CBPF-NF-039/97

A Phenomenological Analysis of Non-resonant Charm Meson Decays

I. Bediaga, C. Göbel

Centro Brasileiro de Pesquisas Físicas,
Rua Dr. Xavier Sigaud 150
22290 – 180 – Rio de Janeiro, RJ, Brazil

R. Méndez–Galain

Instituto de Física,
Facultad de Ingeniería,
CC 30
CP 11000 Montevideo, Uruguay

We analyse the consequences of the usual assumption of a constant function to fit non-resonant decays from experimental Dalitz plot describing charmed meson decays. We first show, using the $D^+ \to \bar{K}^0\pi^+\pi^0$ decay channel as an example, how an inadequate extraction of the non-resonant contribution could yield incorrect measurements for the resonant channels. We analyse how the correct study of this decay will provide a test for the validity of factorization in D meson decays. Finally, we show how form factors could be extracted from non-resonant decays. In particular we discuss the form factor that can be measured from the $D_s^+ \to \pi^-\pi^+\pi^+$ decay. We emphasize its relevance for the study of the decay $\tau \to \nu_\tau 3\pi$ and the extraction of the a_1 meson width.

Key-words: Weak decays; Charm meson; Dalitz plot.

PACS: 13.25.Ft.

I. INTRODUCTION

Many body charm meson decays seem to be largely dominated by intermediate resonances. Experimental data have been studied using the powerful Dalitz plot technique which brings information on both the kinematics and the dynamics of the decay [1].

In a D meson three body decay, the intermediate resonant channels and the direct non-resonant one contribute to the final state. The Dalitz plot can thus present a complex interference of all these contributions. To extract them from the plot, one has to use appropriate fitting functions for each channel. Since the discovery of D mesons, data are fitted using a Breit–Wigner function for each resonance amplitude [2] while the non-resonant (NR) contribution has usually been considered as a phase space independent, constant function.

In a recent paper [3], we have shown that the last hypothesis cannot be safely considered for the study of D meson decays as it proceeds via a weak interaction. Indeed, in weak interactions at the partonic level helicity plays a central role; thus one could expect the amplitude of the reaction to have important variations within the phase space. In ref. [3], the NR contribution to $D^+ \to K^-\pi^+\pi^+$ decay has been evaluated using factorization and an effective hamiltonian for the partonic interaction [4]. According to this calculation, the NR contribution does have significant variations along the phase space of the reaction.

To extract data from the Dalitz plot, an adequate parametrization of the NR contribution is crucial. A correct extraction of the NR contribution could yield important information on the physics involving the decay; particularly, it can bring direct measurements of some form factors. Moreover, using an inadequate parametrization for the NR contribution, the whole decay pattern could be wrong: we could be ascribing to a given resonance those variations corresponding to the NR part.

The purpose of this work is to present two examples concerning these ideas. First, we will analyse the decay $D^+ \to \bar{K}^0\pi^+\pi^0$, since its $\bar{K}^*(892)^0\pi^0$ partial decay width seems to be too large. Second, we will show how the $D_s^+ \to \pi^-\pi^+\pi^+$ decay is particularly well suited to extract a form factor which is relevant in τ and a_1-meson physics.

II. THE $D^+ \to \bar{K}^0\pi^+\pi^0$ DECAY

The resonant decay $D^+ \to \bar{K}^*(892)^0\pi^+$ has been measured in two different ways, according to the detected final state: $B(D^+ \to \bar{K}^{*0}\pi^+) \times B(\bar{K}^{*0} \to \bar{K}^0\pi^0)$, which is extracted from the Dalitz plot of the decay $D^+ \to \bar{K}^0\pi^+\pi^0$; and $B(D^+ \to \bar{K}^{*0}\pi^+) \times B(\bar{K}^{*0} \to K^-\pi^+)$, extracted from the decay $D^+ \to K^-\pi^+\pi^+$. MarkIII reported [5] an "apparent discrepancy" between the two measurements: $B(D^+ \to \bar{K}^{*0}\pi^+) = (5.9 \pm 1.9 \pm 2.5)\%$ when the final state is $\bar{K}^0\pi^0\pi^+$ and $B(D^+ \to \bar{K}^{*0}\pi^+) = (1.8 \pm 0.2 \pm 1.0)\%$ when the final state is $K^-\pi^+\pi^+$.

The last measurement has been confirmed by other experiments [6–8] while the decay $D^+ \to \bar{K}^0\pi^+\pi^0$ has only been measured by MarkIII. It is then natural to think on a possible systematic error in the extraction of the $\bar{K}^*(892)^0$ resonance from the $D^+ \to \bar{K}^0\pi^+\pi^0$ Dalitz plot. One possibility is that events coming from the NR contribution to the decay could have been incorrectly considered as originated from the $\bar{K}^*(892)^0$ resonant channel; thus, the latter has been artificially enhanced.

