Self-dual vortices in a Maxwell Chern-Simons model with non-minimal coupling

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ABSTRACT

We find self-dual vortex solutions in a Maxwell-Chern-Simons model with anomalous magnetic moment. From a recently developed N=2–supersymmetric extension, we obtain the proper Bogomol'nyi equations together with a Higgs potential allowing both topological and non-topological phases in the theory.

Key-words: Vortices; Self-duality; Magnetic moment.

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1   Introduction

A few years ago, it was proposed a Maxwell-Chern-Simons (MCS) gauge theory, with an additional magnetic moment interaction[1] for which Bogomol'nyi-type self-dual equations can be derived and vortex-like configurations appear, whenever suitable relationships among the parameters of the model are obeyed [2]. An important issue that comes about is the claim of a relation between the property of self-duality and the N=2–supersymmetric extension of the model, accomplished by means of a relationship between the central charge of the extended model and the existence of topological quantum numbers [3]. Although a fundamental reason for this connection has not been given so far in the literature, in certain cases it appears to be unavoidable to find the $N = 2$–supersymmetric extension of a given bosonic model in order to obtain the proper Higgs potential and self-dual conditions compatible with the Euler-Lagrange equations.

In this regard, we have succeeded in deriving an N=2–Maxwell-Chern-Simons model with anomalous magnetic moment [4]. Our strategy consisted in the formulation of an N=1–D=4–gauge model with a BF-term, free of constraints on the coupling constants[1]. Upon a convenient dimensional reduction of the component-field action from (1+3) to (1+2) dimensions, we set out an N=2–D=3 Maxwell-Higgs model with a Chern-Simons term and magnetic moment interaction with the matter sector. Adopting this viewpoint, we raised the possibility of freely handling the parameters of the model and, remarkably, it enabled us to obtain topological self-dual solutions, even in the critical regime mentioned above. This is to be compared with previous attempts where just a ϕ^2–Higgs potential has been considered so as to find self-dual solutions [8].

In the present paper, we derive the proper self-dual equations and the Higgs potential needed to allow topological as well as non-topological vortices in a non-minimally coupled MCS model; this is our main result. We perform a gauge-independent calculation which permits a suitable handling of the energy functional, leading to self-dual solutions to the equations of motion in both the symmetric and asymmetric phases of the model (Sections 2 and 3). In Section 4, we discuss the properties of system for the critical value of the magnetic coupling. The analysis of the self-dual solutions and a wide variety of soliton configurations are presented in Section 5. Finally, in Section 6, we draw our General Conclusions.

2   The Lagrangian

In Ref.[4] we have put forward the N=2–susy Lagrangian including the bosonic model we are going to analyse here. In component-field form, it exhibits the proper non-minimally coupled MCS extension needed for our main purpose of finding a topological phase in the bosonic theory. For the sake of a better understanding let us quote below the full expression of the N=2-susy Lagrangian in terms of components

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[1]This is to be compared with the procedure of Ref.[5] which, in turn, relies on a special choice of parameters, in order to have an extended supersymmetry built up directly in D=3 dimensions. Similar constraints have also been needed in order to find an N=2–susy extension of the Maxwell Higgs model [6] and of the Chern-Simons Higgs model [7].

$$\mathcal{S}_{MCS}^{N=2} \;\; = \;\; \int d^3x \left\{ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\kappa}{2}\varepsilon^{\mu\nu\alpha}A_\mu\partial_\nu A_\alpha + 2\Delta^2 + \frac{1}{2}\partial_\mu M\partial^\mu M - 2\kappa M\Delta \right.$$

$$+ \frac{1}{2}\overline{\Lambda}_-(i\!\not{\partial} + \kappa)\Lambda_- + e^{2gM}\left[\nabla_\mu\varphi\nabla^\mu\varphi^* - (eM + 2g\Delta)^2 \varphi\varphi^* \right.$$

$$+ \frac{1}{4}(eM + 2g\Delta)\overline{X}_+X_+ + \frac{i}{8}(\overline{X}_-\!\not{\nabla}X_- + \overline{X}_+\!\not{\nabla}X_+)$$

$$- \frac{ig}{2}\left(\overline{\Lambda}_+\!\not{\nabla}\varphi X_- - h.c.\right) + \frac{g}{2}(eM + 2g\Delta)\left(\overline{\Lambda}_+X_-\varphi + h.c.\right)$$

$$+ \frac{ig^2}{2}\partial_\mu M\left(\overline{X}_-\gamma^\mu\Lambda_+\varphi^* - \overline{\Lambda}_+\gamma^\mu X_-\varphi\right) - i\frac{g^2}{2}\varphi^*\varphi\left(\overline{\Lambda}_+\!\not{\partial}\Lambda_+ + \overline{\Lambda}_-\!\not{\partial}\Lambda_-\right)$$

$$- \frac{g^2}{e}\left(\frac{1}{2}(\overline{\Lambda}_+\gamma^\mu\mathcal{J}_\mu\Lambda_+ - \overline{\Lambda}_-\gamma^\mu\mathcal{J}_\mu\Lambda_-) - \overline{\Lambda}_+\Lambda_+ e(eM + 2g\Delta)\varphi\varphi^*\right)$$

$$- \varphi\varphi^*\left(2e\Delta - 2ge\overline{\Lambda}_+\Lambda_+ + g^2\partial_\mu M\partial^\mu M\right)$$

$$\left. \left. + \frac{e}{2}(\varphi\overline{\Lambda}_+X_- + \varphi^*\overline{X}_-\Lambda_+) + \left| S - \frac{g}{2}\overline{X}_-\Lambda_- - \frac{g^2}{2}\varphi\overline{\Lambda}_+\Lambda_- \right|^2 \right] \right\} \tag{1}$$

where

$$\nabla_\mu\varphi = (\partial_\mu - ieA_\mu - igF_\mu)\,\varphi. \tag{2}$$

The origin of all the fields appearing in equation eq.(1) has been carefully justified in [4]; we refer the reader to this reference for the details. Here, we are basically concerned with the bosonic sector of the theory, so we will focus our attention to a particular piece as we discuss in what follows.

