CBPF-NF-034/94

Direct Electrogravitational Couplings and the Behavior of Primordial Large-Scale Magnetic Fields

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Abstract

A comprehensive approach is suggested to describe the origin and the evolution of large-scale magnetic fields. Vorticity-dependent fluctuactions acting on a primordial charged plasma account for field generation, and the properties of gauge-invariant (but conformally non-invariant) couplings of electromagnetic and graviational fields in a FRW background are applied in order to supply a source-independent, gravity-driven mechanism of conductance induction in the course of the reheating phase of inflationary cosmic scenarios. In consequence, the description of the behavior of large-scale primordial magnetic fields is complemented so as to cover the whole post-inflationay history of the Universe..

Key-words: Cosmology; Magnetic field; Vorticity.

PACS numbers: 98.60.jk – 98.80.Bp – 04.40+c

1 INTRODUCTION

The occurrence of magnetic fields on galactic scales is a well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledge [1]. In the last decades, the suggestion that they could have had a cosmological origin has been considered by a number of authors [2]. Of course, such idea implies that some physical process took place at a hot, remote era of the cosmic history, so as to account for the generation of a primordial magnetic field. Notwithstanding the difficult matter of specifying the precise nature of that process, one is also led to ponder about how would such field behave in the course of the subsequent evolution of the Universe. In the context of the standard Hot Big-Bang (HBB) cosmology a tentative attempt to solve these problems was carried by Harrison [3], and recently they were addressed, among other authors, by Turner and Widrow, to whose article [4] we report the reader for additional references on this subject.

In Ref. [3], Harrison showed that vorticity effects acting upon a primordial charged plasma could induce the generation of a primordial "seed" magnetic field. However, this leads to a further difficulty: how to explain the occurence of such (cosmic) rotational motion in an otherwise homegeneous and isotropic Friedman-Robertson-Walker (FRW) scenario? This drawback of Harrison's proposal was taken by some authors as serious enough to motivate the abandonment of such type of field generating models altogether or, at least, to reinforce the need of alternative conceptions, in addition to charged plasma dynamo action, to achieve a more proper description of such phenomena [4].

In their paper, for instance, Turner and Widrow investigated if *inflation* could account for the generation as well as the maintenance of such a primordial "seed" magnetic field. They demonstrated that at both the radiation-dominated (RD) and matter-dominated (MD) eras of the standard FRW evolution the Universe would behave as a highly conducting plasma and therefore allow for the persistence of a conjectural pristine magnetic field **B**. However, if the occurrence of a previous inflationary phase – and consequently of a reheating (RH) phase also [5] – is admitted, the analysis of the **B** field behaviour becomes far more complex. In effect, for most of the inflationary era Turner and Widrow have succeeded in accomplishing such analysis, concluding that the Universe could not have exhibited a good conductance during this period due to the absence of charged plasma (note that this fact could contribute to the survival of an eventual prior magnetic field in spite of the exponential expansion associated to inflation, once in this case the magnetic flux is not necessarily conserved).

With respect to the RH phase, however, they found that no unambiguous, model-independent conclusion could be achieved, once the computation of the **B** field behavior became fairly sensitive to each specific matter-energy content which is supposed to drive the cosmic evolution on that occasion. They argued, nevertheless, that a seemingly good description of this period, in terms of a fluid endowed with a density analogous to that of incoherent matter (i.e., $\rho \sim a^{-3}$, where a is the expasion factor of the Friedman geometry), could be provided by a source model involving coherent scalar field perturbations.

