The Fuzzy Analog of Chiral Diffeomorphisms in higher dimensional Quantum Field Theories*

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Abstract

Our observation that the chiral diffeomorphisms allow an interpretation as modular groups of local operator algebras in the sense of Tomita and Takesaki allows us to conclude that the higher dimensional generalizations are certain infinite dimensional groups which act in a "fuzzy" way on the operator algebras of local quantum physics. These actions do not require any spacetime noncommutativity and are in complete harmony with causality and localization principles.

The use of an appropriately defined isomorphism reprocesses these fuzzy actions into partially geometric actions on the holographic image and in this way tightens the relation with chiral structures and makes recent attempts to explain the required universal structure of a would be quantum Bekenstein law in terms of Virasoro algebra structures more palatable.

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1 Introduction

Chiral quantum field theory owes most of its analytic accessibility to the presence of a (centrally extended) covering of the diffeomorphism group of the circle. It is well-known that the peculiarity of two-dimensional conformal automorphisms of Minkowski spacetime shows up in the classical calculation of conformal symmetries $x \to x'$ which apart from an x-dependent factor $F(x)$ leave the Minkowski-metric invariant $ds'^2 = F(x)ds^2$.

These classical arguments have however no bearing on symmetries of a purely quantum origin, i.e. on symmetries which do not enter local quantum physics via quantization as Poincaré- or conformal- symmetry, but rather require the noncommutative quantum physical structure for their very existence. In the standard setting of quantum field theory such symmetries would correspond to "fuzzy" transformations in test function space $f \to f^t$ which are not representable by diffeomorphism T

$$A(f) = \int f(x)A(x)d^d x \to A^t(f) \equiv A(f^t) \tag{1}$$
$$\exists\ no\ T\ with\ f^t(x) = f(Tx)$$

The best mathematical way to describe these symmetries is however via the use of local operator algebras in the setting of algebraic QFT. Of course in order to describe a physical symmetry the transformation must satisfy certain causality restriction and be unitarily implementable with respect to a reference state (usually the vacuum) in the sense of a Wigner symmetry.

There is a difference between the present "fuzziness" and that occurring in the presently popular noncommutative spacetime theories (related to noncommutative geometry)[1]. In the present case the pointlike geometric action of the Poincaré or conformal group is maintained since the indexing by spacetime regions in a net of noncommutative algebras remains still classical. The fuzziness refers to the modular analogs of spacetime transformations which may have some geometric interpretation in terms of the infinite dimensional geometry of certain real subspaces of a complex Hilbert space (in the absence of interactions such investigations have been carried out in [25][3]) but which cannot be encoded into diffeomorphisms of spacetime.

Instead of starting an axiomatic discussion of which fuzzy transformations are physically admissible, we find it more natural to go directly into medias res and present the only known mechanism which produces such fuzzy transformations: the modular localization theory of local quantum physics. It attaches a unitary one-parametric symmetry $\Delta^{it}_{\mathcal{O},\Omega}$ to each standard pair $(\mathcal{A}(\mathcal{O}), \Omega)$ of an operator algebra which in the standard formulation of QFT would be affiliated with generating smeared fields (with testing functions with support in the causally closed region $\mathcal{O}$) together with a cyclic and separating vector Ω.[2]

[1] The motivation for both the algebraic formulation and the noncommutative modification of QFT is similar: the avoidance of the ultraviolet problems. Whereas the algebraic approach tries to achieve this by a reformulation which avoids the singular field-coordinatizations of the standard method, the noncommutative formalism amounts to a step into the potentially interesting conceptual "blue yonder" [1].

This transformation does respect the causal horizon of $\mathcal{O}$ and maintains causal disjointness of $\mathcal{O}$ with $\mathcal{O}'$ but apart from this compatibility with causality it acts in the above described fuzzy manner. For certain special localization region of algebras, namely in case of massive theories for a noncompact wedge region W instead of a compact double cone (and for massless theories which lead to conformal invariance even in case of a double cone $\mathcal{O}$) together with Ω=vacuum, the modular transformation $\Delta_{\mathcal{O},\Omega}^{it}$ becomes a local Lorentz boost (or a special conformal transformation). I that case one learns that in Poincaré (or conformal) transformations well-known from the quantization of Noether's theorem can also be generated from a finite number of modular symmetries; in fact for $d > 1+1$ they are the only geometric modular symmetries. In fact the inverse mapping of the holographic isomorphism in the illustration (see appendix)) would automatically carry chiral symmetries which are not pure diffeomorphisms but also involve charges (as e.g. chiral loop groups) into fuzzy symmetries of the massive d=1+1 model. In this paper we will limit our attention to diffeomorphisms.

In this work we would like to argue that the infinite dimensional modular group G generated by the fuzzy $\Delta_{\mathcal{O},\Omega}^{it}$ in the common Hilbert space H is the physically correct substitute for a nonexistent infinite dimensional geometrically acting diffeomorphism group. In order for such a claim to be make sense we must first show that the well-known chiral diffeomorphism group (whose infinitesimal generators obey the Virasoro algebra) have a modular origin.

By taking into account symmetries related to charges in charged sectors we believe that these modular ideas can be generalized in the direction of fuzzy analogs of loop group symmetries, but this will not be the subject of this note. Knowing the Doplicher-Roberts theory [4], which explains the standard internal group symmetry of particle physics in terms of (apparently quite unrelated with group theory) causality and spectral properties of local observables, the idea that symmetries are rooted in their representation theory via localizable states is less outrageous as it appears at first sight.

In the next section we will confirm the validity of this modular interpretation of chiral diffeomorphisms in the case of the abelian current model. In particular we will construct a vector ψ in the Hilbert space which, together with chiral algebra $\mathcal{A}(0,1)$ localized in the interval $[0,1]$, generates a diffeomorphism subgroup $D_2(\lambda)$ with 4 fixed points which leaves ψ invariant and is in fact the Tomita-Takesaki modular group of $(\mathcal{A}(0,1),\psi)$. This should be compared with the well-known fact [2] that the standard Moebius dilation $D(\lambda)$ with the two fixed points $0,\infty$ is the modular group of the standard pair $(\mathcal{A}(0,\infty),\Omega)$ where from now on Ω denotes the vacuum state vector. The generator of this $D_2(\lambda)$ dilation together with those of the modular generated Möbius group generate the full system of L_n-generators $n \in \mathbb{Z}$.

The third section generalizes this observation to arbitrary chiral theories.