To support this hypothesis, we are presenting here a calculation for the NR part of the decay $D^+ \to \bar{K}^0\pi^+\pi^0$ which shows up precisely an important bump near the $\bar{K}^*(892)^0$ peak.

The calculation is based in factorization [10] and in an effective Hamiltonian [4,11] for the partonic interaction as in ref. [3].

The effective partonic Hamiltonian is [4,11]

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \cos^2\theta_c [a_1 : (\bar{s}c)(\bar{u}d) : + a_2 : (\bar{s}d)(\bar{u}c) :] \tag{1}$$

where $(\bar{q}q')$ is a short-hand notation for $\bar{q}\gamma^\mu(1-\gamma_5)q'$. The coefficients a_1 and a_2 characterize the contribution of the effective charged and neutral currents respectively, which include short-distance QCD effects. Figure 1 shows the six diagrams contributing to the amplitude $\mathcal{M}^{NR}_{D^+\to\bar{K}^0\pi^+\pi^0}$. Using factorization one obtains the following decomposition [12] for the hadronic amplitude of the non-resonant decay

$$\begin{aligned}\mathcal{M}^{NR}_{D^+\to\bar{K}^0\pi^0\pi^+} = \frac{G_F}{\sqrt{2}} \cos^2\theta_c [& a_1 \langle \bar{K}^0\pi^0|\bar{s}c|D^+\rangle\langle\pi^+|\bar{u}d|0\rangle \\ & + a_2\langle\bar{K}^0|\bar{s}d|0\rangle\langle\pi^0\pi^+|\bar{u}c|D^+\rangle \quad + \quad a_1\langle\bar{K}^0|\bar{s}c|D^+\rangle\langle\pi^0\pi^+|\bar{u}d|0\rangle \\ & + a_2\langle\bar{K}^0\pi^0|\bar{s}d|0\rangle\langle\pi^+|\bar{u}c|D^+\rangle] \ .\end{aligned} \tag{2}$$

Following ref. [3], the four contributions can be written as

$$\langle\bar{K}^0\pi^0|\bar{s}c|D^+\rangle\langle\pi^+|\bar{u}d|0\rangle = \frac{1}{\sqrt{2}} f_\pi m_\pi^2 F_4^{\bar{K}^0\pi^0}(m^2_{\pi^0\pi^+}, m^2_{\bar{K}^0\pi^+}) \quad , \tag{3}$$

$$\langle\pi^0\pi^+|\bar{u}c|D^+\rangle\langle\bar{K}^0|\bar{s}d|0\rangle = \frac{2}{\sqrt{2}} f_K m^2_{\bar{K}^0} F_4^{\pi^+\pi^0}(m^2_{\bar{K}^0\pi^+}, m^2_{\bar{K}^0\pi^0}) \quad , \tag{4}$$

$$\begin{aligned}\langle\bar{K}^0|\bar{s}c|D^+\rangle\langle\pi^0\pi^+|\bar{u}d|0\rangle = \frac{2}{\sqrt{2}} \{ & F^1_{DK}(m^2_{\pi^0\pi^+}) f^+_{\pi^0\pi^+}(m^2_{\pi^0\pi^+}) (m^2_{\bar{K}^0\pi^0} - m^2_{\bar{K}^0\pi^+}) \quad + \\ & [F^1_{DK}(m^2_{\pi^0\pi^+}) f^+_{\pi^0\pi^+}(m^2_{\pi^0\pi^+}) - F^0_{DK}(m^2_{\pi^0\pi^+}) f^0_{\pi^0\pi^+}(m^2_{\pi^0\pi^+})] \frac{(m^2_D - m^2_{\bar{K}^0})(m^2_{\pi^+} - m^2_{\pi^0})}{m^2_{\pi^0\pi^+}} \} \ ,\end{aligned} \tag{5}$$

and

$$\begin{aligned}\langle\pi^+|\bar{u}c|D^+\rangle\langle\bar{K}^0\pi^0|\bar{s}d|0\rangle = \frac{1}{\sqrt{2}} \{ & F^1_{D\pi}(m^2_{\bar{K}^0\pi^0}) f^+_{K\pi}(m^2_{\bar{K}^0\pi^0}) (m^2_{\pi^0\pi^+} - m^2_{\bar{K}^0\pi^+}) \quad + \\ & [F^1_{D\pi}(m^2_{\bar{K}^0\pi^0}) f^+_{K\pi}(m^2_{\bar{K}^0\pi^0}) - F^0_{D\pi}(m^2_{\bar{K}^0\pi^0}) f^0_{K\pi}(m^2_{\bar{K}^0\pi^0})] \frac{(m^2_D - m^2_{\pi^+})(m^2_{\bar{K}^0} - m^2_{\pi^0})}{m^2_{\bar{K}^0\pi^0}} \} \ .\end{aligned} \tag{6}$$

