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Constraints on the generalized Chaplygin gas from supernovae observations*

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Abstract

We explore the implications of type Ia supernovae (SNIa) observations on flat cosmological models whose matter content is an exotic fluid with equation of state, $p = -M^{4(\alpha+1)}/\rho^{\alpha}$. In this scenario, a single fluid component may drive the Universe from a nonrelativistic matter dominated phase to an accelerated expansion phase behaving, first, like dark matter and in a more recent epoch as dark energy. We show that these models are consistent with current SNIa data for a rather broad range of parameters. However, future SNIa experiments will place stringent constraints on these models, and could safely rule out the special case of a Chaplygin gas ($\alpha = 1$) if the Universe is dominated by a true cosmological constant.

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1 Introduction

According to the standard cosmological scenario (ΛCDM, QCDM) that has emerged at the end of the last century, the Universe is dominated by two unknown components with quite different properties: pressureless cold dark matter (CDM), which is responsible for the formation of structures, and negative-pressure dark energy, that powers the accelerated expansion. There are several candidates for these two components. For the CDM, the leading particle candidates are the axion and the neutralino, two weakly interacting massive particles. The preferred candidates for dark energy are vacuum energy - or a cosmological constant Λ - and a dynamical scalar field (quintessence) [1]. At the cosmological level, the direct detection of each of these two components involves observations at different scales. Since it is not supposed to cluster at small scales, the effect of dark energy can only be detected over large distances, where the accelerated expansion is observed. On the other hand, the CDM can be detected by its local manifestation on the motion of visible matter or through the bending of light in gravitational lensing.

An interesting question that arises is: could this two phenomena - accelerated expansion and clustering - be different manifestations of a single component? In principle the answer is yes, if, for instance, the Universe is dominated by a component with an appropriate exotic equation of state (EOS). We will generically refer to any kind of such unifying dark matter-energy component as UDM. [1]

The above question has been addressed in some works recently [2, 3, 4, 5]. For instance, in Ref. [5] it was investigated the possibility that a tachyonic field, with motivation in string theory, could unify dark energy and dark matter and explain cosmological observations in small and large scales. Here we investigate observational limits on a simple realization of UDM: a fluid with the following equation of state [3, 6, 7, 8],

$$p = -\frac{M^{4(\alpha+1)}}{\rho^{\alpha}}. \tag{1}$$

The particular case $\alpha = 1$ is known as Chaplygin gas and its cosmological relevance, as an alternative to quintessence, has been pointed out in [8]. In [3], it has been shown that the inhomogeneous Chaplygin gas represents a promising model for dark matter-energy unification. Some possible motivations for this scenario from the field theory point of view are discussed in [8, 3, 7]. The Chaplygin gas appears as an effective fluid associated with d-branes [8, 9]. The same EOS is also derived from a complex scalar field with appropriate potential and from a Born-Infeld Lagrangian [3]. More recently, by extending the work of Bilić *et al.* [3], Bento *et al.* [7] also discussed the particle physics motivation for the EOS (1). The fluid given by this EOS is sometimes called generalized Chaplygin gas (GCG).

It is interesting to notice that this model can also be obtained from purely phenomenological arguments, by requiring that an exotic fluid unifies the dark-matter/dark-energy behavior as a function of its density and that it is stable and causal [6]. The simplest EOS satisfying this criteria is given by eq. (1).

Let us consider the homogeneous case of the GCG Universe. The energy conservation can be written as

$$d\rho = -3\,(\rho + p)\,\frac{da}{a}, \tag{2}$$

where a is the scale factor. By solving this equation we may express the energy density in terms of the scale factor:

$$\rho = M^4 \left[B\left(\frac{a_0}{a}\right)^{3(\alpha+1)} + 1\right]^{1/(\alpha+1)}, \tag{3}$$

where a_0 is present value of scale factor and B is an integration constant. When $a/a_0 \ll 1$, we have $\rho \propto a^{-3}$ and the fluid behaves as CDM. For late times, $a/a_0 \gg 1$, and we get $p = -\rho = -M^4 = const.$ as in the cosmological constant case. There is also an intermediate phase where the effective EOS is $p = \alpha\rho$ [8]. Once we have ρ as a function of the scale factor it is simple to find the Hubble parameter. Since

[1]Following the current jargon, another possible denomination for UDM would be "quartessence" since in this scenario we have only one additional component, besides ordinary matter, photons and neutrinos, and not two like in ΛCDM and QCDM.

observations of anisotropies in the cosmic microwave background (CMB) indicate that the Universe is nearly flat [10], here we restrict our attention to the zero curvature case. We also neglect radiation, that it is not relevant for the cosmological tests we discuss in this work.

