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Birefringence of Gravitational Waves

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Abstract

We show that a large class of non-linear pure spin two field theories of gravity presents the phenomenon of birefringence. Such property of the gravitational waves is absent in General Relativity. Thus it can help to discriminate among all existing theories of gravity. Astrophysical and cosmological scenarios where gravitational waves interact with the gravitational field of a compact source are proposed for testing its occurrence in nature.

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I. INTRODUCTION

It is rather well-known that non-linear theories of electromagnetism allow for the phenomenon of birefringence. The origin for this can be traced to the fact that the photon, in non-linear electrodynamics, propagates in spacetime as if its metric structure were no more Minkowskian but instead an effective Riemannian one, the structure of which depends on the background electromagnetic field. The general formulation of this has been shown recently [2]. Thus, one should state that nonlinear electrodynamics deals with two metrics: the background one and another effective one that is felt only by the photons. We emphasize that this duplication phenomenon of the metric structure of spacetime is just a pure electromagnetic effect. Although it is not mandatory, its appearance serves to simplify the description of the photon propagation (in the regime of high frequency.)

Here we would like to analyse the corresponding problem:

- Does non-linear gravity show up a similar phenomenom of birefringence?

It is well-known that General Relativity does not present birefringence for the gravitational waves. Due to the recent renewed interest in the field theory of gravity, this question should be treated in a more wide context and be analysed in terms of massless spin-two formalism. In other words, we will solve the following question:

- Do the characteristics of non-linear spin two field theories show up the phenomenom of birefringence?

We will pursue this issue and shall prove that the phenomenon of birefringence is typical for almost all nonlinear spin-two field theories.

Anticipating the conclusion we will draw later on concerning the existence of two distinct metrics for the propagation of waves of the nonlinear theory, we stress that a bi-metric formulation occurs for any theory of integer spin fields and, as we shall see below, it occurs also for the case of spin two field theories of gravity, since the crucial property is not its spin but its nonlinearity.

Nonetheless, a warn should be put forward here. The relativistic community is well abreasted of the presence of two metrics in some sort of two-tensor gravity theories. This is not the case here since the extra, effective metric that appears in nonlinear theories –be of spin 1 or 2– depends on the structure of the background field. We examine here only those theories of gravity that are described uniquely in terms of only one spin-two field, which we will represent by $\varphi_{\mu\nu}$.

II. NOTATION AND DEFINITIONS.

We define a three-index tensor $F_{\alpha\beta\mu}$, which we will call the **gravitational field**, in terms of the symmetric standard variable $\varphi_{\mu\nu}$ (which will be treated as the potential) to describe a spin-two field, by the expression[4]

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[4]We are using the anti-symmetrization symbol $[x, y] \equiv xy - yx$ and the symmetrization symbol $(x, y) \equiv xy + yx$. Note

$$F_{\alpha\beta\mu} = \frac{1}{2}(\varphi_{\mu[\alpha;\beta]} + F_{[\alpha}\gamma_{\beta]\mu}) \tag{1}$$

where F_α represents the trace

$$F_\alpha = F_{\alpha\mu\nu}\gamma^{\mu\nu} = \varphi_{,\alpha} - \varphi_{\alpha\mu;\nu}\gamma^{\mu\nu}. \tag{2}$$

From the above definition it follows that $F_{\alpha\beta\mu}$ is anti-symmetric in the first pair of indices and obeys the cyclic identity, that is

$$F_{\alpha\mu\nu} + F_{\mu\alpha\nu} = 0 \tag{3}$$

$$F_{\alpha\mu\nu} + F_{\mu\nu\alpha} + F_{\nu\alpha\mu} = 0. \tag{4}$$

The most general non-linear theory must be a function of the invariants one can construct with the field. There are three of them which we represent by M,N and W, that is:

$$\begin{aligned} M &\equiv F_{\alpha\mu\nu}\, F^{\alpha\mu\nu} \\ N &\equiv F_\mu\, F^\mu \\ W &\equiv F_{\alpha\beta\lambda} \overset{*}{F}{}^{\alpha\beta\lambda} \doteq \frac{1}{2} F_{\alpha\beta\lambda}\, F^{\mu\nu\lambda}\, \eta^{\alpha\beta}{}_{\mu\nu}. \end{aligned} \tag{5}$$

We will deal here only with the two invariants $U \equiv M - N$ and W. The reason for this rests on the linear limit. Indeed, in order to obtain the standard Fierz linear theory —as it is required of any candidate to represents the dynamics of spin-two— the invariants M and N[5] should appear only in the combination U. This is the case, for instance in Einstein General Relativity theory.

