CBPF-NF-038/99

EPR paradox revisited

R. Klippert*

Centro Brasileiro de Pesquisas Físicas – CBPF/CNPq
22290-180 – Rio de Janeiro, RJ – Brazil

Abstract

In a seminal paper from 1935 Einstein, Podolsky and Rosen produced one of the most powerful weapon against the unpredictability of the world ensured by quantum mechanics. The recent production of entangled states, with all its possible future applications in quantum computation, re-open the possibility of testing EPR states on physical grounds. The present work intends to present a challenge to the wedding of classical (special) relativity with quantum mechanics, the so called *relativistic quantum mechanics*. Making use of the same apparatus devised in EPR, it is shown that non local quantum states are incompatible with either their possibility of being measured or else with Lorentz invariance (or even with both).

PACS numbers: 03.65.-w, 42.35.Ja, 03.65.Bz

The quantum indeterminism of the fundamental physical laws have challenged the positivist physicists from its very beginning. A large number of plausible claims were thus devised till the midst to this century, which intended to make it clear either an incompleteness of that theory, or else some kind of implyed inconsistency at the classical level. One of the most known amongst them is the 'EPR paradox' [1]. In a refined form, that work pretended to send faster-than-light information between classical observers by making use of non local (entangled) quantum states, in blatant contradiction with the Special Relativity Theory. After a great work on this specific "paradox", it was realized that no information at all was being forwarded by any observer. Particularly, it was due to Bell [2] a formal proof against local (non contextual) hidden variables, later confirmed by experimentation.

Most of the scientific community now regards EPR's work as a philosophical *a priori*, without deeper connections with the 'real' world of physics, *i.e.*, with **measurements**. Notwithstanding, recent works on quantum optics [3] have claimed to having produced and measured macroscopically correlated states ("Schrödinger's cat states"), which are essentially the same as EPR states. Such states refer to the quantum entanglement of the polarizations of the two photons produced by particle-antiparticle annihilation. Simple quantum mechanics yields that these polarizations must be opposite to one another, but only after a suitable measure can one talk about which they are. Perhaps an EPR reasoning could be useful.

Let us consider a given entangled state of two photons as described above. It is usually represented as $|\psi\rangle = (|1,+\rangle\,|2,-\rangle - |1,-\rangle\,|2,+\rangle)/\sqrt{2}$, where the numbers may label distinct regions of space (*e.g.*, the paths of the two photons), while the $\pm$ labels indicate the polarization mode (with no reference to which direction[1] this polarization refers to). The relative minus sign means a singlet state, corresponding to zero total polarization of the quantum system.

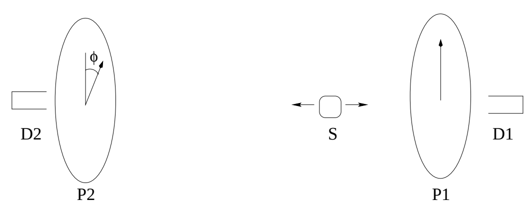

FIG. 1. A source S of entangled photons and the two polarizer/photo-detectors. The first measure at $D1$ is undertaken far before the second one (at $D2$). The azimuthal alignment between the polarizers $P1$ and $P2$ may be arbitrarily set.

If one of such correlated photons hits a polarizer ($P1$) then, provided nothing had happened with the other photon, the polarizer will thus define the polarization direction for such a photon as its own axis, and therefore for the other photon as well (as they form a single system). A

*Electronic mail: `klippert@lafex.cbpf.br`

[1]We will discuss here only linear polarization modes, for which a 'direction' makes sense. Elliptical and circular polarization modes could similarly be considered, with no essential modification on the argument.

suitable photo-detector system ($D1$) can be placed right after the polarizer to provide a classical information of measurement of this photon (which may even include a measurement of whether a $+$ or a $-$ state was found to be there). That the other photon of such an arranged system would end up with the opposite polarization is a physical statement, which can be measured afterwards by a similar polarizer $P2$ and photo-detector $D2$ systems. See figure 1.

Sources of very weak beams are available, allowing one to make sense in describing what happens[2] when a single pair of entangled photons are to be detected. Let us consider this case in order to simplify the analysis. Coincidence on the $D1$ and $D2$ "classical" measurements can therefore be found if and only if the azimuthal angle ϕ between the axes of the two polarizers is not a right angle.

