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A Simple Path Integral Proof of the One-Dimensional Electron Localization

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Abstract

We give a simple path integral proof of the eletron localization phenomena in one-dimension, at a large limit for the eletron mass.

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One of the most interesting problem in solid state physics is the appearence of the localization in the eletron wave function in the presence of a white noise random potential [1]

Our aim in this comment is to present a simple proof for the above cited localization phenomena, by using the path integral framework of Ref.[2], at a large limit for the eletron mass.

Let us start our analysis by considering the stationary state one-dimensional Schroedinger equation in the presence of a random potential

$$(E + \frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \phi(x))\Psi(x,E) \ = \ 0 \tag{1}$$

Here the potential $\phi(x)$ describing the impurities is a white-Gaussian noise potential with correlation function

$$\langle \phi(x_1)\phi(x_2)\rangle \ = \ \gamma\delta(x_1 - x_2) \tag{2}$$

where $\gamma > 0$ is the disorder strenght.

We are interested in the disordered averaged normalized two-point correlation of the probability amplitude distribution

$$\langle \chi(x)\rangle_\phi \ = \ \langle \Psi(x,E,[\phi])\Psi^*(0,E,[E])\rangle_\phi \tag{3}$$

where the brakets $[\phi]$ are introduced to emphasize the functional character of the eletron wave function on the random potential and $\langle ...\rangle$ denotes the ensemble average over all its realizations. Let us consider the leading pure-phase W.K.B eletron wave function functional form

$$\Psi(x,E,[\phi]) \ = \ e^{\frac{i}{\hbar}S(x,E,[\phi])}. \tag{4}$$

According to the Schroedinger equation, Eq.(1), the W.K.B leading phase $S(x,E,[\phi])$ satisfy the Hamilton-Jacobi random equation

$$E + \frac{1}{2m}(\frac{dS}{dx})^2 + \phi(x) \ = \ 0 \tag{5}$$

The ensemble average of $\phi(x)$ can be written as a Gaussian functional integral (see Eq.(3))

$$\langle e^{\frac{i}{\hbar}S(x,E,[\phi])}e^{-\frac{i}{\hbar}S(0,E,[\phi])}\rangle_\phi = \int \mathcal{D}[\phi(x)]e^{-\frac{1}{2\gamma}\int_{-\infty}^{+\infty}dx\phi(x)^2}(e^{\frac{i}{\hbar}S(x,E,[\phi])}e^{-\frac{i}{\hbar}S(0,E,[\phi])}). \tag{6}$$

Next, following along Refs.[2], we use the following functional path integral representation for the integrand of the above equation, i.e,

$$e^{\frac{i}{\hbar}S(x_1,E,[\phi])}e^{\frac{-i}{\hbar}S(x_2,E,[\phi])} = \frac{\int \mathcal{D}^F[s(x)]det_F[\frac{1}{m}\frac{dS}{dx}\frac{d}{dx}]\delta_F(E+\frac{1}{2m}(\frac{ds}{dx})^2-\phi(x))e^{\frac{i}{\hbar}s(x_1)}e^{-\frac{i}{\hbar}s(x_2)}}{\int \mathcal{D}^F[s(x)]\delta_F(E+\frac{1}{2m}(\frac{ds}{dx})^2-\phi(x))} \tag{7}$$

Let us remark that at the large eletron mass limit $m\to\infty$, we have the following functional determinant identity

$$\lim_{m\to\infty}\ln det_F[\frac{1}{m}\frac{ds(x)}{dx}\frac{d}{dx}] = \lim_{m\to\infty}\ln det_F[\frac{d}{dx}+\frac{1}{m}(\frac{ds}{dx}-m)\frac{d}{dx}] \sim \ln det_F[\frac{d}{dx}] = 0 \tag{8}$$

It is a straightforward evaluation to get the result

$$\langle e^{\frac{i}{\hbar}S(x_1,E,[\phi])}e^{\frac{-i}{\hbar}S(x_2,E,[\phi])}\rangle_\phi = \frac{\int \mathcal{D}[s(x)]e^{-\frac{1}{2\gamma}\int_{-\infty}^{+\infty}dx(E+\frac{1}{2m}(\frac{ds}{dx})^2)^2}e^{\frac{i}{\hbar}s(x_1)}e^{-\frac{i}{\hbar}s(x_2)}}{\int \mathcal{D}[s(x)]e^{-\frac{1}{2\gamma}\int_{-\infty}^{+\infty}dx(E+\frac{1}{2m}(\frac{ds}{dx})^2)^2}}. \tag{9}$$

We note that at the large eletron mass, we get, already, the exponential localization behaviour for the observable, Eq.(3), i.e,

$$\langle\chi\rangle_\phi = e^{-\frac{\gamma m}{E}|x|}. \tag{10}$$

Next $\frac{1}{m}$ corrections coming from the quartic self-coupling $\frac{1}{4m^2}(\frac{dS}{dx})^4$ and the functional determinant do not modify the localization behaviour obtained above.

At this point it is worth to compare our result with the cumbersome path integral study of chapter 10 of Ref.[1].

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