From the Friedmann equation with $k = 0$ we have

$$H^2(z) = H_0^2 \left[\Omega_M^* (1+z)^{3(\alpha+1)} + (1-\Omega_M^*)\right]^{1/(\alpha+1)}, \tag{4}$$

where $z = a_0/a - 1$ is the redshift, and we have conveniently defined $\Omega_M^* = B/(B+1)$, or equivalently

$$B = \frac{\Omega_M^*}{1-\Omega_M^*}. \tag{5}$$

Further, we also have

$$M^4 = \rho_{c0} (1-\Omega_M^*)^{\frac{1}{(\alpha+1)}}, \tag{6}$$

where ρ_{c0} is the present value of the critical density. For these models the deceleration parameter can be written as

$$q = -\frac{\dot{H}}{H^2} - 1 = \frac{\frac{\Omega_M^*}{2} - (1-\Omega_M^*)(1+z)^{-3(1+\alpha)}}{\Omega_M^* + (1-\Omega_M^*)(1+z)^{-3(1+\alpha)}}, \tag{7}$$

and the redshift z_*, at which the Universe started its accelerating phase is given by,

$$1 + z_* = \left(\frac{2(1-\Omega_M^*)}{\Omega_M^*}\right)^{\frac{1}{3(\alpha+1)}}. \tag{8}$$

An accelerating Universe at present time ($q_0 < 0$) implies that $\Omega_M^* < 2/3$, and from (5) we have $0 < B < 2$; the lower limit follows from the fact that we assume $\Omega_M^* > 0$. Moreover, if α is not very close to -1, from (6), we obtain $M \sim 10^{-3}$ eV. It would be desirable that a fundamental theory, aimed to describe the UDM, sheds some light on the origin of this mass scale. Thus, at this point this model is not free of some tuning. However, once the origin of the above mass scale is explained, the so called dark matter-energy "coincidence problem" is not present in this scenario.

In a GCG Universe, if the parameter α is positive, the adiabatic sound velocity, ${c_s}^2 = dp/d\rho = -\alpha p/\rho$, is real and therefore, the fluid component is stable. If α is negative and there is only adiabatic pressure fluctuations, they accelerate the collapse producing instabilities that turn the model for structure formation unacceptable [11, 12]. Moreover, to obey causality, the sound velocity in this medium has to be less or equal than the light velocity. Since the maximum allowed sound velocity of this fluid (which occurs in the regions where $p \to -\rho$) is given by $\sqrt{\alpha}$, this condition imposes $\alpha \leq 1$. The Chaplygin gas, $\alpha = 1$, is the extreme case, where the sound velocity can be nearly the speed of light. The case $\alpha = 0$ is equivalent to ΛCDM and is, of course, well motivated. In this paper, we discuss the GCG model from a phenomenological point of view. Hence, although we are aware that most likely $0 \leq \alpha \leq 1$, we also include in our analysis the region where α is negative, but larger than -1. If $\alpha = -1$ we obtain a de Sitter Universe. The situation $\alpha < -1$ seems unphysical, since the energy density of UDM would be increasing with the expansion of the Universe. In fact, as we shall see, age constraints can safely exclude regions in the parameter space with very negative values of α.

In the forthcoming section we will see what constraints to the model described above are set by present and future SNIa observations. Recently, some constraints from SNIa on related models where obtained in Ref. [13]. The work presented here differs from [13] in the following aspects: a) Following the idea of unification, we have not included an additional dark matter component and we have considered the more general case in which α is not necessarily equal to unity. b) When analyzing current SNIa data we perform a Bayesian approach in which the intercept is marginalized c) We also investigate the predicted constraints on the models from future SNIa observations.

2 Type Ia Supernovae Experiments

The luminosity distance of a light source is defined in such a way as to generalize to an expanding and curved space the inverse-square law of brightness valid in a static Euclidean space,

$$d_L = \left(\frac{L}{4\pi\mathcal{F}}\right)^{1/2} = (1+z)\int_0^z \frac{dy}{H(y)}. \tag{9}$$

In (9) L is the absolute luminosity and $\mathcal{F}$ is the measured flux.

