

A New Version of the Dirac's Æther and Its Cosmological Applications

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Abstract

We propose a new formulation for the Æther of Dirac based on a lagrangian approach. We analyse how the presence of a particular self-interaction term in the lagrangian lead us to a description of the æther as being a medium with conductivity which is governed by macroscopic Maxwell equations with a polarization tensor $M_{\alpha\beta}$ depending on the vector potential. These results are then applied to the analysis of the amplification of the primordial magnetic induction in a curved background of Friedmann's geometry.

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1 Introduction

The original version of the Dirac's \mathcal{A} ether was presented long ago as a result following his formulation of the so-called "New Electrodynamics" (NE) [1]. In the NE, Dirac has adopted a radical approach for the discussion of a classical electron in the presence of an electromagnetic field. The main idea was to use the spurious degrees of freedom associated to the gauge potential to describe the electron. In [1] Dirac showed how this could be done by a judicious choice of gauge condition, e.g. $A^2 = k^2$. In fact, introducing this as a gauge fixing term in the Lagrangian ¹ $L = -\frac{1}{4}F^2 + \frac{\lambda}{2}(A^2 - \kappa^2)$, we find $\partial_\nu F^{\nu\mu} = J^\mu \equiv \lambda A^\mu$. Therefore, we obtain a four current depending on A_μ that is suitable for the description of a classical electron interacting with an electromagnetic field. In this approach we observe that the gauge condition doesn't intend to eliminate spurious degrees of freedom of the gauge potential. Instead of that, it acquires a deep physical meaning as the condition that allow us to describe the right physics without having to introduce any extra fields.

One of the striking consequences of these results were discussed in a subsequent paper of Dirac [2] and led to the \mathcal{A} ether model. In fact, by using quantum mechanical arguments Dirac argued it would be possible to consider an \mathcal{a} ether provided we interpret its four velocity v at each point as a quantity subjected to uncertainty relations. Then, the \mathcal{a} ether four velocity instead of being a well-defined quantity, would rather be probabilistically distributed over a range of values. The wave function describing the \mathcal{a} ether would be such as to assure a velocity distribution with all velocities being equally probable. Admitting the \mathcal{a} ether velocity as defining a point in a hyperboloid with equation $v_0^2 - \vec{v}^2 = 1$, $v_0 > 0$, Dirac could relate it to the gauge potential (satisfying $A^2 = k^2$) by $\frac{1}{k}A_\mu = v_\mu$ and from the interpretation given to the NE, he concluded that v would be the velocity an electric charge would flow if placed in the \mathcal{a} ether. The four velocity v , being defined through all points of spacetime, justify its interpretation as a meaningful physical quantity, the \mathcal{a} ether velocity.

Following Dirac, many authors have tried to develop a realistic model for the Dirac's \mathcal{A} ether by considering the \mathcal{a} ether as a vacuum with conductivity $\sigma \approx 10^{-13}/s$ [3, 4]. They have also found the photon as having a non-null mass $m_\gamma = \sigma h/c^2 \approx 10^{-48}g$ (which is below laboratories estimates for the limit of the photon mass). In these models, the basic equations for the \mathcal{a} ether were the Maxwell equations in a conducting medium without

¹With λ a lagrange multiplier and k a constant.

density of charge and current. The intensity of the electric field \vec{E} , and the magnetic induction \vec{B} ² are related to the electric displacement \vec{D} , and the magnetic intensity \vec{H} by the relations $\vec{D} = \epsilon\vec{E}$, $\vec{H} = \frac{1}{\mu}\vec{B}$ which suppose the æther as a homogeneous medium polarized isotropically and linearly. Besides that, it is implicit in [3, 4] that all their analysis is considered in a flat spacetime and from a reference frame at rest relative to the æther.

Our present work is a continuation of an investigation initiated in a preceding paper [5] where we have studied the amplification of the cosmic magnetic induction for the Proca electromagnetic field in the Dirac's Æther. There the equations used were the Proca's equations in a conducting media. Here, we wish to study the Dirac's model from another point of view, extending his original idea but maintaining the physical motivation for the application in a curved background. One of our interests is to analyse how the primordial cosmic magnetic induction has been amplified, as the Universe evolves, for a model that is neither Proca nor the purely Maxwell electrodynamics (but that includes this last one as a particular case). In the literature of the Dirac's Æther, none of the models presented in [3, 4, 6] discusses a possible framework for the study of electromagnetic phenomena in the large scale structure. They all take for granted the basic Maxwell theory. Therefore, any application to a large scale scenario is expected to be modeled within the Maxwell formulation. Since it is unknown if under the influence of a curved background the electromagnetic field presents a different behaviour than that foreseen by the Maxwell theory, it is reasonable to test alternative approaches that may provide a suitable description for the electromagnetic field at the large scale. Our model intends to offer such an alternative.

