CBPF-NF-039/96

Damage Spreading in the Potts Ferromagnet: Evidence for a New Dynamical Phase

by

Ladário da Silva†, *Francisco A. Tamarit* ‡ and *Aglaé C. N. de Magalhães*†*

†Centro Brasileiro de Pesquisas Físicas - CBPF
Rua Dr. Xavier Sigaud, 150
22290-180 – Rio de Janeiro, RJ – Brazil

‡Fa.M.A.F. , Universidad Nacional de Córdoba
Ciudad Universitaria, 5000 Córdoba, Argentina

ABSTRACT

We study, through the damage spreading method, the dynamical behaviour of the square lattice 3-state Potts ferromagnet within the heat-bath dynamics. We find that this very simple model has a new dynamical chaotic phase with unusual features. We obtain that its upper boundary occurs, within the error bars, at the same critical temperature as that of the static Ising one, although the estimated critical exponent $z \simeq 1.54$ is much smaller than the values reported in the literature for the Ising model. The temporal behavior of the damage at this transition suggests that it may be in the directed percolation universality class.

Key-words: Damage spreading, Potts model, dynamical phase transitions, heat-bath dynamics.

PACS number: 64.60.Ht, 75.10.Hk, 02.70.Lq

* E–mails: ladario@cat.cbpf.br, tamarit@fis.uncor.edu and aglae@cat.cbpf.br

In recent years, there has been an increase of interest in the study of dynamical phase transitions and their universality classes. One of the techniques largely employed in this investigation is the damage spreading method [1–13], which consists basically in measuring the time evolution of the Hamming distance or damage $D(t)$ between two initially different configurations of a sample of a system subjected to a specific dynamics and the same thermal noise. From the behaviour of the sample average of the damage $\langle D(t) \rangle$ as a function of the initial damage, the temperature and of any other relevant parameter, one can obtain a dynamical phase diagram, which usually depends on the employed dynamics (heat-bath, Glauber, Metropolis, etc).

In the case of the Ising ferromagnet subjected to a heat-bath stochastic process, it was found, for both dimensions $d = 2$ [2, 3] and $d = 3$ [1, 9], a two-phase structure with a damage transition temperature which agrees, within the obtained error bars, with the corresponding static critical temperature. In more complex systems which contains competing interactions [1, 2, 6], dilution [5], continuous [4] or non-trivial symmetries (Z_q for $q \geq 5$) [7], it has been found three or more dynamical phases where a few of them have no clearly known static equivalent. In the $3D$ Ising spin glasses [1, 6] and in the 2D XY model [4], it was obtained 3 different regimes: a high-temperature one ($T \geq T_1$) where $< D(t) >= 0$; an intermediate one ($T_2 \leq T < T_1$) where $< D(t) > \neq 0$ and it does not depend on the initial value $D(0)$, and a low-temperature regime ($T < T_2$) where $< D(t) > \neq 0$ and it depends on $D(0)$. In both systems the coincidence with the equilibrium critical temperature occurs at the lower transition T_2.

In this letter we study, for the first time as far as we know, the spread of damage of a very simple model, namely the 3-state Potts ferromagnet (for a review on the Potts model, see [14]) which has been reported in the literature to have an *unique* dynamical transition at the equilibrium critical temperature $k_B T_c(q = 3)/J = [\ln(1 + \sqrt{3})]^{-1}$. We show herein that this model presents a *three-phase* structure whose upper transition unexpectedly coincides, within the error bars, with the *static Ising* critical temperature.

Let us associate to each site i of the square lattice a Potts variable σ_i which can assume 3 integer values ($\sigma_i = 0, 1$ and 2) and consider the Potts ferromagnet model described by the following Hamiltonian:

$$\mathcal{H} = -J \sum_{<ij>} \delta(\sigma_i, \sigma_j) \, , \qquad (\sigma_i = 0, 1, 2), \qquad (1)$$

where $J > 0$ is the ferromagnetic coupling constant and $\delta(\sigma_i, \sigma_j)$ is the Kronecker delta function. The sum is over all the nearest-neighbor spins $< ij >$ of the square lattice.

