Uniqueness of Inverse Scattering Problem in Local Quantum Physics*

Bert Schroer
FU-Berlin, Institut für Theoretische Physik,
Arnimallee 14, 14195 Berlin, Germany
presently: CBPF Rio de Janeiro
email: schroer@cbpf.br

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Abstract

It is shown that the a Bisognano-Wichmann-Unruh inspired formulation of local quantum physics which starts from wedge-localized algebras, leads to a uniqueness proof for the scattering problem. The important mathematical tool is the thermal KMS aspect of localization and its strengthening by the requirement of crossing symmetry for generalized formfactors.

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1 Inverse Problem in LQP

Most inverse problems have their origin in classical physics where they result from the question to what extend scattering (asymptotic) data allow a reconstruction of local data. The universality and importance of the problem inspired Marc Kac to the aphorism "How to hear the shape of a drum" which refers to Weyl's problem associated with geometric reconstructions from the asymptotic distribution of eigenvalues of the Laplace-Beltrami operator. In its present use it incorporates a wide range of problems concerning the partial or complete determination of local data from a seemingly weaker asymptotic input. In local quantum physics also the problem of how to re-construct the full content of QFT from its "observable shadow" i.e. the so-called DR-theory [1] may be viewed as an inverse problem.

The inverse problem in the present context is the question to what extends the data contained in a physically admissible (unitary, crossing symmetry,...) S-matrix determines a quantum field theory. Since it is easy to see that an S-matrix cannot be uniquely related to one field but rather is shared by very big equivalence classes of local fields, it is clear that the first step in such an investigation is to formulate QFT in a way that two isomorphic theories whose different appearance is only due to the use of different "field-coordinatizations" are easily recognizable as being one and the same. This is not possible or rather extremely cumbersome in the standard approach based on pointlike fields. Fortunately there exists such a framework which pays due attention to fields belonging to the same local equivalence class (fields from the same Borchers class) and which therefore generate the same system of local algebras as the given field: algebraic quantum field theory (AQFT) or local quantum physics (LQP) [2]. Its relation to standard QFT is similar to that of coordinate-based differential geometry to its more modern coordinate-free intrinsic formulation. Here as there one retains all the underlying principles and only introduces additional more elegant concepts to implement them. If one wants to emphasize the different methods one often uses the AQFT or LQP instead of QFT (a name as "intrinsic" or "field-coordinatization-free" QFT would appear a bit clumsy).

After this reformulation one may hope that the inverse problem is well-defined and admits at most one solution in terms of nets of algebras i.e. only one LQP-model. Using the powerful mathematical tool of Tomita-Takesaki theory (and the closely related crossing symmetry) as adapted in recent years to LQP [11], we show that if a solution exists at all, it is necessarily unique.

This result generalizes previous special findings concerning the modular derivation of the wedge localization interpretation of the Zamolodchikov-Faddeev algebra in d=1+1 in which case one also is able to control a good part of the existence problem [3][4]. The rapidity-dependent Z-F operators are a special case of "(vacuum) **p**olarization-**f**ree-**g**enerators" (PFG) for the wedge-localized algebra and in this special setting the crossing symmetry as formulated in the LSZ theory is a consequence of the more fundamental thermal aspect of wedge localization. In the general case the PFG's have very unwieldy domain properties [5] and we are presently only able to show uniqueness of the associated field theory by using both modular localization properties of wedge algebras as well as momentum space crossing properties of the generalized formfactors.

2 The use of modular theory

The new tool on which the proof relies in an essential way is modular theory. Since even in the setting of QFT "modular" occurs with different meanings, we will briefly define its present use in the sequel. The more common use is that of modular invariance in chiral conformal field theory. Although this is not its present meaning, a future connection to this causality-related classification tool for certain families of 2-dimensional local models to the present also locality-based use of the (Tomita-Takesaki) modular theory in local quantum physics is by no means ruled out.