Let us consider the purely bosonic part of the susy Lagrangian of eq.(1)

$$\mathcal{L} \;\; = \;\; -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}G\partial_\mu M\partial^\mu M + e^{2gM}\nabla_\mu\varphi\nabla^\mu\varphi^* + \frac{\kappa}{2}A_\mu F^\mu$$

$$+ \left\{2\Delta^2 - 2\kappa M\Delta + \eta\Delta - e^{2gM}|\varphi|^2\left[(eM + 2g\Delta)^2 + 2e\Delta\right]\right\}, \tag{3}$$

where

$$G(\varphi) = 1 - 2g^2 e^{2gM}|\varphi|^2 \tag{4}$$

and the term $\eta\Delta$ corresponds to the Fayet-Iliopoulos term included in the susy Lagrangian in order to allow spontaneous breaking of gauge invariance [9].

The equation of motion for the auxiliary Δ–field gives

$$\Delta = \frac{e}{4G}\left(2e^{2gM}|\varphi|^2 - v^2 + \frac{2\kappa}{e}M + 4ge^{2gM}|\varphi|^2 M\right) \tag{5}$$

where for convenience we have written $\eta = -ev^2$. Substitution of the above in eq.(3), gives the following Higgs-type potential

$$U = \frac{e^2}{8G}\left(2e^{2gM}|\varphi|^2 - v^2 + \frac{2\kappa}{e}M + 4ge^{2gM}|\varphi|^2 M\right)^2 + e^2 M^2 e^{2gM}|\varphi|^2 \tag{6}$$

which depends on two fields: a real (M) and a complex (φ) scalar. Upon elimination of the auxiliary field Δ, we may work with the following Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^2_{\mu\nu} + \frac{1}{2}G\partial_\mu M \partial^\mu M + e^{2gM}\nabla_\mu\varphi\nabla^\mu\varphi^* + \frac{\kappa}{2}A_\mu F^\mu - U \tag{7}$$

which shall play a central role in the present discussion. Let us first define the currents

$$\begin{aligned} H_\mu &= -\frac{ie}{2}\left(\varphi^* D_\mu\varphi - \varphi D_\mu\varphi^*\right) \\ \mathcal{J}_\mu &= -\frac{ie}{2}\left(\phi^*\nabla_\mu\phi - \phi\nabla_\mu\phi^*\right) \end{aligned} \tag{8}$$

where ϕ is a complex scalar parametrized in terms of M and φ as given below

$$\phi = \sqrt{2}e^{gM}\varphi \tag{9}$$

As we shall discuss in the next section, the scalar ϕ will be identified as the physical field in terms of which the vortices will be specified. Now, the equation of motion for the gauge field can be written as

$$\partial_\mu F^{\mu\rho} + \kappa F^\rho = \mathcal{J}^{\,\rho} + \frac{g}{e}\varepsilon^{\mu\nu\rho}\partial_\mu\mathcal{J}_\nu \tag{10}$$

where the time component determines the modified "Gauss Law"

$$\partial_i E_i + \kappa B + \frac{g}{e}\varepsilon_{ij}\partial_i\mathcal{J}_j + \mathcal{J}_0 = 0. \tag{11}$$

The gauge invariant modes are now short-range due to the mass term resulting from eq.(11). Hence, the first term has a vanishing space integral. On the other hand, the third term results in a line integral taken at infinity which also vanishes for finite energy configurations. Therefore, it can be seen from the remaining piece that the charge of the vortex solutions is related to non-zero magnetic fluxes by

$$Q = \kappa\Phi_B, \tag{12}$$

where $\Phi_B \equiv -\int d^2x\,B$.

3   The self-dual equations of motion

The energy functional is given by

$$\begin{aligned} \mathcal{E} &= \int d^2x \left\{ \frac{1}{2}G\left(B^2 + E^2\right) + \frac{1}{2}G\partial_0 M\partial_0 M + \frac{1}{2}G\partial_i M\partial_i M \right. \\ &\quad \left. + e^{2gM}D_0\varphi D_0^*\varphi + e^{2gM}D_i\varphi D_i\varphi^* + U \right\} \end{aligned} \tag{13}$$

which can be reorganized as

$$
\begin{aligned}
\mathcal{E} \;=\; & \int d^2x \left\{ \frac{1}{2}G\left(B \mp \frac{e}{2G}\left(2e^{2gM}|\varphi|^2 - v^2 + \frac{2\kappa}{e}M + 4ge^{2gM}|\varphi|^2 M\right)\right)^2 \right. \\
& \pm eB\left(e^{2gM}|\varphi|^2 - \frac{1}{2}v^2 + \frac{\kappa}{e}M + ge^{2gM}|\varphi|^2 M\right) + \frac{1}{2}G\left(E_i \pm \partial_i M\right)^2 \\
& \mp GE_i\partial_i M + e^{2gM}\left(|D_0\varphi \mp ieM\varphi|^2 \pm 2MH_0 + |(D_1 \pm iD_2)\varphi|^2\right. \\
& \left.\left. \pm \frac{1}{e}\varepsilon_{ij}\partial_i H_j \mp eB|\varphi|^2\right)\right\}
\end{aligned}
\tag{14}
$$

Notice that the non-minimal term from the ∇_μ–derivative, though not explicitly written in the above equation, has an effect which is implicit through G. The terms linear in F_μ are not present in the energy because they are topological (metric-independent).