For a source of absolute magnitude M, the apparent bolometric magnitude $m(z)$ can be expressed as

$$m(z) = \mathcal{M} + 5\,\log D_L, \tag{10}$$

where $D_L = D_L(z, \alpha, \Omega_M^*)$ is the luminosity distance in units of H_0^{-1}, and

$$\mathcal{M} = M - 5\,\log H_0 + 25 \tag{11}$$

is the "zero point" magnitude (or Hubble intercept magnitude).

In our computations we follow the Bayesian approach of Drell, Loredo and Wasserman [14] (see also [15]) and we direct the reader to these references for details. We consider the data of fit C, of Perlmutter *et al.* [16], with 16 low-redshift and 38 high-redshift supernovae. In our analysis we use the following marginal likelihood,

$$\mathcal{L}(\alpha, \Omega_M^*) = \frac{s\sqrt{2\pi}}{\Delta\eta}\, e^{-\frac{q}{2}}. \tag{12}$$

Here

$$\begin{aligned} q(\alpha, \Omega_M^*) \;=\; & \sum_{i=1}^{16} \frac{(-5\log D_L - n_i + m_{Bi}^{corr})^2}{\sigma_{low,i}^2} + \\ & \sum_{i=1}^{38} \frac{\left(-5\log D_L - n_i + m_{Bi}^{eff}\right)^2}{\sigma_{high,i}^2}, \end{aligned} \tag{13}$$

where

$$\begin{aligned} n_i(\alpha, \Omega_M^*) \;=\; & s^2\Big(\sum_{i=1}^{16} \frac{5\,\log D_L(z_i, \alpha, \Omega_M^*) - m_{Bi}^{corr}}{\sigma_{low,i}^2} + \\ & \sum_{i=1}^{38} \frac{5\log D_L(z_i, \alpha, \Omega_M^*) - m_{Bi}^{eff}}{\sigma_{high,i}^2}\Big), \end{aligned} \tag{14}$$

$$s^2 = \left(\sum_{i=1}^{16} \frac{1}{\sigma_{low,i}^2} + \sum_{i=1}^{38} \frac{1}{\sigma_{high,i}^2}\right)^{-1}, \tag{15}$$

$$\sigma_{low,i}^2 = \sigma_{m_{B,i}^{corr}}^2 + \left(\frac{5\,\log e}{z_i}\,\sigma_{z_i}\right)^2 \tag{16}$$

and

$$\sigma^2_{high,i} = \sigma^2_{m^{eff}_{B,i}} + \left(\frac{5\ \log e}{z_i}\ \sigma_{z_i} \right)^2 . \tag{17}$$

The quantities m_B^{corr}, m_B^{eff}, $\sigma_{m_B^{corr}}$, $\sigma_{m_B^{eff}}$ and σ_z are given in Tables 1 and 2 of Perlmutter *et al.* [16].

The results of our analysis for the GCG Universe are displayed in Fig. 1. In this figure we show 68 and 95 confidence level contours, in the (α, Ω_M^*)-plane. We observe that current SNIa data constrain Ω_M^* to the range $0.15 \lesssim \Omega_M^* \lesssim 0.4$, but do not strongly constrain the parameter α in the considered range. Other tests may impose further constraints. For instance, in Ref. [17] it shown that CMB alone, imposes $T_0 = 14 \pm 0.5$ Gyr (1σ) for the age of the Universe. If we also assume the HST Key Project result, $H_0 = 72 \pm 8$ [18], and that H_0 and T_0 measurements are uncorrelated, we obtain for the product H_0T_0, the following range: $0.79 < H_0\ T_0 < 1.27$, at the 2σ confidence level. The central value occurs at $H_0T_0 = 1.03$. In Fig. 1, we also display the contours $H_0T_0 = 0.79$ and $H_0T_0 = 1.27$. As remarked before, we can see that negative values of α close to -1, are disfavored. We have also checked that, keeping all other parameters fixed, the position of the first Doppler peak decreases as α increases. It would be interesting to investigate the constraints imposed by cosmic microwave observations on GCG models, but we leave this for future work.