Consider the x_0-x_1 wedge W_0 in d-dimensional Minkowski spacetime which is defined by taking convex combination of the two light rays generators (±1,1,0..0) and on which the affiliated Lorentz boost $\Lambda_{W_0}(\chi)$ acts as an automorphism. Let us change the parametrization by the factor 2π and write the unitary representation in QFT in terms of a selfadjoint K boost generator as

$$\Delta^{it} \equiv U(\Lambda_{W_0}(\chi = 2\pi t)) = e^{2\pi i t K} \tag{1}$$

We then may introduce an unbounded positive operator $\Delta^{\frac{1}{2}}$by "analytic continuation in t" which in functional calculus terms means that we are restricting the Hilbert space to those vectors ψ which upon action with Δ^{it} lead to a vector valued function $\psi(t) \equiv \Delta^{it}\psi$ which is continuous in $-i\pi \leq Imt \leq 0$ and analytic on the open strip $-i\pi < \theta < 0$. In addition to the wedge related boost, we also consider the antiunitary (since it involves time reversal) reflection along the edge

$$\begin{aligned} J &\equiv U(r) \\ r &: x_0, x_1 \rightarrow -x_0, -x_1 \end{aligned} \tag{2}$$

which up to a π-rotation around the e_1-axis is identical to the TCP transformation. Since this transformation commutes with the boost Δ^{it}, its antiunitarity leads to the commutation relation (always on the relevant domains)

$$\begin{aligned} JK &= -KJ \\ J\Delta^{\frac{1}{2}} &= \Delta^{-\frac{1}{2}}J \end{aligned} \tag{3}$$

on the respective domains of definition. This in turn yields

$$\begin{aligned} S^2 &\subset 1 \\ for\ S &\equiv J\Delta^{\frac{1}{2}} \end{aligned} \tag{4}$$

i.e. we encounter the rare case (not even to be found in extensive textbooks on mathematical physics as that by Reed-Simon) of an unbounded antilinear operator which is involutive on its domain. In a moment we will see that it is just this somewhat exotic property which enables the encoding of spacetime geometric information concerning quantum localization into abstract domain properties.

It is the content of a theorem (the Bisognano-Wichmann theorem [2]) that this operator is Tomita's famous S-involution for the operator algebra which the local fields generate if one restricts the smearing functions to have support in the wedge. Namely in terms of the affiliated von Neumann algebra (which is the setting of the Tomita-Takesaki theory) $\mathcal{A}(W_0)$, the S fulfills [2][11]

$$SA\Omega = A^*\Omega \tag{5}$$

where Ω is the vacuum vector and the star denotes the hermitian adjoint in operator algebras. This is a special case of the Tomita Takesaki modular theory whose prerequisite is the existence of a von Neumann algebra in "general position" i.e. a pair ($\mathcal{A}$,Ω) with Ω being a cyclic and seperating vector for $\mathcal{A}$[1]. From this input Tomita and Takesaki derive:

- S defined by (5) is a densely defined closed antilinear involution whose polar decomposition $S = J\Delta^{\frac{1}{2}}$ leads to an antiunitary reflection J (abstract generalization of a TCP reflection) and modular dynamics Δ^{it} (abstract generalization of a Hamiltonian)

- The J, Δ^{it} have the following significance with respect to the operator algebra

$$\begin{aligned} AdJ\mathcal{A} &= \mathcal{A}' \\ Ad\Delta^{it}\mathcal{A} &= \sigma_t(\mathcal{A}) \end{aligned} \tag{6}$$

  here as in the sequel the upper dash on an operator algebra denotes its commutant and the modular unitary Δ^{it} implements the modular group $\sigma_t(\cdot)$ which different from the former only depends on the state $\omega(A) = (\Omega, A\Omega)\,,\ A \in \mathcal{A}$ and not its implementing vector Ω.

- A necessary and sufficient condition for the standardness (cyclicity+separating property) of the pair $(\mathcal{A}, \Omega)$ is the thermal KMS property in terms of the state ω is: there exists a 2π-open-strip analytic function (continuous in the closed strip) $F_{A,B}(z)$ with

$$\begin{aligned} F_{A,B}(t) &\equiv \omega(\sigma_t(A)B) \\ \omega(B\sigma_t(A)) &= lim_{z\to t+i}F_{A,B}(z) \end{aligned}$$

The above theorem of B-W may now be rephrased as saying that those operator wedge algebras which are generated by covariant fields do have a geometric modular theory. In more recent times there have been successful attempts to establish these geometric modular aspects of wedge algebras directly in the seemingly more general setting of algebraic QFT which avoids the use of fields already at the start [12].

