

An Astrophysical Test-Bed for the NDL Gravity Theory Using Gravitational-Wave and Neutrino Bursts from Local Supernovae Explosions

H. J. Mosquera Cuesta^{1,2}, M. Novello¹ and V. A. De Lorenci³

¹Centro Brasileiro de Pesquisas Físicas, Laboratório de Cosmologia e Física Experimental de Altas Energias
Rua Dr. Xavier Sigaud 150, Cep 22290-180, Rio de Janeiro, RJ, Brazil

²Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, Miramare 34014, Trieste, Italy

³Instituto de Ciências - Escola Federal de Engenharia de Itajubá, Av. BPS 1303, Pinheirinho 37500-000, Itajubá,
MG, Brazil

e-mail: herman@lafex.cbpf.br :: novello@lafex.cbpf.br :: lorenci@efei.br

Abstract

The Novello-DeLorenci-Luciane (NDL) theory of gravity predicts that gravitational waves follow geodesics of a modified (effective) geometry with a speed lower than the velocity of light. Here a prospective astrophysical test of this prediction is proposed. We point out that the future measurements of gravitational waves in coincidence with a non-gravitational process as a neutrino burst (and potentially a gamma-rays burst) will prove useful to discriminate among all the existing theories of gravitation.

I. INTRODUCTION

A succesful theory of gravity should be able to correctly predict the way this interaction occurs in all process in nature. Einstein's theory of gravitation has till now passed all of the tests in this concern. However, it encompasses an implicit statement concerning the way gravity-gravity interaction develops when compared to gravity-nongravitational energy interactions. General relativity stands on the *equivalence principle*, which states that any sort of matter including massless fields like the photon, interacts with gravitational fields fundamentally inasmuch as the same manner. This statement allows to interpret all the gravitational interactions, including gravity-gravity as well (this one having no any experimental or observational foundation), as due to changes in the space-time geometry induced by the presence of matter fields $g_{\mu\nu} = \gamma_{\mu\nu} + \varphi_{\mu\nu}$. However, if one dismisses the assumption that the gravitational energy should encompass the hypothesis of universality of the equivalence principle, i. e., Einstein equivalence principle does not apply to free falling "gravitons", a field theory of gravity in which the gravity-gravity interaction occurs in a rather different way compared to gravity-nongravity can be formulated [1].

The Novello-DeLorenci-Luciane (NDL) theory of gravitation has been recently introduced [1]. It was shown it incorporates essentially all the ingredients general relativity endowes [1], and in this vein it resembles Einstein theory as far as the first post-Newtonian approximation for solar system tests is concerned, and also for the radiative solution up to the quadrupole formula level. It has been demonstrated that the most striking prediction of the NDL theory is related to the velocity of propaga-

tion of gravitational perturbations [2]¹. In Ref. [2] was shown that gravitational waves (GWs) travels in the null cone of an effective geometry with a speed lower than the velocity of light, the one for GWs to travel in Einstein's theory.

The GWs dispersion relation in the NDL theory reads

$$k_\mu k_\nu [\gamma^{\mu\nu} + \Lambda^{\mu\nu}] = 0, \quad (1)$$

where

$$\Lambda^{\mu\nu} = 2 \frac{L_{UU}}{L_U} [F^{\mu\nu\beta} F_{(\alpha\beta)}^\nu - F^\mu F^\nu], \quad (2)$$

where L_U and L_{UU} corresponding, respectively, to the first and second derivative of the Lagrangian of the theory respect to the invariant U , defined below. In Eq.(2) the 3-tensor

$$F_{\mu\alpha\beta} = \frac{1}{2} (\varphi_{\mu[\alpha;\beta]} + F_{[\alpha\gamma\beta]\mu}) \quad (3)$$

is interpreted as the gravitational field, which is constructed upon the standard variable $\varphi_{\mu\nu}$ identified as usual with the gravitational potential in the theory. Its trace, Eq.(2), is defined as

$$F_\mu = F_{\mu\alpha\beta} \gamma^{\alpha\beta} = \varphi_{,\alpha} - \varphi_{\alpha\mu;\nu} \gamma^{\mu\nu}. \quad (4)$$

Here symbols $,$ and $;$ stands for ordinary and covariant derivative, respectively, with the Minkowski background.

