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SPIN ONE ISING MODEL WITH COMPETING INTERACTIONS ON THE BETHE LATTICE

by

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ABSTRACT

The phase diagram of the spin one Ising model with competing interactions between first and second nearest neighbors generations, single ion anisotropy and quadrupolar interactions is studied. Modulated phases, multiphase point and reentrances are observed and discussed. For high competing single ion anisotropy the paramagnetic phase reaches a finite region of the ground state.

Key-words: Blume-Emery-Griffiths model; Competing interactions; Modulated phases; Bethe lattice.

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We study the phase diagram of the spin one Ising model on the Bethe lattice of general coordination number $z = q+1$ (q is the connectivity) with competing first and second nearest neighbors interactions, quadrupolar interaction, single-ion anisotropy and under an external magnetic field. A mean field analysis of a similar model has been considered by [1]. They studied the phase diagram for a particular plane in the parameter space finding several modulated phases with reentrant behavior and the existence of a Lifshitz point. The present work provides the analysis of the phase diagram in the entire parameter space. In our approach (2) the partition function, the local magnetization, the non-critical density and the pair correlation function, involving sites deep in the interior of the lattice, are obtained exactly as a function of the fixed point solutions (attractors) of a set of coupled recursion relations of appropriated effective fields. The reduced model Hamiltonian describing the system is

$$-\beta\mathcal{H} = K_1\sum_n S_n S_{n+1} + K_2\sum_n S_n S_{n+2} + K\sum_n S_n^2 S_{n+1}^2 + D\sum S_i^2 + B\sum_i S_i \qquad (1)$$

where β is the inverse of temperature, n labels the generation shells of the Bethe lattice and $S_i = \pm 1, 0$ are the spin variables. The first and third sums are over all pairs of nearest neighbors spins while the second sum is restricted to the pairs of second nearest neighbors spins belonging to the same branch. K_1, K_2, K, D and B are the coupling constant of the first, second nearest neighbors interactions, quadrupolar interaction, single-ion anisotropy and external magnetic field respectively.

Following [2] we define appropriate effective fields obeying the following discrete mapping relating three consecutive generations

$$
\begin{aligned}
a_{n+1} &= B-D+K_1+K+F_1(a_n,b_n)+G^+_{n-1}\\
b_{n+1} &= -B-D-K_1+K+F_1(c_n,d_n)+G^-_{n-1}\\
c_{n+1} &= B-D-K_1+K+F_{-1}(a_n,b_n)+G^+_{n-1}\\
d_{n+1} &= -B-D+K_1+K+F_{-1}(c_n,d_n)+G^-_{n-1}\\
f_{n+1} &= B-D+F_0(a_n,b_n)+G^+_{n-1}\\
g_{n+1} &= -B-D+F_0(c_n,d_n)+G^-_{n-1}
\end{aligned}
\qquad (2)
$$

where

$$F_S(x_n,y_n) = q\ \log\left[\frac{1+e^{K_2S+x_n}+e^{-K_2S+y_n}}{1+e^{K_2S+f_n}+e^{-K_2S+h_n}}\right] \qquad (3)$$

$$G^{\pm}_n = q^2\ \log\left[\frac{1+e^{\pm K_2+f_n}+e^{\mp K_2+h_n}}{1+e^{f_n}+e^{h_n}}\right] \qquad (4)$$

The zero field phase diagram in the $(t, \alpha, \delta, \gamma)$ space, where $t=1/K_1$ is the reduced temperature, $\alpha=-K_2/K_1$, $\delta=-D/K_1$ and $\gamma=K/K_1$ are the competing parameters, can be constructed numerically by solving (2) and then taking B=0. The ferromagnetic (F) and the paramagnetic phases are identified each one by one single stable fixed point with $a^*\neq b^*\neq c^*\neq d^*\neq f^*\neq h^*$ and $a^*=d^*, b^*=c^*, f^*=h^*, a^*\neq b^*\neq f^*$ respectively. The antiphases <2,2> and <3,3>, of generations like ++-- and +++--- respectively, are characterized by sequences of four fixed points with $a^*_n=d^*_{n+2}$, $b^*_n=c^*_{n+2}$, $f^*_n=h^*_{n+2}$, $a^*_n\neq b^*_n\neq f^*_n$, and six fixed points with $a^*_n=d^*_{n+3}$, $b^*_n=c^*_{n+3}$, $f^*_n=f^*_{n+3}$, $a^*_n\neq b^*_n\neq f^*_n$ respectively. Modulated phases (M) of higher order are described by sequences of finite numbers of fixed points (limit cicle) and incommensurate phases (or long period commensurate phases) by quasicontinuous or continuous attractors. In this work we are primarily interested to study the phase diagram in the subspace with $\alpha > 0$, $\delta > 0$ and $\gamma = 0$.