[1] Physicist, who independently developped these concepts, often (especially in chiral conformal field theory) talk about the unique "operator-statevector relation" $A \longleftrightarrow A\Omega$; the correct mathematical backing is the reference to the Reeh-Schlieder theorem [14]. It is often overlooked that the relation is not universal but depends on the chosen $\mathcal{A}(\mathcal{O})$.

The validity of the KMS condition with the modular group acting geometrically as the Lorentz-boost is sufficient for establishing that also J acts geometrically i.e. that the von Neumann commutant is localized in the geometrically opposite wedge W' (Haag duality) $\mathcal{A}(W) = \mathcal{A}(W')'$.

The Poincaré group generates from one standard wedge algebra $(\mathcal{A}(W_0), \Omega)$ a net of wedge algebras $(\mathcal{A}(W), \Omega)_{W \subset \mathcal{W}}$. In order to extract sufficient physical informations one needs nets for smaller compact causally closed regions. A net of double cones D may be defined in terms of intersections

$$\mathcal{A}(D) \equiv \cap_{D \subset W} \mathcal{A}(W) \tag{7}$$

In order to achieve our goal we must be able to relate the wedge algebra with the scattering operator S_{sc}. This is possible in the LSZ framework of QFT because although the representation theory of the connected Poincare-group for the incoming (outgoing) free fields is the same as for the interacting Heisenberg fields, this is not so for the reflections involving time reversal. In particular the J in (3) which represents the wedge reflection in the presence of interactions is different from its interaction-free asymptotic counterpart [3] J_{in}

$$J = S_{sc} J_{in} \tag{8}$$

This implies that in the characterization of the wedge-localized (dense) subspace:

$$\begin{gathered} H(W) = H_R(W) + iH_R(W) \qquad (9) \\ H_R(W) = real\ subspace \{\psi | S\psi = \psi\} \\ S(\psi_1 + i\psi_2) = \psi_1 - i\psi_2,\ S = S_{sc} S_{in} \end{gathered}$$

the position of the dense subspace $H(W)$ inside the total Hilbert space depends in a subtle way on the interaction through S_{sc}. The domain of $\Delta^{\frac{1}{2}}(1)$ is now encoded more concretely in terms of a complex dense space $H(W)$ whose real and imaginary part are vectors in a closed real subspace $H_R(W)$. These real closed subspaces encode the full spatial aspect of wedge localization. With the help of the graph of S one may even introduce a topology in terms of which the dense subspace becomes a Hilbert space in its own right[2], but all these spatial concepts are still removed from the task of characterizing a wedge algebra uniquely in terms of the scattering matrix. The reason is the following. The algebra-state vector relation $A \longleftrightarrow A\Omega$ is not universal but changes with the algebra, even in the family of wedges. In particular the spatial modular theory permits the existence of two different algebras with the same wedge as long as they remain indistinguishable in their one-time action on the vacuum.

Connes has given a criterion [13] which allows to obtain from the spatial modular theory to an algebra with the same modular objects. This is achieved by controlling certain properties of so-called facial subcones of a natural cone associated $\mathcal{P}(\mathcal{A}(W))$ with $H_R(W)$. But one presently lacks a physical

[2] $H(W)$ with the S-graph norm may be called the thermal Hilbert space, because it offers a natural description of the (Hawking-Unruh) thermal aspects of the vacuum upon its restriction to the wedge algebra.

foundation and control for such a procedure. Fortunately for our interest in uniqueness (falling short of an actual construction), such a difficult mathematical road can be bypassed in terms of an additional physical assumption: crossing symmetry of particle matrixelements (formfactors) of wedge localized operators. For this we need to remind the reader of a bit of scattering theory adapted to the algebraic framework.