We have introduced the 3 invariants $m^2_{\bar{K}^0\pi^+} \equiv (p_{\bar{K}^0} + p_{\pi^+})^2$, $m^2_{\bar{K}^0\pi^0} \equiv (p_{\bar{K}^0} + p_{\pi^0})^2$ and $m^2_{\pi^0\pi^+} \equiv (p_{\pi^0} + p_{\pi^+})^2$ and use has been made of the identity

$$m^2_{D^+} + m^2_{\bar{K}^0} + m^2_{\pi^+} + m^2_{\pi^0} = m^2_{\bar{K}^0\pi^+} + m^2_{\bar{K}^0\pi^0} + m^2_{\pi^0\pi^+} \ . \tag{7}$$

The $1/\sqrt{2}$ factors come from the π^0 wave functions. The ten form factors originate from the hadronic matrix elements [3]; we will be back to them later on.

Diagrams (a), (b) and (e) of Fig. 1 exhibit an external light pseudoscalar meson (P). This yields a contribution proportional to $f_P m_P^2$ as one can see from eqs. (3) and (4). The

other diagrams, i.e., (c), (d) and (f), produce contributions proportional to m_D^2 as one can see from eqs. (5) and (6), together with eq. (7). Thus the two first contributions in eq. (2) can be safely neglected in favor of the last two. Moreover, the second term in eq. (5) can be neglected as it is proportional to $(m_{\pi^+}^2 - m_{\pi^0}^2)$.

The eight form factors entering in eqs. (5) and (6) are written as

$$F_{DP}^J(q^2) = \frac{F_{DP}^J(0)}{(1 - q^2/m_{DP,J}^2)} \tag{8}$$

where $J = 0$ or 1 and $P = K$ or π, and

$$f_{AB}^i(q^2) = f_{AB}^i(0)\ (1 + \lambda_{AB}^i q^2/m_\pi^2) \tag{9}$$

where $i = +$ or 0 and $AB = K\pi$ or $\pi^+\pi^0$. According to the remark above, only six of them contribute to our calculation. Five of these form factors have been either measured from semileptonic decays, or calculated using lattice QCD or different quark models. There are no major discrepancies in the literature [13]: $F_{D\bar{K}^0}^1(0) = 0.75 \pm 0.1$, $m_{D\bar{K}^0}^1 = 2.0 \pm 0.2$; $F_{D\pi}^1(0) = 0.75 \pm 0.15$, $m_{D\pi}^1 = 2.1 \pm 0.2$; $F_{D\pi}^0(0) = 0.75 \pm 0.15$, $m_{D\pi}^0 = 2.2 \pm 0.2$; $f_{\bar{K}^0\pi^0}^+(0) = 0.7 \pm 0.1$, $\lambda_{\bar{K}^0\pi^0}^+ = 0.028 \pm 0.002$; $f_{\bar{K}^0\pi^0}^0(0) = 0.7 \pm 0.1$, $\lambda_{\bar{K}^0\pi^0}^0 = 0.004 \pm 0.007$. The sixth form factor entering in our calculation, $f_{\pi^+\pi^0}^+(q^2)$, has neither been measured nor obtained using lattice calculations. One can only hint $f_{\pi^+\pi^0}^+(0)$ calculating [14] the decay width $\pi^+ \to \pi^0 e^+ \nu_e$ and comparing it with experiment, to get $f_{\pi^+\pi^0}^+(0) = 1.4$.

Finally, the only measurement we have for the effective parameters a_1 and a_2 come from the fit of two body charm meson decays. It has been found [15] $a_1 = 1.26 \pm 0.10$ and $a_2 = -0.51 \pm 0.10$.

We have performed a Monte Carlo simulation of the NR contribution to the decay $D^+ \to \bar{K}^0\pi^+\pi^0$ using eqs. (2), (5) and (6). Figure 2 presents the Dalitz plot corresponding to the NR contribution to the decay, according to the calculation presented above. Figure 3 shows the density of events as a function of the invariant variable $m_{\bar{K}^0\pi^0}^2$. It presents a pronounced bump centered in $m_{\bar{K}^0\pi^0}^2 \approx 0.65$ GeV^2. The figures shown have been obtained using the central values of all the parameters presented above. The bump remains unchanged under variation of these parameters within the region allowed by experiment and lattice calculation and a very large variations for the unknown parameters defining the $f_{\pi^+\pi^0}^+(q^2)$ form factor. The bump is also unchanged within a large variation of the ratio a_2/a_1.