Now, the search of the Bogomol'nyi bound for the energy yields the proper self-dual equations in a natural way [2]

$$
\begin{aligned}
B \mp \frac{e}{2G}\left(2e^{2gM}|\varphi|^2 - v^2 + \frac{2\kappa}{e}M + 4ge^{2gM}|\varphi|^2 M\right) &= 0 \\
E_i \pm \partial_i M &= 0 \\
D_0\varphi \mp ieM\varphi &= 0 \\
(D_1 \pm iD_2)\varphi &= 0.
\end{aligned}
\tag{15}
$$

Using the following identities

$$
\begin{aligned}
\frac{1}{e}e^{2gM}\varepsilon_{ij}\partial_i H_j &= \frac{1}{2e}\varepsilon_{ij}\partial_i \mathcal{J}_j + \frac{1}{2e}\partial_i E_i - \frac{g}{e}\varepsilon_{ij}\left(\partial_i M\right)\mathcal{J}_j - 2g^2 e^{2gM}|\varphi|^2 E_i\partial_i M \\
2e^{2gM}H_0 &= \mathcal{J}_0 - ege^{2gM}|\varphi|^2,
\end{aligned}
$$

integrating by parts and dropping surface terms, one finally gets

$$
\begin{aligned}
\mathcal{E} \;=\; & \frac{ev^2}{2}|\Phi_B| + \int d^2x \left\{ \pm M\left(\mathcal{J}_0 + \frac{g}{e}\varepsilon_{ij}\partial_i\mathcal{J}_j + \kappa B + \partial_i E_i\right) + \frac{1}{2}G\partial_0 M\partial_0 M \right. \\
& \left. \pm \frac{1}{2e}\varepsilon_{ij}\partial_i\mathcal{J}_j \pm \frac{1}{2e}\partial_i E_i\right\}.
\end{aligned}
$$

The last two terms vanish whenever integrated over the whole space; so, using the Gauss law and considering static configurations, the lower bound to the energy is clearly attained.

To close this section, let us express the self-dual equations and the Higgs potential in terms of the ϕ–field; in so doing, we shall get expressions that are more useful for our future purposes. Thus eqs. (15) and (6) read

$$
\begin{aligned}
B \mp \frac{e}{2G}\left(|\phi|^2 - v^2 + \frac{2\kappa}{e}M + 2g|\phi|^2 M\right) &= 0 \\
A_0 \pm M &= 0 \\
\nabla_1\phi \pm i\nabla_2\phi &= 0
\end{aligned}
\tag{16}
$$

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[2]See Ref.[10] for a general approach to Bogomol'nyi equations in other models.

and

$$U = \frac{e^2}{8G} \left(|\phi|^2 - v^2 + \frac{2\kappa}{e} M + 2g|\phi|^2 M \right)^2 + \frac{e^2}{2} M^2 |\phi|^2, \tag{17}$$

which, for $g = 0$, gives the Higgs potential of the minimal MCS model as given by the supersymmetric Lagrangian found in ref.[2], as expected. Notice that the system has two degenerate minima, a symmetric phase for which $|\phi| = v$, $M = 0$ and an asymmetric phase where $\phi = 0$, $M = ev^2/2\kappa$.

4   The critical magnetic coupling

Let us now analyze a very special value of the magnetic coupling, namely,

$$g_c = -e/\kappa, \tag{18}$$

for which the equations of motion (10) reduce down to first order, looking analogous to the pure CS model's. This choice yields fractional statistics describing anyons [1]. Remarkably enough, this is the value that has to be fixed in order to obtain an N=2–MCS non-minimal theory, when working from the outset in D=3 [5]. In that case only a symmetric, ϕ^2, Higgs potential has been found, yielding just non-topological solutions [8].

Hence, for $g = g_c$ one has

$$\mathcal{J}_\mu = \kappa F_\mu \tag{19}$$

whose time-component reads

$$\kappa \left(1 - \frac{e^2}{\kappa^2} |\phi|^2 \right) B = e^2 A_0 |\phi|^2. \tag{20}$$

We will now show that, in our model, we can make such a special choice, $g = g_c$, without constraining the potential to a symmetric phase. We shall also find topological vortices, in contrast to previous attempts (we drop the subscript $_c$ in what follows).

Using $A_0 = \mp M$ [see eq.(16)] and defining γ by means of $\kappa = \gamma ev$ ($\gamma \geq 1$ to have positive definite energy configurations) we get

$$B = \mp \gamma ev M \frac{|\phi|^2}{\gamma^2 v^2 - |\phi|^2}. \tag{21}$$

On the other hand, self-duality relations provide also

$$B = \pm \frac{ev^2}{2} \frac{|\phi|^2 - v^2}{\gamma^2 v^2 - |\phi|^2} \gamma^2 \pm \gamma ev M. \tag{22}$$

Thus, for v defined as the maximum value of $|\phi|$, eqs. (21) and (22) give

$$\begin{aligned} M &= -\frac{(|\phi|^2 - v^2)}{2\gamma v} \\ B &= \pm \frac{e}{2} \frac{|\phi|^2 (|\phi|^2 - v^2)}{(\gamma^2 v^2 - |\phi|^2)}. \end{aligned} \tag{23}$$

Notice that the M–field has decoupled from the other components, i.e., it can be written in terms of just the Higgs field ϕ. Another interesting feature of the bosonic model just found is that it still presents together both topological and non-topological phases

$$U = \frac{e^4}{8} \frac{|\phi|^2 (|\phi|^2 - v^2)^2}{\kappa^2 - e^2 |\phi|^2} \tag{24}$$

Note that for $\kappa \to \infty$ ($\gamma >> 1$), it behaves like the Higgs potential typical of a pure CS model [7], as expected.