In the fourth section we show that among the infinitely many fuzzy transformations in higher dimensional QFT which are of modular origin, there is an important subset which agrees with the above diffeomorphism but only if we "holographically reprocess" the original theory. The necessary holography is more radical than the better known (but very special) AdS-CQFT holography an aspect which it shares with the recently proposed concept of "transplantation" [5]. In particular there is no lower dimensional asymptotic region for which the pointlike correlation functions of the original theory coalesce with those of the holographic image [6]. Whereas in the AdS-CQFT isomorphism, as a result of its shared high symmetry, one needs the algebraic reprocessing (which avoids field coordinatizations) mainly in the *unique determination of the higher dimensional AdS_{d+1} theory in terms of its lower dimensional $CQFT_d$ holographic image [7]*, in more general less symmetric cases, in particular when the holographic image is localized on the causal horizon of a higher dimensional region, the algebraic method becomes *indispensable for both directions* of the holographic processing [8][9].

For this reason the algebraic quantum field theory setting of this paper is crucial and we hope to convince the reader that this conceptually novel way of looking at QFT is more than just a pastime of some specialists who are less than satisfied with the standard way of doing QFT.

The transition from standard QFT to AQFT is similar to that of geometry in coordinates to the modern intrinsic coordinate-free formulation with the pointlike fields corresponding to the use of coordinates. But contrary to coordinate-free geometry, which still uses coordinate patches in the formulation of manifolds and allows to derive all results in principle by (often clumsy) calculation in coordinates, it is less clear that every algebraically defined QFT allows a complete description in terms of pointlike field generators. For chiral theories and presumably also for higher dimensional conformal field theories the algebraic and pointlike formulations are known to be equivalent [21]. The importance of the algebraic method would not be diminished if pointlike field descriptions exist on both sides of the holography, since the field coordinatization of the original theory turns out to be radically different from that of its holographic image.

We illustrate these points in an appendix by d=1+1 examples of massive models. It should be already clear from our use of terminology that the holography in this paper is primarily treated as quantum phenomenon in Minkowski space. In a way it supersedes lightray/lightfront quantization and the $p \to \infty$ frame method in a way which will be explained later.

The enhancement of quantum symmetries to (generally "fuzzily acting") infinite symmetry groups is also very interesting from a mathematical viewpoint. The quest of the mathematicians as to what constitutes the most useful generalization of finite dimensional Lie group theory to infinite dimensions has found a successful answer in the theory of loop groups (more generally Kac-Moody algebras) and associated diffeomorphism groups of the circle (Virasoro algebras). However the physical use of these mathematical findings is restricted to low dimensional field theory where they make a perfect match with the causality aspects of current or energy-momentum operators [10]. Therefore it is natural to ask whether the causality and localization principles, together with some other mathematical vehicle which does not confine them to low spacetime dimensions, could not produce a more general framework of symmetry- and group generalizations. Our tentative answer is that, thanks to the existence of the Tomita-Takesaki modular theory [11], this seems to be indeed possible.

Physicists are used to distinguish *inner symmetries* related to charges of particles from *spacetime symmetries* (often called outer)[2]. In the first case the generalization has led to the relevance of braid groups, knot theory, superselection sector theory, endomorphism- and subfactor- theory. An important mathematical step was the study of Jones inclusions and the ensuing basic construction [12] which is intimately related to modular theory. For the generalized spacetime symmetries, the important step was to first unravel the modular origin of the geometric symmetries (Poincaré, conformal) and then to understand in which sense the infinitely many fuzzy modular symmetries can be seen as an answer to the above question. In this study the role of the Jones basic construction and subfactors is replaced by the notion of *modular inclusions [13].*

Our present proposal is belonging to this second kind of symmetry generalization which turns out to be closely related to a conceptually concise formulation of holography. As already mentioned before, the kind of holography required is closer related to the older lightray/lightfront quantization (the causal horizon of a (Rindler) wedge) [14] than it is to the extremely special AdS→CQFT holographic projection at spatial infinity. In fact it replaces the old light cone quantization which was extremely formal, and had in addition to those problems which it shares with the canonical quantization (restriction to canonical operator dimensions which creates a clash with genuine interactions) additional causality problems of its own in that it remained obscure in what sense the original local QFT can be reconstructed from the data of light cone quantization (or from the p→ ∞ frame method).

2 A mathematically controllable illustration

Let us first investigate the modular origin of the diffeomorphism group of the circle for the Weyl algebra. Following [15] we represent these diffeomorphism in terms of the standard dilation $D(\lambda)$ sandwiched between stretching and its inverse compression transformation

$$\begin{aligned}
&\xi : (0,1) \rightarrow (0,\infty)\ ,\ \xi(x) := \frac{2x}{1-x^2} \qquad (2)\\
&with\ inverse\ \xi^{-1}_{(0,1)}(x) = \frac{\sqrt{x^2+1}-1}{x} = -\frac{1}{x} + \sqrt{1+\frac{1}{x^2}}\\
&D_2(\lambda)x = \left(\xi^{-1}D(\lambda)\xi\right)(x) = -\frac{1-x^2}{2\lambda x} + \sqrt{1+\frac{\left(1-x^2\right)^2}{4\lambda^2 x^2}}\\
&D_2(\lambda) : (0,1) \rightarrow (0,1),\ extendible\ to\ diff.\ of\ R_{comp}
\end{aligned}$$

The ξ-transformations are the Cartesian versions of the circular transformations $z \rightarrow z^2, z \rightarrow \sqrt{z}$ which although they are themselves not diffeomorphism of the circle do "lift" the ordinary Möbius transformations $Moeb$ with two fixed points to its quasisymmetric (in the mathematical sense of complex function

[2]Outside of the standard approach to QFT one canot expect a sharp separation between these two symmetries.

theory) analog $Moeb_2$ with the $D_2(\lambda)$ the analog of $D(\lambda)$ having four fixed points instead of two. In the Cartesian description the $\lambda \to \infty$ limits are somewhat easier to handle.