¹A new more stringent test of the NDL theory predictions concerning the birefringence of the GWs will be addressed elsewhere [3]. It is shown there that birefringence of GWs is a peculiar characteristic of almost all non-linear theories of gravity *except general relativity*, and in particular of the NDL.

Thence the discontinuities of the gravitational fields propagates in a modified geometry which changes the background geometry $\gamma^{\mu\nu}$ (the Minkowski metric) into an effective one

$$g_{eff}^{\mu\nu} \equiv \gamma^{\mu\nu} + \Lambda^{\mu\nu}, \quad (5)$$

which has dependence on the fields $F_{\alpha\beta\mu}$ and its dynamics. The overall characteristic of the new geometry is determined by the non-linear character of the lagrangian on which the theory is based. Then the GWs velocity in the NDL reads

$$v_k^2 = 1 - \frac{1}{2b^2} \frac{1}{[1 + (k/b^2)\mathcal{L}]^2} Z^{\mu\nu} \frac{k_\mu}{|\vec{k}|} \frac{k_\nu}{|\vec{k}|}, \quad (6)$$

with the velocity of light $c = 1$ in geometric units. Here we define

$$Z^{\mu\nu} = F^{\mu(\alpha\beta)} F_{(\alpha\beta)}^\nu - F^\mu F^\nu. \quad (7)$$

In the expression for the velocity of the GWs, Eq.(6), the Born-Infeld type Lagrangian density

$$\mathcal{L} = \frac{b^2}{k} \left[\sqrt{1 - \frac{U}{b^2}} - 1 \right], \quad (8)$$

with b a constant and $k \equiv \frac{8\pi G_N}{c^4}$, is the most general functional of the invariant of the theory U . The quantity U , the dynamical parameter of the NDL theory, is defined in terms of the two fundamental invariants of the theory, A and B , as

$$U \equiv A - B \quad (9)$$

with

$$A \equiv F_{\mu\alpha\beta} F^{\mu\alpha\beta} \quad (10)$$

and

$$B \equiv F_\mu F^\mu. \quad (11)$$

Note that in the linear regime $\mathcal{L}(U) = U$. We then obtain the standard weak-field limit as it should be for any massless spin-2 theory of gravity, including general relativity. The reader can see Ref. [2] for a more detailed discussion of the NDL gravitation.

Thus a crucial test of the NDL theory, and consequently a potential discriminator among the existing theories of gravity, could be an exact determination of the velocity of propagation of the GWs themselves. This is an issue which is expected to be accomplished with the advent of the new generation of GW detectors such as the interferometers LIGO, VIRGO, GEO-600, and the TIGAs resonant-mass omni-directional observatories [4]. Below we suggest a prospective astrophysical experimental test of the NDL theory involving the detection of GWs

in coincidence with a neutrino burst from a supernova explosion, including collapsars or hypernovae events.

We stress that the future detection of the GWs themselves (at least for one detector) is unable to provide the looked for discriminator criteria to settle this issue in the light of Einstein's and NDL theories. Therefore, a non-gravitational astrophysical or cosmological process is called for, and the expected neutrino bursts from both the *deleptonization* process in the supernova core and the gamma-ray burst surge accompanying the GWs in a hypernova event may prove useful.

It worths to quote that other tests have been suggested by Bianchi et al. [5] to be used to discriminate among the different theories of gravitation already proposed. The idea is that the spin content of any gravity theory can be extracted by relating the measurements of the gravitational wave excited spheroidal eigen-modes to the Penrose-Newman parameters. Because the resonant-mass detector toroidal modes (a sphere in their study case) cannot be excited by any metric GW they can be used as a veto. However, we stress that this method to constrain the more correct theory of gravity cannot prove useful to settle the issue between the NDL and Einstein's theory because it only works under the assumption that both radiations propagate at the speed of light, a property that is clearly not endowed for both theories. In this vein the Bianchi et al. method [5] is a limited one. Thence, it remains that in order to really discriminate between the NDL and general relativity an additional astrophysical or cosmological *non-gravitational* process must be involved. That is why we suggest that the concomitant production and quasi-simultaneous detection of GWs and neutrino (and most likely gamma-rays) bursts from a local supernova event may prove a more stringent procedure to pinpoint the issue.

