosen to be renormalized.

Finally in table III we present the figures for the thermal correlation length exponent for the spin-glass-ferromagnetic transition ν_{TO} and the concentration correlation length exponents ν_{CC} and ν_{CO} as function of the hypercubic lattice dimensionalities as given by eqs.(20), (24) and (25) respectively. We note that both ν_{TO} and ν_{CO} decrease with the lattice dimensionality toward to zero while $\nu_{CC} \to 0.555...$ as $d \to \infty$.

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FIGURE CAPTIONS

Figure 1: Schematic representation of the simplest clusters with (a) $N'=1$ and $i=1...z$ and (b) $N=2$ and $j=1...(z-1)$.

Figure 2: General phase diagram for the D-vector model on the Bethe lattice with coordination number $(q+1)$, plotted as a function of temperature ($t_n=th_n(DJ/kT)$) against the concentration of ferromagnetic bonds ($p=(x+1)/2$). The paramagnetic (P), ferromagnetic (F) and spin-glass phases met at the multicritical point C ($t_c=x_c=1/\sqrt{q}$). The transition from the (SG) to the (F) phases at $T=0$ ($t_n=1$) occurs for concentration $p_o=[(q+1)/2q]^{1/2} > p_c=(1+1/\sqrt{q})/2$.

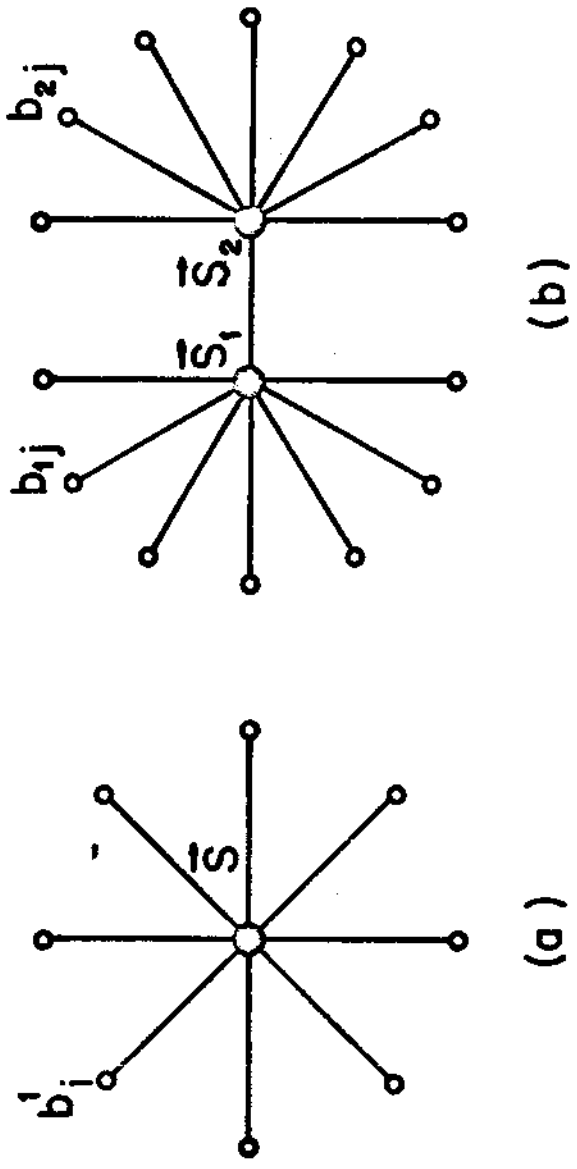

Fig. 1

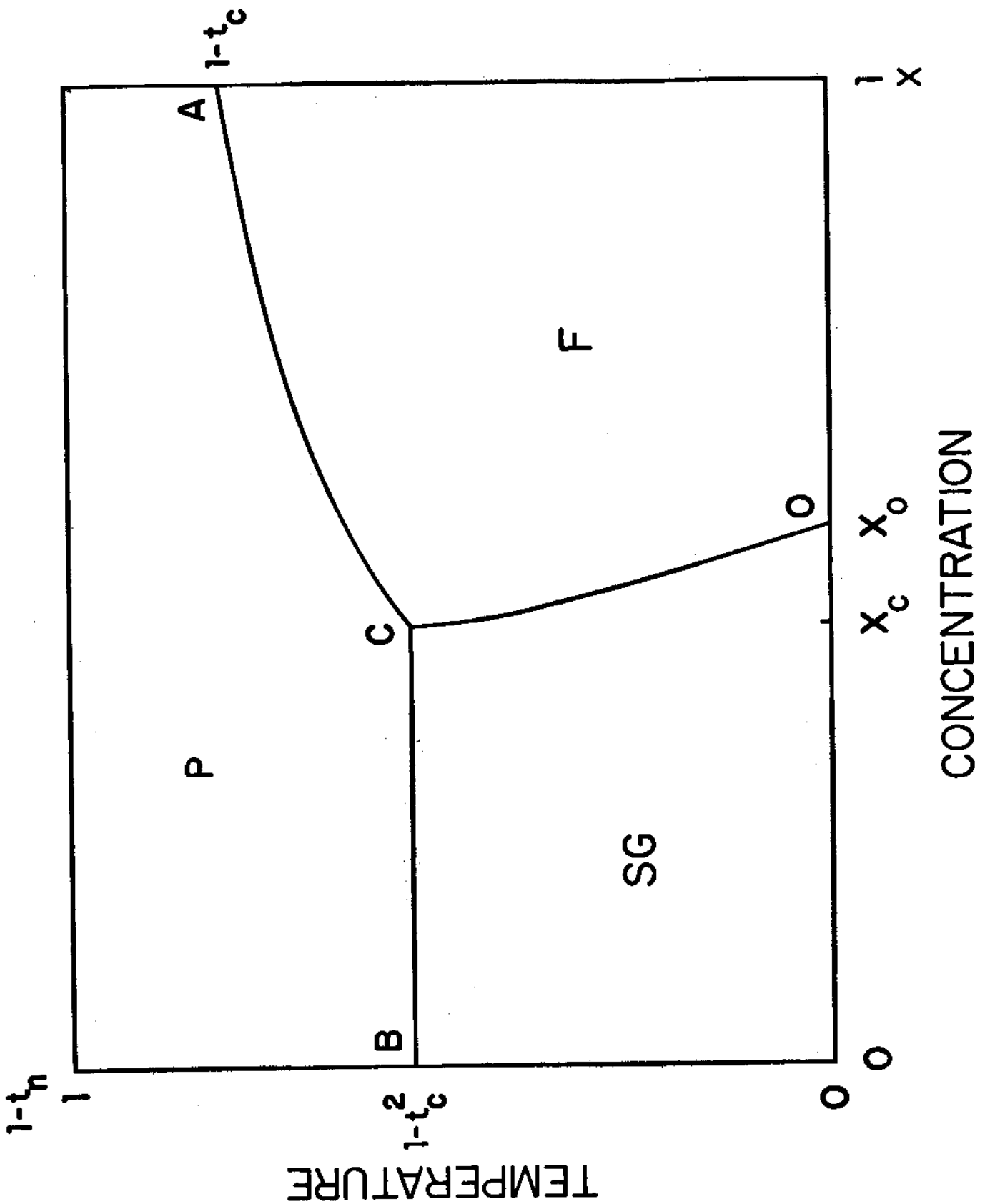
1-t_h
1
$\frac{2}{1-t_c}$
0
0
x_c
x_o
1
1 - x
A
B
C
O
P
F
SG
TEMPERATURE
CONCENTRATION

