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Anti-symmetric rank two Tensor Matter Field on Superspace for $N_T = 2$

*Wesley Spalenza[a,b,1], **Wander G. Ney[a,b,c] and ***J.A. Helayel-Neto[a,b,1]

[a]Centro Brasileiro de Pesquisas Físicas - (CBPF) - Rio de Janeiro - RJ
[b]Grupo de Física Teórica José Leite Lopes - (GFT) - Petrópoles - RJ
[c]Centro Federal de Educação Tecnológica - (CEFET) - Campos dos Goytacazes - RJ

*wesley@cbpf.br, **wandergn@hotmail.com, ***helayel@cbpf.br.

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Abstract

In this work, we discuss the interaction between anti-symmetric rank-two tensor matter and topological Yang-Mills fields. The matter field considered here is the rank-2 Avdeev-Chizhov's tensor matter field in a suitably extended $N_T = 2$ SUSY. We start off from the $N_T = 2$, $D = 4$ superspace formulation and we go over to Riemannian manifolds. The matter field is coupled to the topological Yang-Mills field. We show that the two actions are obtained as a $Q-$exact forms and which allow us to write the energy-momentum stress tensor as $Q-$exact observables.

Key-words: Topological Theories; Anti-symmetric rank-2 Tensor Matter Fields

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1 Introduction

Topological field theories such as Chern-Simons theory and BF-models in gauge theories, probe space-time in its global structure, and this aspect has a significative relevance in quantum field theories. On the other hand there is several reserch in term of antisymmetric rank-2 tensor fields that can be put into two categories: gauge fields or matter fields. In recent years, Avdeev Chizhov [1, 2, 3] proposed a model where the antisymmetric tensor field is a matter.

In a recent work [4], Geyer-Mülsch presented a formulation until then unknown in the literature, which is a construction of the Avdeev-Chizhov action described in the topological formalism [5]. This was built for $N_T = 1$ and generalized for $N_T = 2$. Known the properties of the anti-symmetric rank-two tensor matter field theory, also called Avdeev-Chizhov's field [6], the supersymmetric properties and characteristics presented also in [7]; following this formalism we shall write this action in the superfield formalism, presented by Horne [8] in topological theories as a Donaldson-Witten topological theories [9, 5].

Our goal in this work is to discuss the interaction between matter and topological Yang-Mills fields as presented by Geyer-Mülsch [4] for $N_T = 1$ and $N_T = 2$. The matter field considered here is the rank-2 tensor matter field as a complex self-duality condition [6]. Thus, we write this field now as an anti-symmetric rank-two tensor matter superfield in $N_T = 2$ SUSY in the superspace formalism, founded also in [7]. The matter field is coupled to the topological Yang-Mills connection by means of the Blau-Thompson action. We write the Yang-Mills superconnection as a $2-$superform in a superspace with four bosonic dimensions spacetime described by Grassmann-odd coordinates and two fermionic dimensions described by Grassmann-even coordinates and construct the action in a superfield formalism following the definitions by Horne [8]. Then, we go over to Riemannian manifolds duely described in terms of the vierbein and the spin connection, where we taking the gravitation as background. We introduce and discuss the Wess-Zumino gauge condition induced by the shift supersymmetry better detailed in [10]. Then, we arrive at a topological invariant action as the sum of the Avdeev-Chizhov's action coupled to the topological super-Yang-Mills action; both actions are obtained as $Q-$exact forms, and of the energy-momentum stress tensor $Q-$exactness as a observables.

2 The $N_T = 2$ Super-conection, Super-curvature and Shift Algebra

Let us now consider the Donaldson-Witten theory, whose space of solutions is the space of self-dual instantons, $F = *F$. To follow our superfield formulation, we shall proceed with the definition of the action of Horne [8] and Blau-Thompson [12, 13]. The $N_T = 2$ superfield conventions are the ones of [10]. The superfields superconnection and its associated superghosts are given as below:

$$\hat{A} = \hat{A}^a T_a,\ \hat{C} = \hat{C}^a T_a \tag{2.1}$$

whose the generators belonging the Lie algebra:

$$[T_a, T_b] = i f_{ab}{}^c T_c. \tag{2.2}$$

Expliciting the superforms (2.1) in components superfields, we have

$$\hat{A} = A(x_\mu, \theta_I) + E_I(x_\mu, \theta_I) d\theta^I,\ \hat{C} = C(x_\mu, \theta_I), \tag{2.3}$$

with $I = 1, 2$; in components, we have:

$$A(x, \theta) \quad = \quad a(x) + \theta^I \psi_I(x) + \frac{1}{2}\theta^2 \alpha(x), \tag{2.4}$$

$$E_I(x,\theta) = \chi_I(x) + \theta^I\phi_{IJ}(x) + \frac{1}{2}\theta^2\eta_I(x), \quad (2.5)$$

$$C(x,\theta) = c(x) + \theta^I c_I(x) + \frac{1}{2}\theta^2 c_F(x). \quad (2.6)$$