3 Uniqueness from KMS-thermality and crossing

It has been shown that any vector ψ which is in the domain of the positive analytically contimued standard L-boost (1) $\Delta_W^{\frac{1}{2}}$ has a unique relation to an (generally unbounded) operator $F_{\psi,\mathcal{A}(W)}$ affiliated with $\mathcal{A}(W)$ with

$$F_{\psi,\mathcal{A}(W)}\Omega = \Psi,\ F^*_{\psi,\mathcal{A}(W)}\Omega = S_W\Psi$$

But this famous state-vector-operator relation depends crucially on the standard pair $(\mathcal{A}(W),\Omega)$. If the same scattering data would allow for another wedge algebra $\mathcal{B}(W)\neq\mathcal{A}(W)$, the same vector $\Psi\in H(W)$ is associated with another operator $\Psi = F_{\psi,\mathcal{B}(W)}\Omega$. $H(W)$ contains all those in- or out- n-particle vectors which are in the domain of $\Delta_W^{\frac{1}{2}}$ which form a dense set.

Let us assume that we are dealing with a state-vector of the special form $\Psi = A\Omega, A\in\mathcal{A}(W)$. With respect to the $\mathcal{B}(W)$ algebra there exists a unique affiliated densely defined closed operator F with

$$\begin{aligned} A\Omega &= F\Omega \qquad (10)\\ F\eta&\mathcal{B}(W) \end{aligned}$$

where in the last line we used the standard η notation for an operator affiliated with $\mathcal{B}(W)$. This forces in particular the inner products with the n-particle out state vectors to be the same

$$^{out}\left\langle p_n...p_1\left|A\right|\Omega\right\rangle = {}^{out}\left\langle p_n...p_1\left|F\right|\Omega\right\rangle \qquad (11)$$

For those operators $A\in\mathcal{A}(W)$ which are localized in a double cone $\mathcal{A}(\mathcal{O})\subset\mathcal{A}(W)$ the LSZ-formalism and onshell analytic continuation lead to the crossing symmetry (see appendix)

$$\begin{aligned} &^{out}\left\langle p_1,p_2,...p_l\left|A\right|q_1,q_2...q_k\right\rangle^{in} = \qquad (12)\\ &\underset{z\rightarrow\theta-i\pi}{a.c.}\ {}^{out}\left\langle p_1,p_2,...p_{l-1}\left|A\right|q_1,q_2...q_k,\bar{p}_l(\theta)\right\rangle^{in} + c.t = \\ &:^{out}\left\langle p_1,p_2,...p_{l-1}\left|A\right|q_1,q_2...q_k,-\bar{p}_l\right\rangle^{in} + c.t. \end{aligned}$$

whereb the last equality defines the meaning of a formfactor with a negative momentum. Crossing symmetry is a momentum space property within LSZ scattering theory. It says that in multiparticle matrix elements of operators between outgoing bra- and incoming ket- particle states as above, one can flip a particle in the incoming ket state to the outgoing bra if one also converts it into an antiparticle (charge conservation) at an analytically continued real point $p \rightarrow -p$ on the complex physical mass shell (energy-momentum conservation). The bar on the momentum denotes the antiparticle and in order to indicate the analytic continuation we have chosen the momentum p to be of the form $p = m(ch\theta, sh\theta, 0, 0)$ (i.e. affiliated with the standard wedge, which can always be achieved by L-covariance), so that the analytic continuation corresponds to $\theta \rightarrow i\pi - \theta$. The *c.t.* contraction terms consist of a $\langle p_l | q_i \rangle$ δ-function multiplied with a lower $l+k-2$ particle formfactor. Note that the crossing of the S-matrix itself arises for $A = 1$. The most prominent crossing relation is that for elastic scattering which involves the simultaneous flip of two particles with one from the incoming and one from the outgoing configuration (the flip of only one is not compatible with the onshell energy-momentum conservation) The last line is an abuse of notation since an analytic continuation is generally not implementable as an operator or Hilbert space operations. Crossing is not a Wigner symmetry in state space but rather a property which involves analytic continuation. In order to justify it, there has to be at least strip analyticity in the rapidity θ of the momentum to be crossed so that the forward particle mass shell can be analytically connected with the antiparticle backward shell. Starting from the matrixelement (11) the successive application of the crossing property (12) allows to obtain arbitrary matrixelements of A between bra-out and ket-in particle states by starting from a special situation with only the vacuum on one side.