Table I: Ferromagnetic thermal correlation length exponent ν_{TA} as function of the hypercubic lattice and spin dimensionalities. (d=(q+1)/2).

	method	d=2	d=3	d=4	d=5
D=1	MFRG (a)	0.7827	0.6813	0.6431	0.6228
	MKRG (b)	0.669			
	RSRG	1.1486 (c)	0.8705 (d)	0.8426 (d)	
	ϵ-expansion (e)	0.99 ± 0.04	0.6310±0.0015		
	field theory (f)		0.630±0.0015		
	series (g)		0.64		
	data (h)		0.625±0.003		
	data (i)		0.65±0.02		
	exact	1 (j)			1/2 (k)
D=2	MFRG (a)	0.8125	0.6901	0.6473	0.6253
	field theory (f)		0.669±0.002		
	series (ℓ)		0.673±0.006		
	data (m)		0.675±0.001		
D=3	MFRG (a)	0.8307	0.6954	0.6498	0.6268
	MKRG (n)		1.40		
	series (o)		0.7025±0.010		
	field theory (f)		0.705±0.003		
	data (i)		0.70±0.002		
D=4	MFRG (a)	0.8425	0.6989	0.6515	0.6277
D=5	MFRG (a)	0.8507	0.7014	0.6527	0.6284
D=∞	MFRG (a)	0.8924	0.7156	0.6597	0.6236

(a) present work
(b) Yeomans and Stinchcombe[22]
(c) Tsallis and Levy[23]
(d) Martin and Tsallis[24]
(e) Le Guillon and Zinn-Justin[25]
(f) Le Guillon and Zinn-Justin[26]
(g) Fischer and Burford[27]
(h) Chang et al.[28]
(i) Als-Nielsen[29]
(j) Wu[30]
(k) Pfeuty and Toulouse[31]
(ℓ) Pfeuty et al.[32]
(m) Mueller et al.[33]
(n) Stinchcombe[34]
(o) Ritchie and Fischer

Table II: Spin-glass thermal correlation length exponent as function of hypercubic lattice and spin dimensionalities ($d=(q+1)/2$).

	method	d=2	d=3	d=4	d=5
D=1	MFRG (a)	0.9350	0.7638	0.7000	0.6228
	Monte Carlo (b)		1.3±0.1		
	Monte Carlo (c)		1.0∿1.5	0.6	
	Monte Carlo (d)		1.2±0.1		
D=2	MFRG (a)	1.0855	0.8249	0.7374	0.6253
	defect energy (e)	1.09±0.05	2.2±0.05		
	MKRG (e)	1.08	2.3		
D=3	MFRG (a)	1.1770	0.8622	0.7602	0.7094
	DWRG (f)	0.714±0.015	1.54±0.019		
	Monte Carlo (g)		1.14		
D=4	MFRG (a)	1.2309	0.8860	0.7750	0.7201
D=5	MFRG (a)	1.2642	0.9019	0.7853	0.7276
D=∞	MFRG (a)	1.3814	0.9718	0.8343	0.7652
	defect energy (e)	0.65±0.02	1.01±0.02		

(a) present work
(b) Ogielski[8,36]
(c) Wang and Swendsen[37]
(d) Ogielski and Morgenstern[16]
(e) Morris et al.[14]
(f) McMillan[19]
(g) Olive et al.[12]

Table III: Concentration correlation length exponent ν_{CC} and ν_{CO} and thermal correlation length ν_{TO} as function of the hypercubic lattice dimensionality (d=(q+1)/2).

d	2	3	4	5	6
ν_{CC}	0.7291	0.6642	0.6348	0.6179	0.6069
ν_{CO}	0.4969	0.3374	0.2564	0.2069	0.1735
ν_{TO}	0.2006	0.1002	0.0668	0.0501	0.0401

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