The aim of the present paper is to forward these investigations and suggest a more complete scheme to describe the evolution of cosmic magnetic fields. According to our proposal, vorticity fluctuations acting on a primordial charged plasma in the course of an ultra-relativistic or "stiff matter" (SM) phase give rise to "seed" magnetic fields, while the theory of direct electrogravitational couplings (in which the conformal invariance of electromagnetism is broken) supplies a mechanism of conductance induction which is both independent of the particular choice of source structure and efficient enough to provide a good conductance for the Universe throughout the RH phase. On the other hand, during both the RD and the MD eras the effective contribution of such mechanism to the total conductance is very small, and thus no substancial changes of the results previously obtained with regard to these eras are implied. Consequently, the combination of vortical charged plasma processes and conformally-broken electrodynamics would ensure a simple, effective primordial field generation as well as a good conducting behavior for the entire post-inflationary evolution of the Universe. Thus, in a sense, the approaches followed by Harrison and by Turner and Widrow are made to converge in the present proposal.

It seems interesting to remark at this point, furthermore, that the activity and efficiency of such conductance induction mechanism do not depend, in fact, of the conjectured occurrence of an inflationary era; as we shall see later on, it suffices that the Universe showed a non-constant value of the scalar curvature during the chosen epoch. In this way, one can consider conductance generation in the case of non-singular configurations also [6].

This paper is arranged as follows: in Section 2 we discuss two approaches currently used to promote the coupling of electromagnetic fields to gravity – the so-called Minimal and Direct (or Non-Minimal) coupling procedures. In Section 3 a important set of non-minimal Lagrangians is classified, and some assumptions are made in order to establish a suitable representation of electrogravitational processes in a FRW background. Section 4 is dedicated to a short survey of some interesting consequences of our modified electromagnetic Lagrangian. In Section 5 the subject of primordial vorticity fluctuations of a FRW background is discussed, and it is shown that vortical effects upon a primordial charged plasma can give rise to a suitable "seed" magnetic field. A detailed description follows of a gravity-driven mechanism of conductance induction, which is then applied to the problem of magnetic field behavior in inflationary eras. Section 6 contains somes additional comments on related problems.

2 Minimal and Direct Couplings With Gravity

Consider the problem of describing the dynamical evolution of an electromagnetic field in the presence of gravitation. According to the general programme of field theory in curved spaces, two principles may be chosen to account for the coupling of physical fields to gravity; let us briefly discuss both.

In his early work on the theory of General Relativity, Einstein [7] formulated in a very precise manner a simple and straightforward prescription for the generalisation of the results of Special Relativity in flat Minkowski space to curved spacetimes. In the case of electro-gravity interactions, for instance, it suffices to substitute the requirement of a Minkowski spacetime structure for a Riemmanian one, and to rewrite Maxwell's equations in a covariant fashion, Then, from the well-known Lagrangian given by

$$L = -\frac{1}{2}\sqrt{-g}\; F_{\mu\nu}F^{\mu\nu} \tag{1}$$

where $g = \det[g_{\mu\nu}]$ and $F_{\mu\nu} \equiv A_{\mu,\nu} - A_{\nu,\mu}$, $A_\mu(x)$ being the electromagnetic 4-potential, one obtains the generalised Maxwell equations:

$$F^{\mu\nu}{}_{;\nu} = 0 \tag{2.a}$$

$$F_{\mu\nu;\alpha} + F_{\nu\alpha;\nu} + F_{\alpha\mu;\nu} = 0 \quad . \tag{2.b}$$

Note that according to this recipe functionals of the curvature do *not* appear explicitly in the dynamics of Maxwell's field $F_{\mu\nu}$. This is, in fact, a common feature of all dynamical schemes derived by means of Einstein's procedure – hence, it is called the *Hypothesis* of *Minimal Coupling* (HMC).