It is not difficult to find a state of the current algebra which is invariant under $Moeb_2$, one only has to modify the vacuum two point function of the current by a factor[3]

$$\langle j(x)j(y)\rangle_2 = \langle j(x)j(y)\rangle_0 \frac{\left(1+x^2\right)\left(1+y^2\right)}{\left(1+xy\right)^2} \tag{3}$$

one easily checks that thanks to this additional factor the new expectation value is invariant under $Moeb_2$, in particular under $D_2(\lambda)$. But since the state is not faithful on algebras whose localization includes opposite intervals, we have to restrict the algebra generated by currents localized in the interval $(0,1)$. Whereas the transformation $D_2(\lambda)$ is globally well-defined, the state $\langle j(x)j(y)\rangle_2$ is only faithful and normal on subalgebras which are localized between two fixed points of $D_2(\lambda)$. In order to have a formulation in terms of bounded operators one may pass to the Weyl algebra in which case the above formula defines a state $\langle\cdot\rangle_2$ on the von Neumann algebra

$$\begin{aligned} \mathcal{A}(0,1) &\equiv alg\left\{W(f)|\, suppf \subset [0,1]\right\} \\ W(f) &= e^{ij(f)},\ j(f) = \int f(x)j(x)dx \\ \langle W(f)\rangle_2 &= e^{-\frac{1}{2}\langle j(f)j(f)\rangle_2} \end{aligned} \tag{4}$$

The invariance of this algebra and the state under $D_2(\lambda)$-transformations is expressed by the identity

$$\begin{aligned} &\int \frac{f_{\lambda,2}(x)g_{\lambda,2}(y)}{\left(x-y+i\varepsilon\right)^2}\frac{\left(1+x^2\right)\left(1+y^2\right)}{\left(1+xy\right)^2}dxdy = \\ &\int \frac{f(x)g(y)}{\left(x-y+i\varepsilon\right)^2}\frac{\left(1+x^2\right)\left(1+y^2\right)}{\left(1+xy\right)^2}dxdy \\ &f_{\lambda,2}(x) \equiv f(D_2(\lambda)x) = \left(D_2^{-1}(\lambda)\circ f\right)(x),\ suppf \subset [0,1] \end{aligned} \tag{5}$$

The intermediate steps of the calculation can be found in [15] page 146.

For the following it is convenient to rewrite the state in terms of the vacuum state and the previously introduced map ξ^{-1}

$$\begin{aligned} &\int \frac{f(x)g(y)}{\left(x-y+i\varepsilon\right)^2}\frac{\left(1+x^2\right)\left(1+y^2\right)}{\left(1+xy\right)^2}dxdy = \\ &\int \frac{\left(\xi\circ f\right)(x)(\xi\circ g)(y)}{\left(x-y+i\varepsilon\right)^2}dxdy,\ \ \left(\xi\circ f\right)(x) \equiv f(\xi^{-1}(x)) \\ &or\ \ \langle j(f)j(g)\rangle_2 = \langle j(\xi\circ f)j(\xi\circ g)\rangle \end{aligned} \tag{6}$$

In this way of writing the KMS property of the new state with respect to the action of $D_2(\lambda)$ on $\mathcal{A}(0,1)$

[3] The following calculations are identical to those in section 3 of [15]. The use of these calculations for the determination of the *modular theory of multi-intervals* is however incorrect since the state $\langle\cdot\rangle_2$ ceases to be faithful on large algebras containing oppositely localized intervals. This is the reason for the present restriction of test functions.

becomes obvious and follows from the identity [15]

$$\left\langle j(D_2^{-1}(\lambda)\circ f)j(g)\right\rangle_2 = \left\langle j(D^{-1}(\lambda)\circ\xi\circ f)j(\xi\circ g)\right\rangle \tag{7}$$
$$supp f\subset[0,1]\,,\ supp\xi\circ f\subset[0,\infty]$$

together with the vacuum KMS property of the right hand side [19]. The right hand side has the strip analytic properties in $t\to t+i$ where $\lambda=e^{2\pi t}$ and the values on the upper rim of the strip are related to those of the lower by the KMS relation. With the KMS prerequisite for the action of $D_2(\lambda)$ on $\mathcal{A}(0,1)$ in the state $\omega_2\equiv\langle\cdot\rangle_2$ being fulfilled, one only has to find a globally $D_2(\lambda)$-invariant vector $|\psi_2\rangle$ in H which implements ω_2

$$\langle A\rangle_2 = \langle\psi_2\,|A|\,\psi_2\rangle \tag{8}$$
$$A\in\mathcal{A}(0,1)$$

For this purpose we use the fact that the faithful state ω_2 on the algebra $\mathcal{A}(0,1)$ has a unique vector implementation in the natural cone $\mathcal{P}_\Omega$ of the standard pair $(\mathcal{A}(0,1),\Omega)$ [20] with a vector ψ_2 which is automatically separating with respect to $\mathcal{A}(0,1)$ as a result of the faithfulness of ω_2 on $\mathcal{A}(0,1)$. This globally defined vector inherits the invariance under the $D_2(\lambda)$ action from ω_2 and hence $D_2(\lambda=2\pi t)$ is the unitary modular operator $\Delta^{it}_{(\mathcal{A}(0,1),\psi_2)}$ of $(\mathcal{A}(0,1),\psi_2)$. The expectation values of operators localized outside the interval $(0,1)$ are only known after an explicit calculation of ψ_2[4]. They are certainly not given by the extension of the state ω_2 (4) outside $\mathcal{A}(0,1)$.

Since the automorphism induced by the Ad action of D_2 on the test functions is identical to that of $\frac{1}{2}(L_2-L_{-2})$ in standard notation and the commutator with L_0 is proportional to the sum L_2+L_{-2} and hence one obtains all the generators of $Moeb$ and $Moeb_2$ through commutators. It has been shown that $Moeb$ has a modular origin [13][16]. From this, together with the fact that all the $L_{\pm n}$ for $n>2$ are obtainable iteratively from $L_{\pm1}$ and $L_{\pm2}$, follows our claim that the diffeomorphism group of the chiral current-Weyl algebra can be obtained by modular methods. This is very interesting since modular methods to not suffer from restrictions in spacetime dimensions as those methods which use the structure of the chiral energy momentum tensor or the chiral current algebra.

In order to elevate this model observation into a new tool of local quantum physics, we still have to overcome to hurdles. First we should generalize this observation to arbitrary chiral theories and second we should understand the higher dimensional analog.

3 Extension to general chiral theories

We now apply similar considerations to general chiral theories. For such models it has been shown that there is not much difference between the pointlike and the algebraic viewpoint; algebraic nets always may

[4] Most of the calculations in modular theory are to prove existence and structural properties whereas explicit constructions are usually ver hard.

be considered as being generated by pointlike fields [21]. For simplicity of presentation we will use the better known pointlike formulation.

In order to show the modular origin of the $D_2(\lambda)$-symmetry we study (for simplicity) the sequence of Wightman functions of the $D_2(\lambda)$ transforms of a (primary) field $D_2(\lambda)\ \phi$ with dimension d and show that the limit $\lambda \to \infty$ exists and is $D_2(\lambda)$ invariant. This can be used as an alternative method to show the existence of unitary implementation of the $D_2(\lambda)$-automorphism.

