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Dirac's Æther in Curved Spacetime-II: The *Geometric Amplification* of the Cosmic Magnetic Induction

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Abstract

We search for an amplification mechanism of the seed cosmic magnetic induction by studying a new version of the Dirac's æther in a curved cosmological background. We find that an amplification takes place if the scale factor $R(t)$ varies with the cosmic time, which brings to the magnetic field the effect of a *geometric amplification*.

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1 Introduction

Information from our universe comes vastly from the propagation of light throughout the cosmic medium. However, until now the origin of the cosmic magnetic induction (CMI) in astronomical objects remains unknown (see [1] for a comprehensive review). In fact, no theory has completely succeeded in explaining the evolution of the CMI, from its generation in the early universe to the present values observed in a multitude of cosmic scales. In stars, magnetic fields range from 10^8 T in the interiors of neutron stars to values of ≈ 1 T in sunspots, and down to 10^{-7} T in protostellar objects. It is believed that in stellar interiors the standard dynamo action, in combination with convective motions and the reconnection of the field lines, is able to explain the range of values observed. In galaxies, however, magnetic dynamos leave many unanswered questions [2], the most immediate of them is the rather long characteristic time-scales over which they could operate. For instance, the interstellar medium in our Galaxy has an ordered field of about 2×10^{-10} T, superposed upon which there is a random component of about 1-2 times this value [3]. Considering that the period of rotation of the interstellar gas about the Galactic center is $\approx 2.5 \times 10^8$ years, there would have been at most 50 complete rotations of this gas about the center. The differential rotation of the ionized gas in the interstellar medium results in the stretching and amplification of the magnetic field in the disk. Therefore, any primordial magnetic field would be tightly wound up. The problem is that this mechanism is not enough to produce an ordered field. The winding up of the field lines would result in tightly wound tubes of magnetic flux running in opposite directions. It is then necessary to have a mechanism that is able to reconnect the lines of force in order to create the large-scale uniform field.

As we proceed to even larger structures, evidence for large scale magnetic fields are seen. In the intracluster medium in cluster of galaxies, fields of $\approx 10^{-10}$ T have been derived from the diffuse synchrotron radio emission observed from a number of clusters of galaxies, as well as from the observation of depolarization of the emission of extended radio sources by the surrounding intracluster medium. In the intergalactic medium between clusters of galaxies, we are only able to set an upper limit for the magnetic field of 10^{-13} T, a value provided from the lack of polarization of the emission reaching us from distant radio sources. However, it is widely believed that magnetic inductions are present in the universe whatever the scale we look for, and new technological developments are in their way to confirm this (e.g., the SIRTF infrared satellite and the SOAR optical telescope). Whether present-day field values were built up when the first galaxies formed remains questionable, but even if this is the case, the process requires the existence of a very week seed field that was slowly amplified over cosmic time.

It has been shown that it is possible to generate a seed CMI in a plasma with no fields present at the recombination time supposing that there are only variations in the pressure of the electrons in the plasma. This effect is known as the Biermann battery (see [2]). The flow of electrons to lower pressure regions results in a charged unbalanced plasma, which produces an electric field opposing the flow of electrons. As a result, the flow stops and an eletric and magnetic field (emf) is created. This emf cannot drive a current though, since the integral around any closed loop in the case of a linear gradient is zero. If, however, there are variations in the electron density throughout the plasma, different emfs can be induced in different regions and then currents flow in the plasma creating a magnetic field. Nevertheless, this process saturates at about 10^{-25} T, since it is limited by the self-inductance of the current loop itself. How then to reconcile this extremely low value with the upper limit found today for the intergalactic medium between clusters?

Recently, one special mechanism has been studied in which the amplification of the seed CMI is understood as being caused by the expansion of the cosmological background [4]. This is called *geometric amplification* because the only agent responsible is the geometric scale factor $R(t)$. One of the advantages of this approach is that it does not rule out other models, while keeping the amplification factor obtained for the seed CMI within the observational constraint.

The Dirac's æther is a kind of cosmic conducting medium with a very small conductivity that does not violate the experimental limits which confirms the maxwellian theory in terrestrial laboratories. In the first paper [5], we studied the equations of the Dirac's æther coupled to the Proca field in the background of an Einstein static universe. Here we search for a *geometric amplification* of the seed CMI by applying to the cosmic medium a recent version of the Dirac's æther model [6, 7]. This new version of the Dirac's æther maintains the most important features of the original Dirac's model with aditional advantages, e.g., a new interpretation of the 4-velocity as the velocity of the different parts of the æther relative to a generic observer, inertial or not.