5   Analysis of the self-dual solutions

Assuming maximal (rotational) symmetry, we take the following ansatz to find self-dual vortices

$$\phi(r, \theta) = v R(r) e^{in\theta} \tag{25}$$

$$A(r) = \frac{\widehat{\theta}}{er} [a(r) - n] \tag{26}$$

where R and a are real functions of r, and n an integer indicating the topological charge of the vortex. Then, the magnetic field reads

$$B = \frac{1}{er} a' \tag{27}$$

and the flux is

$$\Phi_B = \frac{2\pi}{e} [a(0) - a(\infty)] \tag{28}$$

Now, since in polar coordinates one has

$$\partial_1 \pm i\partial_2 = e^{\pm i\theta} \left(\partial_r \pm \frac{i}{r} \partial_\theta \right), \tag{29}$$

eq. (16) can be written as

$$\left(1 - \frac{R^2}{\gamma^2} \right) \frac{dR}{dr} \mp \frac{a}{r} R = 0 \tag{30}$$

$$\frac{1}{r} \frac{da}{dr} \mp \frac{R^2 (R^2 - 1)}{(\gamma^2 - R^2)} = 0 \tag{31}$$

where we have used (23) and redefined $r \to \frac{\sqrt{2}}{ev} r$.

The natural boundary conditions at infinity result from the requirement of finite energy, while a non-singular behavior determines the values at the origin.

In the *topological* phase, $R(\infty) = 1$ and $a(\infty) = 0$ for nontrivial vorticity n. Then, for large r the asymptotic form of the topological vortices is given by

$$R(r) \simeq 1 - \frac{\sqrt{2}}{2} d \, \gamma K_0 \left(\frac{\sqrt{2}}{\gamma^2 - 1} \gamma r \right) \tag{32}$$

$$a(r) \simeq d \, r K_1 \left(\frac{\sqrt{2}}{\gamma^2 - 1} \gamma r \right) \tag{33}$$

where d is a constant whose value is determined by the form of the solutions at the origin. Also at the origin, one must expect non-singular fields, implying $R(0) = 0$ and $a(0) = n$. Hence, the magnetic flux is quantized as follows

$$\Phi_B = \frac{2\pi}{e}\left[a(0) - a(\infty)\right] = \frac{2\pi}{e}n. \tag{34}$$

Now, we can combine eqs. (30,31) to produce a second order equation

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right) = \frac{(R')^2}{R}\frac{1 + R^2/\gamma^2}{1 - R^2/\gamma^2} - \frac{1 - R^2}{\gamma^2\left(1 - R^2/\gamma^2\right)^2}R^3 \tag{35}$$

so that the behavior of the solutions for small values of r, where $(1 \pm R^2) \simeq 1$, can be approximated by a Liouville-type function

$$R(r) \simeq \frac{2N\gamma}{r}\left[\left(\frac{r}{r_0}\right)^N + \left(\frac{r_0}{r}\right)^N\right]^{-1} \tag{36}$$

where N and r_0 are arbitrary constants. Upon substitution of the above expression in eq.(30), we find

$$a(r) \simeq -1 + N\frac{1 - \left(\frac{r}{r_0}\right)^N}{1 + \left(\frac{r}{r_0}\right)^N}, \tag{37}$$

so that using $a(0) = n$ we obtain $N = n + 1$. It implies that, near the origin, the form of the vortex is power-like

$$\begin{aligned}
R(r) &\simeq \gamma c_n r^n, \\
a(r) &\simeq n - c_n r^{n+1}
\end{aligned} \tag{38}$$

where $c_n = 2(n + 1)/r_0^{n+1}$. This last relation is obtained by expanding eq.(36) around $r = 0$; however, the precise numerical values of the c_n constants are determined by the shape of the fields at infinity, rather than by their behavior at the origin. Indeed, we have numerically solved the self-dual equations of motion by means of an iterative procedure, giving a tentative value for c_n which is corrected each time by imposing that both limits, $R \to 1$ and $a \to 0$, hold together at infinity. For illustration, we quote some of the results in Fig. 1 and Fig. 2, for the cases $n = 1, 2, 3$. Notice the ring-type structure of the topological vortices (see. Fig. 2 and Fig. 3). This profile is analogous to the pure CS magnetic field shape [11]. The $n < 0$ configurations are related to the $n > 0$ ones by the transformation $a \to -a$ and $R \to R$.

Table 1. Several values of c_n for $n = 1, 2, 3$ and $\gamma = 1.5, 2.0, 4.0$ [see eq.(38)].

$\gamma \setminus n$	1	2	3
1.5	1.7948×10^{-1}	2.0538×10^{-2}	1.4206×10^{-3}
2.0	1.0664×10^{-1}	9.1194×10^{-3}	4.7256×10^{-4}
4.0	2.8118×10^{-2}	1.1979×10^{-3}	3.1004×10^{-5}

Now, let us analyze the *non-topological* sector. In this case v is no longer a relevant parameter, but we can use the same ansatz as in eqs.(25,26) with $v = \kappa/e$ ($\gamma = 1$). Now, the system of differential equations gets simplified by

$$
\begin{aligned}
\frac{1}{r}\frac{da}{dr} &= \mp R^2 \\
\frac{dR}{dr} &= \pm \frac{aR}{r\left(1 - R^2\right)},
\end{aligned}
\tag{39}
$$

but it still admits soliton solutions. These are analogous to those found with the ϕ^2 potential considered in [8], although in our case the symmetric phase of the potential arises from $U = \frac{e^2}{8}|\phi|^2\left(\frac{\kappa^2}{e^2} - |\phi|^2\right)$ and the soliton structures are of course not identical. It should be mentioned that in the present situation a nontrivial vacuum value, $|\phi|^2 = \frac{\kappa^2}{e^2}$, is not physically meaningful as the magnetic flux Φ_B together with the energy would be divergent. Note also that for large r, where $(1 - R^2) \simeq 1$, the second order equation for the field $R(r)$ becomes again Liouville's type so that asymptotic solutions look as (36) and (37) in contrast to the asymptotic behavior of topological solutions as given in eqs.(32) and (33). Since we need $R(\infty) = 0$ to have finite energy configurations, from (37) we obtain $N^\infty = -(a_n^\infty + 1)$, where $a(\infty) = a_n^\infty$.