Our numerical simulations are implemented on square lattices of linear size L ($N = L^2$ sites) submitted to periodic boundary conditions. We consider two replicas A and B with different initial configurations $\{\sigma_i^A(t)\}$ and $\{\sigma_i^B(t)\}$ at time $t = 0$ and let them evolve through a sequential Monte Carlo heat-bath process (in fully vectorized code). We use the same sequence of random numbers for updating the spins. Consequently if two configurations become equal at time t, they will remain identical for all later times, and we can introduce the survival probability $P(t)$ of two replicas A and B being still different at time t. We define the Hamming distance $D(t)$ between the configurations $\{\sigma_i^A(t)\}$ and $\{\sigma_i^B(t)\}$ at time t as:

$$D(t) = \frac{1}{N}\sum_{i=1}^{N}[1 - \delta(\sigma_i^A(t), \sigma_i^B(t))] \tag{2}$$

where the sum is over all the N sites. In order to average $D(t)$ over thermal fluctuations we repeat the simulations for M samples, obtaining thus the average damage $< D(t) >$.

We use herein three different sets of initial configurations for the two replicas, namely:

(a) ordered along distintic states: $\{\sigma_i^A(0) = 0, \forall i\}$ and $\{\sigma_i^B(0) = 1, \forall i\}$ ($D(0) = 1$);

(b) configuration $\{\sigma_i^A(0)\}$ is random and configuration $\{\sigma_i^B(0)\} = \{\sigma_i^A(0)\}$ except for 50% of the spins which are randomly chosen and given any of the two other possible states with equiprobability ($D(0) = 1/2$);

(c) same as in (b) except for 5% of chosen spins that are different ($D(0) = 0.05$).

In figure 1 we show the survival probability $P(t)$ as a function of the temperature T at different times t with initial conditions (c). ¿From this we clearly observe *three* distinct behaviors: i) a low temperature one (for $T < T_2$, with $T_2 \simeq 1.0$) where P varies sensibly with T; ii) an intermediate temperature one (for $T_2 \leq T < T_1$, with $T_1 \simeq 1.2$) where P remains maximum ($P = 1$); iii) a high temperature one (for $T \geq T_1$) where P vanishes.

The sudden increase of P near T_2 as well as the abrupt fall of P near T_1 signal the existence of dynamical transitions at T_2 and T_1, respectively. A similar behavior, for an initial $D(0) = 1$, has been observed by Leroyer and Rouidi [7] in the p-state clock model ($5 \leq p \leq 10$) for the two higher transition temperatures (see their fig. 9). But in our case, we have also checked that in the intermediate phase $P(t)$ equals 1 for *any* initial damage (including an infinitesimal one). Therefore we can say that this phase is fully *chaotic* in the sense that even two configurations infinitesimally close at $t = 0$ will always become separated by a *finite* distance. This is in contrast with the behavior found for the $3D$ Ising ferromagnet[1] where, for all temperatures, P decreases as the initial damage decreases.

The three temperature regimes can be also seen in the plot of $< D(t = 10000) >$ as a function of T, for different initial conditions, exhibited in fig. 2. We observe in such a simple and discrete model the same behavior as that found for the spin glass [1, 6] and XY model [4] described above . We have also checked the loss of initialization dependence in the intermediate phase for several other values of $D(0)$ including one single spin flip.

Since the finite time and size effects are more serious near the transition temperatures T_1 and T_2, we followed the finite size scaling procedure [3, 4, 9] in order to get more reliable estimates of these transition temperatures. For this, we computed the following quantities for each sample s $(s = 1, 2, ..., M)$ of linear size L at the temperature T:

$$\tau_1(L,T,s) = \frac{\sum_t t D_s(t)}{\sum_t D_s(t)} \tag{3}$$

$$\tau_2(L,T,s) = \frac{\sum_t t^2 D_s(t)}{\sum_t D_s(t)} \tag{4}$$

and the ratio

$$R(L,T,s) = \frac{\tau_2(L,T,s)}{\tau_1^2(L,T,s)} \tag{5}$$

where the samples were iterated until the damage D_s for the s-th sample has vanished (near 200000 MCS for the largest systems close to the critical temperature). τ_1 and τ_2 are measures of characteristic times for two configurations to meet.

One expects, from finite size scaling arguments, that the sample average $< R(L,T) >$ becomes independent of L at the transition temperature T_1 and for a sufficiently large size [3, 4, 9]. The plot of the curves $< R(L,T) >$ for different values of L shown in Fig. 3 leads to the following estimate

$$\frac{k_B T_1}{J} = 1.13 \pm 0.01 \tag{6}$$

which is unexpectedly close to the exact static critical temperature $[\ln(1+\sqrt{2})^{-1}] = 1.13459...$ of the Ising ferromagnet.