However, in recent years an alternative procedure for coupling physical fields with gravity has been explored in the literature at an increasing rate. In the subject of spontaneous symmetry breaking mechanisms in gauge field theories, for instance, dynamical equations of motion are derived from functional expressions including both scalar field and curvature contributions [8]. Once the mixing with curvature terms is a general characteristic of the dynamics of physical fields coupled in such a direct manner to gravity, this second coupling principle may be assigned to the *Hypothesis of Direct Coupling to the Curvature* (HDCC). Given the purposes of the present paper, we will restrict our attention to a particularly simple case of an HDCC-based description of the electrogravitational interaction, which concerns the introduction, in the electrodynamical action, of additional Lagrangian terms which are of the first order in the curvature and of the second order in the electromagnetic field (for a more thorough examination of this kind of HDCC Lagrangians the reader is referred to Refs. [6, 9]). In the next following sections we will provide a short comparison of both coupling schemes, and examine some physically interesting consequences of HDCC electrodynamics in a FRW background.

3 HDCC Electrodynamics

It is well known that in the conventional (HMC-based) Einstein-Maxwell approach the resulting electrogravitational action is unique and consists of a sum of separate contributions corresponding to the electromagnetic and gravitational fields:

$$L_{EM} = \sqrt{-g}\; R - \frac{1}{2}\sqrt{-g}\; F_{\mu\nu}F^{\mu\nu}\ , \tag{3}$$

in which the first term leads to the Einstein-Hilbert action for pure gravity. In contrast, in the HDCC cases mentioned above the Lagrangian of the electromagnetic field does contain some functional of the curvature, and one may come to deal with seven possibilities; according to their behavior with respect transformations, these candidates can be divided into two classes, as follows:

First class:

$$\begin{aligned} L_1 &= RA_\mu A^\mu \\ L_2 &= R_{\mu\nu}A^\mu A^\nu \end{aligned}$$

Second class:

$$\begin{aligned} L_3 &= R F_{\mu\nu} F^{\mu\nu} \\ L_4 &= R F_{\mu\nu} \overset{*}{F}{}^{\mu\nu} \\ L_5 &= R_{\mu\alpha} F^{\mu}{}_{\lambda} F^{\lambda\alpha} \\ L_6 &= R_{\alpha\beta\mu\nu} F^{\alpha\beta} F^{\mu\nu} \\ L_7 &= R_{\alpha\beta\mu\nu} F^{\alpha\beta} \overset{*}{F}{}^{\mu\nu} \end{aligned}$$

(The asterik represents the dual operation.)

Lagrangians of the first class are gauge dependent, but have the proper dimensionality. Indeed, in the action

$$S_I = \int \sqrt{-g}\,(\lambda_1 L_1 + \lambda_2 L_2)\, d^4x \tag{4}$$

constants λ_1 and λ_2 have no dimension. Second class Lagrangians, in turn, are gauge invariant, but they all require coupling constant λ_i such that $\dim \lambda_i = (length)^2$. The introduction of such dimensional parameters in electrodynamics may be attributed to the influence of the curved background, and therefore their precise values should depend on fundamental constants such as, e.g., the fine structure constant $\alpha = e^2/\hbar c$ and/or the gravitational constant k. A far more interesting justification on the other hand, takes into account the occurrence of tidal effects in photons (due to quantum one-loop vacuum polarisation) which imply the introduction of second class curvature terms in the equations of motion of quantum electrodynamics in curved spaces [10].

In principle, the whole set of HDCC terms should contribute to the dynamics of the electromagnetic field. However, if the conservation of certain symmetries is imposed, then the spectrum of acceptable candidates may be further restricted. If gauge invariance is to be preserved, for instance, first class terms L_1 and L_2 are eliminated; if parity conservation is required as well, then L_4 and L_7 (which involve the dual of Maxwell's field) are rejected. Once the Weyl conformal tensor vanishes in a conformally-flat FRW geometry, the remaining trio L_3, L_5, and L_6 is reduced to a pair. Moreover, at early times the cosmic evolution it is reasonable to suppose that our primordial electromagnetic field can be treated as a test-field (i.e., its feedback upon spacetime curvature is negligible); since the Friedman conformal factor is time-dependent only, then either L_3 or L_5 can be chosen at will. We will therefore assume that electromagnetic processes in a FRW background are properly described by the addition of a L_3 type term to the customary extended Maxwell Lagrangian Eq. (1) and hereafter consider the functional expression