At the origin, non-singular soliton configurations must satisfy $nR(0) = 0$ and $a(0) = n$. Let us now distinguish between the following two possibilities.

On the one hand, non-vorticity ($n = 0$) implies that $R(0) = b_0$ is a continuous parameter, restricted between 0 and 1. This restriction is to guarantee the validity of eq.(35) for all r, namely to avoid singularities.

When $b_0 \to 0$, we may assume that Liouville's approximation is valid for all r, and then we can employ (37) to calculate a_0^∞ by using just the value at the origin $a(0) = 0$. It provides $(a_0^\infty)_{\min} = -2$ as an analytical result. When $b_0 \to 1$, we can not use (37) at the origin any longer, and we have to perform numerical calculations yielding $(a_0^\infty)^{\max} = -1.83$ (see Figs. 4,5,6 and Table 2). In Figs. 4-6 it can be seen the flux-tube structure of the $n = 0$ solitons, with the maximum value of the magnetic field at the origin.

On the other hand, non-trivial vorticity ($n \neq 0$) implies that, at the origin, $R(r)$ can only be zero. Then we can use (37) so that again $N = n + 1$. Thus, for $r \simeq 0$ we have

$$
\begin{aligned}
R(r) &\simeq b_n r^n \tag{40} \\
a(r) &\simeq n - b_n r^{n+1}
\end{aligned}
$$

but in contrast to the topological case, the constants are now continuous parameters bounded as $0 < b_n \leq b_n^{\max}$. For $b_n \simeq 0$, $R(r) \ll 1$ and Liouville's approximation is valid everywhere, hence, we are able to analytically obtain a lower bound for $a(\infty)$ namely, $(a_n^\infty)_{\min} = -(n + 2)$. On the other hand, by numerical investigation we can obtain the maximum values $b_n^{\max}$ and accordingly $(a_n^\infty)^{\max}$, as we illustrate in Table 2.

Thus, in the non-topological phase the magnetic flux is not quantized but instead it is bounded for each vortex number; the width of the band shrinks as n is increased, varying continuously between

$$
2(n - (a_n^\infty)_{\max}) \leq \frac{e}{\pi}\Phi_B \leq 4(n + 1).
\tag{41}
$$

Table 2. Values of b_n^{max} and $-(a_n^\infty)_{\mathrm{max}}$ for $n = 0, 1, 2, 3, 4, 5$.

n	b_n^{max}	$-(a_n^\infty)^{\mathrm{max}}$
0	1.0000	1.8300
1	3.4648×10^{-1}	2.5484
2	5.9971×10^{-2}	3.5210
3	6.2389×10^{-3}	4.5112
4	4.6720×10^{-4}	5.5069
5	2.6900×10^{-5}	6.5011

As we show in Table 2, the asymptotic values of the gauge field remain constrained, increasingly close to $-(n + 3/2)$ while the magnetic flux approaches the limit $(4n + 3)\frac{\pi}{e}$ as n grows.

In Fig. 9, we show the ring structure of the non-topological vortices, with the maximum of the magnetic field out of the origin as it happens in the topological phase of the model. Notice that for b_n^{max} the radius of the vortices, together with their distance to the origin, are minimal for a given charge n, while the Higgs field presents a cuspidal profile attaining its maximum value, $R = 1^-$.

6   Conclusions

We have obtained self-dual soliton solutions of a Maxwell-Chern-Simons model with anomalous coupling in both topological and non-topological sectors. To do this, we have focused the bosonic part of an N=2–D=2+1 supersymmetric model –obtained from an N=1–D=3+1 model after dimensional reduction– which enabled us finding a topological phase of this model in D=2+1. As long as we know, it is the first time that topological self-dual vortices are found in a non-minimal MCS model. We have also analyzed the non-topological phase in detail for several values of the parameters and magnetic fluxes. It is worth noticing that in contrast to the works in Ref.[8] the non-topological phase of our model is not given by a simple ϕ^2 Higgs potential but rather as a fourth order function $|\phi|^2(\frac{\kappa^2}{e^2} - |\phi|^2)$. Our non-topological solutions are then different from those presented in [8] although similar in shape.

In order to compare with our results with previous reports we have restricted the analysis to the critical anomalous coupling. Indeed, we have demonstrated that it is possible to obtain a topological phase in the non-minimal MCS model having self-dual vortex solutions, and that the corresponding non-topological phase is not the same as in previous attempts. A natural extension to this work would be relaxing this constraint so as to consider non-critical values of g. Such a general situation should also present topological solitons. Since analytical as well as computational analysis are more involved in that case, it is still under investigation and the results shall be soon reported elsewhere.