The associated supercurvature is defined as

$$\hat{F} = \hat{d}\hat{A} + \hat{A}^2 = (dA + A^2) + (\partial_I A + D_A E_I)\ d\theta^I + \frac{1}{2}(\partial_I E_J + \partial_J E_I + [E_I, E_J])d\theta^I d\theta^J \quad (2.7)$$

which also can be expressed as: $\hat{F} = F + \Psi_I\ d\theta^I + \Phi_{IJ}\ d\theta^I d\theta^J$, whose components read as below:

$$\begin{aligned} F &= f - \theta^I D_a\psi_I + \frac{1}{2}\theta^2(D_a\alpha + \frac{1}{2}\varepsilon^{IJ}[\psi_I,\psi_J]), && (2.8)\\ \Psi_I &= \psi_I + D_a\chi_I + \theta^J(\varepsilon_{IJ}\alpha - \theta^J D_a\phi_{IJ} + \theta^J[\psi_J,\chi_I]) \\ &\quad +\theta^2(\frac{1}{2}D_a\eta_I - \frac{1}{2}\varepsilon^{KJ}[\psi_K,\phi_{IJ}] + \frac{1}{2}[\alpha,\chi_I]), && (2.9)\\ \Phi_{IJ} &= \frac{1}{2}\{\phi_{IJ} + \phi_{JI} + [\chi_I,\chi_J] + \theta^K(\varepsilon_{KI}\eta_J + \varepsilon_{JK}\eta_I + [\chi_I,\phi_{JK}] + [\phi_{IK},\chi_J]) \\ &\quad +\frac{1}{2}\theta^2([\chi_I,\eta_J] + [\eta_I,\chi_J] - \varepsilon^{KL}[\phi_{IK},\phi_{JL}])\}, && (2.10) \end{aligned}$$

where $f = da + a^2$ and the covariant derivatives in a being given by $D_a(\cdot) = d(\cdot) + [a,(\cdot)]$; $(\cdot)$ represents any field which the derivative act upon.

The susy number, s, is defined by attributing -1 to θ. Thus, the supersymmetry generators, Q, have SUSY number 1. The BRST tranformation of the superconnection (2.3) is $s\hat{A} = -\hat{d}\hat{C} - [\hat{A},\hat{C}] = -\hat{D}_{\hat{A}}\hat{C}$ and component superfields, is given by

$$\begin{aligned} &sA = -dC - [A,C] = -D_A C, \\ &sE_I = -\partial_I C - [E_I, C] = -D_I C, \\ &sC = -C^2, \end{aligned} \quad (2.11)$$

which in components take the form:

$$\begin{aligned} &sa = -dc - [a,c] = -D_a c\ , \\ &s\psi_I = -[c,\psi_I] - D_a c_I, \\ &s\alpha = -[c,\alpha] - D_a c_F + \varepsilon^{IJ}[c_I,\psi_J]\,, \\ &s\chi_I = -[c,\chi_I] - c_I, \\ &s\phi_{IJ} = -[c,\phi_{IJ}] - \varepsilon_{IJ}c_F + [\chi_I, c_J]\ , \\ &s\eta_I = -[c,\eta_I] - [c_F,\chi_I] + \varepsilon^{JK}[c_J,\phi_{IK}], \\ &sc = -c^2, \\ &sc_I = -[c,c_I], \\ &sc_F = -[c,c_F] + \tfrac{1}{2}\varepsilon^{IJ}[c_I,c_J]\,. \end{aligned} \quad (2.12)$$

and the super-covariant derivative is decomposed in: $\hat{D}_{\hat{A}} = D_A + d\theta^I D_I$.

The supersymmetry transformations or shift symmetry transformations, are defined as:

$$Q_I A = \partial_I A,\ Q_I E_J = \partial_I E_J,\ Q_I C = \partial_I C,$$

in components, they read as follows:

$$\begin{array}{lll} Q_I a = \psi_I\ , & Q_I\psi_J = -\varepsilon_{IJ}\alpha, & Q_I\alpha = 0, \\ Q_I\chi_J = \phi_{JI}, & Q_I\phi_{Jk} = -\varepsilon_{IK}\eta_J\ , & Q_I\eta_J = 0, \\ Q_I c = c_I, & Q_I c_I = -\varepsilon_{IJ}c_F, & Q_I c_F = 0. \end{array} \quad (2.13)$$

Next, we believe it is interesting to introduce and discuss a sort of Wess-Zumino gauge choice associated to the shift symmetry above, which is the topological BRST transformation. The Wess-Zumino [2] gauge seen in [11, 10], is here defined by the condition

$$\chi_I = 0 \ \ and \ \ \phi_{[IJ]} = 0. \tag{2.14}$$

due to the linear shift in the transformations (2.12) for scalar fields χ_I and ϕ_{IJ} respectively, with parameters given by the ghost fields c_I and c_F. There exists now, only the symmetric field $\phi_{(IJ)}$, that we write from now on, simply as ϕ_{IJ}. This condition is not susy-invariant under Q_I, and it can be defined in terms of the infinitesimal fermionic parameter ϵ^I as

$$\widetilde{Q} = \epsilon^I \widetilde{Q}_I.$$

This operator leaves the conditions invariant, and it is built up by the combinations of Q with the BRST transformations in the Wess-Zumino gauge, such that

$$\widetilde{Q} = (s+Q)|_{c_I=\varepsilon^J\phi_{IJ}\,,\ c_F=\frac{1}{2}\varepsilon^J\eta_J}. \tag{2.15}$$