$$L_{NM} = \sqrt{-g}\left[-\frac{1}{2}\; F_{\mu\nu}F^{\mu\nu} + \lambda\; R\; F_{\mu\nu}F^{\mu\nu}\right]\,, \tag{5}$$

where λ is a coupling parameter with dimensions $[length]^2$, as the basis of our future developments. The corresponding modification of Maxwell's equations Eq. (2a, 2b) is then given by

$$F^{\mu\nu}{}_{;\nu} = 2\;\lambda(R\;F^{\mu\nu})_{;\nu}\,. \tag{6}$$

4 The Lagrangian RF^2

In the sake of completeness, it seems worth pointing out here a number of physically interesting aspects that may be derived from the Lagrangian in Eq. (5). Consider the standard expansion of the electromagnetic potential $A_\mu(x)$ in terms of a small dimensionless parameter ε (such that $\varepsilon \ll \varepsilon^2$):

$$A_\mu = Re[(a_\mu + \varepsilon b_\mu + \varepsilon^2 c_\mu + \cdots)\exp(i\phi/\varepsilon)]\ . \tag{7}$$

Assuming that the propagation of an electromagnetic distubance can be effectively represented by its high-frequency limit (the so-called geometrical optics approximation – which, in a curved spacetime, is valid when the phase ϕ of the disturbance changes much faster than the amplitude variation over a region of typical lenght L such that $L \sim [curvature]^{-2}$), and denoting the variation of the phase ϕ by the vector $K_\mu = \partial_\mu\phi$, the evolution of the decomposition factors a_μ, b_μ, etc. in Eq. (7) can be evaluated for both the HMC and HDCC schemes, as follows: from the extended Maxwell equations Eq. (2a, 2b) one obtains

$$2\ a^\mu{}_{;\lambda}K^\lambda + a^\mu K^\lambda{}_{;\lambda} + iK^2 b^\mu = 0 \tag{8.a}$$
$$(a^2K^\mu)_{;\mu} = 0 \tag{8.b}$$
$$K^2 = 0 \tag{8.c}$$

while for the HDCC modified relation Eq. (6) the resulting equations are

$$(1-2\lambda R)(2a^\mu{}_{;\lambda}K^\lambda + a^\mu K^\lambda{}_{;\lambda} + iK^2b^\mu) - 2\lambda R_{,\lambda}\,(K^\lambda a^\mu - K^\mu a^\lambda) = 0 \tag{9.a}$$
$$[(1-2\lambda R)a^2K^\mu]_{;\mu} = 0 \tag{9.b}$$
$$K^2 = 0 \tag{9.c}$$

where $a^2 = a^\mu a_\mu$, $K^2 = K^\mu K_\mu$. A comparison of both sets of dynamical field equations affords some interesting conclusions, namely:

(i) In both cases, photons follow null geodesics of the background geometry.

This is certainly a significant result, since it explicitly shows that *in the case of a FRW configurations the dynamics derived from Lagrangian* L_{NM} in Eq. (5) is in *agreement with the empirical evidence put forth by comological redshift observations.* It is important to note that in HDCC-based schemes the evolution of the characteristic surfaces of propagation of electromagnetic disturbances does not necessarily coincide with the customary pattern derived from the HMC; in fact, it is only in the case of Lagrangian L_{NM} that such basic property is shared with the minimal couling approach [9].

(ii) In HDCC scenarios the number of photons in the Universe needs not to be conserved.