Acknowledgments

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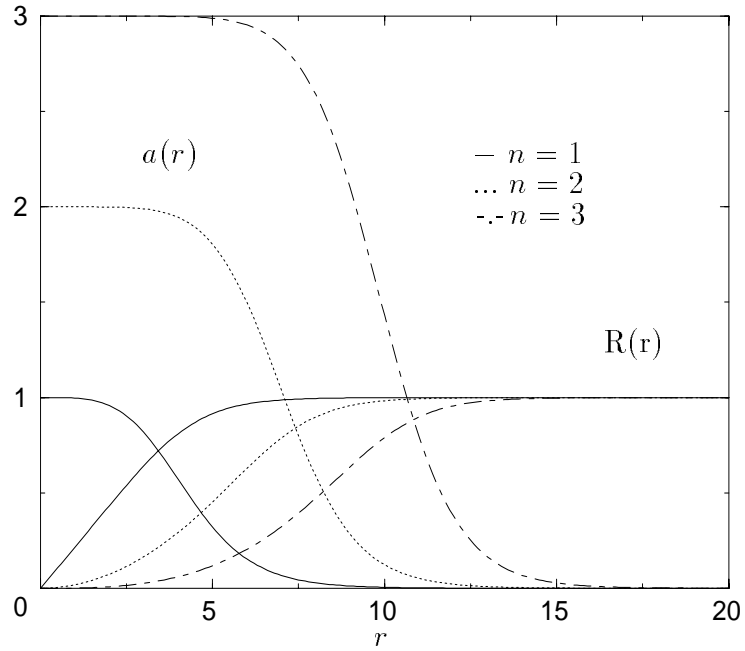
Figure 1: The scalar $R(r)$ and the gauge field $a(r)$ in the topological phase. The values of the c_n constants are fixed by the shape of the fields at infinity: $c_1 = 0.1066$, $c_2 = 9.1190 \times 10^{-3}$, $c_3 = 4.726 \times 10^{-4}$, for $\gamma = 2$.
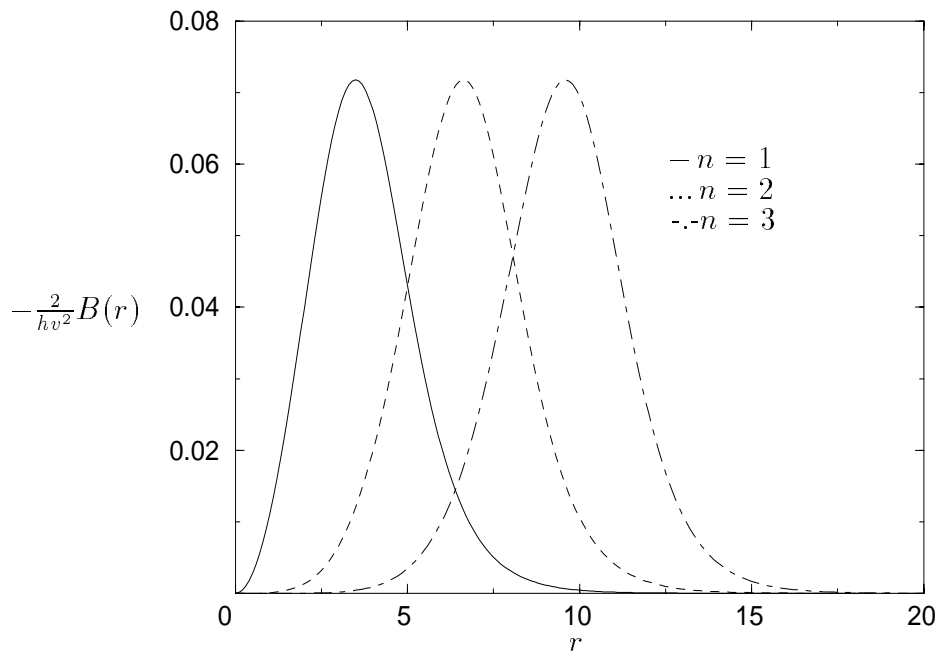