Finally we consider how well the proposed Supernova Acceleration Probe (SNAP) [19], may constrain the parameters α and Ω_M^* . Following previous investigations [20], we assume, in our Monte Carlo simulations, that a total of 2000 supernovae (roughly one year of SNAP observations) will be observed with the following redshift distribution. We consider, 1920 SNIa, distributed in 24 bins, from $z = 0$ to $z = 1.2$. From redshift $z = 1.2$ to $z = 1.5$, we assume that 60 SNIa will be observed and we divide them in 6 bins. From $z = 1.5$ to $z = 1.7$ we consider 4 bins with 5 SNIa in each bin. All the supernovae are assumed to be uniformly distributed with $\Delta z = 0.05$. In our simulations, we assume that the errors in magnitude are Gaussian distributed with zero mean and variance $\sigma_m = 0.16$. This includes observational errors and intrinsic scatter in the SNIa absolute magnitudes. We neglect, in our simulations, uncertainties in the redshift. We also investigated the effect of a redshift dependent systematic error of the kind $\delta m = \pm(0.02/1.5)\ z$. This kind of systematic error slightly shifts the "ellipses" up or down - depending if the signal in δm is plus or minus - but not along the major axis of the "ellipses". We have not considered in this work the systematic effect of lensing [21]. This important effect, is not expected to change qualitatively our conclusions, unless the Universe contains a significant fraction of compact objects [22]. In this case, a more detailed analysis is required [23].

In Fig. 2 we display the results of our simulation assuming a fiducial model with $\Omega_M^* = 0.3$ and $\alpha = 0$. For the figure the Hubble intercept is assumed to be exactly know. In Fig. 3, we considered the case in which the intercept $\mathcal{M}$ is not known, and we marginalized over it following Goliath *et. al.* [20]. In Fig. 4 the fiducial model has $\Omega_M^* = 0.3$ and $\alpha = 1$, and again the intercept is not assumed to be known. From the figures it is clear that SNAP will be able to rule out the Chaplygin gas model ($\alpha = 1$) if the Universe is dominated by a true cosmological constant. Alternatively, if the Universe is dominated by the Chaplygin gas a cosmological constant can be ruled out.

3 Summary

We derived constraints, from current and future SNIa observations, in a scenario where both the accelerated expansion and CDM are manifestations of a single component. We considered the special case of a generalized Chaplygin gas. For the homogenous model, an important difference between UDM and models with Λ or scalar fields is that in the former there is a transformation of effective CDM into effective dark energy that produces the accelerated expansion.

Our results show that the GCG is consistent with current SNIa data, for any value of α in the considered range, although values of $\alpha \sim 0.4$ are favored. If the accelerated expansion is caused by a cosmological constant, than SNAP data should be able to rule out the Chaplygin ($\alpha = 1$) gas and alternatively, if the Universe is dominated by the Chaplygin gas a cosmological constant would be ruled out with high confidence.

For simplicity, we have discussed in this letter the case of a Universe composed of UDM only. Of course, one should also include the baryonic component, whose energy density scales differently from the