Under this condition, the general form of the Lagrangian density is given by:

$$L = L(U, W). \tag{6}$$

In this way, the gravitational action is expressed as:

$$S = \int \mathrm{d}^4x \sqrt{-\gamma}\, L, \tag{7}$$

where γ is the determinant of the flat spacetime metric $\gamma_{\mu\nu}$ written in an arbitrary coordinate system.

From the Hamilton principle we find the following equation of motion in the absence of material sources:

$$\left[L_U\, F^{\lambda(\mu\nu)} + L_W\, \overset{*}{F}{}^{\lambda(\mu\nu)} \right]_{;\lambda} = 0. \tag{8}$$

L_X represents the derivative of the Lagrangian with respect to the invariant X, which may be U or W.

that indices are raised and lowered by the Minskowski background metric $\gamma_{\mu\nu}$. The covariant derivative is denoted by a semicomma ';' and it is constructed with this metric.

[5]A linear term in W does not contribute for the dynamics once it is a topological invariant, as shown in the appendix.

In a previous paper [1] a modification of the standard Feynman-Deser approach of field theoretical derivation of Einstein's general relativity, which led to a competitive gravitational theory was presented. The main lines of such NDL approach can be summarized as follows:

- Gravity is described by a symmetric second rank tensor $\varphi_{\mu\nu}$ that satisfies a non-linear equation of motion;
- Matter couples to gravity in an universal way. In this interaction, the gravitational field appears only in the combination $\gamma_{\mu\nu} + \varphi_{\mu\nu}$, inducing us to define a quantity $g_{\mu\nu} = \gamma_{\mu\nu} + \varphi_{\mu\nu}$. This tensor $g_{\mu\nu}$ acts as an effective metric tensor of the spacetime as seen by matter or energy of any form except gravitational energy;
- The self interaction of the gravitational field break the above universal modification of the spacetime geometry.

III. THE METHOD OF THE EFFECTIVE GEOMETRY

Just for later comparison we resume briefly the propagation of electromagnetic waves in a non-linear regime. As it was shown [2] the non-linear photon propagates in a modified Riemannian geometry characterized by an effective metric $g^{eff}_{\mu\nu}$ which is not the background one[6].

A. Electromagnetic waves

In the case of non-linear electrodynamics, the photon path is kinematically described by

$$g^{\mu\nu}\, k_\mu\, k_\nu = 0, \tag{9}$$

where the effective metric $g^{\mu\nu}$ is given by

$$\begin{aligned} g^{\mu\nu} = L_F\gamma^{\mu\nu} - 4\, &\left[(L_{FF} + \Omega L_{FG})\, F^\mu{}_\lambda F^{\lambda\nu} \right. \\ &\left. + (L_{FG} + \Omega L_{GG})\, F^\mu{}_\lambda F^{*\,\lambda\nu} \right]. \end{aligned} \tag{10}$$

Here the scalar Ω obeys the equation

$$\Omega^2\, \Omega_1 + \Omega\, \Omega_2 + \Omega_3 = 0, \tag{11}$$

with the quantities $\Omega_i,\ i = 1, 2, 3$ being given by

[6]Although the use of this formulation is not mandatory, it simplifies greatly the analysis of the properties of the wave propagation. Besides, we can describe the photon propagation in a frame in which the electromagnetic forces on the photon are eliminated.

$$\Omega_1 = -L_F L_{FG} + 2F L_{FG} L_{GG} + G(L_{GG}^2 - L_{FG}^2), \quad (12)$$

$$\Omega_2 = (L_F + 2G L_{FG})(L_{GG} - L_{FF}) + 2F(L_{FF} L_{GG} + L_{FG}^2), \quad (13)$$

$$\Omega_3 = L_F L_{FG} + 2F L_{FF} L_{FG} + G(L_{FG}^2 - L_{FF}^2). \quad (14)$$

The proof of this was presented in [2].

We now will turn our exam for the case of massless spin-two field. We show that a very similar situation occurs, that is, the gravitational waves propagates in a modified Riemannian geometry characterized by an effective metric $g_{\mu\nu}^{eff}$ which is not the background one.