In order to determine the other transition temperature T_2 below which $< D(t) >$ depends on the initial damage we follow the same procedure used for the XY model in ref. [4]. Consider, thus, three different replicas A, B and C and define the following measure $\Delta(t)$ for comparing the evolution of $\{\sigma_i^A(t)\}$, $\{\sigma_i^B(t)\}$ and $\{\sigma_i^C(t)\}$:

$$\Delta(t) = D_{AC}(t) - D_{AB}(t) \tag{7}$$

whose sample average $< \Delta >$ for a sufficiently long time t plays the role of order parameter for the continuous transition at T_2.

Defining $\tau_1^{(\Delta)}(L,T,s)$, $\tau_2^{(\Delta)}(L,T,s)$ and $R^{(\Delta)}(L,T,s)$ in a similar way to the respective Eqs. 3, 4 and 5 with $D_s(t)$ being replaced by $\Delta_s(t)$ (and performing the sum until $\Delta_s(t)$ vanishes for the first time) we obtain the temperature dependence of the average $< R^{(\Delta)} >$ drawn in fig. 4. These curves cross at a temperature

$$\frac{k_B T_2}{J} = 0.99 \pm 0.01 \tag{8}$$

which, similarly to the $3D$ spin glass [1, 6] and the $2D$ XY model [4], is very close to the exact critical temperature 0.99497... of the 3-state Potts ferromagnet at thermal equilibrium.

We also computed the dynamic critical exponents z_1 and z_2 at the respective transition temperatures T_1 and T_2 (normally defined as $\tau \sim L^z$, where τ is the relaxation time for the dynamics at the transition) by considering the average vanishing time of $< D >$ and $< \Delta >$ similar to the procedure of Wang et al [9]. We obtained that $z_1 \simeq 1.54 \pm 0.02$ and $z_2 \simeq 2.28 \pm 0.03$. Notice that z_2 compares well with the recent value $z \approx 2.196$ computed for the 3-state Potts model from short-time dynamics [15] and with other previous results where $2.1 \leq z \leq 2.8$ (see [16] and references therein, [17]). In contrast, z_1 differs a lot from the values ($1.9 < z < 2.3$) quoted in the literature for the Ising ferromagnet (see [16] and references therein, [8–12]). Notice that the damage spreading technique with the heat-bath dynamics has led, for the Ising model [8–12], to values of z which are consistent with those predicted by other methods. Thus we conclude that, although T_1 is consistent with the static Ising critical temperature, the dynamic transition between the intermediate and high temperature regimes is not in the Ising universality class.

A similar discrepancy in the universality class also happens in the upper transition of spin glasses, where T_1 occurs at the non-frustrated bond percolation and the critical exponents correspond to the standard bond percolation [6, 18]. In fact we have also examined the temporal behavior of $< D(t) >$ for short times ($t << L^z$) at the transition temperature T_1. We have found a power law decay [3] ($< D(t) > \sim t^{-\delta}$) where although we cannot compute a precise value for δ_1, it seems to be compatible with the value $\delta \simeq 0.46$ for the directed percolation (DP) in $2+1$ dimensions [19]. This is in agreement with Grassberger's conjecture [20] since the transition at T_1 does not coincide with the static 3-state Potts one.

In summary, we have studied the spread of damage in the 3-state Potts model within the heat bath dynamics. We have found *three* (instead of two) dynamical phases which are characterized by $< D >$ and $< \Delta >$ as follows: (i) for $T < T_2$, $< D > \neq 0$ and $< \Delta > \neq 0$; (ii) for $T_2 \leq T < T_1$,$< D > \neq 0$ and $< \Delta >= 0$; (iii) for $T \geq T_1$,$< D >=< \Delta >= 0$. In regime (ii) two initial configurations stay different for a very long time (since $P(t) = 1$) and always achieve, for a fixed temperature, the same long-time Hamming distance no matter how close they are at $t = 0$. It is remarkable that the *very simple* pure system studied herein presents a new dynamical chaotic phase with such unusual features. It is even more remarkable that its upper boundary coincides, within the error bars, with the *static Ising* critical temperature, although its z exponent differs from the Ising one. We found that the relaxation time at T_1 is smaller than at T_2(since $z_1 < z_2$) and that the memory effect of the initial damage disappears for $T \geq T_2$. This suggests that, similarly to the $3D$ spin glasses [21, 22], the long-time behavior of the spin auto-correlation function in this unexpected regime decays slower than in the high temperature regime (where it is probably given by a simple exponential decay) but faster than in the low-temperature one (where it could be eventually given by a stretched exponential decay similar to that found for the $2D$ Ising ferromagnet [23, 13]).