The Möbius-invariance of the vacuum can be expressed in terms of observable[5] correlation functions

$$\langle \phi(x_1)...\phi(x_m)\rangle_0 = [g'(x_1)...g'(x_m)]^d \langle \phi(g(x))...\phi(g\,(x_m))\rangle_0\,,\ g \in Moeb \tag{9}$$

Using the noninvariance of the vacuum under $D_2(\lambda)$ we define the following λ-dependent sequence of correlation functions

$$\begin{aligned}\varphi_\lambda(x_1,...,x_m) := [D_2(\lambda)'(x_1)]^d ... [D_2(\lambda)'(x_m)]^d \times \\ \times \langle \phi(D_2(\lambda)\,(x_1))...\phi\,(D_2(\lambda)(x_m))\rangle_0\end{aligned}$$

The following formula for $D_2(\lambda)$ is suitable for taking the limit $\lambda \to \infty$:

$$D_2(\lambda)x = 1 - \frac{1/\xi(x)}{\lambda} + \eta(\lambda,x)\ ,\ \forall x \in (0,1) \tag{10}$$

where η is a smooth function of x for fixed λ and is of order λ^{-2} for fixed x. In particular:

$$\lim_{\lambda\to\infty} \lambda\eta(\lambda,x) = 0\ ,\ \forall x \in (0,1) \tag{11}$$

We also have

$$D_2(\lambda)'x = \frac{\xi'(x)/\xi^2(x)}{\lambda} + \lambda\eta'(\lambda,x)\xi'(x) \tag{12}$$

and

$$\lim_{\lambda\to\infty} \lambda^2\eta'(\lambda,x) = 0 \tag{13}$$

We now can rewrite φ_λ:

$$\begin{aligned}\varphi_\lambda(x_1,...,x_m) := \left[\frac{\xi'(x_1)/\xi^2(x_1)}{\lambda} + \lambda\eta'(\lambda,x_1)\xi'(x_1)\right]^d ... \times \\ \times \left\langle \phi\left(1 - \frac{1/\xi(x_1)}{\lambda} + \eta(\lambda,x_1)\right) ... \right\rangle_0\end{aligned} \tag{14}$$

[5]For fields with anomalous dimensions which live on the covering of the conformal compactification this simple invariance relation is restricted to those conformal transformations which leave infinity invariant [17].

By using translational invariance of vacuum we obtain

$$\begin{aligned}\varphi_\lambda(x_1,...,x_m) &= \left[\frac{\xi'(x_1)/\xi^2(x_1)}{\lambda} + \lambda\eta'(\lambda,x_1)\xi'(x_1)\right]^d ... \times \\ &\times \left\langle \phi\left(-\frac{1/\xi(x_1)}{\lambda} + \eta(\lambda,x_1)\right)...\right\rangle_0 \\ &= \left(\frac{1}{\lambda}\right)^{md}\left[1\xi'(x_1)/\xi^2(x_1) + \lambda^2\eta'(\lambda,x_1)\xi'(x_1)\right]^d ... \times \\ &\times \left\langle \phi\left(\frac{1}{\lambda}\left[-1/\xi(x_1) + \lambda\eta(\lambda,x_1)\right]\right)...\right\rangle_0\end{aligned} \tag{15}$$

Using scaling invariance of vacuum, we get

$$\begin{aligned}\varphi_\lambda(x_1,...,x_m) &= \left[\xi'(x_1)/\xi^2(x_1) + \lambda^2\eta'(\lambda,x_1)\xi'(x_1)\right]^d ... \times \\ &\times \langle \phi\left[-1/\xi_{\tilde{n}}(x_1) + \lambda\eta(\lambda,x_1)\right]...\rangle_0\end{aligned} \tag{16}$$

By taking the limit $\lambda \to \infty$,

$$\begin{aligned}\lim_{\lambda\to\infty}\varphi_\lambda(x_1,...,x_m) &= \left[\xi'(x_1)/\xi^2(x_1)\right]^d ... \left[\xi'(x_m)/\xi^2(x_m)\right]^d \times \\ &\times \langle \phi\left[(-1/\xi(x_1))...\phi\left(-1/\xi(x_m)\right]\right\rangle_0\end{aligned} \tag{17}$$

Again, scaling invariance of vacuum implies

$$\begin{aligned}\lim_{\lambda\to\infty}\varphi_\lambda(x_1,...,x_m) &= \left[\left.\left(\frac{-1}{\xi}\right)'\right|_{(x_1)} ... \left.\left(\frac{-1}{\xi}\right)'\right|_{(x_m)}\right]^d \times \\ &\times \langle \phi_{\alpha_1}\left(-1/\xi(x_1)\right)...\phi_{\alpha_m}\left(-1/\xi(x_m)\right)\rangle_0\end{aligned} \tag{18}$$

Möbius invariance of vacuum implies ($x \to -1/x$ belongs to the Möbius group)

$$\begin{aligned}\lim_{\lambda\to\infty}\varphi_\lambda(x_1,...,x_m) &= \left[\left.(-1/\xi)'\right|_{(x_1)} ... \left.(-1/\xi)'\right|_{(x_m)}\right]^d \xi^{2d}(x_1)...\xi^{2d}(x_m) \times \\ &\times \langle \phi\left(\xi(x_1)\right)...\phi\left(\xi(x_m)\right)\rangle_0\end{aligned} \tag{19}$$

Simplifying, we get the simple expression, analogous to 9,

$$\lim_{\lambda\to\infty}\varphi_\lambda(x_1,...,x_m) = \left[\xi'(x_1)...\xi'(x_m)\right]^d \langle \phi\left(\xi(x_1)\right)...\phi\left(\xi(x_m)\right)\rangle_0 \tag{20}$$

We call φ the state over $\mathcal{A}(0,1)$ defined by the above correlation functions and designed by $\langle ... \rangle_\Phi$ for latter convenience.