The results in terms of component fields are displayed below:

$$\begin{array}{l}
\widetilde{Q}a = -D_a c \ + \epsilon^I\psi_I, \\
\widetilde{Q}\psi_I = -\left[c,\psi_I\right] - \epsilon^J D_a\phi_{IJ} + \epsilon_I\alpha, \\
\widetilde{Q}\alpha = -[c,\alpha] + \varepsilon^{IJ}\epsilon^K\left[\phi_{Ik},\psi_J\right] - \frac{1}{2}\epsilon^I D_a\eta_I, \\
\widetilde{Q}\phi_{IJ} = -[c,\phi_{IJ}] + \frac{1}{2}\left(\epsilon_I\eta_J + \epsilon_J\eta_I\right), \\
\widetilde{Q}\eta_I = -[c,\eta_I] + \varepsilon^{JK}\epsilon^M[\phi_{JM},\phi_{IK}], \\
\widetilde{Q}c = -c^2 + \epsilon^I\epsilon^J\phi_{IJ}.
\end{array} \tag{2.16}$$

in agreement with the transformation found in the works of [14, 13] and the nilpotence is

$$(\widetilde{Q})^2 \propto \delta_{\phi_{IJ}} \tag{2.17}$$

that is a infinitesimal transformation of ϕ_{IJ}. With the result of the previous section, we are ready to write down the Blau-Thompson action, which is the invariant Yang-Mills action, for the topological theory.

3 The Blau-Thompson action

The associated action for $N_T=2$, $D=4$ is the Witten action [8, 14, 15], described in $N_T=2$ by the Blau-Thompson action [12, 13], with gauge completely fixed in terms of the superfield. For the construction of this action, we wish a Lagrange multiplier that couples to the topological super-Yang-Mills so as to manifest its self-duality: $F=*F$. We then define a 2-form-superfield Lagrange multiplier, with the property of anti-self-duality and super-gauge covariant: $sK=-[C,K]$, such that

$$K(x,\theta) = k(x) + \theta^I k_I(x) + \frac{1}{2}\theta^2\kappa(x).$$

We still wish a quadratic term in the last component field of K. Still, we need a 0-form-superfield to complete the gauge-fixing for Ψ_I, which is defined as:

$$H_I(x,\theta) = h_I(x) + \theta^J h_{JI}(x) + \frac{1}{2}\theta^2\rho_I(x). \tag{3.1}$$

To fix the super-Yang-Mills gauge, we define a anti-ghost superfield of C, being a 0-form-superfield of fermionic nature

$$\overline{C}(x,\theta) = \overline{c}(x) + \theta^I\overline{c}_I(x) + \frac{1}{2}\theta^2\overline{c}_F(x), \tag{3.2}$$

[2]This name was given due to likeness of we treat of a linear gauge for ghost scalar fields.

associated to it; we define a 0-form-superfield Lagrange mulptiplier

$$B(x,\theta) = b(x) + \theta^I b_I(x) + \frac{1}{2}\theta^2\beta(x). \tag{3.3}$$

Their BRST tranformations are $s\overline{C} = B,\ sB = 0$, and in components reads

$$\begin{array}{lll} s\overline{c} = b, & s\overline{c}_I = b_I\,, & s\overline{c}_F = \beta, \\ sb = 0, & sb_I = 0, & s\beta = 0. \end{array} \tag{3.4}$$

Therefore the complete Blau-Thompson action on superspace, takes the form

$$S_{BT} = \int d^2\theta\{K * F + \zeta K * D^2_\theta K + \varepsilon^{IJ} H_I D_A * \Psi_J + s(\overline{C} d * A)\}, \tag{3.5}$$

with ζ being constant. In components, we have

$$\begin{aligned} S_{BT} \;\; = \;\; & \int\{\frac{1}{2}\kappa * f + \zeta\kappa * \kappa + \zeta\varepsilon^{IJ}(k * [\eta_I, k_J] + [k_J, \eta_I] * k) - \zeta\phi^{IJ}\phi_{IJ} k * k \\ & -\frac{1}{2}\varepsilon^{IJ} k_I * D_a\psi_J + \frac{1}{2}k * D_a\alpha + \frac{1}{4}k * \varepsilon^{IJ}[\psi_I, \psi_J] \\ & +\varepsilon^{IJ}(\frac{1}{2}\rho_I D_a * \psi_J + \frac{1}{2}h_{JI} D_a * \alpha - \frac{1}{2}\varepsilon^{KL} h_{KI} D_a * D_a\phi_{JL} \\ & +\frac{1}{2}h_I D_a * D_a\eta_J - \varepsilon^{KL} h_I D_a * [\psi_K, \phi_{JL}] - \frac{1}{2}[h_I, \psi_J] * \alpha \\ & -\frac{1}{2}\varepsilon^{KL}[\psi_K, h_I] * D_a\phi_{JL} + \frac{1}{2}\varepsilon^{KL}[\psi_K, h_{LI}] * \psi_J + [\alpha, h_I] * \psi_J) \\ & +\frac{1}{2}bd * B + \frac{1}{2}\varepsilon^{IJ} b_I d * \psi_J + \frac{1}{2}\beta d * a - \frac{1}{2}\overline{c} d * D_a c_F \\ & -\frac{1}{2}\varepsilon^{IJ}\overline{c} d * [\psi_J, c_J] - \frac{1}{2}\overline{c} d * [B, c] + \frac{1}{2}\varepsilon^{IJ}\overline{c}_I d * D_a c_J \\ & +\frac{1}{2}\varepsilon^{IJ}\overline{c}_I d * [\psi_J, c] - \frac{1}{2}\overline{c}_F d * D_a c\}. \end{aligned} \tag{3.6}$$

In the next section, we shall see, the Avdeev-Chizhov action in general Riemannian Manifold with the background metric.

