CBPF-NF-034/95

Inflationary Solutions in Multidimensional Cosmology with Perfect Fluid†

by

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Abstract

A cosmological model describing the evolution of n Ricci-flat spaces ($n > 1$) in the presence of 1-component perfect-fluid and minimally coupled scalar field is considered. When the pressures in all spaces are proportional to the density the Einstein equations are integrated for a large variety of parameters. The solutions with exponential and power-law inflations are obtained.

Key-words: Multidimensional cosmology; Inflation.

PACS number: 04.20.J, 04.60 + n, 03.65. Ge.

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†To appear in the Proc. VIII Swiecka Summer School, Rio, 1995.

1 Introduction

Last years the Kaluza-Klein ideas and superstring theory greatly stimulated the interest to multidimensional cosmology (see, for example, [1]-[22] and references therein).

Classical and quantum multidimensional cosmological models were investigated in our papers starting from [6]-[9]. Some windows to observational effects of extra dimensions were found and analyzed such as possible variations of the effective gravitational constant, its relations with other cosmological parameters [6, 7, 10, 13].

But the treatment of classical models may be only the necessary first step in analyzing the properties of the "Early Universe" and last stages of the gravitational collapse in a multidimensional approach. In quantum multidimensional cosmology we hope to find answers to such questions as the singular state, the "creation of the Universe", the nature and value of the cosmological constant, some ideas about possible "seeds" of the observable structure of the Universe, stability of fundamental constants etc. In the third quantization scheme the problems of topological changes may be treated thoroughly. It should be noted also that the multidimensional schemes may be also used in multicomponent inflationary scenarios [23]-[25] (see for example [21]- [22]).

2 The model

We consider a cosmological model describing the evolution of n Ricci-flat spaces in the presence of the 1-component perfect-fluid matter [15] and a homogeneous massless minimally coupled scalar field. The metric of the model

$$g = -exp[2\gamma(t)]dt \otimes dt + \sum_{i=1}^{n} exp[2x^i(t)]g^{(i)}, \tag{2.1}$$

is defined on the manifold

$$M = R \times M_1 \times \ldots \times M_n, \tag{2.2}$$

where the manifold M_i with the metric $g^{(i)}$ is a Ricci-flat space of dimension N_i, $i = 1, \ldots, n$; $n \geq 2$. We take the field equations in the following form:

$$R^M_N - \frac{1}{2}\delta^M_N R = \kappa^2 T^M_N, \tag{2.3}$$

$$\Box\varphi = 0, \tag{2.4}$$

where κ^2 is the gravitational constant, $\varphi = \varphi(t)$ is scalar field, $\Box$ is the d'Alembert operator for the metric (2.1) and the energy-momentum tensor is adopted in the following form

$$T^M_N = T^{M(pf)}_N + T^{M(\phi)}_N, \tag{2.5}$$

$$(T^{M(pf)}_N) = diag(-\rho, p_1\delta^{m_1}_{k_1}, \ldots, p_n\delta^{m_n}_{k_n}), \tag{2.6}$$

$$T^{M(\phi)}_N = \partial^M\varphi\partial_N\varphi - \frac{1}{2}\delta^M_N(\partial\varphi)^2. \tag{2.7}$$

We put pressures of the perfect fluid in all spaces to be proportional to the density

$$p_i(t) = (1 - \frac{u_i}{N_i})\rho(t), \tag{2.8}$$

where $u_i = const$, $i = 1, \ldots, n$.

We impose also the following restriction on the vector $u = (u_i) \in R^n$

$$< u, u >_* < 0. \tag{2.9}$$

Here bilinear form $< ., . >_*$: $R^n \times R^n \to R$ is defined by the relation

$$< u, v >_* = G^{ij}u_i v_j, \tag{2.10}$$

$u, v \in R^n$, where

$$G^{ij} = \frac{\delta^{ij}}{N_i} + \frac{1}{2-D} \qquad (2.11)$$

are components of the matrix inverse to the matrix of the minisuperspace metric [8, 9]

$$G_{ij} = N_i \delta_{ij} - N_i N_j . \qquad (2.12)$$

In (2.11) $D = 1 + \sum_{i=1}^{n} N_i$ is the dimension of the manifold M (2.2).