Proposition 1 *φ is a positive faithful normal state over $\mathcal{A}(0,1)$ which is invariant under $D_2(\lambda)$ for all $\lambda > 0$.*

Proof. *The local normality follows directly from the limit construction via a sequence of normal states. Positiveness of φ is equivalent to the proposition [18, Theorem 3-3]: for any sequence of test* *functions $\{f_j\}$, $f_j(x_1, ..., x_j)$ being C^∞ functions defined for $x_1, ..., x_j \in (0,1)$ and with $f_j = 0$ except for a finite number of j's, it holds the inequalities*

$$0 \leq \sum_{k,j=0}^{\infty} \int_0^1 ... \int_0^1 dx_1...dx_j dy_1...dy_k \bar{f}_j(x_1, ..., x_j) f_k(y_1, ..., y_k) \times \times \langle \phi(x_j)...\phi(x_1)\phi(y_1)...\phi(y_k)\rangle_\Phi \tag{21}$$

But, if we define the transformed functions

$$\hat{f}_j(x_1, ..., x_j) := f_j(\xi^{-1}(x_1), ..., \xi^{-1}(x_j)) \ \ , \ x_1, ..., x_j \in (0, \infty) \tag{22}$$

we can write the above sum as

$$\begin{aligned} &\sum_{k,j=0}^{\infty} \int_0^1 ... \int_0^1 dx_1...dx_j dy_1...dy_k \bar{f}_j(x_1, ..., x_j) f_k(y_1, ..., y_k) \times \\ &\times \langle \phi(x_j)...\phi(x_1)\phi(y_1)...\phi(y_k)\rangle_\Phi = \\ &= \sum_{k,j=0}^{\infty} \int ... \int dx_1...dx_j dy_1...dy_k \bar{f}_j(x_1, ..., x_j) f_k(y_1, ..., y_k) \times \\ &\qquad \times [\xi'(x_1)...\xi'(y_k)]^d \langle \phi(\xi(x_j))...\phi(\xi(x_1))\phi(\xi(y_1))...\phi((\xi(y_k))\rangle_0 \\ &= \sum_{k,j=0}^{\infty} \int ... \int dx_1...dx_j dy_1...dy_k \bar{f}_j(\xi^{-1}(x_1), ...) f_k(\xi^{-1}(y_1), ...) \times \\ &\times \left(\xi^{-1}\right)'(x_1)... \left(\xi^{-1}\right)'(x_j) \left(\xi^{-1}\right)'(y_1)... \left(\xi^{-1}\right)'(y_k) \times \\ &\times \left[\xi'(\xi^{-1}(x_1))...\xi'(\xi^{-1}(y_k))\right]^d \langle \phi(x_j)...\phi(x_1)\phi(y_1)...\phi(y_k)\rangle_0 \\ &= \sum_{k,j=0}^{\infty} \int ... \int dx_1...dx_j dy_1...dy_k \bar{f}_j(\xi^{-1}(x_1), ...) f_k(\xi^{-1}(y_1), ...) \times \\ &\times \langle \phi(x_j)...\phi(x_1)\phi(y_1)...\phi(y_k)\rangle_0 \\ &= \sum_{k,j=0}^{\infty} \int_0^1 ... \int_0^1 dx_1...dx_j dy_1...dy_k \bar{\hat{f}}_j(x_1, ..., x_j) \hat{f}_k(y_1, ..., y_k) \times \\ &\qquad \times \langle \phi(x_j)...\phi(x_1)\phi(y_1)...\phi(y_k)\rangle_0 \end{aligned} \tag{23}$$

which turns out to be non-negative as a result of vacuum positivity..

From a similar expression, we can see φ is faithful on $\mathcal{A}(0,1)$ since vacuum is faithful on $\mathcal{A}(0,\infty)$. First, note that

$$\xi(D_2(\lambda)x) = \lambda\xi(x)$$

and

$$\xi'(D_2(\lambda)x) = \frac{\lambda\xi'(x)}{D_2(\lambda)'(x)}$$

Then

$$\begin{aligned}&\langle\phi(D_2(\lambda)x_1)...\phi(D_2(\lambda)x_n)\rangle_\Phi = \\&[\xi'(D_2(\lambda)x_1)...\xi'(D_2(\lambda)x_n)]^d \langle\phi(\lambda\xi(x_1))...\phi(\lambda\xi(x_m))\rangle_0 \\&= \left[\frac{\lambda\xi'(x_1)}{D_2(\lambda)'(x_1)}...\frac{\lambda\xi'(x_n)}{D_2(\lambda)'(x_n)}\right]^d \lambda^{-nd}\langle\phi(\xi(x_1))...\phi(\xi(x_n))\rangle_0 \\&= [D_2(\lambda)'(x_1)...D_2(\lambda)'(x_n)]^{-d}\langle\phi(x_1)...\phi(x_n)\rangle_\Phi\end{aligned} \tag{24}$$

■

From the calculations in the above proof, we see that $D_2(\lambda)$ defines an automorphism on the field ϕ

$$[\alpha(D_2(\lambda))\phi](x) = [D_2(\lambda)'(x)]^d \phi(D_2(\lambda)(x)) \tag{25}$$

and the correlation functions $\langle...\rangle_2$ are transformed according to

$$\langle\phi(x_1)...\phi(x_m)\rangle_\Phi = [D_2(\lambda)'(x)...D_2(\lambda)'(x)]^d \langle\phi(D_2(\lambda)(x_1))...\phi(D_2(\lambda)(x_m))\rangle_\Phi \tag{26}$$

Since φ is normal, there exists a unique vector $|\Phi\rangle$ in the natural cone $\mathcal{P}_\Omega$ of the standard pair $(\mathcal{A}(0,1),\Omega)$ representing φ:

$$\varphi(A) = \langle\Phi|A|\Phi\rangle$$

Since φ is faithful for $\mathcal{A}(0,1)$, it turns out that Φ is cyclic and separating for this algebra. With this vector, we can extend φ to the entire algebra $\mathcal{A} = \mathcal{A}(\overline{\mathbb{R}})$.

To realize that the modular group associated to φ and $\mathcal{A}(0,1)$ is given by the action of $D_2(\lambda)$, we have to proceed in exactly the same manner as in the special case of current field, as explained in the previous section.

4 The higher-dimensional case

The easiest way to study the fuzzy counterpart of diffeomorphism outside of chiral models is to look at d=1+1 dimensional massive theories. Let us assume that we know a two-dimensional massive theory in the algebraic formulation i.e. as an inclusion preserving map of spacetime double cones $\mathcal{O}$ into operator algebras acting on one Hilbert space

$$\mathcal{O} \to \mathcal{A}(\mathcal{O}) \tag{27}$$

which fulfill a number of physically motivated requirements among which the two most prominent ones are Einstein causality and covariance with energy-momentum positivity. At this point one may as well imagine that this comes from an algebraic reformulation of the standard formulation in terms of a collection of pointlike covariant Bose fields which generate the net of spacetime indexed observable operator algebras.