This strong stability of the bump in the simulation results shown in Fig. 3 is due to the following: it turns out that if the ratio $|a_2/a_1|$ is not very large (in fact, smaller than about 2.5), then the contribution of eq. (5) largely drives the behavior of the $m_{\bar{K}^0\pi^0}^2$ distribution. Thus, we can write

$$\mathcal{M}_{D^+ \to \bar{K}^0\pi^0\pi^+}^{NR} \propto F_{DK}^{1-}(m_{\pi^0\pi^+}^2) f_{\pi^0\pi^+}^+(m_{\pi^0\pi^+}^2)\ (m_{\bar{K}^0\pi^0}^2 - m_{\bar{K}^0\pi^+}^2) \tag{10}$$

Since the form factors in (10) depend only on $m_{\pi^0\pi^+}^2$, the $m_{\bar{K}^0\pi^0}^2$ distribution of the events is thus almost independent of the various poorly known quantities associated with the decay. For comparison, we present in Figure 4 the $m_{\bar{K}^0\pi^0}^2$ distribution of events when no dynamics is assumed for the NR decay, i.e., $\mathcal{M}^{NR} = \text{const}$. The bump in Figure 3 is thus a robust signature of a factorization-based calculation.

However, as some non-perturbative QCD effects – as final state interactions and soft gluon exchange – have been neglected, it is possible that factorization does not suffice to describe the decay. In the extreme case where the non-perturbative effects dominate the decay, the structure predicted above can be washed out. Thus, the experimental determination of a bump in the NR contribution to the decay $D^+ \to \bar{K}^0\pi^+\pi^0$ centered at $m^2_{\bar{K}^0\pi^0} \approx 0.65$ GeV^2 would be a test for the validity of factorization in D decays.

The bump predicted by this calculation lies near the peak one expects for the Breit-Wigner distribution corresponding to the $\bar{K}^*(892)^0$ resonance. If non-factorizable terms does not completely eliminate the bump, many events originated from the NR decay $D^+ \to \bar{K}^0\pi^+\pi^0$ have been probably incorrectly ascribed to the $D^+ \to \bar{K}^*(892)^0\pi^+$ resonant channel.

For completeness, we present in Figure 5 the $m^2_{K^-\pi^+}$ distribution of events for the NR part of the decay $D^+ \to K^-\pi^+\pi^+$. We use the amplitude we obtained in ref. [3]. There is no bump near the the $\bar{K}^*(892)^0$ squared mass. It looks more like a simple phase space distribution, as that of Figure 4.

Thus, the difference between the NR contributions of the decays $D^+ \to K^-\pi^+\pi^+$ and $D^+ \to \bar{K}^0\pi^+\pi^0$ could explain the different values reported for the decay $D^+ \to K^*(892)^0\pi^+$ according to the final state. It is an example of the indirect consequences of assuming inadequately a constant NR function to fit data.

III. THE $D_S^+ \to \pi^-\pi^+\pi^+$ DECAY

A correct extraction of the NR contribution to a given charmed many body decay could also have other important advantages. As we have shown above, the NR contribution to a given heavy meson decay is written in terms of various form factors. Thus, its correct extraction from the Dalitz plot could also be a way to measure those form factors within the whole phase space of the reaction.

Two problems arise here. First, one has to accept that non-factorizable effects are small, so that the expression of the NR amplitude in terms of the form factors can be simply obtained using factorization hypothesis — as we did above and in ref. [3]. Second, even assuming the validity of factorization, those expressions are products of form factors and it is thus complicated to extract separately each of them.

We present here an example in which these two problems are supposed to be not important. It is the case of the decay $D_s^+ \to \pi^-\pi^+\pi^+$. The main reason is that there is only one diagram contributing to the decay and it is an annihilation diagram. It is shown in Fig. 6. In this case, following Bjorken ideas [16], factorization looks natural. One thus expects the decay to be simply described by:

$$\mathcal{M}^{NR}_{D_s^+ \to \pi^+\pi^+\pi^-} = \frac{G_F}{\sqrt{2}} \cos^2\theta_c a_1 \langle 0|A^\mu|D_s^+\rangle \langle \pi^+\pi^+\pi^-|A_\mu|0\rangle \tag{11}$$

where

$$\langle 0|A^\mu|D_s^+\rangle = -if_{D_s} p_D^\mu \tag{12}$$

and the second matrix element can be decomposed in four form factors [17]. The only remaining term is the axial spin 0 one, i.e., similar to the one appearing in eqs. (3) and (4). One obtains [18]

$$\mathcal{M}_{D_s^+ \to \pi^+\pi^+\pi^-} = -i\frac{G_F}{\sqrt{2}}\cos^2\theta_c a_1 m_{D_s}^2 f_{D_s} F_4(m^2_{\pi^-\pi_1^+}, m^2_{\pi^-\pi_2^+}) \quad (13)$$

where $m^2_{\pi^-\pi_1^+} \equiv (p_{\pi^-} + p_{\pi_1^+})^2$ and $m^2_{\pi^-\pi_2^+} \equiv (p_{\pi^-} + p_{\pi_2^+})^2$.