Figure 2: The magnetic field B as a function of r for $n = 1$, 2, 3 and $\gamma = 2$ in the topological phase. The vortex structure is ring-type like pure CS vortices.
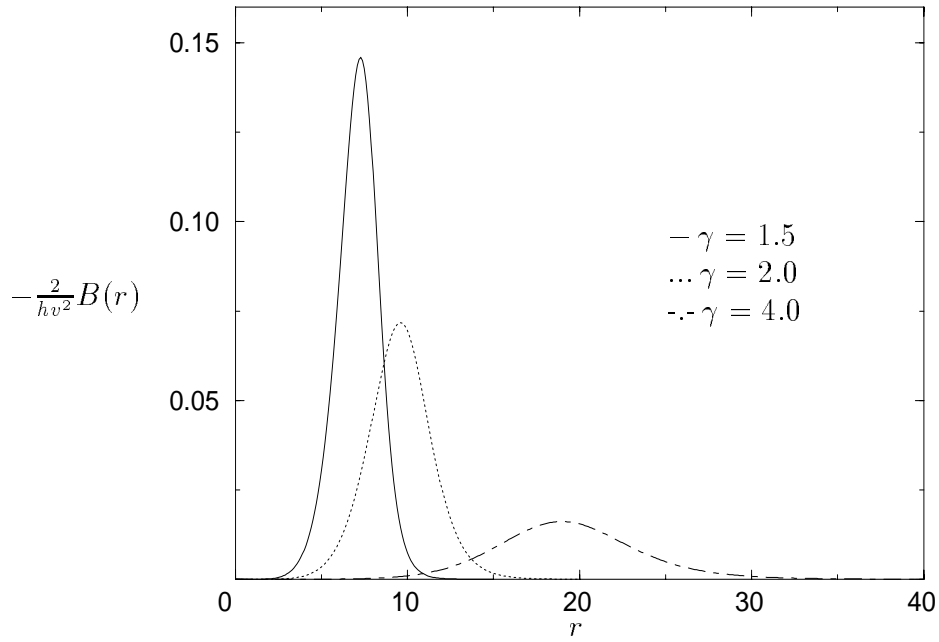
Figure 3: The magnetic field B in the topological phase for $n = 1$ and several values of $\gamma = \frac{\kappa}{hv}$.
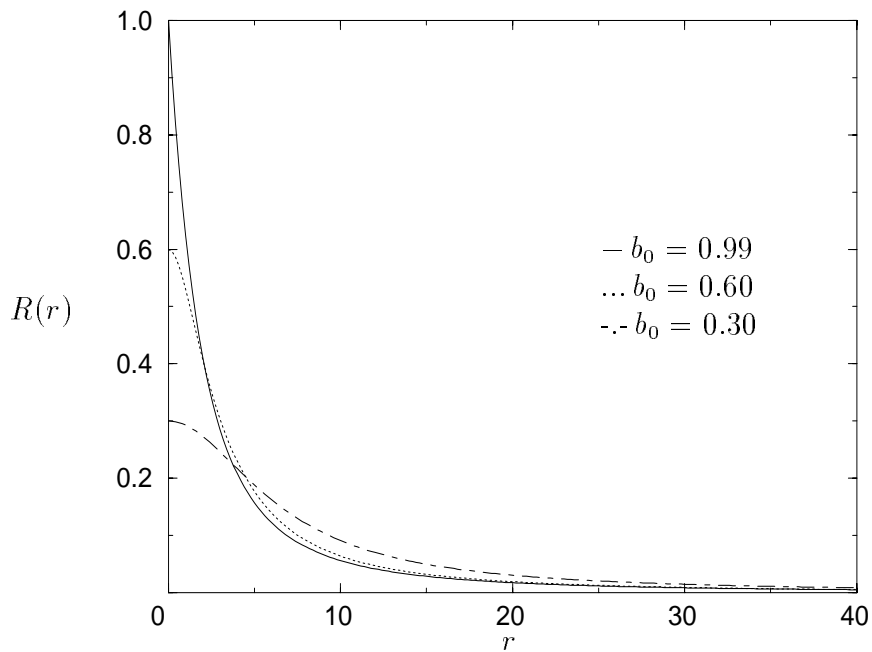
Figure 4: The Higgs field $R(r)$ in the non-topological phase for $n = 0$ and several values of b_0.
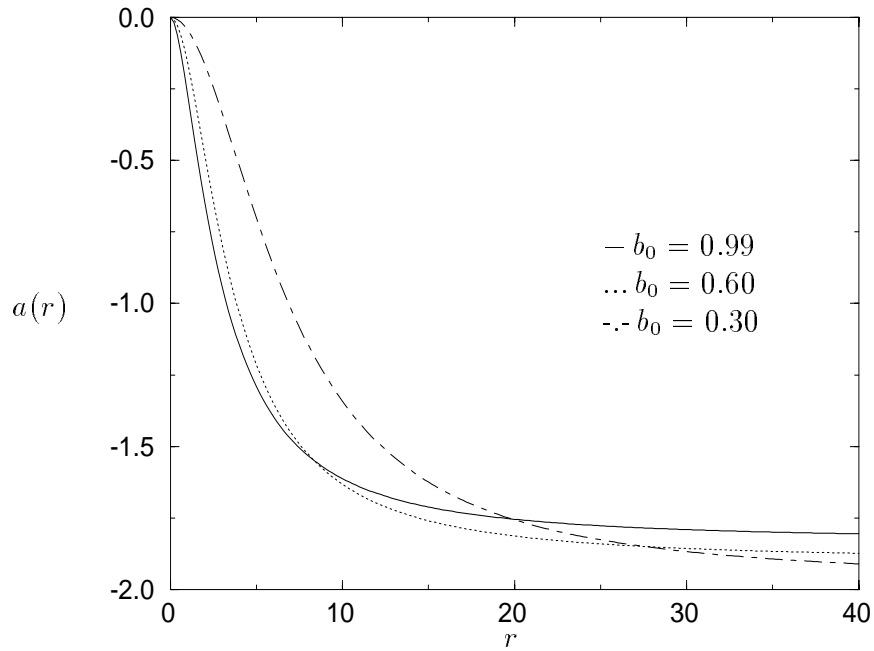
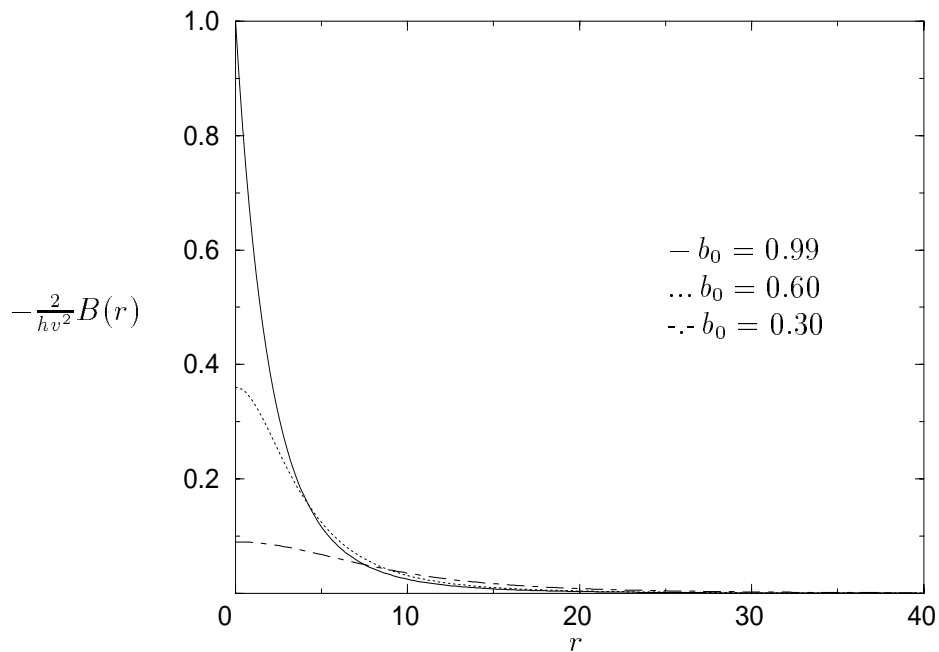Figure 5: The gauge field $a(r)$ in the non-topological phase for $n = 0$ and several values of b_0.