In particular we may want to look at this net of algebras restricted to the right hand wedge $W : x > |t|$. Let $\mathcal{A}(W)$ denote the wedge algebra i.e. the weak limit of the union $\cup_{\mathcal{O}\subset W}\mathcal{A}(\mathcal{O})$ together with its subnet structure. In the setting of classical wave equations the data inside W are uniquely determined in terms of the characteristic data along either its upper or lower light ray horizon. The only exception to this would be the d=1+1 massless theory (classical wave equation) where one needs the data on both bounding horizons (of the Rindler world W), a fact which harmonizes well with the chiral factorization in the quantum field theory. With the help of a lightlike translation $T^{(+)}(a)$ along its upper light ray $R^{(+)}$ we generate a modular inclusion $W_a \subset W$ (Fig) i.e. an inclusion for which the modular group of $\mathcal{A}(W)$ which is the Lorentz boost $\Lambda_{t\text{-}x}(\chi)$ for positive rapidities χ compresses $\mathcal{A}(W_a)$ into itself. The relative commutant $\mathcal{A}(W_a)' \cap \mathcal{A}(W)$ is an algebra localized on the interval $(0, a)$ of the upper lightray (Fig 1, to be found on last page); this is the only localization region which causality associates with this relative commutant. The full upper right lightray algebra localized on the upper horizon $R^{(+)}$ is obtained as the union [24][22]

$$\mathcal{A}(R^{(+)}) = \bigcup_t Ad\Delta_W^{i\tau}(A(W_a)' \cap A(W)) \tag{28}$$
$$\Delta_W^{i\tau} = U(\Lambda_{t\text{-}x}(2\pi\tau))$$

with a similar construction of the net $\mathcal{A}(R^{(-)})$ localized on the lower horizon. It is easy to see that the extension of the halfline net $\mathcal{A}(R^{(+)})$ to the full line $\mathcal{A}(R)$ (using translations into the opposite direction or spatial reflections) is a conformal field theory. The translation and dilation of the 3-parametric Möbius group are inherited from the upper light cone translation $T(a)$ and the Lorentz boost $\Lambda_{t\text{-}x}(2\pi\tau)$ and the existence of the third Möbius-transformation (the "rotation") would follow from the "standardness of the modular inclusion" [23]. A prerequisite for obtaining in this way an algebraic version of a chiral QFT is the nontriviality of the relative commutant from which the nontriviality of the chiral net would follow. In the classical characteristic propagation setting for massive modes it is well known that the data on the upper (or lower) horizon of W determine the wave functions inside W. The counterpart of this on the quantum level is the equality

$$\mathcal{A}(W) = \mathcal{A}(R^{(+)}) \tag{29}$$
$$\curvearrowright A(R^{1,1}) = A(R)$$

(the last line is the equality for the full algebras which follows by using reflections or opposite translations) which has been proven for massive free theories in all spacetime dimensions[6] by using the associated Wigner one particle representation theory. For a general proof we lack sufficient insight into the structure of interacting theories. However for the class of factorizing d=1+1 theories we are able to show this

[6] For $d > 1{+}1$ the notation $\mathcal{A}(R^{(+)})$ stands for the transversally unresolved lightfront. Without the additional transversal resolution the lightfront holography image is incomplete i.e. one cannot reconstruct the transversal localization structure of the original net $\mathcal{A}(W)$ even though the equality (29) continues to hold for the longitudinal part of the net.

equality at least on the level of their formal expansions in terms of bilinear forms (related to formfactors) [24][25]. From this one learns that the validity of this equation is not restricted to canonical or near canonical dimensions of the generating fields.

The chiral conformal theory $\mathcal{A}(R)$ is not a theory of massless objects since otherwise it would be impossible to reconstruct the massive $\mathcal{A}$ theory from its holographic image. A massless chiral theory on the upper horizon would be invariant against a translation $T^{(-)}(a)$ into the lower lightray direction but this is definitely not the case for the present chiral theory (29)

$$\left[T^{(-)}(a), \mathcal{A}(R^{(+)})\right] \neq 0 \tag{30}$$
$$P^{(-)} \cdot P^{(+)} \; \textit{has mass gap}$$
$$P^{(-)},\, P^{(+)} \; \textit{are gapless}$$

where the $P's$ are the infinitesimal generators of the two light cone translations. With other words the chiral theories which originate from holographic projection onto the horizon possess an additional automorphism which is not part of the set of physical spacetime automorphisms for zero mass theories. The prerequisite for an upper/lower chiral conformal theory namely the gaplessness of lightcone translations is always guarantied. In order to recover the full $\mathcal{A}(W)$ theory in higher spacetime dimensions $d > 1+1$ the study of only one modular inclusion is not sufficient. We will present the necessary steps for an invertible holographic imaging somewhere else (for some ideas in this direction see).

Before we use the above d=1+1 holographic projection for the construction of higher modular symmetries it is instructive to contrast the above construction of a horizon-localized theory with the formal "lightcone quantization" or "infinite momentum frame" method. In that case one selects concrete field coordinates and studies what happens to them of one restricts to the lightray without converting them first into the net of algebras which they generate. One runs into problems even if these fields are free fields which are related to the singularities which one encounters if one approaches a light ray distance in pointlike correlation functions of pointlike fields. Whereas some of these problems (at least in the case of low spin) can be avoided by careful application of distribution theory, this is not possible if the ultraviolet singularities are too big. The reprocessing of a field into its associated net of field-coordinatization-free algebras is a much more radical physical step than enhancing mathematical rigor by the use of distribution theory. Our above arguments have shown that the nontriviality of relative commutant algebras in the modular inclusion construction has a much better chance than nontriviality in the face of hard to control ultraviolet behavior. After the holographic net has been constructed one may parametrize this net in terms of pointlike chiral fields; the possibility of generating chiral nets by pointlike fields is guarantied by mathematics [21] and again there is no limitation from short distance behavior. But these chiral fields after holographic projection have little to do with the original fields (assuming that our original net of algebras was generated by massive pointlike fields). This shows why lightcone quantization remained a highly artistic (mathematically hard to control, but with a lot of physical intuitive content) procedure.

What is behind it is "holographic reprocessing" linking QFTs in different spacetime dimensions, whose understanding defies the methods of standard QFT as Lagrangian/functional integral quantization but not their physical principles.