4 Tensorial Matter in a General Riemannian Manifold

To couple the theory above to the Avdeev-Chizhov model, we start describing the Avdeev-Chizhov action through the complex self-dual field φ [6], initially written in the four-dimensional Minkowskian manifold, whose index are: $m, n, \ldots$. We write this action, according to the work of [6]

$$S_{matter} = \int d^4x\{(D^m\varphi_{mn})^\dagger(D_p\varphi^{pn}) + q(\varphi^\dagger_{mn}\varphi^{pn}\varphi^{\dagger mq}\varphi_{pq})\} \tag{4.1}$$

here q is a coupling constant of self-interaction, and the covariant derivative $D^m_a\varphi_{mn} = \partial^m\varphi_{mn} - [a^m, \varphi_{mn}]$; a^m is the Lie-algebra-valued gauge potential and we assume φ_{mn} to belong a given representating of the gauge group G. This action is invariant under the folowing transformations

$$\delta_G(\omega)a_m = D_m\omega,\ \delta_G(\omega)\varphi_{mn} = \varphi_{mn}\omega,\ \delta_G(\omega)\varphi^\dagger_{mn} = -\omega\varphi^\dagger_{mn}, \tag{4.2}$$

with φ given by

$$\varphi_{mn} = T_{mn} + i\widetilde{T}_{mn}, \tag{4.3}$$

which exhibit the properties $\varphi_{mn} = i\widetilde{\varphi}_{mn}$, $\widetilde{\widetilde{\varphi}}_{mn} = -\varphi_{mn}$, where the duality is defined by $\widetilde{\varphi}_{mn} = \frac{1}{2}\varepsilon_{mnpq}\varphi^{pq}$.

For treat this theory, in the general Riemannian Manifold as a topological theory, Geyer-Mülsch [4] rewriting the configurate field in a four-dimensional Riemannian manifold, endowed of the vierbein ${e_\mu}^m$ and a spin-connection ω_μ^{mn}, i.e., the tensorial matter read as $\varphi_{\mu\nu} = {e_\mu}^m {e_\nu}^n \varphi_{mn}$, where the action (4.1) is given by

$$S_{matter} = \int d^4x\sqrt{g}\{(\nabla_\mu\varphi^{\mu\nu})^\dagger(\nabla_\rho{\varphi^\rho}_\nu) + q(\varphi^\dagger_{\mu\nu}\varphi^{\rho\nu}\varphi^{\dagger\mu\lambda}\varphi_{\rho\lambda})\}. \tag{4.4}$$

In this 4-dimesional Riemannian manifold, we find the folowing properties:

$$\sqrt{g}\varepsilon_{\mu\nu\rho\lambda}\varepsilon^{mnpq} = {e_{[\mu}}^m {e_\nu}^n {e_\rho}^p {e_{\lambda]}}^q, \tag{4.5}$$

$${e_\mu}^m {e_\nu}^n g^{\mu\nu} = \eta^{mn}, \ {e_\mu}^m {e_\nu}^n \eta_{mn} = g_{\mu\nu}. \tag{4.6}$$

The covariant derivative in the Riemannian Manifold is now written in terms of the spin-connection:

$$\nabla_\mu = D_\mu + \omega_\mu, \tag{4.7}$$

where $\omega_\mu = \frac{1}{2}{\omega_\mu}^{mn}\sigma_{mn}$, being σ_{mn} the generator of the holonomy Euclidian group $SO(4)$ [?], alson we have: $D_\mu = (D_a)_\mu$.

5 Supersimetrization of the Avdeev-Chizhov Action

¿From now on, we can write the action (4.4) in terms of superfields, mentioning the conventions of the works [10, 8]. The superfield that accommodates the rank-two anti-symmetric tensorial matter field, is similar to the one defined in [7], being now expressed as a linear fermionic. This is defined as a rank two anti-symmetric tensor in the 4-dimensional Riemannian manifold, and with the topological fermionic index I, referring to the topological SUSY index:

$$\Sigma^I_{\mu\nu}(x,\theta) = \lambda^I_{\mu\nu}(x) + \theta^I\varphi_{\mu\nu}(x) + \frac{1}{2}\theta^2\zeta^I_{\mu\nu}(x), \tag{5.1}$$

where $\varphi_{\mu\nu}(x)$ is the Avdeev-Chizhov field. The super-manifold is composed by Riemannian manifold and the $N_T = 2$ topological manifold, because it, is not need to define the super-vierbein and super-spin-connection.