B. The case of gravitational waves

1. One-parameter Lagrangians

Our main purpose in this paper is to investigate the effects of nonlinearities in the equation of evolution of gravitational waves. We will restrict the analysis in this section to the simple class of Lagrangians[7] defined by

$$L = L(U).$$

From the least action principle we obtain the field equation

$$\left[L_U F^{\lambda(\mu\nu)}\right]_{;\lambda} = 0. \quad (15)$$

Applying conditions (3) and (4) (and its covariant derivatives) for the discontinuity of the field equation (15) through Σ we obtain

$$f_{\mu\,(\alpha\beta)}\, k^\mu + 2\,\frac{L_{UU}}{L_U}\,\xi\, F_{\mu\,(\alpha\beta)}\, k^\mu = 0 \quad (16)$$

where ξ is defined by

$$\xi \doteq F^{\alpha\beta\mu}\, f_{\alpha\beta\mu} - F^\mu\, f_\mu. \quad (17)$$

The consequence of such discontinuity in the identity (the analogous of the electromagnetic cyclic condition $F_{\{\mu\nu;\lambda\}} = 0$)

$$F_{\alpha\beta}{}^{\nu}{}_{;\lambda} + F_{\beta\lambda}{}^{\nu}{}_{;\alpha} + F^{\nu\ ;\beta}_{\lambda\alpha} = \frac{1}{2}\{\delta^\nu_\alpha W_{[\beta\lambda]} + \delta^\nu_\beta W_{[\lambda\alpha]} + \delta^\nu_\lambda W_{[\alpha\beta]}\}, \quad (18)$$

with

$$W_{\alpha\beta} \doteq F_\alpha{}^{\varepsilon}{}_{\beta;\varepsilon} - F_{\alpha,\beta} \quad (19)$$

$$W_{[\alpha\beta]} = W_{\alpha\beta} - W_{\beta\alpha}, \quad (20)$$

yields after some algebraic steps

$$\xi\eta_{\mu\nu}\, k^\mu\, k^\nu - 2\, F^{\alpha\mu\nu}\, f_{\beta\mu\nu}\, k^\alpha\, k^\beta + F^{\alpha\beta\lambda}\, f_\alpha\, k_\beta\, k_\lambda + F_\alpha\, f_\beta\, k^\alpha\, k^\beta = 0. \quad (21)$$

From these equations we obtain the propagation equation for the field discontinuities

$$L_U \eta^{\mu\nu} k_\mu k_\nu + 4 L_{UU} (F^{\mu\alpha\beta}\, F^{\nu}{}_{\alpha\beta} - \frac{1}{4} F^{\alpha\beta\mu}\, F_{\alpha\beta}{}^{\nu} - \frac{1}{2} F^\mu F^\nu) k_\mu k_\nu = 0. \quad (22)$$

Expression (22) suggests that one can interpret the self-interaction of the background field $F^{\mu\nu\alpha}$, in what concerns the propagation of the discontinuities, as if it had induced a modification on the spacetime metric $\eta_{\mu\nu}$, leading to the effective geometry

$$g^{\mu\nu}_{\text{eff}} = L_U\, \eta^{\mu\nu} + 4\, L_{UU} (F^{\mu\alpha\beta}\, F^{\nu}{}_{\alpha\beta} - \frac{1}{4} F^{\alpha\beta\mu}\, F_{\alpha\beta}{}^{\nu} - \frac{1}{2} F^\mu F^\nu). \quad (23)$$

A simple inspection of this equation shows that in the particular case of the linear theory the discontinuity of the gravitational field propagates along null paths in the Minkowski background.

2. Two parameter Lagrangians

In this section we will go one step further and deal with the general case in which the effective action depends upon both invariants, that is

$$L = L(U, W). \quad (24)$$

The equations of motion are given by Eq.(8). Our aim is to examine the propagation of the discontinuities in such a case. We have the two basic equations

$$\left(2AF_{\alpha\mu\nu} + AF_{\mu\nu\alpha} + BF^*_{\alpha(\mu\nu)} + 2L_U\, f_{\alpha\mu\nu}\right) k^\alpha = 0 \quad (25)$$

and

$$\left(\xi\delta^\beta_\alpha - 2F^{\beta\mu\nu} f_{\alpha\mu\nu} + F^\beta f_\alpha\right) k^\alpha k_\beta = 0. \quad (26)$$

Following the same procedure as presented in the previous section we arrive at

$$k_\beta k_\alpha [\xi L_U \eta^{\alpha\beta} + A(2F^{\beta\mu\nu} F^{\alpha}{}_{\mu\nu} - \frac{1}{2} F^{\mu\nu\beta} F_{\mu\nu}{}^{\alpha} - F^\alpha F^\beta)] = 0 \quad (27)$$

and

$$k_\beta k_\alpha \quad [(\eta L_U - BU)\eta^{\alpha\beta} + B(2F^{\beta\mu\nu} F^{\alpha}{}_{\mu\nu} + \frac{1}{2} F^{\mu\nu\beta} F_{\mu\nu}{}^{\alpha} - F^\alpha F^\beta - F_\lambda F^{\lambda(\alpha\beta)})] = 0. \quad (28)$$

[7] The NDL theory is contained in this class.