The second problem raised above is naturally solved in this particular decay: the amplitude is proportional to just one form factor; thus, one can directly extract it from the plot.

In two body decays of D mesons, amplitudes proportional to m_D^2 only happen through spectator diagrams while those contributions coming from non spectator diagrams — as the one we are considering here — are proportional to the masses of the final state mesons, and thus less important. Since the amplitude of eq. (13) is proportional to $m_{D_s}^2$, in principle, it is not small. Nevertheless, if one assumes PCAC to be valid – due to the fact that final state quarks are light – one expects this decay to be small, and this can only happen if the F_4 form factor is negligible. However, the validity of PCAC in this context is not clear, as we will see in the following.

The F_4 form factor [19] has never been measured and there are no clear theoretical predictions for it. Some authors [20,21] proposed expressions based in models that are valid only for small values of the squared momentum transfered to the three pions, q^2. However, in the decay $D_s^+ \to \pi^-\pi^+\pi^+$, $q^2 = m_{D_s}^2$.

The measurement of F_4 will have important consequences on the understanding of τ and a_1 meson physics. The a_1 width can be measured through the decay $\tau \to \nu_\tau 3\pi$ but its value turns out to be 2 or even 3 times larger than the value extracted from other measurements [9]. The value of the a_1 width extracted from the decay $\tau \to \nu_\tau 3\pi$ strongly depends on the magnitude of a possible non-resonant decay, which is driven by F_4, i.e. the same form factor involved in the NR decay $D_s^+ \to \pi^-\pi^+\pi^+$.

Experimental measurements [24] of the channel $\tau \to \nu_\tau 3\pi$ cannot distinguish between models predicting a large amount of PCAC breaking [22], i.e. a large F_4, from those predicting a small amount of this breaking [23]. Both a large and a small F_4 are acceptable, but the extracted values of the a_1 width can vary by as much as a factor of two when fitting data using the first or the second kind of model.

Thus, the correct extraction of the NR part of the decay $D_s^+ \to \pi^-\pi^+\pi^+$ can bring a first measurement of the form factor F_4, clarifying the amount of PCAC breaking and then helping to extract the correct value of the a_1 meson width. At present, the existent measurements of the decay $D_s^+ \to \pi^-\pi^+\pi^+$ are not consistent: The branching ratio for the NR decay measured from the E691 experiment [25] is 1.0 ± 0.4 % – implying a large F_4 – while recent results from E687 [26] suggest a value about six times smaller – resulting on a much smaller F_4. They have both been obtained using a constant function to fit the NR contribution.

IV. SUMMARY AND CONCLUSIONS

In this work, we discuss on some of the consequences of our previous claim [3] that NR contributions in D meson decays cannot be fitted with a constant as they usually are. We show here that important physics information is hidden in this contribution which has been

loosely considered up to now. We present two examples to show the information one could obtain if the NR contribution were correctly extracted from the Dalitz plot.

First, we argue that in the decay $D^+ \to \bar{K}^0\pi^+\pi^0$ events produced via the NR channel could have been assumed to be originated from the $\bar{K}^*(892)^0$ resonant contribution. Using a model based in factorization, we showed that NR have a bump near the $\bar{K}^*(892)^0$ squared mass. This bump is very stable within a large variation of some poorly known quantities entering in the calculated amplitude. It is thus a strong prediction of factorization.

Second, we claim here that an adequate extraction of the NR contribution from data could allow us to measure unknown form factors. The $D_s^+ \to \pi^-\pi^+\pi^+$ is particularly interesting: its amplitude can be factorized to give a contribution proportional to the F_4 form factor. This form factor drives the spin zero part of the axial current matrix element describing the decay in three pions. It is very relevant to the decay $\tau \to \nu_\tau 3\pi$. Different model predictions could be tested and the longstanding problem concerning the a_1 meson width will benefit from this crucial information.

Coming experiments on charmed meson decays are expected to measure $D^+ \to \bar{K}^0\pi^+\pi^0$ and $D_s^+ \to \pi^-\pi^+\pi^+$ decays with high statistics. Using a non-constant function for the NR contribution when fitting the decay from its Dalitz plot, it will be possible to extract adequately the NR contributions. This will a) clarify the eventual discrepancy in the $D^+ \to \bar{K}^*(892)^0\pi^+$ decay width, b) test the validity of factorization technique when applied to D meson decays and c) bring a first measurement of the relevant form factor F_4.