This work is organized as follows. In the next section, we describe the model, the symmetries of the proposed field, and solve the equations numerically. In the last section we devote ourselves to the discussion and concluding remarks.

2 Model and Results

In this paper we use the new equations of the Dirac's æther coupled with geometry. For a generic observer (inertial or not) these equations are

$$F^{\mu\nu}_{\ \ ;\mu} + \frac{\sigma}{c}(A^\mu v^\nu - A^\nu v^\mu)_{\ ;\mu} = J^\nu \tag{2.1}$$

where the semicolon denotes the covariant derivative. A^μ is the electromagnetic 4-potential; $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ are the components of the electromagnetic field tensor; v is the 4-velocity of the æther relative to the observer; c is the speed of light; and $J^\nu \equiv (-\sigma/c)v_\mu F^{\mu\nu}$. It is interesting to remember that in a flat space-time an inertial observer moving with the cosmic æther has velocity components $v^\mu_{\ae ther} = (1; 0, 0, 0)$. Others inertial observers have $v^\mu = \Lambda^\mu_\nu\, v^\nu_{\ae ther} = \Lambda^\nu_0$, with the Lorentz transformation Λ relating the observer and the æther frame. These equations (1) differ from the ones of our previous model by the presence of a skew-symmetric term $(A^\mu v^\nu - A^\nu v^\mu)_{\ ;\mu}$ instead of the term $(1/\lambda^2)A^\nu$ (that comes from the Proca term). We adopt a cylindrical coordinate system

$x^\mu = (t;\, \rho,\, \phi,\, \zeta)$ in a Friedmann cosmological background. The metric tensor in all three Friedmann geometries and the field A^μ, with cylindrical symmetry, are given by

$$g_{\mu\nu} = diag\left[R^2(t)(1\,;-1\,,-u^2(\rho)\,,-w^2(\rho))\right], \tag{2.2}$$

$$A^\mu = (0;\,0,\,1,\,0)f(t)/R^2(t), \tag{2.3}$$

where $u(\rho)$ and $w(\rho)$ are functions of the coordinate ρ. Depending on the geometry of the space-time, they will define the type of three-geometry (with constant curvature k_c) under consideration. $f(t)$ is a function to be determined by the field equations. The velocity of observers moving with the cosmic æther is $v_\mu = R(t)\,\delta^0_\mu$.

The field strength $F_{\mu\nu}$ has non-zero independent components F_{02} and F_{12}. In an orthonormal basis, the non-null components of the fields $\mathbf{E}$ and $c\mathbf{B}$ are

$$E_\phi = -\dot{f}u^2/R^2, \qquad cB_\zeta = 2fu\;u'/R^2, \tag{2.4}$$

where the dot means d/dt, the prime is $d/d\rho$, and c is the light velocity. The electric field and the magnetic induction are orthogonal and non-homogeneous, and both depend on t and ρ. Their moduli are

$$|\mathbf{E}| = |u\dot{f}|/R^2\,, \qquad |\mathbf{B}| = 2|fu'|/(cR^2). \tag{2.5}$$

Table 1 shows the curvature k_c of each one of the Friedmann's geometries considered, $R(t)$, $u(\rho)$, $w(\rho)$, and the equation for $f(t)$ that needs to be solved. We are particularly interested in finding solutions for the CMI that provide an explanation for the amplification of the magnetic induction below the the suggested limit of 10^{-13} T for the intergalactic medium between cluster of galaxies.

Let us now integrate numerically the equations in Table 1, from the initial cosmic conformal time $t_i = 0.0890$ to the final time $t_f = 1.6100$. In standard cosmology these values correspond respectively to the final stage of the matter-radiation coupling and our current epoch. $20,000$ integration steps are performed. We assume that the conductivity of the Dirac's æther is $\approx 10^{-19}$/s, α is 10^{26} m, and the initial CMI is $\approx 10^{-25}$ T. Our

Table 1: Curvature k_c, $R(t)$, $u(\rho)$, $w(\rho)$, and equations to be solved.

k_c	$R(t)$	$u(\rho)$	$w(\rho)$	Equation for $f(t)$
0	$(\alpha/2)t^2$	ρ	1	$\ddot{f} - \alpha t\ (\sigma/c)\ f = 0$
$+1$	$\alpha(1-\cos t)$	$\sin\rho$	$\cos\rho$	$\ddot{f} + [4 -\ (\sigma/c)\ \alpha\ \sin t]\ f = 0$
-1	$\alpha(\cosh t - 1)$	$\sinh\rho$	$\cosh\rho$	$\ddot{f} - [4 +\ (\sigma/c)\ \alpha\ \sinh t]\ f = 0$

model also includes a weak initial electric field of magnitude $\approx 10^{-4}$ V/m to be dissipated during the time evolution. These limits are fixed in order to provide a realistic value for the modulus of the CMI that agrees with the one established by the usual theory of the cosmic fields. These initial values do not perturb the gravitational field; from a simple calculation it is evident that the energy-momentum tensor of the electromagnetic and gravitational fields are related by a factor above 10^{10}.