The analytic aspects are evidently inherited by the right hand side in (11). But we have to justify the interpretation in terms of crossing particles from ket into bra's. For this purpose we imagine the vector $F\Omega$ to be weakly approximated by a sequence of $F_n\Omega$ with F_n being operators in the $\mathcal{B}$-algebra which are localized in an increasing family of double cones approaching the standard wedge. According to LSZ scattering theory the crossing relations apply to compactly localized operators in the $\mathcal{B}$-net. Hence the interpretation of the limiting analyticity relation as a particle crossing relation is unavoidable. But this means that crossing symmetry generalizes the equality of $A \in \mathcal{A}(W)$ with $F\eta\mathcal{B}(W)$ to that of A/F formfactors on a dense set of out-in states. This is only possible if F is also bounded and equal to an $A \in \mathcal{A}(W)$.

A direct attempt of a proof using only the KMS property of the $\mathcal{A}(W)$ and $\mathcal{B}(W)$ algebras remains inconclusive because the known domain properties of the afilliated operators $F\eta\mathcal{B}(W)$ are too weak to secure the algebraic aspects of the $B-$KMS property. Although it is easy to choose a dense set of asymptotic n-particle states which are created by $\mathcal{A}(W)$ or $\mathcal{B}(W)$ affiliated operators, the domain justification for applying KMS property to those operators fails. Domain properties of $F's$ are very different from those of smeared pointlike fields, except for the special case of "tempered PFG's" [5][3]. The temperedness restriction allows only elastic interactions in d=1+1. In fact the only known models are those where the Fouriertransforms of the PFG's $G_W(x)$ fulfill a Zamolodchikov-Faddeev algebra which in the simplest case of a selfconjugate particle reads

[3]Temperedness restrictions for nonlocal creation anyonic operators actually appeared first in [7].

$$G_W(x) = \frac{1}{\sqrt{2\pi}} \int (e^{-ipx} Z(\theta) + h.c.) d\theta, \;\; p = m(ch\theta, sh\theta) \tag{13}$$
$$Z(\theta)Z(\theta') = S(\theta - \theta')Z(\theta')Z(\theta)$$
$$Z(\theta)Z^*(\theta') = S^{-1}(\theta - \theta')Z^*(\theta')Z(\theta) + \delta(\theta - \theta')$$

The unitarity of the structure functions $S(\theta)$ is a consequence of the *-algebra property of the $Z's$ and the crossing symmetry of the mixed $A \in \mathcal{A}(\mathcal{W}) -$ G_W-correlation functions follows from the wedge localization and the ensuing KMS property. The $Z's$ have a very simple representation in a bosonic/fermionic Fock space. Each operator A affiliated with $\mathcal{A}(W)$ has a formal power series expansion

$$A = \sum \frac{1}{n!} \int_C ... \int_C a_n(\theta_1, ...\theta_n) : Z(\theta_1)...Z(\theta_n) : \tag{14}$$

where $Z(\theta - i\pi) = Z(\theta)^*$ and each integration path C extends over the upper and lower part of the rim of the strip. The strip-analyticity of the coefficient functions a_n expresses the wedge localization of A. The sharpening to double cone localization by the intersection of wedges leads to meromorphic functions which obey the kinematical pole condition of Smirnov [8]. Expansions like (14) are nothing more than a generating operator for the formfactors i.e. bilinear forms which fall short of being genuine operators with domains and closures. They are analogous to the LSZ expansions of Heisenberg fields into (asymptotic) free fields. For our above uniqueness argument this is enough, for a constructive approach this is insufficient.

It is also interesting to note that despite the close relation between the onshell incoming fields and the onshell $Z(\theta)$, the latter share some features with local Heisenberg fields namely one $Z(\theta)$ can have several particle states (alias bound states) whereas each type of particle requires the introduction of one incoming field.