We will present more detailed results, as well as the influence of external fields in the considered damage spreading in a forthcoming paper.

Acknowledgments We acknowledge C. Tsallis for fruitful discussions and useful comments on the manuscript. E. M. F. Curado is also acknowledged for helpful discussions. We thank CNPq and VITAE Foundation for financial support. Most of the simulations were carried out on a Cray Y-MP2E of the Supercomputing Center at UFRGS (CESUP). Particularly, we thank Denise Grüne Ewald (CESUP) for a generous help on the acquisition of extra computational time.

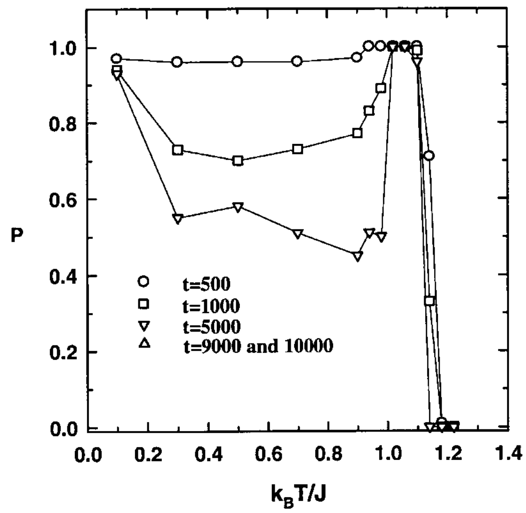

fig. 1

Figure 1: The survival probability P(t) versus temperature for different values of time. The data for t=9000 and 10000 coincide within the used scale. $M = 100$ samples of linear size $L = 64$ with initial configurations (c) (where $D(0) = 0.05$) were examined.

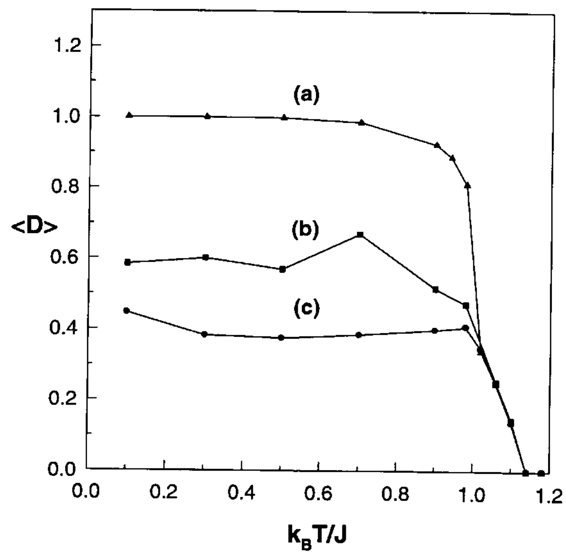

fig. 2

Figure 2: Average damage $< D >$ versus temperature for three different initial damages (a)$D(0) = 1$, (b) $D(0) = 0.5$ and (c) $D(0) = 0.05$. Simulations were performed for $M = 100$ samples of linear size $L = 64$ and $t = 10000$.

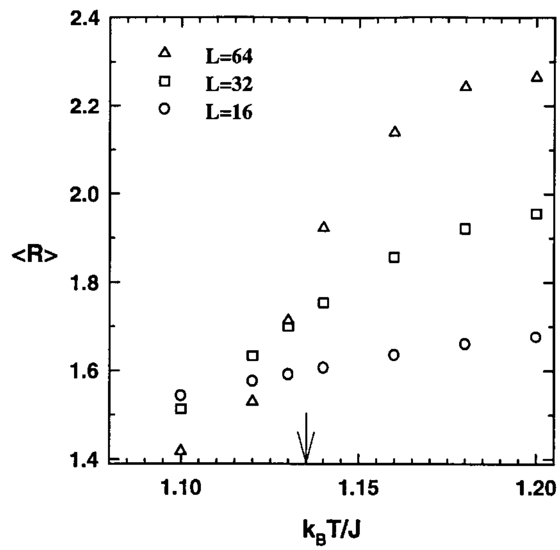

fig. 3

Figure 3: The ratio $< R > = < \tau_2/\tau_1^2 >$ versus temperature for distinct sizes L. The number M of samples used were $16000, 10000$ and 500 for $L = 16, 32$ and 64, respectively. The initial damage was $D(0) = 1$ (set (a) of initial configurations). The error bars are smaller than the symbols. The arrow signals the exact static Ising critical temperature.