The superfield is defined under the SUSY transformations

$$Q_I\Sigma_{\mu\nu J} = \partial_I\Sigma_{\mu\nu J}, \tag{5.2}$$

and in components:

$$\begin{array}{l} Q_I\lambda_{\mu\nu J} = \varepsilon_{IJ}\varphi_{\mu\nu} \\ Q_I\varphi_{\mu\nu} = -\zeta_{\mu\nu I} \\ Q_I\zeta_{\mu\nu J} = 0 \end{array} \tag{5.3}$$

Based on the work of [6], we rewrite the BRST transformations, referring the non-Abelian Avdeev-Chizhov model, in terms of the transformations:

$$\begin{array}{rcl} s\varphi^i_{mn} &=& ic^a(T^a)^{ij}\varphi^j_{mn}, \\ s\varphi^{\dagger i}_{mn} &=& -ic^a\varphi^{\dagger j}_{mn}(T^a)^{ji}, \\ s(\nabla_m\varphi_{mn})^i &=& ic^a(T^a)^{ij}(\nabla_m\varphi_{mn})^j, \\ s(\nabla_m\varphi_{mn})^{\dagger i} &=& -ic^a(\nabla_m\varphi_{mn})^{\dagger j}(T^a)^{ji}, \end{array}$$

where (2.2) is the Lie algebra. We wish to write the BRST−transformation for a supergauge transformation, generalizing the transformations for the Avdeev-Chizhov fields, according to

$$\begin{array}{l} s(\Sigma^I_{\mu\nu}) = iC(\Sigma^I_{\mu\nu}), \\ s(\Sigma^I_{\mu\nu})^\dagger = iC(\Sigma^I_{\mu\nu})^\dagger, \end{array} \tag{5.4}$$

in components, we get:

$$\begin{array}{l} s\lambda^I_{\mu\nu} = ic\lambda^I_{\mu\nu}, \\ s\lambda^{\dagger I}_{\mu\nu} = -ic\lambda^{\dagger I}_{\mu\nu}, \\ s\varphi_{\mu\nu} = ic\varphi_{\mu\nu} + ic^I\lambda_{\mu\nu I}, \\ s\varphi^\dagger_{\mu\nu} = -ic\varphi^\dagger_{\mu\nu} - ic^I\lambda^\dagger_{\mu\nu I}, \\ s\zeta^I_{\mu\nu} = ic\zeta^I_{\mu\nu} - ic^I\varphi_{\mu\nu} + ic_F\lambda^I_{\mu\nu}, \\ s\zeta^{\dagger I}_{\mu\nu} = -ic\zeta^{\dagger I}_{\mu\nu} + ic^I\varphi^\dagger_{\mu\nu} - ic_F\lambda^{\dagger I}_{\mu\nu}. \end{array} \tag{5.5}$$

The super-derivative of the (5.1) is covariant under the BRST−transformation, where now, the covariant super-derivative is

$$\mathcal{D}_\mu(\cdot) = (D_A)_\mu(\cdot) + \omega_\mu(\cdot) = \nabla_\mu(\cdot) + \theta^I[\psi_{I\,\mu}, (\cdot)] + \frac{1}{2}\theta^2[\alpha_\mu, (\cdot)],$$

acoording to (4.7), then gives

$$\begin{array}{rcl} s(\mathcal{D}_\mu\Sigma^I_{\mu\nu}) & = & C\,(\mathcal{D}_\mu\Sigma^I_{\mu\nu}), \\ s(D_I\Sigma^I_{\mu\nu}) & = & C\,(D_I\Sigma^I_{\mu\nu}), \end{array}$$

where we chose here, $s\omega_\mu = 0$.

By now performing BRST−transformations on the components that survive in the $N_T = 2$ Wess-Zumino gauge (2.15), we find:

$$\begin{array}{l} \widetilde{Q}\lambda_{\mu\nu I} = \epsilon^J\varepsilon_{JI}\varphi_{\mu\nu} + ic\lambda_{\mu\nu I}, \\ \widetilde{Q}\lambda^\dagger_{\mu\nu I} = \epsilon^J\varepsilon_{JI}\varphi^\dagger_{\mu\nu} - ic\lambda^\dagger_{\mu\nu I}, \\ \widetilde{Q}\varphi_{\mu\nu} = ic\varphi_{\mu\nu} + i\epsilon^I\zeta_{\mu\nu I} + i\epsilon^I\phi_{IJ}\lambda^J_{\mu\nu}, \\ \widetilde{Q}\varphi^\dagger_{\mu\nu} = -ic\varphi^\dagger_{\mu\nu} - i\epsilon^I\zeta^\dagger_{\mu\nu I} - i\epsilon^I\phi_{IJ}\lambda^{\dagger J}_{\mu\nu}, \\ \widetilde{Q}\zeta_{\mu\nu I} = ic\zeta_{\mu\nu I} - i\epsilon^J\phi_{JI}\varphi_{\mu\nu} + i\epsilon^J\eta_J\lambda_{\mu\nu I}, \\ \widetilde{Q}\zeta^\dagger_{\mu\nu I} = -ic\zeta^\dagger_{\mu\nu I} + i\epsilon^J\phi_{JI}\varphi^\dagger_{\mu\nu} - i\epsilon^J\eta_J\lambda^\dagger_{\mu\nu I}. \end{array} \tag{5.6}$$

in agreement to (2.17).