Although the coefficient functions $S(\theta)$ of the Z-algebra turn out to be the 2-particle scattering matrix, there is no need to know this for the calculations: absence of real particle production, wedge-localitazion and the related KMS property (i.e. spacetime properties) are enough.

The case without the temperedness restriction starts also from formfactors between the dense set of wedge affiliated n-particle-ket-states and the bra-vacuum which according to modular theory must be equal for the two putatively theories. It then bypasses domain issues by using for the apparently stronger assumption of crossing symmetry for the successive movement of particles from the ket to the bra state. The asymptotic states in the $\Delta^{\frac{1}{2}}$ domain have a tensor product structure. If their wedge representatives $F_n\Omega$ would inherit this factorization structure in the form $F_n\Omega = F_{n-1}G\Omega = F_{n-1}\Omega \times G\Omega$ with G being a PFG, then the use of the crossing property would not be necessary since the result would follow from the KMS formula of wedge localization. But I have not been able to derive such a factorization from the known domain properties of wedge affiliated $F's$ in the general nontempered case. The use of PFG's for the construction of wedge algebras seems to be restricted to factorizing models, in more realistic interacting theories they do not seem to be useful generators of wedge algebras.

For a recent discussion of how the wedge algebras $\mathcal{A}(W)$ are related to their holographic projections onto the (upper) horizon $\mathcal{A}(R_+)$ we refer to [6].

4 Related problems, outlook

We have seen that by combining modular theory (which gives mathematical precision to the state-vector-operator relation) with crossing symmetry (which permits to elevate relations involving the vacuum vector to relations between dense sets of scattering states), one obtains a uniqueness argument for the inverse problem in QFT: a physically admissible S-matrix has inspite of the myriads of interpolating fields at most one system of local algebras i.e. at most one field-coordinatization-independent algebraic QFT. For the solution of the existence problem i.e. the explicite construction of a system of algebras (and if desired their possible pointlike generators) from scattering concepts one presently has to assume the temperedness of wedge algebra generating PFG's which limits the constructive approach to d=1+1 factorizing models with the additional benefit of a spacetime interpretation of the ensuing Zamolodchikov-Faddeev algebraic structure.

These uniqueness and existence aspects touch upon age old problems of particle physics which despite the passage of time have lost nothing of their importance. Beginning as far back as Heisenberg's S-matrix proposal [9], there was the desire to avoid the short distance problems of pointlike field theory by advocating a pure S-matrix theory. The main aim was to transfer as much "physical blood" from what one has learned about perturbation of free fields but to avoid that offshell short distance region which relates to coalescing or lightlike spacetime arguments of fields. With the overwhelming success of renormalized QED and the deep conceptual gains in the particle/field realm (LSZ, Haag-Ruelle, Wightman,..), the motivation for a pure S-matrix theory subsided only to reappear in a strengthened form (enriched by analytic properties and crossing) under the heading of "the S-matrix bootstrap"[4] in the 60ies (also the cradle of the Veneziano's dual model and string theory).

The present modular enrichment subjects this old approach to a sympathetic but critical review. Although there is agreement with its basic premise that onshell concepts should play an important role right from the beginning and that locally coupling free fields is not really an intrinsic God-given way of introducing interactions enforced by the underlying principles, it would not subscribe to an abandoning of the causality and spectral principles of QFT. The message would rather be that one should avoid the use of (inevitably singular) pointlike correlation functions in products and integrals over products of correlations (which are the intermediate steps of standard perturbation theory) in favour of multiparticle formfactors of one individual localized operator and postpone the operator control of these bilinear forms

[4] For a recent review see [10].

(particle matric elements) up to the end. After the structure of some wedge localized algebras has been understood in a constructive manner, there is no harm to sharpen the localization of these algebras by forming intersections and even to use pointlike generators. The short distance singularities of pointlike generators belonging to limits of sequences of formfactors with improved localizations are of no harm [16]; their distributional aspects are governed by Wightman's theory of vacuum correlations.