II. CORE-COLLAPSE AND NEUTRINO-DRIVEN SUPERNOVAE EXPLOSIONS

During the precedent three decades most researchers in supernovae physics have explained type-II events as a consequence of neutrinos carrying the huge binding energy of the newly born neutron star. Then neutrinos deposit a portion of their energy in a low density region surrounding the star's core and a fireball of pairs and radiation finally explodes the remainings of the star. In these lines, core-collapse supernovae explosions are one of the most powerful sources of neutrinos ν_e, ν_μ, ν_τ and its antiparticles, and likely the sterile one ν_s . Different theoretical and numerical models of type II supernovae explosions [6–8] have estimated that

$$\Delta E_{total} = 5.2 \times 10^{53} \text{erg} \left(\frac{10 \text{ km}}{R_{NS}} \right) \left(\frac{M_{NS}}{1.4 M_\odot} \right)^2 \quad (12)$$

are carried away by neutrinos. Almost $\sim 10^{58}$ neutrinos of mean energies (10 – 25) MeV are released over a time scale of seconds through the process $\gamma + \gamma \longrightarrow e^+ + e^- \longrightarrow \bar{\nu} + \nu$. Investigations have shown that nearly 99% of the total gravitational binding energy of the protoneutron star can directly be carried away by these neutrinos on their diffusion timescale $\Delta t_{nu} \sim 12$ s after the core bounce $\Delta t_{CB} \sim 1$ ms [6,7]. The remaining energy being radiated in electromagnetic and gravitational waves.

A. Neutrino Production in GRBs

Current models of GRBs predict both ultra high, very high [9] and high energy neutrinos [10] and ultra high energy cosmic rays emissions [9] which may account for the extra-galactic high energy proton flux observed. Next we discuss how the most energetic neutrinos (expected to accompany the GWs burst from a collapsar) are emitted according to the GRBs standard fireball model. The reader can see Ref. [9] for a more complete review of this mechanism. In the GRBs fireball picture the detected γ -rays are produced via synchrotron radiation of ultrarelativistic electrons boosted by internal shocks of an expanding relativistic blast wave (wind) of electron-positron pairs, some baryons and a huge number of photons. The typical synchrotron frequency is constrained by the characteristic energy of the accelerated electrons and also by the intensity of magnetic field in the emitting region. Since the electron synchrotron cooling time is short compared to the wind expansion time, electrons lose their energy radiatively. The standard energy of the observed synchrotron photons

$$E_\gamma^b = \frac{\Gamma \hbar \gamma_e^2 e B}{m_e c} \quad (13)$$

is given by

$$E_\gamma^b \simeq 4 \xi_B^{1/2} \xi_e^{3/2} \left(\frac{L_{\gamma,51}^{1/2}}{\Gamma_{300}^2 \Delta t_{\text{ms}}} \right) \text{MeV}, \quad (14)$$

where $L_{\gamma,51}$ defines the energy released in GRBs with $L_\gamma = 10^{51} L_{\gamma,51}$ ergs $^{-1}$ the standard luminosity of BATSE observed GRBs, $\Delta t = 1 \Delta t_{\text{ms}}$ ms is the typical timescale of variability, $\Gamma = 300 \Gamma_{300}$ the Lorentz expansion factor, and ξ_B corresponds to the fraction of energy carried by the magnetic field

$$4\pi r_d^2 c \Gamma^2 B^2 = 8\pi \xi_B L \quad (15)$$

being L the total wind luminosity, and ξ_e the one electrons carry away. No theory is available to provide specific values for both ξ_B and ξ_e . However, for values near the equipartition the model photons' break energy E_ν^b

is in agreement with the observed one for $\Gamma \sim 300$ and $\Delta t = 1$ ms, as discussed below.

The hardness of the GRBs spectra, which extend to 100 MeV, constrains the wind to have Lorentz factor $\Gamma \sim 300$, while the observed variability of the GRBs flux on a timescale $\Delta t \sim 1$ ms implies that the internal collisions occur at a radius $r_d \sim \Gamma^2 \Delta t$, as due to variability of the central engine on the same timescale. Since most of the BATSE observed GBRs present variability on $\Delta t \leq 10$ ms and the bursters rapid variability is $\Delta t \sim 1$ ms, the implied characteristic dimension of the emitting region is $r_{em} \sim 10^7$ cm, that is, it should be a compact object.