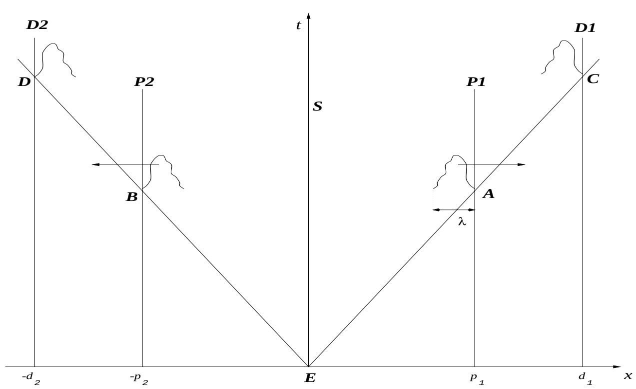

FIG. 2. A spacetime diagram (in two dimensions) of the experiment. The entangled photons are produced at E, and are focalized towards the polarizers. C marks the arriving signal at $D1$, while B refers to the arriving signal at $P2$. And similarly for D and A. Vertical lines represent the world lines of the apparatus (at rest) described in figure 1. We denote by λ the width of the pulse. Note that the distances p_1 and d_1 (as well as $p_2 = p_1$ and $d_2 = d_1$) are not at the same scale.

So far so good. But what to say if the above experiment is changed in such a way that the source S now distances just the same[3] from the polarizers ($p_1 \doteq \overline{S\,P1} = \overline{S\,P2} \doteq p_2$), and also from the detectors ($d_1 \doteq \overline{S\,D1} = \overline{S\,D2} \doteq d_2$)? To avoid interpretation misconceptions, let us consider the main hypothesis: the wave packages describing the two photons are sharp enough to ensure that, when one of the wave fronts hits $D1$ (event C) the wave front of the other one had already hit $P2$ (event B); and similarly for the other one (events D and A, respectively). E denotes the origin of the photon pair at S. See figure 2.

For the construction above, both photons are to be detected at the same time for a convenient choice of the relative orientation ϕ. Moreover, they also hit the polarizers simultaneously — apart from a quantum indeterminacy, whose relevance may vanish if $p_1 \gg \lambda$ and $d_1/p_1 - 1 \ll 1$ for a given λ; we will assume this as a 'classical hypothesis' —. If this is so, both polarizers are to impose to the corresponding photon their polarization axes (the *orientation* of each resulting photon polarization is not predictable, according to quantum mechanics). The pair of photons emerging from the polarizers can no longer present total zero polarization for ϕ/π not an integer. Their eventual detections should confirm that coincidence at $D1$ and $D2$ is now possible even for $\phi = \pi/2$. This possibility translates itself into probability by repeatedly starting the whole process from the very beginning, thus allowing the predicted correlation (*i.e.*, clicks at random) to be tested by experimental data.

One would face a formal difficulty when trying to evolve the quantum state by means of a (deterministic) Schrödinger equation from E to C (and D), due to the simultaneous determination (at the polarizers) of two non commuting observables: the polarization at two different spatial directions. It could be argued that the polarizers themselves should also be considered as measuring apparatuses, despite nothing is effectively being **measured** there[4]. The question will be left aside, its eventual solution being dependent on philosophical basements (which we would like to postpone as far as possible). Concerning classical measures, only the triggers (and the pattern they present) are undoubtedly real.

The experiment provided above can easily be turned into a relativistic formulation, with the only requirements that all A, B, C, D belong to the future-directed causal cone from E, while the connections from B to both A and C are assumed to be spacelike. And similarly for the other "arm" of the apparatus (from event A to both B and D). These requirements are clearly consistent with the previously considered classical hypothesis. Therefore, relativistic quantum mechanics should provide a consistent description of the whole picture. From now on only the choice $\phi = \pi/2$ will be discussed.

Consider now an inertial observer O' (see figure 3) at E in fast motion towards $P1$. O' agrees that the entangled pair of photons were produced by the (moving) source S at E. From Lorentz transformation laws [4], for such observer the time order of the relevant events will be $t'(E) < t'(A) < t'(B) < t'(C) < t'(D)$. This means that, from O''s point of view, $P1$ 'acts' first on the quantum system, thus defining the polarization axis of both photons (as discussed in the context of the first

[2] No ontological "realistic" picture will be assumed, however.
[3] These distances may differ a bit, as it will soon be shown.

[4] Note the experimenter only knows a photon had hit the polarizer after the corresponding detector is fired.

experiment, figure 1); therefore, classical detections are to be expected only at the (moving) detector $D1$, while $D2$ must never trigger[5] (Another inertial observer O'' can also be found which moves fast enough to order the events as $t''(E) < t''(A) < t'(C) < t''(B) < t''(D)$, to whom the detection at $D1$ occurs even before $P2$ can 'start to act' onto the system.) This, of course, predicts a radically different behavior of the possible patterns of the classical information (*i.e.*, which detector fires). After the end of the experiment, this information can be interchanged between the observers — and they clearly should agree, if any physical description from both are to make sense — by any communication procedure.