The bootstrap formfactor approach to factorizing models may serve as an excellent illustration of this new way of thinking about QFT. Here one avoids the technical frontiers between renormalizable/nonrenormalizable interactions by totally bypassing Lagrangian quantization or causal perturbation of free fields. This is achieved for those models by starting with an algebraic structure which avoids pointlike fields in favor of the above Z-operators which are only consistent with wedge-like localization. With other words, the system of wedge algebra $\mathcal{A}(W)$ is constructed before any pointlike field appears on the scene. The next step namly the formation of double cone intersection algebras leads to the so-called kinematical pole relation which relates the lower with the higher formfactors and defines the formfactor spaces for double-cone localized objects. Whereas the Z-algebra generators were PFGs, the double-cone localization leads to the vacuum polarization clouds in form of Z-expansions (14) which extend to infinity. The finite size of the spacetime extension of the double-cone localized [4] operators shows up in form of a Payley-Wiener asymptotic behavior of the meromorphic formfactors and the pointlike fields with their spacetime point in the double cone correspond to a polynomial behavior in certain reduced formfactors [16]. The point which needs to be emphasized here is there is no ultraviolet limitation coming from power-counting and leading to the standard separation into renormalizable and nonrenormalizable coupling; every admissable factorizing S-Matrix leads to power bounded formfactors in terms of a few physical parameters (which are already preempted in the S-matrix). This is in my view the most valuable message of that theoretical laboratory called "factorizing (integrable) theories".

A limitation as that of a Lagrangian field having to carry an operator dimension near the the canonical (free field) dimension does not appear and with it the threat of nonrenormalizability in the sense of too many (infinite) renormalization parameters has disappeared. So the standard separation according to short distance behavior becomes meaningless in this new framework; short distance properties (of what? there is no preferred Lagrangian field coordinate!) are simply not part of the modular program. Instead the remaining question about existence is whether the double cone intersection algebras are nontrivial in the sense of formfactors and whether these formfactors are really coming from closed or bounded operators.

In view of these facts it would appear to be somewhat unreasonable (though presently it cannot be ruled out) to believe that all this happens as a accident of the speciality of the models and that beyond factorization (or beyond tempered PFG's) the Lagrangian point of view continues to define the intrinsic (i.e. fixed by the principles of local quantum physics) borderline of ultraviolet good/bad QFT. The present uniqueness argument of the inverse problem strengthens this suspicion. The existence of a field-coordinate free construction of local quantum physics apparently diminishes the physical role of short distance properties of individual fields by trading the nontrivial existence of a model against the ultraviolet odds with the nontriviality of intersections of wedge algebras. But for a constructive approach beyond factorizing S-matrices based on these new concepts one needs better generators than the nontempered PFGs. Whatever will be the final answer about the intrinsic role (if any) of the short distance problems in the standard approach, the algebraic field-free formalism (which is expected to allow also for perturbative solutions) should eventually teach us whether the power counting in couplings of free fields (augmented by cohomological tricks which lower the short distance powers of interaction polynomials in the intermediate computational steps and bring them into the renormalizable range[5]) is the one set by the physical principles or not. In the latter case one would have to blame the restrictions of the standard approach on the the inapropiate and premature use of "singular field coordinatizations". In view of the often made claim that string theory could be ultraviolett finite, it may be interesting to review these problems in the algebraic setting of QFT. We expect that the results of perturbatively renormalizable fields will be reproduced in this setting and that it will lead to an enlargement similar to what happened in d=1+1 with factorizing models).

It has been known that without the presence of interaction terms i.e. for free higher spin wave equations their does indeed exist a modular approach which, if combined with the Wigner group theoretic characterization of massive and massless particles in terms of irreducible positive energy representations, allows for a powerful connection of group theoretic induced localization with real modular subspaces of the Wigner representation space [3][18]. In this way one may bypass the standard method of first converting the content of Wigners intrinsic description into nonunique covariant wave functions and then using the Cauchy initial value approach in order to characterize the localization-subspaces, which is particularly helpful with increasing spin. In the presence of local perturbations one looses the functorial characterization of nets of local algebras in terms of localized real subspaces. Nevertheless one may view e.g. the constructive approach to factorizable d=1+1 models as an extension of the Wigner program of coming from particles to local observables in the presence of interactions. I believe that it is fruitfull to view the general modular approach as an extension of Wigner's program.