Four kinds of diagrams can be distinguished accordingly to the range of values of δ. For $0 < \delta <$

$q(q+1)/2q+1$ the ground state is composed by the F, <3,3> and <2,2> phases and we note the existence of a multiphase point (Lifshitz point) at finite temperatures where the P, F and M phases met. Figure 1 shows a diagram of this kind. For $\delta = q(q+1)/2q+1$ the P phase met the ground state at the multiphase point $t=0$, $\alpha=1/(2q+1)$ as shown in figure (2). For $q(q+1)/(2q+1)<\delta<q$ the P phase reaches a finite region of the ground state, that increases with the increasing of δ. We also note that the <3,3> - antiphase region decreases with δ until to disappear at $\delta=q$. Figure (3) is a typical diagram for this latter range. However for $\delta>q$ only the F,P and <2,2> phases are present in the ground state. $\delta=q$ marks the disappearance of the M phases at the ground state. Figure (4) shows the (t,α) diagram for $\delta>q$. In figures (2-4) we point out the existence of reentrances in the transition lines of the P phase which are associated with the presence of competing interactions and the degree of freedom of spin variables. We note that the width of reentrances in both directions of the (t,α) plane increases with competing single-ion anisotropy.

The $q\to\infty$ limit of the present model can be worked out giving rise to a reduced mapping of two coupled recursion equations. For $\gamma=0$ this mapping is further reduced to one single recursion equation as studied in details in reference [3]. The structure of the modulated phases as well as the existence of tricritical points in both q finite and $q\to\infty$ limits are now being in study. We acknowledge Tania Tomé and Silvio Salinas for helpfull discussion and to the CNPq, FINEP and CAPES for granting support.

Figure Captions:

Fig. 1: (α,t) - diagram for $\delta=0.6$ and $q=2$. $\alpha=-K_2/K_1$ $t=1/K_1$ and $\delta=-D/K_1$.

Fig. 2: (α,t) - diagram for $\delta=1.2$ and $q=2$. $\alpha=-K_2/K_1$, $t=1/K_1$ and $\delta=-D/K_1$.

Fig. 3: (α,t) - diagram for $\delta=1.8$ and $q=2$. $\alpha=-K_2/K_1$, $t=1/K_1$ and $\delta=-D/K_1$.

Fig. 4: (α,t) - diagram for $\delta=2.4$ and $q=2$. $\alpha=-K_2/K_1$, $t=1/K_1$ and $\delta=-D/K_1$.