One of us (RMG) would like to thank the Fermilab Theoretical Physics Department, where part of this work has been done. Two of us (IB and CG) want to thank the CNPq (Brazil) for financial support.

---

[1] E. Byckling and K. Kajantie; *Particle Kinematics* (John Wiley & Sons, New York, 1973).
[2] J.D. Jackson; *Nuovo Cimento* **34**, 1644 (1964).
[3] I. Bediaga, C. Göbel, and R. Méndez–Galain; *Phys. Rev. Lett.* **78**, 22 (1997).
[4] M. Bauer, B. Stech and M. Wirbel; *Z. Phys.* **C34**, 103 (1987).
[5] J. Adler *et al.*, MarkIII Collab.; *Phys. Lett.* **B196**, 107 (1987).
[6] Alvarez *et al*, NA14 Collab.; . *Zeit. fur Phys* **C50**, 11 (1991).
[7] J.C. Anjos *et al.*, E691 Collab.; *Phys. Rev.* **D48**, 56 (1993).
[8] P.L. Frabetti *et al.*, E687 Collab.; *Phys. Lett.* **B331**, 217 (1994).
[9] Particle Data Group, Review of Particle Properties; *Phys. Rev.* **D54**, 1 (1996).
[10] D. Fakirov and B. Stech; *Nucl. Phys.* **B133**, 315 (1978);
N. Cabibo and L. Maiani; *Phys. Lett.* **B73**, 418 (1978).
[11] A.J. Buras, J.-M. Gérard, and R. Rückl; *Nucl. Phys.* **B268**, 16 (1986).
[12] Contributions of the two diagrams (b) and (e) are identical; same for those of diagrams (c) and (f); this explains the factor of 2 in eqs. (4) and (5).
[13] See for example the lattice results in: K.C. Bowler *et al*, UKQCD Collab.; *Phys. Rev.* **D51**, 4905 (1995).

A summary of the experimental results and model predictions can be found in ref. [9].
[14] L.B. Okun; *Leptons and Quarks* (North-Holland, Amsterdam, 1982).
[15] A. Buras, preprint MPI-Ph/94-60 (1994);
T.R. Browder and K. Honscheid; *Prog. Part. Nucl. Phys.* **35**, 81 (1995).
[16] J. Bjorken, Nucl. Phys. (Proc. Suppl.) **B11**, 325 (1989).
[17] J.H. Kühn and E. Mirkes; *Z. Phys.* **C56**, 661 (1992).
[18] M. Gourdin, Y.Y. Keum, and X.Y. Pham; *Phys. Rev.* **D53**, 3687 (1996).
[19] F_4 is also known in the literature as F_s, for scalar.
[20] S. Weinberg; *Phys. Rev. Lett.* **17**, 336 (1966).
[21] G. Colangelo, M. Finkemeier, R. Urech; *Phys. Rev.* **D54**, 4403 (1996).
[22] N. Isgur, C. Morningstar and C. Reader; *Phys. Rev.* **D39**, 1357 (1989);
R. Decker, M. Finkemeier and E. Mirkes; *Phys. Rev.* **D50**, 6863 (1994).
[23] J.H. Khün and A. Santamaria; *Z. Phys* **C48**, 445 (1990).
[24] R. Akers *et al*, OPAL Collab.; *Z. Phys.* **C67**, 45 (1995).
[25] J.C. Anjos *et al*, E691 Collab.; *Phys. Rev. Lett.* **62**, 125 (1989).
[26] P.L. Frabetti *et al*, E687 Collab.; FERMILAB-Pub-97/045-E.