In this paper we have concentrated on the easier uniqueness problem, leaving the more difficult construction problem (outside the temperedness) for the future. Of course we hope that we still are able to avoid the explicit use of crossing symmetry and will be able to use instead the spacetime related KMS condition. We nourish the hope that a better understood modular construction program may reveal whether or not the present borderlines between physically useful and less useful (renormalizable/nonrenormalizable) theories is really the true frontier marked by the underlying principles or only the limitation of the ad hoc implementation of interactions by locally coupling free fields in Fockspace.

[5]We refer to the BRS-like cohomological representation of massive spin Wigner one particles spaces which allows an alternative presentation of massive selfinteracting vectormesons to the standard Higgs mechanism [17].

4.1 Appendix: Crossing symmetry with LSZ reduction

Crossing has been first observed in Feynman perturbation before it was derived in the LSZ scattering theory. Its formal aspects are easily obtained from the LSZ asymptotic convergence

$$
\begin{aligned}
lim_{t\to\mp\infty} A^{\#}(f_t)\Phi &= A^{\#}(f)_{in/out}\Phi,\ A^{\#} = A \ or \ A^{*} \qquad (15)\\
A(f_t) &= \int f_t(x)U(x)AU^{*}(x)d^4x,\ A \in \mathcal{A}(\mathcal{O})\\
f_t(x) &= \frac{1}{(2\pi)^2}\int e^{i(p_0-\omega(p))t-ipx} f(\vec{p})d^4x,\ \omega(p) = \sqrt{\vec{p}^2+m^2}
\end{aligned}
$$

which can be derived on a dense set of states and shown to lead to the well-known reduction formulas .

$$
\begin{aligned}
&{}^{out}\left\langle q_1, q_2, ...q_m \left| F \right| p_1, p_2...p_n \right\rangle^{in} |_{conn} = \qquad (16)\\
&-i\int {}^{out}\left\langle q_2, ...q_m \left| K_y TFA^{*}(y) \right| p_1, p_2...p_n \right\rangle^{in} d^4y e^{-iq_1 y} =\\
&-i\int {}^{out}\left\langle q_1, q_2, ...q_m \left| K_y TFA(y) \right| p_2...p_n \right\rangle^{in} d^4y e^{ip_1 y} \ =
\end{aligned}
$$

Here the time-ordering T between the original operator $F \in \mathcal{A}(\mathcal{O})$ and the interpolating Heisenberg field $A(x)$ resp. $A^{*}(x)$ appears if one reduces a particle from the bra- or ket state [15]. For the definition of the time ordering of a fixed finitely localized operator F and a field with variable localization y we may use $TFA(y) = \theta(-y)FA(y) + \theta(y)A(y)F$, however as we place the momenta on-shell, the definition of time ordering for y near $locF$ is irrelevant. Each such reduction is accompanied by another disconnected contribution in which the creation operator of an outgoing particle say $a_{out}^{*}(q_1)$ changes to an incoming $a_{in}(q_1)$ acting on the incoming configuration (and the opposite situation i.e. $a_{in}^{*}(p_1) \to a_{out}(p_1)$). These terms (which contain formfactors with one particle less in the bra- and ket- vektors) have been omitted since they do not contribute to generic nonoverlapping momentum contributions and to the analytic continuations. Under the assumption that there is an analytic path from $p \to -p$ (or $\theta \to \theta - i\pi$ in the wedge adapted rapidity parametrization) the comparison between the two expressions gives the desired crossing symmetry: a particle of momentum p in the ket state within a formfactor is indistinguishable from a bra antiparticle at momentum -p (here denoted as -p̄).

In order to obtain that required analytic path it is convenient to pass from time ordering to retardation

$$TFA(y) = RFA(y) + \{F, A(y)\} \qquad (17)$$

The unordered (anticommutator) term does not have the pole structure on which the Klein-Gordon operator K_y can have a nontrivial on-shell action and therefore drops out. The application of the JLD spectral representation puts the p-dependence into the denominator of the integrand of an integral representation where the construction of the analytic path proceeds in a completely analog fashion to the derivation of crossing for the S-matrix [19][15].

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