Having arrived at a faithful (isomorphic) holographic image, it is not difficult to guess what the looked for fuzzy analogs of the chiral diffeomorphisms are. Certainly the conformal rotation acts on the $\mathcal{A}(R)$ net geometrically whereas its action on the two-dimensional massive net $\mathcal{A}(R^{1,1})$ is fuzzy. But according to the previous section there is the infinite group of chiral diffeomorphisms which.

As a result of the necessity to obtain as well the transversal net structure, the higher dimensional holography is much more involved and will not be discussed here.

5 Outlook

In this short note we tried to draw attention to a new point of view which may be important for a better understanding of QFT as well as for a possible mathematical enrichment in the area of infinite dimensional Lie groups. The physical causality and localization principles of local quantum physics make a perfect natural match[7] which do not directly act on spacetime but rather on test function spaces of smeared fields or nets of von Neumann algebras in a way which is fuzzy but nevertheless *compatible with causality and localization requirements*. We argued that this is the correct analog of the diffeomorphisms in chiral theories by using appropriately defined holographic images.

The important mathematical tool in all these considerations is the modular theory of Tomita-Takesaki adapted to the field theoretic nets of local von Neumann algebras. These are the same methods as those which show the thermal aspects of the vacuum state upon restriction to local algebras among which the Unruh-Hawking illustration for wedge-localized algebras is the most prominent example. Our findings do not only generalize the spacetime Noether symmetries of the Lagrangian formalism, *but they also make the recent (somewhat ad hoc) use of the chiral diffeomorphism algebras in connection with attempts at the derivation of a sufficiently generic (i.e. model independent) quantum Bekenstein area law more palatable [26].* In fact they suggest to look for a "localization entropy" in standard QFT which could be the Minkowski space local quantum physics analog which preempts the existence of a sufficiently generic quantum explanation of the Bekenstein area law in the more geometric setting of curved spacetime and eventually may lead to clues about quantum gravity.

Whereas the thermal aspect of wedge- or double- cone localized algebras has a very easy explanation in terms of the AQFT setting, the localization- entropy discussion is much more subtle [25]. By the very nature of the local von Neumann algebras (hyperfinite type III_1) their (von Neumann) entropy is always infinite and one needs the formalism of "split inclusion" in order to arrive at the conjecture that perhaps the restriction of the vacuum state to the local algebra including a "collar" of thickness δ around

[7] In fact they are linked in such an inexorable fashion that any cutoff- or noncommutative geometry- modification would wreck both sides.

the boundary may be finite and for small δ proportional to the area of the boundary. In this case the mechanism for the localization entropy would be similar to the vacuum polarization effect in partial Noether charges in the region where the smearing function decreases from its value one to zero.

¿From such a localization entropy with area proportionality towards a finite geometric entropy in the classical sense of Bekenstein and Hawking there may be still a long way. But what seems to be very intriguing already now is the observation that attempts of AQFT at generalizations of symmetries, encoding of spacetime into algebraic relations, thermal and entropy aspects of localization, holography and transplantations of QFTs point use the same modular concepts as the ones in the present work.

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5.1 Appendix: Examples from d=1+1 holography

Before presenting some examples it may be helpful to remind the reader that although in d=1+1 massive theories lightray restrictions as well as zero mass limits of fields lead both to chiral theories one should avoid confusing these limits. In order to avoid leave aside infrared divergences consider a (selfconjugate) free field with a halfinteger "Lorentz-spin"

$$\psi(x) = \frac{1}{\sqrt{2\pi}} \int \left(e^{ipx} u(\theta) a(\theta) + e^{-ipx} v(\theta) b^*(\theta) \right) d\theta = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \tag{31}$$

$$u(\theta) = \sqrt{m} \begin{pmatrix} e^{-\frac{1}{2}\theta} \\ e^{\frac{1}{2}\theta} \end{pmatrix}, \ v(\theta) = \sqrt{m} \begin{pmatrix} e^{-\frac{1}{2}\theta} \\ -e^{\frac{1}{2}\theta} \end{pmatrix}$$

$$p = m(ch\theta, sh\theta)$$

The zero mass limit for the first component requires

$$lim_{m\to 0} e^{\frac{1}{2}(lnm - \theta)} < \infty \tag{32}$$

$$\curvearrowright \theta = \hat{\theta} + lnm \to \hat{\theta} - \infty$$

i.e. θ is driven towards (negative) infinity. The c-number factors in the rapidity integral can be rewritten in terms of the reduces rapidity $\hat{\theta}$ only

$$e^{ipx} \sqrt{m} e^{-\frac{1}{2}\theta} \to e^{ie^{\hat{\theta}} x_+} e^{-\frac{1}{2}\hat{\theta}} \tag{33}$$

but there is no limit for the annihilation/creation operators in the Fock space $a(\theta) \to a(\hat{\theta} - \infty)$. Although there is no zero mass limit in the original Fock space, such a limit makes perfect sense for the two-point function (and hence for all correlation functions). Its operator reconstruction gives a radical change of Fock space: instead of the original $a^{\#} - b^{\#}$ degrees of freedom one now has the well-known *left-right mover doubling*.

The lightray restriction on the other and leads to a chiral theory without this doubling. Parametrizing the x in terms of Lorentz-length $\sqrt{\pm x^2}$ and rapidity χ, and letting the $ln\sqrt{\pm x^2}$ approach $-\infty$ and $\chi \to \infty$ (which leaves the argument of the operators unchanged), we obtain a chiral field without doubling. The 2-dimensional massive field may be reproduces by applying a lightray translation in the opposite direction.

We also may consider the last limit in the spirit of the more rigorous holographic setting. In that case one would start from the definition of the wedge localized algebra which can be generated by smeared fields with smearing functions supported in W

$$\psi_+(f),\ suppf \in W \tag{34}$$

and fulfill a modular boundary condition in their strip-analytic (rapidity-parametrized) Fouriertransforms which characterize the real subspace H_R of the previous section. The modular theory for Fermions is the same up to an additional "twist" operator K in $S = JK\Delta^{\frac{1}{2}}$ which accounts for the difference between the geometric and the algebraic opposite (in the sense of the commutant). The same algebra can be generated with the second component $\psi_-(g)$; one only has to pay attention to the slightly different analytic characterization of H_R in terms of the $g's$. In this case the upper horizon algebra defined as in the previous section can also be generated by smearing the chiral lightray fields obtained in the limit $ln\sqrt{-x^2} \to -\infty$, $\chi \to \hat{\chi} + \infty$ with test functions $f(x_+)$ $suppf \in R_+$ whose rapidity-parametrized Fouriertransforms with the modular boundary conditions define the same spaces as those in (34). With other words not only the algebras (29) are identical, but already the generators are equal. It is very important to stress that the direct construction of such lightray-restricted fields only works if the operator dimension of the field is close to its free-field value. More precisely the two-point Kallen-Lehmann spectral function must fulfill

$$\int \rho(\kappa^2) d\kappa^2 < \infty \tag{35}$$

which is the as the well known finiteness of the inverse wave function renormalization constant of the field in the canonical formalism. In addition to this restriction there is an infrared restriction on the smearing function for small momenta in the case of s=0

$$\int_0^\infty \frac{\bar{f}(p)g(p)}{p} dp < \infty$$

which restricts the localizing test functions in x space to those which are a derivative of a localized function (so that only the current i.e. the derivative of the would be scalar field makes sense). In free field theory the above restriction process is therefore only applicable to the free field itself, but not to its composites which constitute the Borchers equivalence class of field-coordinatizations.