Indeed, it is a straightforward consequence of Eq. (8b) – hence, of HMC – that the number N of photons contained in a volume V, given by the formula

$$N = \int a^2\omega dV \tag{10}$$

(where the frequency ω is defined, with respect to an observer endowed with a 4-velocity V_μ, by $\omega = K_\mu V^\mu$, as usual) is constant of motion. Now, in a FRW universe the photon number density $n = N/V$ satisfies the evolution equation

$$\dot{n} + n\theta = 0 \tag{11}$$

(in which the expansion factor $\theta = \dot{V}/V$ measures volume variation with cosmic time, the dot representing time derivative) and accordingly the total number of photons is conserved, $\dot{N} = 0$. However, for the HDCC case a completely different result is obtained, once for a FRW configuration Eq. (9b) yields

$$\dot{n}(1 - 2\lambda R) + n\theta = 2\lambda n(\dot{R} + R\theta) \ . \tag{12}$$

Therefore, it is a direct consequence of Eq. (9b) – hence, of HDCC – that *the gravitational field can in effect modify the total number of photons existing in the Universe.* This remarkable feature is due to the breaking of the conformal invariance of electromagnetism (see below) and evidenlty demands further consideration, once it is in apparent contradiction with the presumed constancy of the ratio n_γ/n_B between the number densities of photons and baryons comprised in the Universe (which, in the context of the standard HBB model, is understood as a "fundamental constant" presently estimated to be of the order of 10^9) – one of the aspects of the so-called "baryon asymmetry problem" of standard cosmology [11]. Such variability is related to the fact that, in contrast with HMC-based scenarios, within the HDCC approach photons cannot be treated as idealistic, point-like objects due to tidal effects which take place in the interaction with gravity [10]; in consequence they acquire a gravity-induced chemical potential which is no longer required to vanish. Thus even in the case of a homogeneous, conformally-flat FRW configuration there may occur a gravity-driven contribution to the photonic chemical potential which, in cosmic periods of intense curvature, cannot be neglected. Elsewhere, we studied the thermodynamical properties of such a directly coupled photon gas and showed that they converge to the black-body behavior suggested by current observations [12].

In spite of its extensive use in a variety of current research domains, nevertheless the HDCC approach has not acquired the status of a standard coupling procedure yet, due to the esteem traditionally bestowed upon the HMC method. In Turner and Widrow's article quoted above, for instance, HDCC is invoked with the definite purpose of changing the well-known invariance properties of Maxwell's thoeory with respect to conformal transformations.

Indeed, within the HMC approach Maxwell's field $F_{\mu\nu}(x)$ is conformally invariant, which means that it is kept unchanged under a conformal transformation of the metric tensor $g_{\mu\nu}(x)$ such as

$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \Omega^2(x) g_{\mu\nu} \ , \tag{13}$$

in which $\Omega(x)$ is a scalar function. Now, in their attempt to accomplish for the existence of a significant primordial magnetic field, those authors have come to realize that "...conformal invariance of electromagnetism mut be broken to produce appreciable primeval magnetic flux" (Ref. [4], page 2744); thus, they were led to investigate the subject of non-conformally invariant electrodynamical descriptions in curved backgrounds. They studied, in particular, two approaches in which the breaking of conformal invariance is attained through the adoption of a HDCC-based interaction scheme [4].

We would like at this point to call attention upon a result that has passed unnoticed, seemingly, to those authors; namely, that one of the consequences that may stem from the adoption of certain types of direct coupling is the possibility of inducing a significant conductance coefficient σ (i.e., $\sigma/H \gtrsim 1$, where $H = 3\theta$ is the Hubble parameter) precisely at the problematic RH phase of the cosmic evolution (Section I). This issue will be addressed at the next section.

5 Field Generation and a Gravity-Driven Mechanism of Conductance Induction

According to Harrison [3], the effects of cosmic vorticity upon a primordial charged plasma could give rise, via dynamo action, to magnetic fields of a suitable magnitude ("seed" fields) so as to provide for the origin of the large-scale fields observed today. However, once there is very scarce – if any – evidence for a global rotation of the Universe in the present records, there arises the problem of fitting Harrison's idea to the standard homogeneous and isotropic (hence, irrotational) FRW picture of the cosmological evolution. Notwithstanding other possible means of generating primodial "seed" fields – for instance, the breaking of conformal invariance of quantum electrodynamics due to the trace anomaly [13] – here we will explore the theory of vorticity perturbations of the FRW geometry in order to supply a sound support to Harrison's argument.