In these expressions we have set

$$A \doteq 2\,(\xi\, L_{UU} + 2\zeta\, L_{UW}),$$

$$B \doteq 2\,(\xi\, L_{UW} + 2\zeta\, L_{WW}),$$

and ζ is defined by

$$\zeta \doteq F^{\alpha\beta\mu}\, f^{*}_{\alpha\beta\mu}. \tag{29}$$

A lot of simplifications can be made if we take into account the following identities:

$$F^{*\beta\mu\nu} F^{*}{}_{\alpha\mu\nu} - F^{\beta\mu\nu} F_{\alpha\mu\nu} = -\frac{1}{2} M \delta^{\beta}_{\alpha}$$

$$F^{\alpha\mu\nu} F^{*}{}_{\beta\mu\nu} = -\, F^{\alpha *}{}_{\beta\lambda} F^{\lambda} + \frac{1}{4} F^{\mu\nu\alpha} F^{*}{}_{\mu\nu\beta}.$$

In order to simplify our equations it is worth to define the quantity $\Omega \doteq \zeta/\xi$. The quantity Ω is then given by the algebraic expression

$$\Omega_{\pm} = \frac{L_{WW} - L_{UU} \pm \sqrt{\Delta}}{2 L_{UW}}, \tag{30}$$

where

$$\Delta \doteq (L_{UU} - L_{UW})^2 + 4 (L_{UW})^2.$$

Thus, in the general case we are concerned here, the graviton path is kinematically described by

$$g^{\mu\nu}\, k_{\mu}\, k_{\nu} = 0, \tag{31}$$

where the effective metric $g^{\mu\nu}$ is given by

$$g^{\mu\nu} = L_U \eta^{\mu\nu} + 2\,(L_{UU} + \Omega_{\pm} L_{UW}) \left(2 F^{\beta\mu\nu} F^{\alpha}{}_{\mu\nu} - \frac{1}{2} F^{\mu\nu\beta}\, F_{\mu\nu}{}^{\alpha} - F^{\alpha}\, F^{\beta} \right). \tag{32}$$

Thus the last equation confirms that the *propagation of gravitational waves also exhibit the birefringence phenomenon.* Besides, it is easy to check that when the Lagrangian does not depend on the invariant W, expression (32) reduces to the form (23), as it is the case in the original version of the NDL theory of gravity [1].

IV. ASTROPHYSICAL TEST FOR GRAVITATIONAL WAVES BIREFRINGENCE

As discussed in the previous sections the NDL theory of gravity predicts the existence of birefringence of the gravitational waves. Next we envisage a potential astrophysical context in which such a prediction may be confronted.

The MACHO Collaboration has announced recently that astronomical observations of starfields in our galaxy, using the Hubble Space Telescope (HST) and ground based telescopes, have provided compelling evidence for the existence of stellar-mass (~ 6 M$_{\odot}$) black holes (BHs) adrift among the stars comprising the Milky Way [3]. The two observations (1996 and 1998) revealed a subtle brightning of a background star produced by the microlensing gravitational enhancement of the light it emits due to the passage of an invisible object in between the star and Earth. A detalied analysis of the data ruled out white dwarfs or neutron stars as the lensing invisible source, and strongly points towards dark stellar-mass objects (i. e., black holes) as the magnification sources since ordinary (massive) stars would be so bright to outshine the background star. These observations could have been supplemented by the discovery of multiple images of the lensed star but unfortunately the HST angular resolution is about two orders of magnitude larger than the minimum required for resolving (observing the separation of) a pair of images from it induced by the BH bending angle [4]

$$\hat{\alpha}(r_0) = 2 \int_{r_0}^{\infty} \frac{e^{\mu/2} dr}{\left[\frac{r^4}{b^2} e^{-\nu} - r^2\right]^{1/2}} - \pi. \tag{33}$$

Here r_0 is the passage distance from the lensing object, μ and ν are the metric fields of a static spherically symmetric object, and b is the impact parameter

$$b = r_0 e^{-\nu(r_0)/2}. \tag{34}$$

In addition to these effects the starlight (radio waves, for instance) should undergo a time-delay respect to a pulse traveling in a region free of gravitation which may be measured by precise timing or throughout polarization patterns from the star. This effect, the Shapiro-Delay, which is induced by the motion of the binary, is due to the light travel through changing gravitational fields, and it was predicted to exist for the case of binary radio pulsars. [5,6]