In the presence of interactions one cannot expect pointlike fields which generate the chiral holographic projection to be simply related to the original field. It is known that the Zamolodchikov-Faddeev nonlocal generalization of the free field algebra (we only look at the simplest case for which the so-called Sinh-

Gordon Z-F algebra is an example)

$$F(x) = \int \left(Z(\theta)e^{-ipx} + Z^*(\theta)e^{ipx}\right) d\theta \tag{36}$$
$$Z(\theta)Z(\theta') = S(\theta-\theta')Z(\theta')Z(\theta)$$
$$Z(\theta)Z^*(\theta') = S^{-1}(\theta-\theta')Z^*(\theta')Z(\theta) + \delta(\theta-\theta')$$

where the $Z^{\#}s$ can be explicitly represented in terms of nonlocal rapidity space expressions in $a^{\#\prime}s$ Fock space operators. It has been shown that although the $F(x)$ are non-causal (non-local), they still carry the notion of wedge-localization i.e. $F(f)$ is affiliated with the wedge algebra $\mathcal{A}(W)$ if and only if the structure functions of the Z-algebra fulfill the properties of an admissable 2-particle elastic S-matrix i.e. besides unitarity the all important crossing symmetry including a bootstrap bound state structure. In fact it is believed that the spacetime concept of d=1+1 tempered polarization-free generators [27] (PFG's) and the momentum space Z-F algebraic structure are completely equivalent. A general $\mathcal{A}(W)$-affiliated operator can be shown to be of the form

$$A = \sum \frac{1}{n!} \int_C ... \int_C a_n(\theta_1, ...\theta_n) : Z(\theta_1)...Z(\theta_n) : \tag{37}$$

where the a_n have a simple relation to the various formfactors of A (including bound states) whose different in-out distributions of momenta correspond to the different contributions to the integral from the upper/lower rim of the analytic strip bounded by the contour C, which are related by crossing [25]. Note that when we write down such sums without discussing their convergence related to their operator aspects we are only dealing with bilinear forms i.e. matrix elements between a dense set of multi-particle state vectors of "would-be operators". The affiliation to double cones (Fig 2) is obtained by restricting the space of wedge affiliated objects by requiring their commutance with the spatially translated (to the right inside W) generators $F_a(f)$

$$[A, F_a(f)] = 0 \ \forall f \ suppf \subset W \tag{38}$$
$$F_a(f) = U(a)F(f)U^*(a)$$

This condition defines the bilinear forms A affiliated with $\mathcal{A}(C_a) = \mathcal{A}(W_a)' \cap \mathcal{A}(W)$ which for a =right spacelike is a double cone C_a with one apex in the origin and the other at a. The condition on the coefficients is the famous "kinematical pole relation" of Smirnov [28], a recursive condition which reads

$$a_{n+2}(\vartheta + i\pi + i\varepsilon, \vartheta, \theta_1, \theta_2....\theta_n) \simeq \frac{1}{\varepsilon}\left[1 - \prod_{i=1}^{n} S(\vartheta - \theta_i)\right] a_n(\theta_1, \theta_2....\theta_n) \tag{39}$$

with an additional Paley-Wiener type condition on the meromorphic $a_n s$ which encodes the geometry of C_a. Pointlike fields obey the same relation but with a polynomial behavior which distinguishes the individual fields after having split off certain standard formfactors which are the same for all fields in the same sector.

The important point in all these technicalities is now the fact that if we would have taken a lightlike a on the upper horizon of W as was done for the definition of an interval algebra (Fig 1)

$$A(R(0,a)) = \mathcal{A}(W_a)' \cap \mathcal{A}(W)$$

in the holographic formalism of section 4, the condition would also had led us to (39). This underlines the equality (29) of the wedge algebra with its holographic image . But the would be pointlike fields affiliated with $\mathcal{A}(W)$ have no restrictions onto the upper horizon, rather the chiral fields affiliated with $\mathcal{A}(R_+)$ have to be determined anew according to [21] using the Möbius symmetry of the holographic image. Even more, the spacetime indexing of the holographic net (and hence the physical interpretation) is very different (relatively fuzzy) to that of the original net. Realistic lightfront holography implies a violent "scrambling", but it is still possible to reconstitute the original net structure. In this respect the very mild AdS-CQFT holography is somewhat atypical.

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light-like translated wedge:

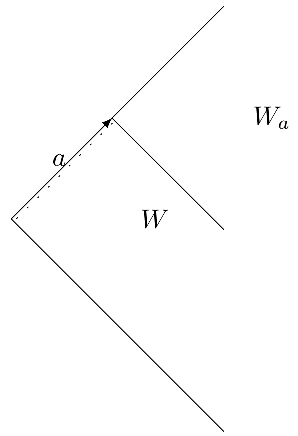

*Fig.1: Standard wedge translated by a light-like vector a.
The broken line indicates the localization of
relative commutant.*

space-like translated wedge:

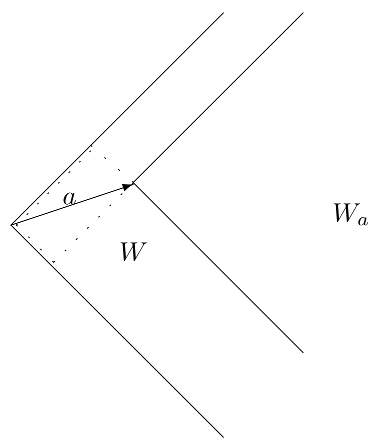

*Fig.2: Standard wedge translated by a space-like vector a.
The localization of the relative commutant is the
double cone indicated by broken lines.*