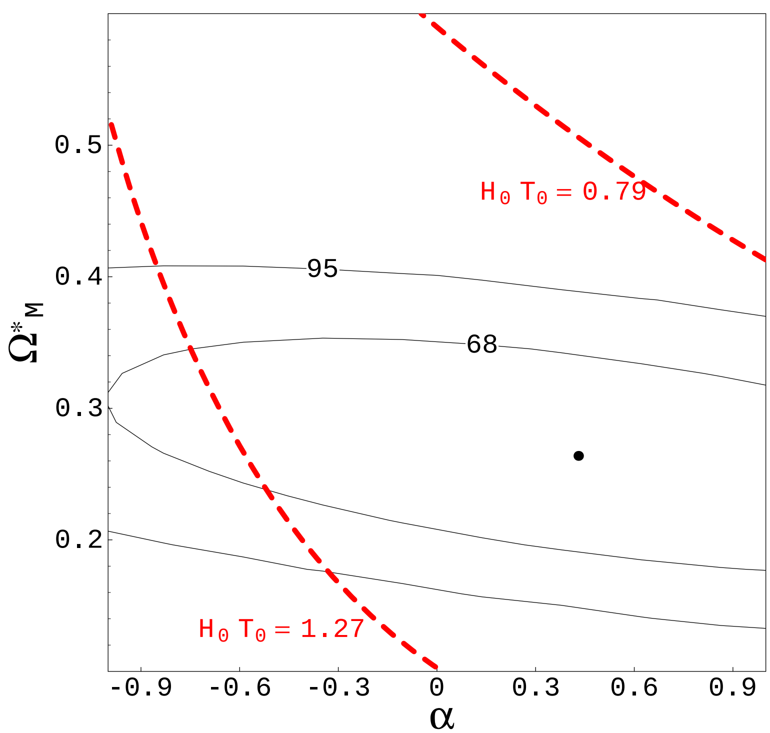

Figure 1: In the figure 68 and 95 confidence level contours, in the (α, Ω^*_M)-plane, are displayed. For the figure we use fit C, of Perlmutter *et al.* [16]. The point in the figure, with coordinates $(0.43, 0.26)$, represents the best fit value. Constraints from the age of the Universe give $0.79 < H_0\,T_0 < 1.27$ (at the 2σ confidence level), the dashed lines represent these two limits.

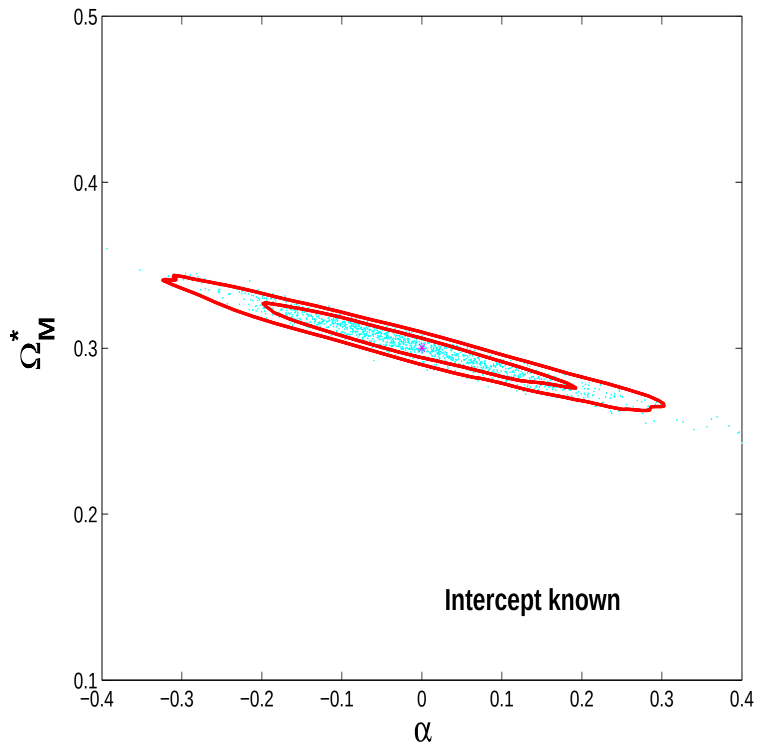

Figure 2: Predicted 68 and 95 confidence level contours for the SNAP mission are shown. We considered a fiducial model with $\Omega_M^* = 0.3$ and $\alpha = 0$. For the figure the Hubble intercept is supposed to be known.