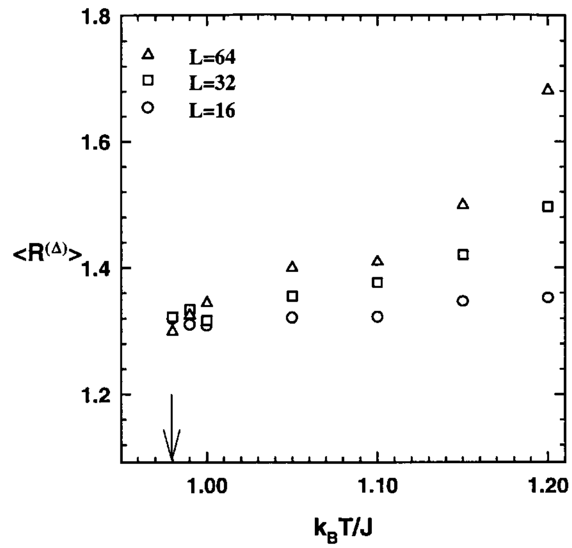

fig. 4

Figure 4: The ratio $< R^{(\Delta)} > = < \tau_2^{(\Delta)}/(\tau_1^{(\Delta)})^2 >$versus temperature for different sizes. We used $M = 10000, 1000$ and 250 for the respective sizes $L = 16, 32$ and 64. The replicas A and B are in the inital configuration (b), while the third replica C is obtained from A by changing the state of each spin to one of the two other possible states with equiprobability (hence, $D_{AB} = 1/2$ and $D_{AC} = 1$). The error bars are smaller than the symbols. The arrow signals the exact static critical temperature $T_c(q = 3)$.

References

[1] B. Derrida and G. Weisbuch, Europhys. Lett. **4**, 657 (1987)

[2] M. N. Barber and B. Derrida, J. Stat. Phys. **51**, 877 (1988)

[3] A. Neumann and B. Derrida, J. Phys. France **49**, 1647 (1988)

[4] O. Golinelli and B. Derrida., J. Phys. **A22**, L939 (1989)

[5] O. Golinelli, Physica **A167**, 736 (1990)

[6] L. de Arcangelis: in "Correlations and Connectivity ", edited by H.E. Stanley and N. Ostrowsky (Kluver Acad. Publ.,1990)

[7] Y. Leroyer and K. Rouidi, J. Phys. **A24**,1931 (1991)

[8] D. Stauffer, J. Phys. **A26**, L599 (1993)

[9] F. Wang, N. Hatano and M. Suzuki, J. Phys. **A28**, 4543 (1995)

[10] P. Grassberger, Physica **A214**, 547 (1995)

[11] U. Gropengiesser, Physica **A215**, 308 (1995); Physica **A217**, 227 (1995) (erratum)

[12] F. Wang and M. Suzuki, Physica **A223**, 34 (1996)

[13] P. Grassberger and D. Stauffer, to be published

[14] F. Y. Wu, Rev. Mod. Phys. **54**, 235 (1982); C. Tsallis and A. C. N. de Magalhães, Phys. Rep. **268**, 305 (1996)

[15] L. Schülke and B. Zheng, Phys. Lett **A204**, 295 (1995)

[16] S. Tang and D. P. Landau, Phys. Rev. **B36**, 567 (1987)

[17] M. Aydin and M. C. Yalabik, J. Phys **A21**, 769 (1988);**17**, 2531 (1984) and **18**,1741 (1985)

[18] L. de Arcangelis, A Coniglio and F. Peruggi, Europhys. Lett. **14**, 515 (1991)

[19] P. Grassberger, J. Phys. **A22**, 3673 (1989) and **A28**, L67 (1995)

[20] P. Grassberger, J. Stat. Phys. **79**,13 (1995)

[21] A. T. Ogielsky, Phys. Rev. **B32**,7384 (1985)

[22] I. A. Campbell, Phys. Rev. **B33**, 3587 (1986)

[23] D. A. Huse and D. S. Fisher, Phys. Rev **B35**, 6841 (1987)