Figure 6: The magnetic field $B(r)$ in the non-topological phase for $n = 0$ and several values of b_0. The structure looks as a flux tube.
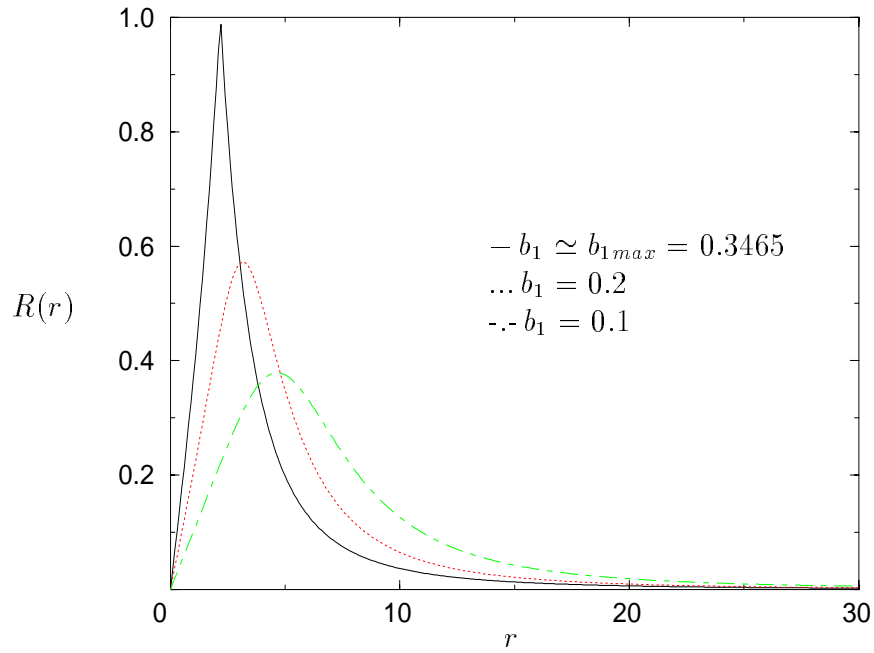
Figure 7: The Higgs field $R(r)$ in the non-topological phase for $n = 1$ and several values of b_1.
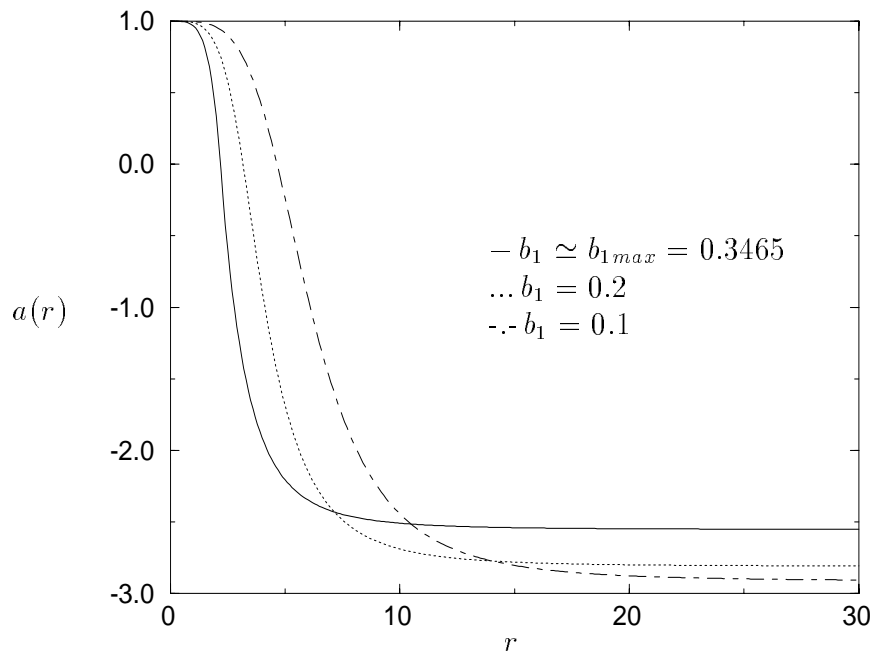

Figure 8: The gauge field $a(r)$ in the non-topological phase for $n = 1$ and several values of b_1.
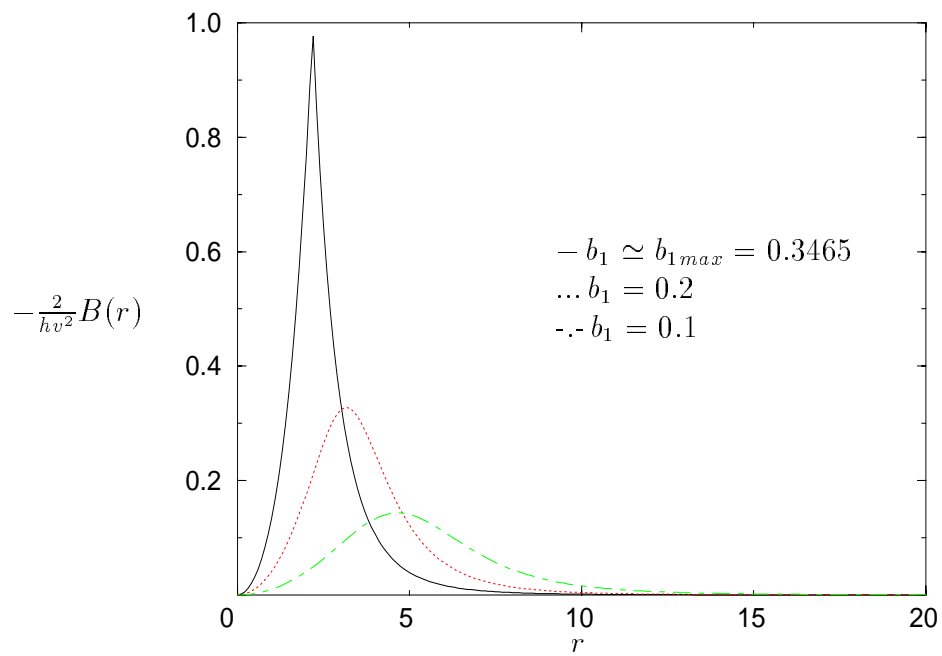
Figure 9: The magnetic field $B(r)$ as a function of r for $n = 1$ and several values of b_1 in the non-topological phase. The vortices are ring type as in the topological sector.