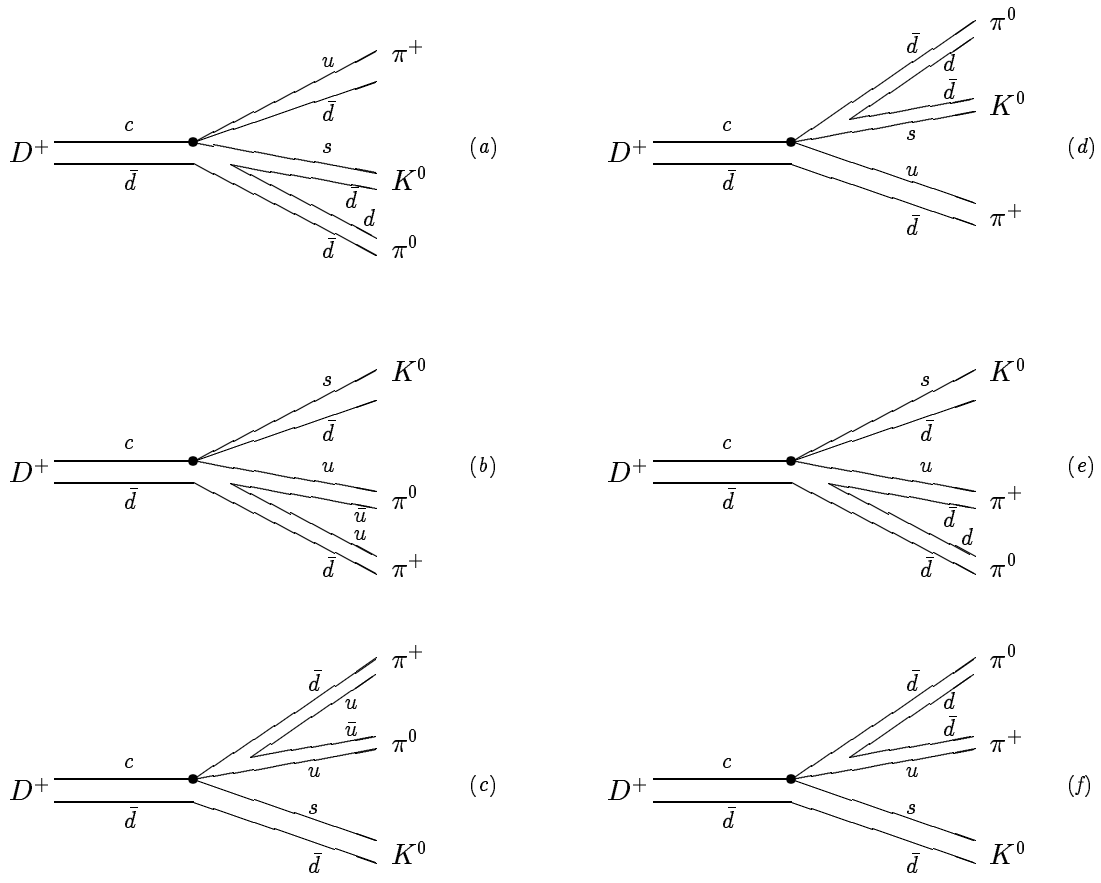

FIG. 1. The six diagrams contributing to the decay $D^+ \to \bar{K}^0\pi^0\pi^+$ according to the effective Hamiltonian of equation (1).