In Table 2, we display a small ensemble of points that gives us a qualitative view of the amplification phenomenon in terms of the quantities $\mathcal{E}(t) \equiv |\mathbf{E}/u| = |\dot{f}|/R^2$ and $\mathcal{B}(t) \equiv |\mathbf{B}/u'| = 2|f|/(cR^2)$, that for simplicity we will also refer as the electric field and magnetic induction.

These results are very similar to the ones of our latter work. Comparing the initial and the final values of the fields for each geometry the data show an amplification of the $\mathcal{B}$ field of the order $\approx 10^{+6}$, and an overall reduction of the electric field of the order $\approx 10^{-5}$. It should be noticed that these results are determined not only by the evolution of the function $f(t)$ (which constrains the field equations) but also by the direct contribution of the geometry as given by the scale factor, $R(t)$, that is present in both mathematical expressions for $\mathcal{E}$ and $\mathcal{B}$. It is the interchange between the gravitational and electromagnetic fields that imposes, as the Universe evolves, the decrease of the electric field and the amplification of the $\mathcal{B}$ field.

Table 2: Some Results of the Numerical Integration

t	Flat ($k_c = 0$)		Elliptic ($k_c = +1$)		Hyperbolic ($k_c = -1$)	
	$log\vert \mathcal{E} \vert$	$log\vert c\mathcal{B} \vert$	$log\vert \mathcal{E} \vert$	$log\vert c\mathcal{B} \vert$	$log\vert \mathcal{E} \vert$	$log\vert c\mathcal{B} \vert$
0.0890	-4.0004	-14.6999	-3.9998	-14.6988	-4.0009	-14.6999
0.1000	-4.2028	-5.8604	-4.2021	-5.8597	-4.2036	-5.8611
0.2000	-5.4069	-6.0606	-5.4040	-6.0577	-5.4098	-6.0635
0.3000	-6.1112	-6.4859	-6.1048	-6.4795	-6.1178	-6.4925
0.5000	-6.9986	-7.0838	-6.9807	-7.0657	-7.0166	-7.1018
0.8000	-7.8147	-7.6621	-7.7690	-7.6158	-7.8607	-7.708
1.0000	-8.2018	-7.9419	-8.1308	-7.8695	-8.2734	-8.0138
1.2000	-8.5177	-8.1723	-8.4156	-8.0678	-8.6204	-8.2753
1.5000	-8.9034	-8.4557	-8.7441	-8.2913	-9.0624	-8.6154
1.6000	-9.0147	-8.5378	-8.8333	-8.3505	-9.1049	-8.7191

3 Conclusion

In all cases studied here, our results show the desired amplification of the initial CMI, together with the reduction of the electric field. The new version of the Dirac's æther incorporates in a natural way the 4-velocity (first pointed out by Dirac in 1951 [8] which is interpreted as the velocity of the æther relative to an observer. This allow us to adapt our description to any observer, inertial or not. In the case of a curved background, an observer with $v_\alpha = R(t)\delta^0_\alpha$ will see the same phenomenon of amplification of the magnetic induction and reduction of the electric intensity that had already been observed in [4], in the context of a Proca electromagnetic field in a Dirac æther.

The geometric relations between the electromagnetic field and the metric tensor involves the scale factor $R(t)$. The amplification of the $\mathcal{B}$ field in this model is determined by the coupling between gravitational and electromagnetic fields as in the usual theory of electromagnetism in a curved background. Larger couplings between the electromagnetic and the gravitational fields could lead to an even faster and/or more intense interchange, as can be seen in [9]. As the *geometric amplification* describes how the expansion of the universe influences the electromagnetic fields, there is also the possibility that electromagnetic fields influence the expansion of the universe [10]. Our results confirm once again the strict relations between the electromagnetic and gravitational phenomena.

The obtained geometric amplification is most probably superseded by the conventional dynamo effect in the interior of stars, in the intragalactic medium, and even in the scales of galaxies. However, in scales of clusters and larger, where the dynamo action most probably fails, the geometric amplification of the seed CMI may be the only important effect to be considered. We believe that in the near future technological advances will be able to detect these fields and confirm our results.

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