We build up rank two anti-symmetric tensorial matter field on superspace formulation, leaving the superfield with the same properties as shown in [7], this is invariant under gauge transformations (5.6) and SUSY transformations. The kinetic term is proposed as

$$S_{kin} = \int d^4x d^2\theta \sqrt{g}\varepsilon^{IJ}\{(\mathcal{D}_\mu\Sigma^{\mu\nu}_I)^\dagger(\mathcal{D}_\rho\Sigma^\rho{}_{\nu\,J})\}$$

In components, we get:

$$\begin{array}{rcl} S_{kin} & = & \displaystyle\int d^4x\sqrt{g}\{\frac{1}{2}(\nabla_\mu\varphi^{\mu\nu})^\dagger(\nabla_\rho\varphi^\rho{}_\nu) + \frac{1}{2}\varepsilon^{IJ}(\nabla_\mu\lambda^{\mu\nu}_I)^\dagger(\nabla_\rho\zeta^\rho{}_{\nu\,J}) \\ & & \displaystyle +\frac{1}{2}\varepsilon^{IJ}(\nabla_\mu\zeta^{\mu\nu}_I)^\dagger(\nabla_\rho\lambda^\rho{}_{\nu\,J}) + (\nabla_\mu\varphi^{\mu\nu})^\dagger[\psi^I_\rho, \lambda^\rho{}_{\nu\,I}] \\ & & +[\psi_{\mu\,J}, \varphi^{\dagger\mu\nu}](\nabla_\rho\varphi^\rho{}_\nu) + \varepsilon^{IJ}(\nabla_\mu\lambda^{\mu\nu}_I)^\dagger\left([\alpha_\rho, \lambda^\rho{}_{\nu\,J}] + [\psi_{\rho\,J}, \varphi^\rho{}_\nu]\right) \\ & & +\varepsilon^{IJ}\left([\alpha_\mu, \lambda^{\dagger\mu\nu}_I] + [\psi_{\mu\,J}, \varphi^{\dagger\mu\nu}]\right)(\nabla_\rho\lambda^\rho{}_{\nu\,J})\} \end{array} \tag{5.7}$$

The interaction term, it has a small contribution of having second derivative, in the Grassmann coordinates, it should also be invariant front the gauge transformations (5.6) and supersymmetry. We defined this as

$$S_{int} = \int d^4x d^2\theta \sqrt{g}\{\varepsilon^{IJ}\varepsilon^{LM}(\Sigma_{\mu\nu\, I})^\dagger D^K(\Sigma^{\rho\nu}_J)(\Sigma^{\mu\lambda}_L)^\dagger D_K(\Sigma_{\rho\lambda\, M})\} \tag{5.8}$$

where $D_K(\cdot) = \partial_K(\cdot) + [E_K, (\cdot)]$, in components

$$\begin{aligned} S_{int} &= \frac{1}{2}\int d^4x\sqrt{g}\{\varphi^\dagger_{\mu\nu}\varphi^{\rho\nu}\varphi^{\dagger\mu\lambda}\varphi_{\rho\lambda} - \varepsilon^{IJ}[(\lambda^\dagger_{\mu\nu\, I}\zeta^{\rho\nu}_J + \zeta^\dagger_{\mu\nu\, I}\lambda^{\rho\nu}_J)\varphi^{\dagger\mu\lambda}\varphi_{\rho\lambda} \\ &-\varphi^\dagger_{\mu\nu}\varphi^{\rho\nu}(\lambda^{\dagger\mu\lambda}_I\zeta_{\rho\lambda\, J} + \zeta^{\dagger\mu\lambda}_I\lambda_{\rho\lambda\, J})] + \varepsilon^{IJ}\varepsilon^{KL}[\lambda^\dagger_{\mu\nu\, I}\zeta^{\rho\nu}_J(\lambda^{\dagger\mu\lambda}_K\zeta_{\rho\lambda\, L} + \zeta^{\dagger\mu\lambda}_K\lambda_{\rho\lambda\, L}) \\ &+\zeta^\dagger_{\mu\nu\, I}\lambda^{\rho\nu}_J(\lambda^{\dagger\mu\lambda}_K\zeta_{\rho\nu\, L} + \zeta^{\dagger\mu\lambda}_K\lambda_{\rho\nu\, L}) + \lambda^\dagger_{\mu\nu\, I}\lambda^{\rho\nu}_J[\eta_L, \lambda^{\mu\lambda}_K]\varphi_{\rho\lambda} && (5.9)\\ &-\lambda^\dagger_{\mu\nu\, I}\varphi^{\rho\nu}_J\eta_L\lambda^{\dagger\mu\lambda}_K\lambda_{\rho\lambda} + \varphi^\dagger_{\mu\nu}\lambda^{\rho\nu}_J\eta_I\lambda^{\dagger\mu\lambda}_K\lambda_{\rho\lambda\, L} - \lambda^\dagger_{\mu\nu\, I}\lambda^{\rho\nu}_J\eta_L\lambda^{\dagger\mu\lambda}_K\varphi_{\rho\lambda} && (5.10)\\ &-\lambda^\dagger_{\mu\nu\, I}\lambda^{\rho\nu}_J\eta_K\varphi^{\dagger\mu\lambda}\lambda_{\rho\lambda\, L} + \lambda^\dagger_{\mu\nu\, I}\lambda^{\rho\nu}_J\phi^{MN}\phi_{MN}\lambda^{\dagger\mu\lambda}_K\lambda_{\rho\lambda\, L}]\} && (5.11) \end{aligned}$$