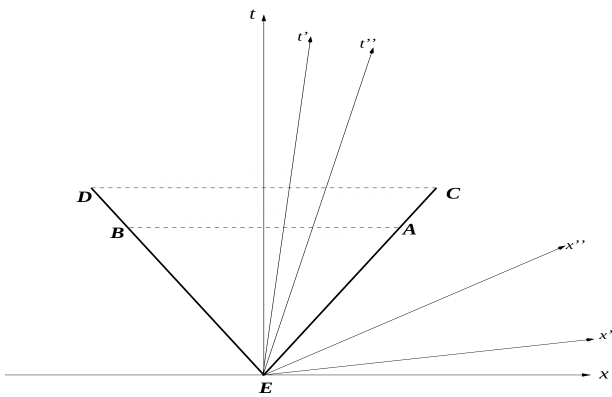

FIG. 3. Possible time orders for the same events of figure 2. Drawn are the paths of the two photons until their eventual detections at $D1$ and $D2$. We also show the 'orthogonal' axes of an observer O' at E fastly moving towards $P1$, who describes A occurring first than B. An even faster observer O'' is included, to whom also C occurs first than B.

The situation becomes even worser when one considers an observer $\tilde{O}'$ (or else $\tilde{O}''$) at E moving towards $P2$ with the same speed than O' (O'') relative to the inertial system O (*e.g.*, the lab). As the situation is completely symmetric, $\tilde{O}'$ ($\tilde{O}''$) would expect $D2$ could fire, but $D1$ must not trigger[6]. A large number of detections are (in principle) required in order to provide a clear distinction between the predictions of O and either O' (O'') or $\tilde{O}'$ ($\tilde{O}''$), but a single trigger of an ideal detector would suffice to put either O' (O'') or $\tilde{O}'$ ($\tilde{O}''$) into a real trouble.

To summarize, any trigger at any ideal detector would be impossible to be consistently explained by a suitably chosen inertial observer. The only result of such an experience which can still make sense (to all observers) is no detection at all — but note this would violate Born's probabilistic interpretation as well —. Seems to us that, on the grounds of the above, one should choose between rejecting (at least) one of the following:

- Causal consistency;
- Classical Lorentz transformations;
- Quantum theory of measurement;
- Entangled states.

In author's personal opinion, the weaker concept is the latter.

ACKNOWLEDGMENTS

The author would like to thank the LAFEX group at CBPF, particularly to M. Novello and J. M. Salim, for their hospitality and comments, and for useful discussions while this work was done.

[1] A. Einstein, B. Podolsky and N. Rosen, *Phys. Rev.* **47**, 777-780 (1935).
[2] J. S. Bell, *Physics* **1** (3), 195-200 (1964).
[3] J. I. Kim *et al*, *Phys. Rev. Lett.* **82** (24), 4737-4740 (1999); M. Brune *et al*, *Phys. Rev. Lett.* **77** (24), 4878-4890 (1996); L. Davidovich *et al*, *Phys. Rev.* **A 53** (3), 1295-1309 (1996).
[4] We assume that the moving polarizers (and detectors) interact with light roughly the same as if they were at rest. For otherwise, the difference could sign their absolute state of motion, in contradiction with Special Relativity.
[5] J. S. Freedman and J. F. Clauser, *Phys. Rev. Lett.* **28** (14), 938-941 (1972); E. S. Fry and R. C. Thompson, *Phys. Rev. Lett.* **37** (8), 465-468 (1976); A. Aspect *et al*, *Phys. Rev. Lett.* **47** (7), 460-463 (1981).
[6] J. F. Clauser *et al*, *Phys. Rev. Lett.* **23** (15), 880-884 (1969).

[5] This would be strictly true for ideal polarizers (transmittance efficiency for polarized light parallel $\epsilon_M^i = 1$ and perpendicular $\epsilon_m^i = 1$ to the polarizer axis, where $i = 1, 2$ denotes the polarizer). Realistic cases are reported of polarizers whose efficience ratio ϵ_m/ϵ_M ranges from 0.050 to 0.025 [5], but this may reach values of 10^{-5} [6]. The corresponding value should apply to the expected ratio of the counters $\#(D2)/\#(D1)$, thus making it as small as possible for suitable devices.

[6] See footnote 5, interchanging $D1$ with $D2$.