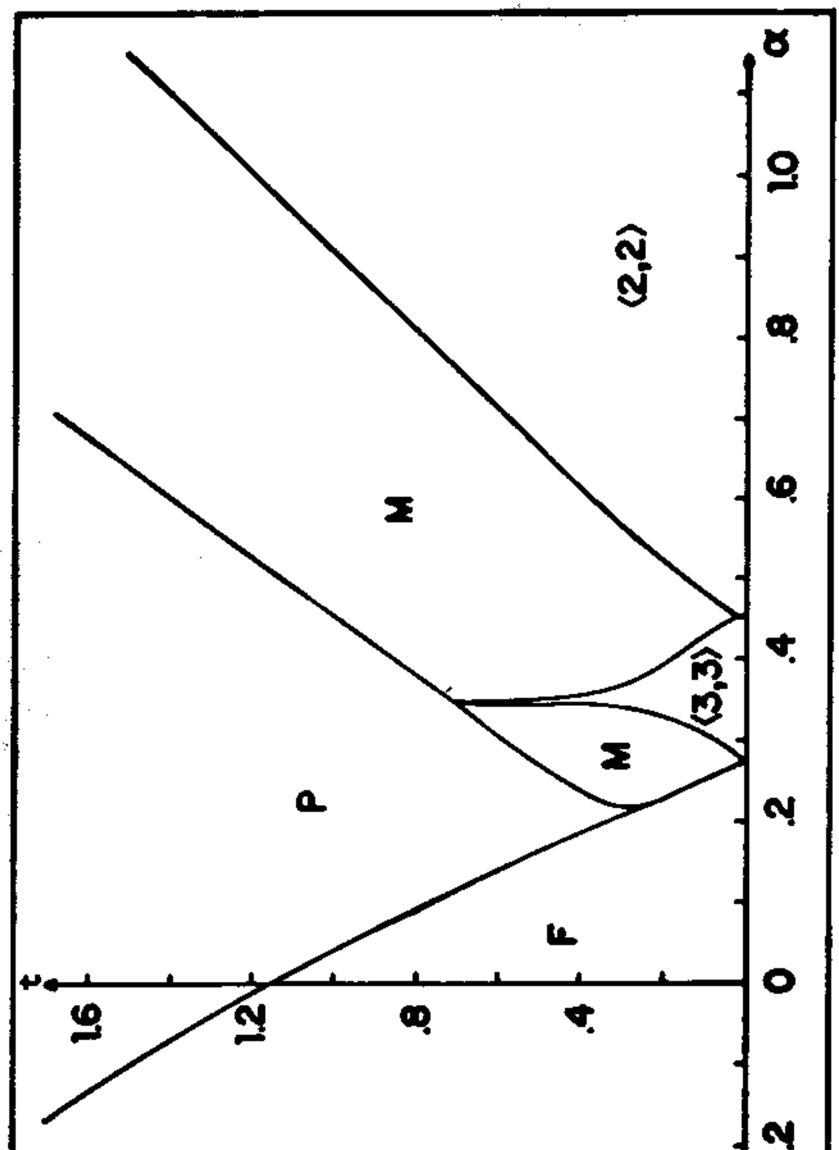

Figure 1

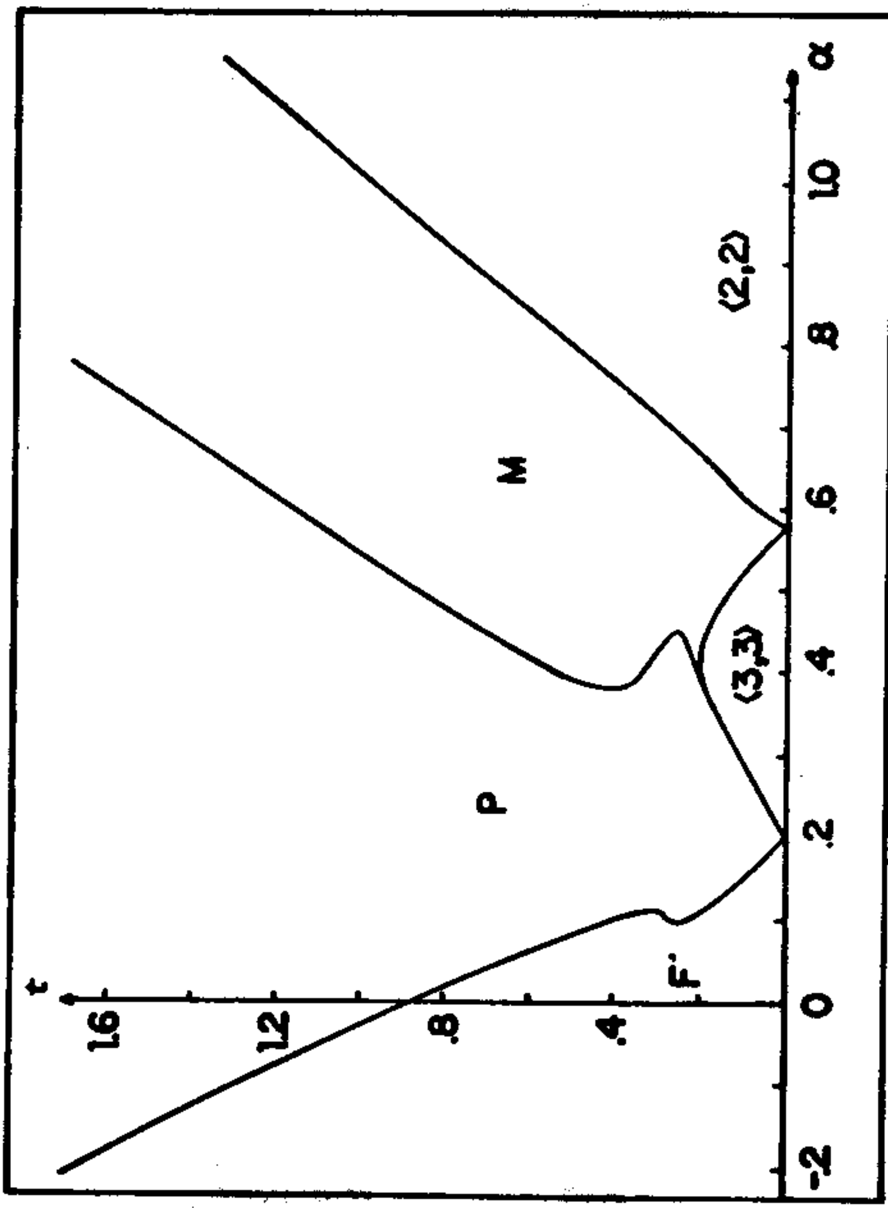