In the case of an arbitrary fluid in a curved background, the evolution of vorticity is ruled by

$$h_\alpha^{\ \gamma} h_\beta^{\ \delta} (\omega_{\gamma\delta})^{\cdot} + \sigma_{\alpha\rho}\omega_\beta^{\ \rho} + \omega_{\alpha\rho}\sigma_\beta^{\ \rho} - 1/2 h_\alpha^{\ \gamma} h_\beta^{\ \sigma} \dot{V}_{[\gamma||\delta]} + 2/3\ \theta\ \omega_{\alpha\beta} = 0 \qquad (14)$$

in which the vorticity, shear, acceleration, expansion and derivative along the observer's trajectory are given, for an arbitrary velocity field V_α respectively by [14]

$$\omega_{\alpha\beta} = h_\alpha^{\ \gamma} h_\beta^{\ \delta} V_{[\gamma||\delta]}\ , \qquad (15.a)$$

$$\sigma_{\alpha\beta} = h_\alpha^{\ \gamma} h_\beta^{\ \delta} V_{(\gamma||\delta)} - \theta/3 h_{\alpha\beta}\ , \qquad (15.b)$$

$$a^\alpha = V^\alpha_{\ ||\beta} V^\beta\ , \qquad (15.c)$$

$$\theta = V^\alpha_{\ ||\alpha}\ , \qquad (15.d)$$

$$\dot{X}_\alpha = X_{\alpha||\beta} V^\beta\ . \qquad (15.e)$$

The evolution of vorticity perturbations in a FRW background is then given by

$$\dot{\omega}_{\alpha\beta} - 1/2\ h_\alpha^{\ \gamma} h_\beta^{\ \delta} a_{[\gamma||\delta]} + 2/3\ \theta\omega_{\alpha\beta} = 0\ . \qquad (16)$$

It is well known that such vortex perturbations are extremely sensitive to the choice of the equation of state of the cosmic background fluid. It has been argued, for instance, that the expansion of the Universe acts as a damping effect upon primordial vorticity

[15]. This is in fact true in the usual case of presureless fluid, and also for all types of fluids satisfying equations of state $p = \lambda\rho$ for $\lambda < 1/3$. However, the case of radiation ($\lambda = 1/3$) is a limiting one, once the growth of perturbations is still inhibited but the damping effect of expansion is not strong enough to eliminate them altogether; hence, vorticity is preserved. For values of λ greater than 1/3 the vorticity increases according to [14]

$$\omega = \omega_0 a^{3\lambda-1} , \tag{17}$$

where a is the expansion factor of the FRW geometry. For a "stiff matter" fluid ($\lambda = 1$), for instance, vortex fluctuations grow quadratically. In such an extreme regime, any small fluctuation can provide the required conditions for the growing of primordial vorticities and, in consequence, of "seed" magnetic also. This enables us to argue in favour of Harrison's idea of primeval field generation.

Let us then proceed to the proof that Lagrangian L_{NM} in Eq. (5) may indeed provide a non-null, effective conductance coefficient. Rewritting the HDCC equation of motion Eq. (6) one obtains

$$F^{\mu\nu}{}_{;\nu} = F^{\mu\nu}\partial_\nu[\ln(1 - 2\lambda R)] . \tag{18}$$

As stated before, here the electromagnetic field is to be considered as a test-field, and Eq. (18) is to be examined in the scope of spatially homogeneous and isotropic FRW cosmology described by the line element

$$ds^2 = dt^2 - a^2(t)d\Sigma^2 . \tag{19}$$

Once the FRW scalar curvature R depends on the cosmic time only, from Eq. (18) one obtains