In the line of this microlensing effect of starlight by a BH, analogously a gravitational wave (GW) signal from a galactic background source, a compact binary pulsar like PSR J1141-6545 (5 hours period), PSR 1534+12 or PSR 1913+16 should also be lensed (splitted) when passing near a massive compact dark object such as the MACHO Collaboration BHs. Since both theoretical accurate estimates [7] and observational statistical inferences [8] of the abundance of galactic neutron star-neutron star binaries and coalescence rates of them are more promissing than earlier calculations, the following astronomical configuration looks a target to search for. Let us think for a while that a galactic but distant binary radio pulsar is perfectly aligned with the lensing object (a Schwarszchild BH) and the Earth. A GW pulse is emitted from the binary, passes by the lens and is detected at Earth. Then according to general relativity both the polarization modes h_{+} and $h_{\times}$

of the (linearized) GW signal will undergo deflection and time delay when flying-by the lens as in the case for light waves, but both will arrive to the detector at the same time, that is, there will be no time lag because in GR GWs travel at the speed of light and there is no birefringence effects on their propagation. Nevertheless, a phase lag for them in GR is predicted to be exactly $\pi/4$ radians. This dephasing is expected to be measured by the new GWs detectors [9]. As expected the signal power should be enhanced (enlargening of the GW amplitude) in a forseeable manner.

Notwithstanding, in the NDL theory of gravity the existence of birefringence of the gravitational waves as described above will induce not only a rather different time delay but a phase lag too in the arriving GW signals, due to the different velocity of propagation v_k for different spatial directions

$$v_k^2 = 1 - \frac{1}{2b^2}\frac{1}{[1+(k/b^2)L]^2} Z^{\mu\nu}\frac{k_\mu}{|\vec{k}|}\frac{k_\nu}{|\vec{k}|}, \tag{35}$$

with the velocity of light $c = 1$ in geometric units (see Ref. [1] for further details and definitions). This property may be tested with data collected with the forthcoming generation of GWs observatories such as LIGO, VIRGO, GEO-600, TIGAs, etc. cross-correlated with data from neutrinos, gamma-ray bursts and cosmic rays detectors [10]. We have shown above that each polarization mode of the GW in the NDL theory is velocity-dependent (upon direction and magnitude). Then, the radiation component traveling at the lens left-hand side (from our point of view) will be affected in a different way compared with the right-handed component due to this global birefringence dependence. Thus the detected signals will be accordingly time lagged and phase-modulated in a way not mimicking GR, and such effects may be measured futurely. The above astrophysical scenario also works for a gravitational radiation source at the other side of our galaxy intervened by the Milky Way central black hole candidate Sagitarius A*.

Moreover, if the lens BH is a Kerr type one then the *frame dragging* (Lense-Thirring effect) induced by the BH spin would dramatically accentuate these effects, and it turns out that its observational verification will be a reachable endeavour in the days to come. Besides, we stress that an analogous situation should occur on cosmological distance scales in the case a cosmic string (spinning or not) lies between a pair of coalescing galactic-core black holes behind it and the GWs detectors. The GW signal will undergo almost the same phenomena of multiple imaging, time lag and dephasing (all these quantities depending on the specific location and relative orientation of the source and detector respect to the lensing object as predicted by the NDL theory), and exemplified above for the local (galactic) microlensing sources discovered by the MACHO Collaboration. However, it is expected the separation of the double "GWs" images to be independent of how close the GWs graze the string, and it will exhibit no magnification effect as it does in the BH case. Such a feature is only exhibed by lensing effects around cosmic strings. An analytical treatment of these prospective experimental tests of the NDL gravity theory will be tackle in a forthcoming work.

V. APPENDIX: TOPOLOGICAL INVARIANT

From the definition Eq.(1) it follows that the quantity

$$Q \equiv \int F^*_{\alpha\beta\mu}\, F^{\alpha\beta\mu}$$

is a topological invariant. Indeed, we can re-write this under the form

$$Q = \int \partial_\nu \; \left(\eta^{\alpha\beta\mu\nu}\, F_{\alpha\beta\lambda}\, \varphi^\lambda_\mu\right).$$

Thus, the linear term W in Eq.(5) does not contribute in the total Lagrangian

$$L = C_1(F_{\alpha\beta\mu}F^{\alpha\beta\mu} - F_\mu F^\mu) + C_2 F^*{}_{\alpha\beta\mu}F^{\alpha\beta\mu} = C_1 U + C_2 W \tag{36}$$

since the term containing the dual tensor is simply a total derivative.

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