Figure 2

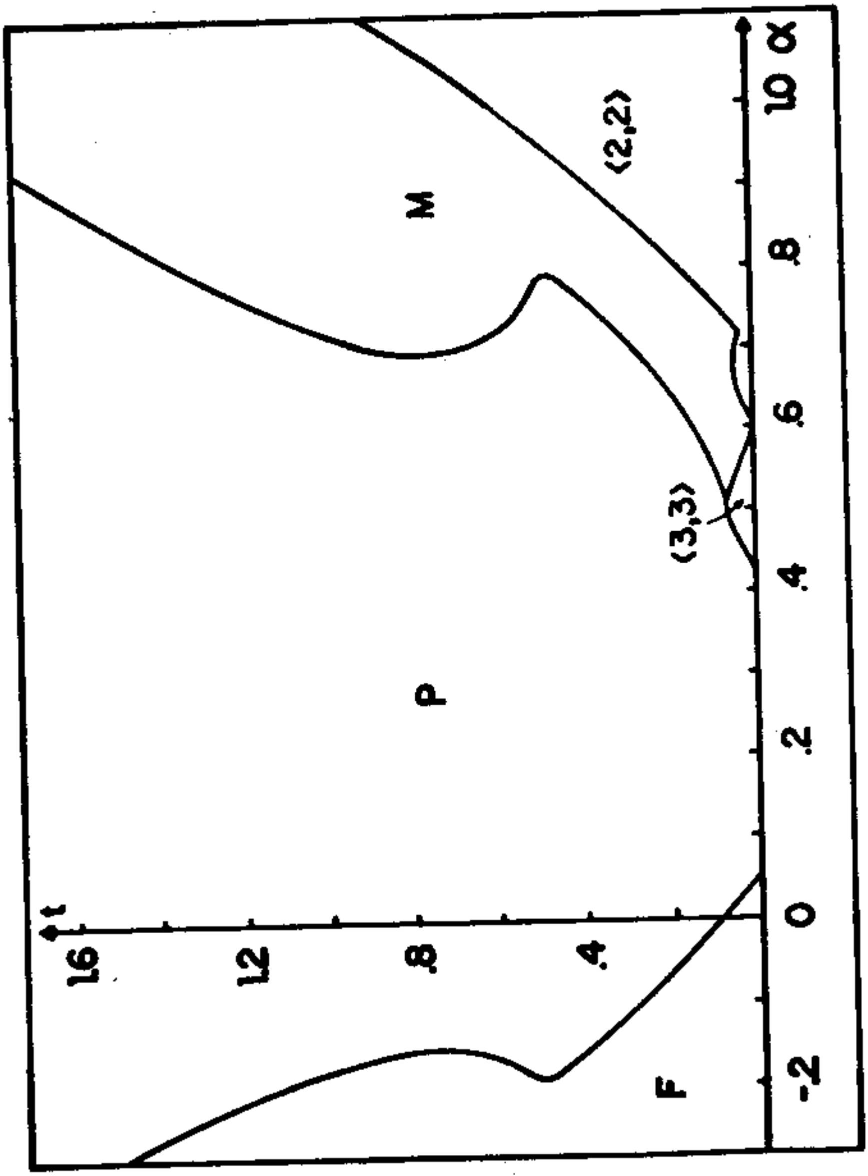

Figure 3