The total action is being determinad for: $S_{Kin} + qS_{Int}$, such that

$$S_{AC} = -\int d^2\theta\sqrt{g}\{\varepsilon^{IJ}(\mathcal{D}_\mu\Sigma^{\mu\nu}_I)^\dagger(\mathcal{D}_\rho\Sigma^\rho{}_{\nu\, J}) + q\varepsilon^{IJ}\varepsilon^{LM}(\Sigma_{\mu\nu\, I})^\dagger D^K(\Sigma^{\rho\nu}_J)(\Sigma^{\mu\lambda}_L)^\dagger D_K(\Sigma_{\rho\lambda\, M})\} \tag{5.12}$$

where q is a quartic coupling constant self-duality. In components we have the Avdeev-Chizhov action more coming terms of the supersimetrization of the model

$$\begin{aligned} S_{AC} &= \int d^4x\sqrt{g}\{\frac{1}{2}(\nabla_\mu\varphi^{\mu\nu})^\dagger(\nabla_\rho\varphi^\rho{}_\nu) + \frac{1}{2}\varepsilon^{IJ}(\nabla_\mu\lambda^{\mu\nu}_I)^\dagger(\nabla_\rho\zeta^\rho{}_{\nu\, J}) \\ &+\frac{1}{2}\varepsilon^{IJ}(\nabla_\mu\zeta^{\mu\nu}_I)^\dagger(\nabla_\rho\lambda^\rho{}_{\nu\, J}) + (\nabla_\mu\varphi^{\mu\nu})^\dagger[\psi^I_\rho, \lambda^\rho{}_{\nu\, I}] \\ &+[\psi_{\mu\, J}, \varphi^{\dagger\mu\nu}](\nabla_\rho\varphi^\rho{}_\nu) + \varepsilon^{IJ}(\nabla_\mu\lambda^{\mu\nu}_I)^\dagger\left([\alpha_\rho, \lambda^\rho{}_{\nu\, J}] + [\psi_{\rho\, J}, \varphi^\rho{}_\nu]\right) \\ &+\varepsilon^{IJ}\left([\alpha_\mu, \lambda^{\dagger\mu\nu}_I] + [\psi_{\mu\, J}, \varphi^{\dagger\mu\nu}]\right)(\nabla_\rho\lambda^\rho{}_{\nu\, J}) \\ &+q(\varphi^\dagger_{\mu\nu}\varphi^{\rho\nu}\varphi^{\dagger\mu\lambda}\varphi_{\rho\lambda} - \varepsilon^{IJ}[(\lambda^\dagger_{\mu\nu\, I}\zeta^{\rho\nu}_J + \zeta^\dagger_{\mu\nu\, I}\lambda^{\rho\nu}_J)\varphi^{\dagger\mu\lambda}\varphi_{\rho\lambda} \\ &-\varphi^\dagger_{\mu\nu}\varphi^{\rho\nu}(\lambda^{\dagger\mu\lambda}_I\zeta_{\rho\lambda\, J} + \zeta^{\dagger\mu\lambda}_I\lambda_{\rho\lambda\, J})] + \varepsilon^{IJ}\varepsilon^{KL}[\lambda^\dagger_{\mu\nu\, I}\zeta^{\rho\nu}_J(\lambda^{\dagger\mu\lambda}_K\zeta_{\rho\lambda\, L} + \zeta^{\dagger\mu\lambda}_K\lambda_{\rho\lambda\, L}) \\ &+\zeta^\dagger_{\mu\nu\, I}\lambda^{\rho\nu}_J(\lambda^{\dagger\mu\lambda}_K\zeta_{\rho\nu\, L} + \zeta^{\dagger\mu\lambda}_K\lambda_{\rho\nu\, L}) + \lambda^\dagger_{\mu\nu\, I}\lambda^{\rho\nu}_J[\eta_L, \lambda^{\mu\lambda}_K]\varphi_{\rho\lambda} \\ &-\lambda^\dagger_{\mu\nu\, I}\varphi^{\rho\nu}_J\eta_L\lambda^{\dagger\mu\lambda}_K\lambda_{\rho\lambda} + \varphi^\dagger_{\mu\nu}\lambda^{\rho\nu}_J\eta_I\lambda^{\dagger\mu\lambda}_K\lambda_{\rho\lambda\, L} - \lambda^\dagger_{\mu\nu\, I}\lambda^{\rho\nu}_J\eta_L\lambda^{\dagger\mu\lambda}_K\varphi_{\rho\lambda} \\ &-\lambda^\dagger_{\mu\nu\, I}\lambda^{\rho\nu}_J\eta_K\varphi^{\dagger\mu\lambda}\lambda_{\rho\lambda\, L} + \lambda^\dagger_{\mu\nu\, I}\lambda^{\rho\nu}_J\phi^{MN}\phi_{MN}\lambda^{\dagger\mu\lambda}_K\lambda_{\rho\lambda\, L}])\} && (5.13) \end{aligned}$$

that is conformal invariant. Therefore the total gauge invariant action, is write as: $S_{AC} + S_{BT}$.