In the acceleration region protons (the fireball baryon load) are also expected to be shocked. Then their *photo-meson* interaction with observed burst photons should produce a surge of neutrinos almost simultaneously with the GRBs via the decay $\pi^+ \leftrightarrow \mu^+ + \nu_\mu \leftrightarrow e^+ + \nu_e + \bar{\nu}_\mu + \nu_\mu$. The neutrino spectrum in the fireball driven explosion follows the observed γ -rays spectrum, which approximates the broken power-law

$$\frac{dN_\gamma}{dE_\gamma} \propto E_\gamma^\beta, \quad (16)$$

with $\beta \sim 1$ for low energies and $\beta \sim 2$ for high energies compared to the observed *break energy* $E_\gamma^\beta \sim 1$ MeV, where β changes. The interaction of protons accelerated to a power-law distribution

$$\frac{dN_p}{dE_p} \propto E_p^{-2} \quad (17)$$

with the fireball photons results in a broken power-law neutrino spectrum

$$\frac{dN_\nu}{dE_\nu} \propto E_\nu^{-\beta}, \quad (18)$$

with $\beta = 1$ for $E_\nu < E_\nu^b$, and $\beta = 2$ for $E_\nu > E_\nu^b$. Thus the neutrino break energy E_ν^b is fixed by the threshold energy of photons for *photo-production* interacting with the dominant ~ 1 MeV fireball photons, and reads

$$E_\nu^b \simeq 5 \times 10^{14} \Gamma_{300}^2 \left(\frac{E_\gamma^b}{1 \text{MeV}} \right)^{-1} \text{eV}. \quad (19)$$

The normalization of the ν -flux is determined by the efficiency of pion production. The energy portion lost via pion production by protons producing the neutrinos above the break energy is essentially independent of the energy and is expressed as

$$f_\pi = 0.20 \left[\frac{L_{\gamma,51}}{\left(\frac{E_\gamma^b}{1 \text{MeV}} \right) \Gamma_{300}^4 \Delta t_{\text{ms}}} \right]. \quad (20)$$

Thus, the hardness of the γ -rays spectrum and its intrinsic time variability lead to determine Δt and Γ from

BATSE observations. Both quantities being also constrained by the observed photons' characteristic energy break ~ 1 MeV. Thence, the detection of any species of neutrinos in association with the GWs signal observed will yield in an highly accurate estimate of the time-delay in between, and through an analysis of such a time lag to clarify the issue of the GWs velocity in the light of the NDL and Einstein's theories, as we show below.

On the other hand, during the core-collapse of supernova the time-varying anisotropic distribution of density gradients in the proto-neutron star translates into the equivalent of a changing quadrupole mass-tensor whose dynamics induces emission of gravitational wave bursts [11]. Because the NDL theory agrees with general relativity upto the first post-Newtonian order, we can compute the amplitude of the GW signal as

$$h_{ij} = \frac{2G}{c^4 D} \frac{d^2 Q_{ij}}{dt^2} \longrightarrow h \sim 10^{\{-18\}\{-19\}} \quad (21)$$

for distances as far as the Large Magellanic Cloud $D \sim 55$ kpc. Here Q_{ij} defines the mass quadrupole tensor. Since the GWs do not couple to any other form of energy they stream away from the SN core whereas ordinary neutrinos in principle do not. This interaction induces a time-delay in the neutrino propagation respect to light, or equivalently to GWs in the Einstein theory of gravitation. We suggest that such time lag can be used also to test the prediction of the NDL theory that GWs travel at a speed lower than the corresponding one for light.

III. COLLAPSARS, NEUTRINO AND GRAVITATIONAL-WAVE BURSTS: A TEST OF THE NDL THEORY

The just described picture for driving supernovae explosions is by now being considered unable to explain the observational fact that some supernovae appear to require more energy (an order of magnitude higher) than is provided by the current mechanism based on neutrino transport [12]. Moreover, the trend in gamma-ray burst (GRBs) modelers is converging on a scenario in which a massive presupernova star (and its final explosion as a "hypernova") is the leading candidate [12]. This new paradigm *the collapsars*: supernovae explosions in which a stellar mass black hole, formed previously to the star final disruption, is the central engine for the GRBs, is supported by the fact that some supernovae have been found to be associated with GRBs events. The abrupt fallback ($\Delta T_{acc} \leq 10^{-3}$ s)² of a surrounding accretion disk, remnant of the failed supernova previous stage, triggers

²This timescale will define also the main characteristic frequency of the GW signal emitted.

the emission of strong GRBs most likely accompanied by GWs and neutrino bursts. In our view this model comprises the necessary non-gravitational astrophysical processes ³ ($\gamma + \nu$ bursts) through which we can stringently test the NDL theory concerning the velocity of propagation of GWs. For more details on the collapsar mechanism we address the reader to Ref. [12], and references therein.