3 Classical solutions

We get the following non-exceptional solutions of the field equations (2.3) and (2.4) [27]

$$g = -(\prod_{i=1}^{n} (a_i(\tau))^{2N_i - u_i}) d\tau \otimes d\tau + \sum_{i=1}^{n} a_i^2(\tau) g^{(i)}, \qquad (3.1)$$

$$a_i(\tau) = A_i [\sinh(r\tau/T)/r]^{2u^i/<u,u>_*} [\tanh(r\tau/2T)/r]^{\beta^i}, \qquad (3.2)$$

$$\exp(\kappa\varphi(\tau)) = A_\varphi [\tanh(r\tau/2T)/r]^{\beta_\varphi}, \qquad (3.3)$$

$$\kappa^2 \rho(\tau) = A \prod_{i=1}^{n} (a_i(\tau))^{u_i - 2N_i}, \qquad (3.4)$$

$i = 1, \ldots, n$; where $r = \sqrt{A/|A|}$, $T = (\frac{1}{2}|A < u, u >_* |)^{-1/2}$. $A_i, A_\varphi > 0$ are constants and the parameters β^i, β_φ satisfy the relations

$$\sum_{i=1}^{n} u_i \beta^i = 0, \qquad \sum_{i,j=1}^{n} G_{ij} \beta^i \beta^j + (\beta_\varphi)^2 = -4/ < u, u >_* . \qquad (3.5)$$

Here $\tau > 0$ for $A > 0$ and $0 < \tau < \pi T$ for $A < 0$.

For positive energy density ($A > 0$), see (3.4), we have a family of exceptional solutions with the constant real scalar field [15]

$$g = -(\prod_{i=1}^{n} (a_i(\tau))^{2N_i - u_i}) d\tau \otimes d\tau + \sum_{i=1}^{n} a_i^2(\tau) g^{(i)}, \qquad (3.6)$$

$$a_i(\tau) = \bar{A}_i \exp[\pm 2u^i \tau/(T < u, u >_*)], \qquad (3.7)$$

$$\varphi(\tau) = const, \qquad (3.8)$$

and $\rho(\tau)$ is defined by (3.4). Here $\bar{A}_i > 0$ ($i = 1, \ldots, n$) are constants, and T is defined as in (3.1-3.4).

We note that for $A > 0$ the solution (3.7) with the sign $''+''$ is an attractor for the solutions (3.2).

Inflationary solutions. First we consider the case

$$< u^{(\Lambda)} - u, u >_* \neq 0, \qquad (3.9)$$

where $u_i^{(\Lambda)} = 2N_i$ correspond to the cosmological term. The solution (3.6), (3.7) in synchronous time parametruization reads as

$$g = -dt_s \otimes dt_s + \sum_{i=1}^{n} a_i(t_s) g^{(i)}, \qquad (3.10)$$

$$a_i(t_s) = A_i t_s^{\nu^i}, \qquad (3.11)$$

$$\kappa^2 \rho = \frac{-2 < u, u >_*}{< u^{(\Lambda)} - u, u >_*^2 t_s^2}. \qquad (3.12)$$

where

$$\nu^i = 2u^i / < u^{(\Lambda)} - u, u >_* . \qquad (3.13)$$

$i = 1, \ldots, n$. Thus, formulas (3.10)-(3.13) and $\varphi = const$ describe exceptional solutions for the case (3.9). We call these solutions as the power-law inflationary solutions.

The solution is a self-similar one.

Now we consider the case

$$< u^{(\Lambda)} - u, u >_* = 0. \qquad (3.14)$$

In this case

$$\kappa^2 \rho = const \tag{3.15}$$

and

$$a_i(t_s) = \bar{A}_i \exp[\mp \frac{u^i}{\sqrt{-<u,u>_*}} \frac{t_s}{T_0}], \tag{3.16}$$

where

$$T_0 = (2\kappa^2 \rho)^{-1/2}. \tag{3.17}$$

The relations (3.10), (3.15)-(3.17) and $\varphi = const$ describe the exponential-type inflation for the case (3.14). In the special case $u = u^{(\Lambda)}$ (cosmological constant case) this solution was considered in [14].

The corresponding quantum solutions were considered in [27]. Applying the arguments considered in [16] one may show that the ground state wave function

$$\Psi_0^{(HH)} = I_0 \left(\frac{\sqrt{2|A|}}{q} \exp(qz^0) \right), \qquad A < 0, \tag{3.18}$$

$$J_0 \left(\frac{\sqrt{2A}}{q} \exp(qz^0) \right), \qquad A > 0, \tag{3.19}$$

satisfies the Hartle-Hawking boundary condition. Here $2q = \sqrt{-<u,u>_*}$ and $\exp(qz^0) = \prod_{i=1}^n a_i^{u_i/2}$ is quasivolume.

4 Some Examples

Let us consider the isotropic case when pressures in all spaces are equal. Then

$$u_i = hN_i = \frac{h}{2} u_i^{(\Lambda)} , \tag{4.1}$$

$$p_i = (1-h)\rho = p \tag{4.2}$$

For this case

$$<u,u>_* = -h \frac{D-1}{D-2} < 0 \tag{4.3}$$

$$\text{if } h \neq 0 \quad \text{or} \quad p \neq \rho .$$

The cosmological constant corresponds to $h = 2$, and the dust-like matter to $h = 1$.