The $Q-$exactness of the total action above is also contempled for $N_T = 2$ SUSY as in [4], this because the fermionic volume element $Q^2 \propto Q_1Q_2$, that means the exactness in the charge Q_1, Q_2 of this action. This proof for $N_T = 1$ and N_T any, is given in the works [10]. Acoording to Blau-Thompson in their review [16], the energy-momentum stress tensor $\Theta_{\mu\nu}$ is also $Q-$exact,

$$\mathcal{O} = \langle 0|\Theta_{\mu\nu}|0\rangle = \langle 0|\frac{2}{\sqrt{g}}\frac{\delta}{\delta g^{\mu\nu}}(S_{BT} + S_{AC})|0\rangle = \langle 0|Q\,\Upsilon_{\mu\nu}|0\rangle \tag{5.14}$$

guaranteeing the topological nature of the theory, where we shall just use the Avdeev-Chizhov kinetic term, because the interaction term carries the coupling constant q, which is irrelevant in the obtaining of the theory's observables [4].

Concluding Remarks

The main goal of this paper is the settlement of a topological superspace formulation for the investigation of the coupling between the rank-2 Avdeev-Chizhov matter field and Yang-Mills fields, given us a $Q-$exact Energy-momentum observables, reminding that a lot of observables class, and it is what we are trying to do in the moment, classify them [18].

It is worthwhile to draw the attention here to the shift symmetry to which we detect the ghost caracter of the Avdeev-Chizhov field. This seems to be a non-trivial remark. On the other hand, it is known that there appears a ghost mode in the spectrum of excitations of our tensor matter field. The connection between these two observations remain to be clarified. The fact that the Avdeec-Chizhov field manifest itself as a ghost guide future developments in the guest of a consistent mechanism to systematically decouple the unphysical mode mentioned above.

We are also trying to embed the tensor field in the framework of gauge theory with Lorentz symmetry breaking [17]. We expect that this breaking may select the right ghost mode present among the two spin 1 components of the Avdeev-Chizhov field.

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Appendix

A Conventions

The topological fermionic index: $I = 1, 2$, is lowered and rised by the anti-symmetric Levi-Civita tensor: $\varepsilon_{IJ}, \varepsilon^{IJ}$, with $\varepsilon^{12} = -\varepsilon_{12} = 1$. The $\theta-$coordinates definitions: $\theta^I = \varepsilon^{IJ}\theta_J,\ \theta_I = \varepsilon_{IJ}\theta^J$, the quadratic form are:

$$\theta^2 = \theta^I\theta_I = -\theta_I\theta^I,\ \theta^I\theta^J = -\frac{1}{2}\varepsilon^{IJ}\theta^2,\ \theta_I\theta_J = \frac{1}{2}\varepsilon_{IJ}\theta^2,$$

with $\varepsilon_{IK}\varepsilon^{KJ} = \delta_I{}^J$. The derivatives in the θ^I coordinates, are defined by

$$\partial_I = \frac{\partial}{\partial\theta^I},\ \partial^I = \frac{\partial}{\partial\theta_I}\ \ and\ \ \partial_I\theta^J \stackrel{Def}{=} \delta_I{}^J, \tag{A.1}$$

$$\partial_I f(x,\theta) = \varepsilon_{IJ}\partial^J f(x,\theta),$$

with $f(x,\theta)$ a any superfunction. Deriving the $\theta-$coordinates, gives

$$\partial^I\theta^J = -\varepsilon^{IJ},\ \ \partial_I\theta_J = -\varepsilon_{IJ} \tag{A.2}$$

A superfield is expanded of the following form

$$F(x,\theta) = f(x) + \theta^I f_I(x) + \frac{1}{2}\theta^2 f_F, \tag{A.3}$$

obeying the transformation $Q_I F(x,\theta) \stackrel{Def}{=} \partial_I F(x,\theta)$. In components is waited that

$$Q_I f = f_I \ ; \ \ Q_I f_J = -\varepsilon_{IJ} f_F \ ; \ \ Q_I f_F = 0. \tag{A.4}$$

Caracteristics table of the superconnection fields:

$Charge \backslash Geometric\, fields$	ϵ^I	a	ψ^I	α	χ^I	ϕ^{IJ}	η^I	c	c^I	c_F
s	-1	0	1	2	1	2	3	0	1	2
g	1	0	0	0	0	0	0	1	1	1
p	0	1	1	1	0	0	0	0	0	0
P_{grs}	$+$	$-$	$+$	$-$	$-$	$+$	$-$	$-$	$+$	$-$

(A.5)

where have s: susy number, g: ghost number, p: degree form, P_{grs}: Grassmann parity.

B Topological Grassmannian integration

The definition of integration in this topological SUSY representation is

$$\int d\theta^I \stackrel{Def}{=} \partial_I. \tag{B.1}$$

This result is applied to a superfunction $f(x,\theta)$, so that the volume element is

$$\int d^2\theta f(x,\theta) \stackrel{Def}{=} \frac{1}{4}\varepsilon^{IJ}\partial_I\partial_J f(x,\theta), \tag{B.2}$$

therefore the square of the supersymmetric charge operator (shift operator), is definide by:

$$Q^2 = Q^I Q_I = \partial^I \partial_I = 4\int d^2\theta,$$

which is a volume element too.

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