Thus, let us assume for a while that the gravitational radiation (including the GW burst produced during the neutrino outburst [11]) travels at the speed of light. Because of the observational evidence that neutrinos actually oscillate [14], which implies they endow a mass; and consequently cannot travel at the speed of light, we can use the analogous expression for computing the neutrino time delay compared to photons emanating from the heavy neutrino radiative decay channel, to estimate their proper time delay with respect to the gravitational radiation surge generated at core bounce. Then the time delay for the neutrinos (emitted simultaneously with the burst of GWs) to arrive to the neutrino telescope is expressed as [15–17]

$$\Delta T_{GWs \leftrightarrow \nu_s} = 0.515 \text{ s} \left(\frac{D}{10 \text{ kpc}} \right) \left(\frac{m_\nu^2}{100 \text{ eV}^2} \right) \left(\frac{100 \text{ MeV}^2}{E_\nu^2} \right), \quad (22)$$

where E_ν represents the neutrino energy, D the source distance to Earth and m_ν the neutrino mass. Since there is a network (SNEWS⁴) of neutrino detectors currently running that are sensitive to the prompt core-collapse supernova neutrino bursts in our galaxy [18], which can include futurely the new generation of GWs observatories already near completion [4], the appropriate timing of both signals ($\nu + \text{GWs}$) will provide the time-of-flight lag in between, i. e., the neutrino time delay will directly be established by both the observations [19], provided the source pinpointing by both capabilities be settled.

Thus, for a 10 kpc distance, e. g., to the galactic center; for instance, the expected neutrino time lag should be: $\Delta T_{GWs \leftrightarrow \nu_s} = 0.515$ s, for a (ν_e) neutrino mass ≤ 10 eV, and energy ≤ 10 MeV, as in SN1987A. Thus the comparison between measured and theoretical time-of-flight

³It is certainly possible that no detectable gamma-ray burst at all is released at the onset of the supernova explosion. Irrespectively of the emission of that radiation, one can take some chance to determine the source distance independently because the source position may be accurately pinpointed by the concomitant operation of three or more GW interferometric detectors [20,13] by using the time lag among the detectors and triangulation techniques. Of course, the SNEWS network can also help to settle down this issue.

⁴The SuperNova Early Warning System.

delay will lead to a highly accurate estimate of the GWs velocity. An inferred mismatch between both timescales (expected and measured) may signal that the GWs speed as predicted by Einstein theory is not the correct one. This fact would positively point towards the NDL prediction as a more plausible explanation, since alternative theories as scalar-tensor gravity or other bi-metric gravitational theories predict that GWs travel at the speed of light, too.

IV. CONCLUSIONS

Above we have shown how the almost simultaneous emission of GWs, GRBs and ν s in a single astrophysical event may provide the non-gravitational processes that may turn the discrimination between general relativity and the NDL theory of gravity a reachable task in the near future. Prospective timing (detection) of such bursts from a unique source on the sky may prove powerful to settle the discrepancy between both theories in what concerns to the velocity of propagation of GWs. In this sense, the new generation of gravitational-wave observatories such as LIGO, VIRGO and GEO-600, together with the SNEWS neutrino network and the GRBs new detectors, and potentially the ultra high energy cosmic rays observatory AUGER, may prove useful. Moreover, because the neutrino energy can be measured by the time it gets the neutrino telescope and the source distance can be reliably estimated as discussed above, then from Eq.(22) the mass of the neutrino responsible for the observed event will be determined or stringently constrained by means not explored earlier. This will yield an innovative manner to check the threshold set to the neutrino mass by SuperKamiokande neutrino detector contained events.

Overall, for extra-galactic sources the detection of GW bursts by three or more gravitational radiation interferometric detectors, together with the redshift determination of the host distant galaxy⁵ where the associated bursts of gamma-rays and neutrinos will come from [21], will lead to a very accurated estimate of the source distance D and location. This is a key piece in order to use the time-delay equation to settle the dispute between both theories.

⁵In the case of closer sources primary or second distance indicators may be used.

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