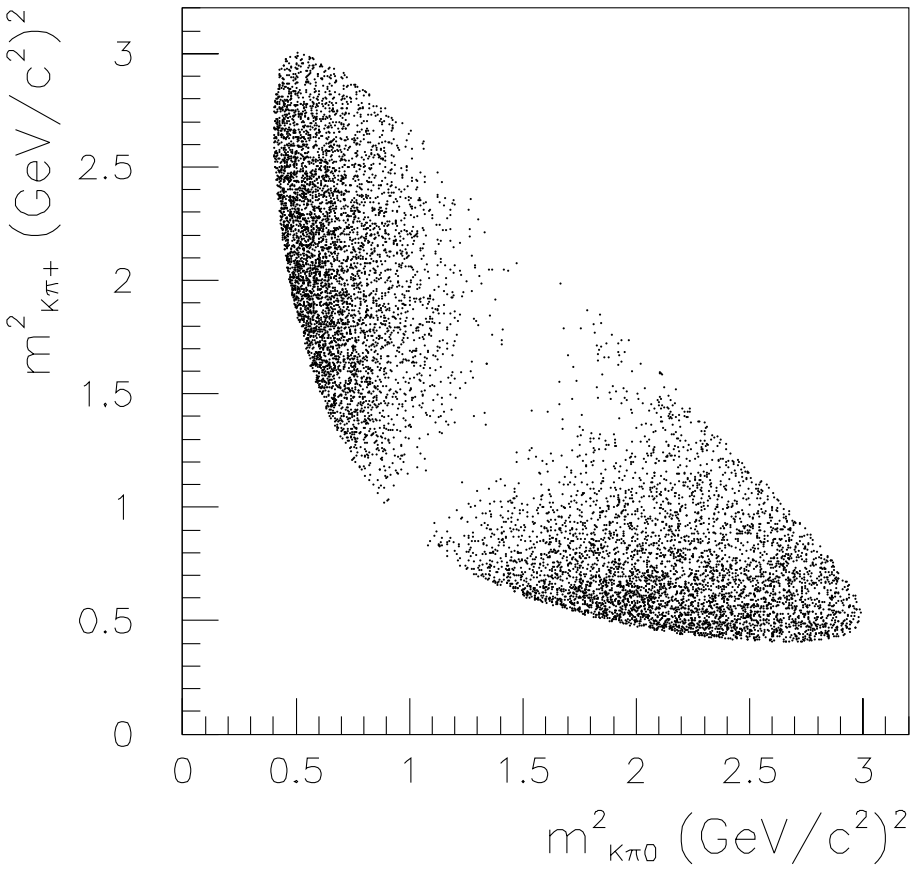

FIG. 2. The Dalitz plot for the NR decay $D^+ \to \bar{K}^0\pi^+\pi^0$. It has been obtained with Monte Carlo simulation weighted by $|\mathcal{M}_{D^+\to K^0\pi^+\pi^0}|^2$ as in equations (2), (5) and (6).

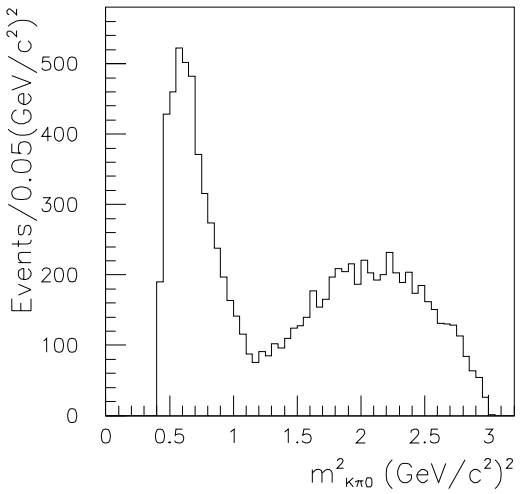

FIG. 3. The $m^2_{\bar{K}^0\pi^0}$ density distribution for the NR decay $D^+ \to \bar{K}^0\pi^+\pi^0$. It has been obtained with Monte Carlo simulation weighted by $|\mathcal{M}_{D+\to K^0\pi+\pi^0}|^2$ as in equations (2), (5) and (6).

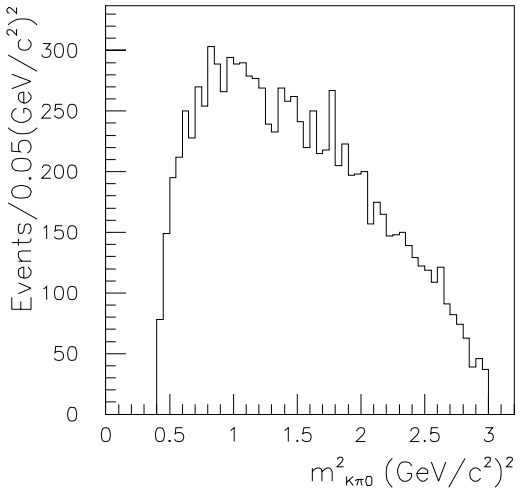

FIG. 4. Similar as Fig. 2, but for a flat decay, i.e., $|\mathcal{M}|^2 = \text{const}$.

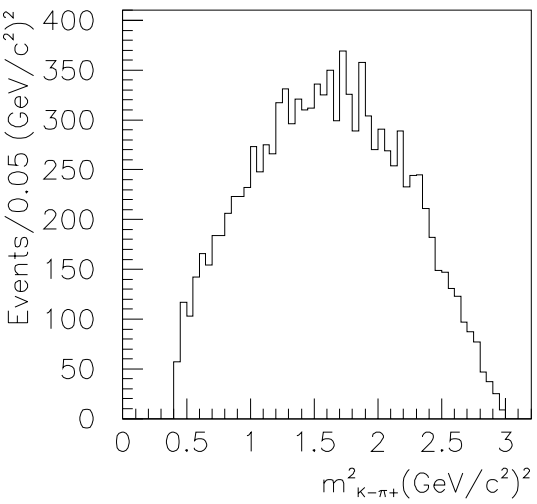

FIG. 5. Similar as Fig. 2, but for the decay $D^+ \to K^-\pi^+\pi^+$, using the model calculation developed in ref. [3].

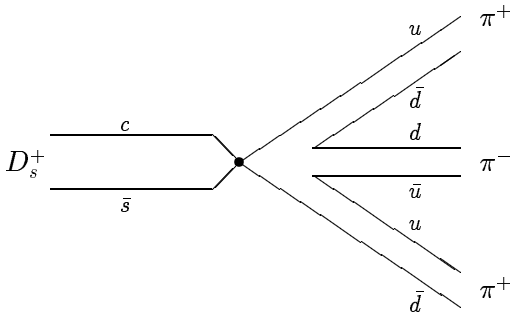

FIG. 6. The annihilation diagram dominating the $D_s^+ \to \pi^-\pi^+\pi^+$ decay.