Then,

$$u^i = G^{ij} u_j = h/(2-D) , \tag{4.4}$$

$$\nu^i = 2/h(D-1) = \nu$$

We see that for $h > 0$ (or $p < \rho$) we have according to (3.11) the isotropic expansion and for $h < 0$ ($p > \rho$) the isotropic contraction. We may calculate also for this isotropic case

$$< u^{(\Lambda)} - u, u >_* = \frac{1}{4} (2-h) < u^{(\Lambda)}, u^{(\Lambda)} >_* , \tag{4.5}$$

which for $h = 2$ is equal to zero.

Accordingly, we have the power-law (in general) and the exponential law ($h = 2$) inflations here as well.

Acknowledgement

This work was supported in part by the Russian Ministry of Science and by the CNPq grant for V.N.M. during his stay at CBPF.

References

[1] P.G.O.Freund, *Nucl. Phys. B* **209** (1982), 146.

[2] D.Sahdev, *Phys. Lett. B* **137** (1984) 155.

[3] E.Kolb, D.Linkley and D.Seckel, *Phys. Rev. D* **30** (1984), 1205.

[4] M.Gleiser, S.Rajpoot and J.G.Taylor, *Ann. Phys. (NY)* **160** (1985), 299.

[5] U.Bleyer and D.-E.Liebscher, *Annalen d. Physik (Lpz)* **44** (1987), 81.

[6] V.D.Ivashchuk and V.N.Melnikov, Nuovo Cimento **B102** (1988) 131.

[7] K.A.Bronnikov, V.D.Ivashchuk and V.N.Melnikov, Nuovo Cimento **B102** (1988) 2O9.

[8] V.D.Ivashchuk, V.N.Melnikov, *Phys. Lett. A* **135** (1989), 465.

[9] V.D.Ivashchuk, V.N.Melnikov and A.I.Zhuk, *Nuovo Cimento B*, **104** (1989), 575.

[10] S.B.Fadeev, V.D.Ivashchuk and V.N.Melnikov, In *Gravitation and Modern Cosmology*, Plenum N.-Y., 1991, p. 37.

[11] U.Bleyer, D.-E.Liebscher and A.G.Polnarev, *Class. Quant. Grav. 8* (1991), 477.

[12] V.D.Ivashchuk, *Phys. Lett. A* **170** (1992) 16.

[13] V.N. Melnikov. In *Cosmology and Gravitation.* Ed. M. Novello. Ed. Frontieres, Singapore, 1994, p. 147.

[14] V.D.Ivashchuk, V.N.Melnikov, *Teor. and Mat. Fiz.* **98** (1994), 312 [in Russian].

[15] V.D.Ivashchuk and V.N.Melnikov, Multidimensional cosmology with m-component perfect fluid. *Preprint* RGA 002/94; gr-qc/ 9403063; *Int. J. Mod. Phys. D* **3** (1994), 795.

[16] U.Bleyer, V.D.Ivashchuk, V.N.Melnikov and A.I.Zhuk, *Nucl. Phys. B* **429** (1994), 177.

[17] V.R.Gavrilov, V.D.Ivashchuk and V.N.Melnikov, Integrable pseudo-euclidean Toda-like systems in multidimensional cosmology with multicomponent perfect fluid. *Preprint* RGA 002/94; gr-qc/ 9403063; to appear in *J. Math. Phys.* (1995).

[18] U.Bleyer and A.Zhuk, *Gravitation and Cosmology,* **1** No 2 (1995) 106-118.

[19] V.D.Ivashchuk, A.A.Kirillov and V.N.Melnikov, *Pis'ma ZhETF* **60** No 4 (1994), 225; *Izv. Vuzov, Fizika* No 11 (1994) [in Russian].

[20] V.D.Ivashchuk and V.N.Melnikov, *Class. Quan. Grav.*. **12** (1995) 809.

[21] M.Gasperini and G.Veneziano, *Phys. Rev. D* **50** (1994), 2519.

[22] C.Angelantonj, L.Amendola, M. Litterio and F.Occhionero, String cosmology and inflation, *Preprint* FERMILAB-Pub-94/315-A, 1994.

[23] A.Linde, *Rep. Prog. Phys.* **47** (1984) 925.

[24] A.A.Starobinsky, *Pis'ma Zh.ETF* **42** (1985), 124.

[25] D.Polarski and A.A.Starobinsky, *Phys. Rev. D*, **50** (1994), 6123.

[26] E.Kolb and M.Turner, "The Early Universe", Addison-Wesley, Reading, MA, 1990.

[27] V.D.Ivashchuk and V.N.Melnikov, Multidimensional classical and quantum cosmology with perfect fluid. *Preprint* RGA 002/95; hep-th/ 9503223; *Gravitation and Cosmology,* **1** No 2 (1995) 133-148.