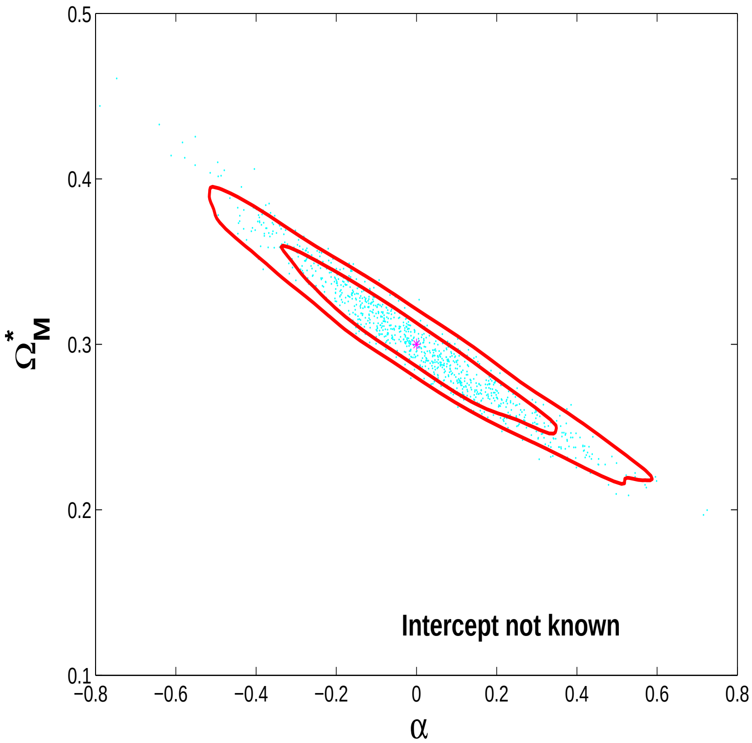

Figure 3: Predicted 68 and 95 confidence level contours for the SNAP mission are shown. We considered a fiducial model with $\Omega_M^* = 0.3$ and $\alpha = 0$. For the figure the Hubble intercept is not supposed to be known.

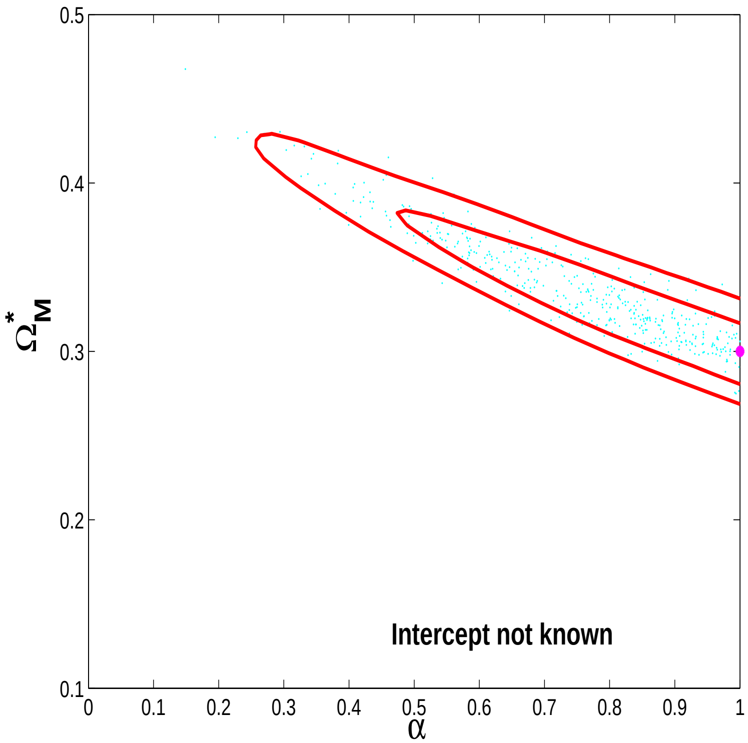

Figure 4: Predicted 68 and 95 confidence level contours for the SNAP mission are shown. We considered a fiducial model with $\Omega_M^* = 0.3$ and $\alpha = 1$. For the figure the Hubble intercept is not supposed to be known.

UDM. When baryons are included in the Hubble parameter the picture does not change, although some details do. For instance, if we introduce Ω_b and perform the analysis with the current supernovae data, the results for Ω_M^* stay almost unchanged, but the best fit value for α decreases ($\alpha \sim 0.15$ for $\Omega_b \sim 0.04$, instead of $\alpha \sim 0.4$ for $\Omega_b = 0$). Also, the age constraints on α are weaker. For instance, for $\Omega_b = 0.04$ we can exclude negative values of α close to -1 only for $\Omega_M^* \lesssim 0.3$. In the case of the data expected from SNAP, we redid the analysis of the preceding section for $\Omega_b = 0.04 \pm 0.004$, assuming a Gaussian distribution. We marginalized over Ω_b and noticed that the contours increase only slightly.

The GCG seems to be a promising model for unifying dark matter and dark energy. More generically, the idea of UDM ("quartessence") has to be explored further, both from the particle physics point of view - to provide a fundamental theory to it -, as well as from the observational side, to constrain UDM models guiding us to unveil its nature.

Note added: After this manuscript was submitted for publication, another paper using the GCG and SNIa observations appeared on the web [24]. Their results are similar to ours, although they do not set constraints on the parameter α of the GCG equation of state.

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