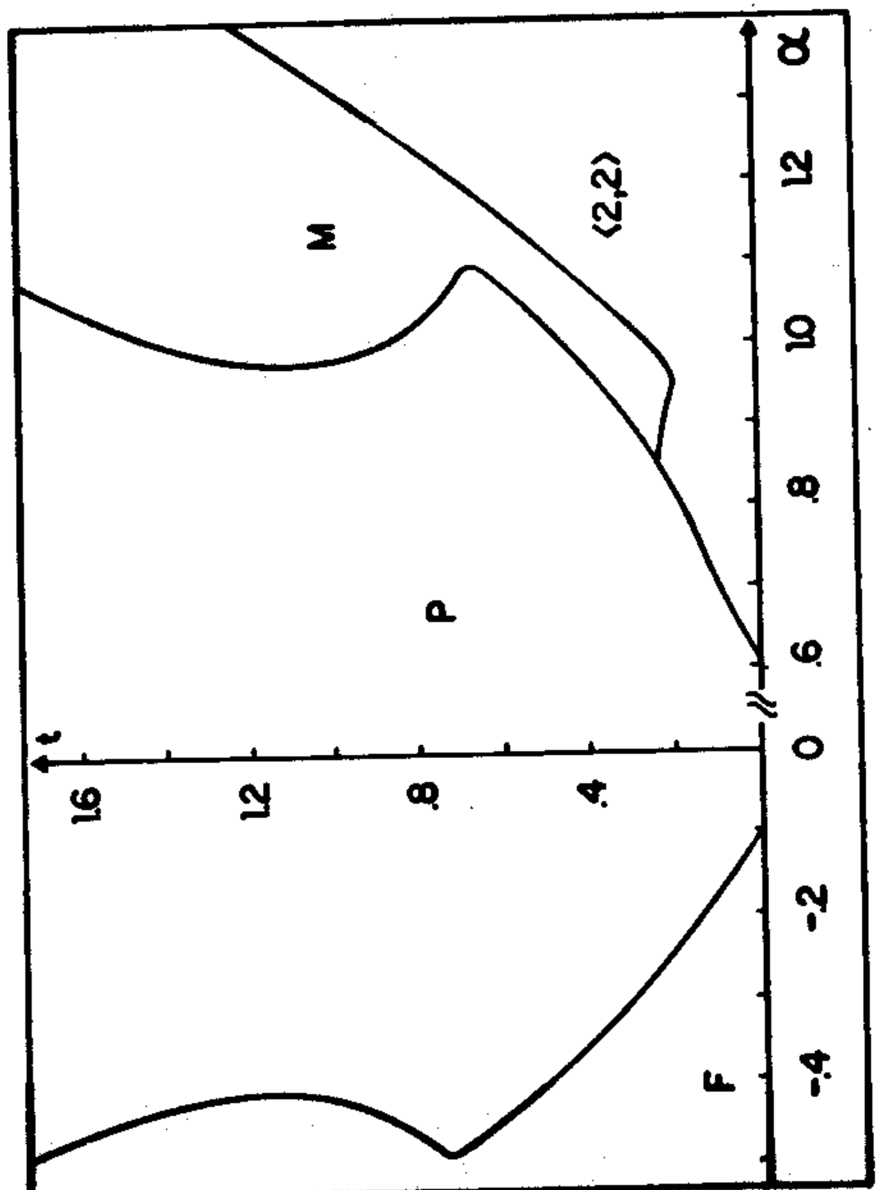

Figure 4

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