$$F^{\mu\nu}{}_{;\nu} = \sigma_{eff} E^\mu . \tag{20}$$

in which the electric field E^μ is defined, with respect to an observer endowed with 4-velocity $V_\nu = \delta^0_\nu$, by $E^\mu \equiv F^{\mu\nu}V_\nu = F^{\mu 0}$, and according the effective conductance σ_{eff} is given by

$$\sigma_{eff} = d/dt[\ln(1 - 2\lambda R)] . \tag{21}$$

We conclude that *HDCC-based dynamics expressed by Eq. (18) induces a non-vanishing contribution to the overall conductance of the Universe whenever the scalar curvature R does not behave as a constant.* It then follows that neither in an eventual de Sitter-like inflationary phase ($R = const.$) nor in the RD phase ($R = 0$) such mechanism provides any substancial contribution to the cosmic conductance. The ensuing standard MD phase, in turn, occurs so late in the time-scale of the global cosmic evolution that σ_{eff} becomes in fact negligible – as we will see next.

In order to accomplish the demonstration of our initial statements, let us then turn to the investigation of the behavior of σ_{eff} in relation to a RH phase subsequent to a conjectural De Sitter-type inflationary era. According to Turner and Widrow's arguments quoted above (Section 1), and equation of state analogous to that of incoherent matter may be chosen to describe this phase, and making use of Eq. (21) one obtains in this case that

$$\sigma_{RH} = \frac{16\lambda}{t(3t^2 - 8\lambda)} . \tag{22}$$

To evaluate the actual efficiency of this conductance-inducing mechanism one needs to quantify the coupling parameter λ. In the literature, two values have been used often, both associated to some sort of "fundamental length" derived from combinations of basic constants of physics – and in fact it seems difficult to devise other non-arbitrary, physically sensible values distinct from the following:

$$(i) \qquad \lambda_{PL} = [L_{PL}]^2 = 10^{-66} cm^2 \, , \tag{23}$$

where L_{PL} is Planck's length;

$$(ii) \qquad \lambda_{DH} = \frac{1}{m_e^2} \, \frac{5\alpha}{720\pi} \sim 10^{-26} cm^2 \, , \tag{24}$$

which Drummond and Hathrell calculated in accordance with the idea that the introduction of non-minimaly coupled interaction terms is ultimately an effective consequence of the occurence of virtual electron loops which give the photon a "size" proportional to the Compton wavelength of the electron [10].

Taking as typical parameters of the inflatrionay era the temperature $T \sim 10^{14} GeV$ and the time scale $t \sim 10^{-23}$ sec, and using Drummond and Hathrell's value λ_{DH}, it is then easy to obtain from Eq. (17) that $\sigma_{RH}/H \gtrsim 1$ for the subsequent RH phase, that is, in the course of this phase the Universe effectively displays a good conducting behavior. On the other hand, in the case of the much later MD phase the induction of conductance through such mechanism evidently becomes almost insignificant. In the sake of completeness, in the "stiff matter" case mentioned above the same reasonings lead to $\sigma_{SM} = 4\sigma_{RH}$.

6 Concluding Remarks

The subject of direct couplings of physical fields to spacetime curvature is a rewarding one when the need comes to overcome the limitations of standard procedures, and it is being progressively sanctioned by its increasing usage in a variety of research domains. Here we applied an HDCC-based approach to the problem of generating and mantaining a primordial magnetic field in the course of the different phases of standard inflationary cosmological evolution. The results obtained above demonstrate that in the scope of the HDCC interaction scheme proposed here it is in fact possible to induce an appreciable amount of conductance during the unsettling RH phase, while no significant contribution occurs for the remaining eras; in this way, previous efforts on this matter could be complemented and extended to the entire post-inflationary stage of the evolution of the Universe. Once the problem of the origin of a "seed" magnetic field is clearly related to the choice of initial conditions for the cosmological evolution, one may wonder about the behavior of such field in the context of the quantum creation of a non-singular Universe.

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