Although we are not concerned with the relevant question about the structure of the æther as a conducting medium [6], it is implicit in our formulation that there is an inertial reference frame in which the æther is at rest and consequently the field of velocities of the points of the æther write as $v_{\text{æther}}^\mu = (1, 0, 0, 0)$. We refer it as the æther frame. The main point of our research is based on the proposition of an action for the Æther of Dirac which may be able to reproduce some characteristics of previous models. We propose then the action $\mathcal{S} = \int dx \left(-\frac{1}{4}F^2 + \sigma v_\alpha F^{\alpha\mu} A_\mu \right)$ with v being the æther's velocity relative to a generic observer (inertial or not). For inertial observers, $v^\mu = \Lambda_\nu^\mu v_{\text{æther}}^\nu = \Lambda_0^\mu$

²Here we will also refer to the intensity of the electric field \vec{E} and the magnetic induction \vec{B} as simply electric and magnetic field.

³ is still constant and we obtain from the equations of motion a four-current in the form $J^\mu = -\sigma v^\mu \partial_\nu A + \sigma v_\nu \partial^\mu A^\nu$, that is understood as being induced in the æther by the presence of the electromagnetic field. Therefore, it defines a polarization tensor $M_{\alpha\beta}$ from which we obtain the vectors of polarization and magnetization of the medium. We will see in section **2.4** that in the æther reference frame the form assumed by this 4-current allow us to define the electric displacement vector as $\vec{D} = \vec{E} + \sigma \vec{A}$ while the magnetic intensity coincides with the magnetic field, $\vec{H} = \vec{B}$, with the resulting equations being similar in form with the macroscopic Maxwell equations in a medium. This coincides with the results of [3, 4] but with a particular difference: $\vec{D} \neq \epsilon \vec{E}$. This shows that in our model the æther cannot be thought of as an isotropic medium. Moreover, in a generic reference frame moving relative to the æther, the vectors of electric displacement \vec{D} and magnetic intensity \vec{H} will depend on v . Our treatment offers an extension of the results of [3, 4] in the sense that our equations incorporate the situation of reference frames that are not at rest relative to the æther.

In our approach the basic electromagnetic fields are the vectors \vec{E} and \vec{B} which are components of the tensor $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$. The equations of motion (3) are determined directly from our action, with the Jacobi identity $F_{\mu\nu,\alpha} + F_{\nu\alpha,\mu} + F_{\alpha\mu,\nu} = 0$ completing the set of equations involving the electromagnetic field. In the traditional formulation of classical electrodynamics the non-homogeneous Maxwell equations in a conducting medium are obtained introducing a second tensor $H^{\mu\nu}$ with components \vec{D} and \vec{H} which satisfy $\partial_\mu H^{\mu\nu} = -j^\nu$. Compared to our formalism we don't have to introduce by hand any tensor $H^{\mu\nu}$. Here it is the interaction term $v_\alpha F^{\alpha\mu} A_\mu$ that forces the appearance of additional terms (depending on A_μ) in the equations for \vec{E} and \vec{B} which allow us to identify as \vec{D} and \vec{H} . Both the classical electrodynamics in vacuo (31-34) and in conducting media (35-38) are obtained as a particular case of our model for an observer that is at rest relative to the æther. For observers that move slowly relative to the æther and/or for phenomena in which the æther conductivity is sufficiently small we are expected to have the interaction term giving a small contribution to the Maxwell equations and therefore the model may qualitatively agree with the same predictions of the traditional formulation of classical electrodynamics.

This work is organized as follows. Considering an observer moving with constant velocity relative to the æther, in Section **2.1** we discuss the lagrangian of the model,

³with Λ the Lorentz transformation relating the observer and the æther frame.

identify the 4-current and a constraint that appears associated to its conservation; in Section **2.2** we analyse a global gauge invariance of the action, we show that in the æther frame the conservation of the Noether charge coincides with the Gauss law, we interpret the interaction term $v_\alpha F^{\alpha\mu} A_\mu$ as $\tilde{J} \cdot A$ with $\tilde{J}^\mu \equiv v_\alpha F^{\alpha\mu}$ a conserved 4-current, and we propose a mass term that preserves the global symmetry; in Section **2.3** we show that our action admits a local gauge invariance; in Section **2.4** we show that in the æther frame we obtain macroscopic Maxwell equations with $\vec{D} = \vec{E} + \sigma \vec{A}$ and $\vec{H} = \vec{B}$. Considering the case of a curved background we will have, in general, $v \neq cte$; in Section **3** we establish the material relations involving the fields; in Section **4** the equations of the model in a cosmological background are solved and the solutions are discussed. The last section is devoted to concluding remarks.

Notations For Flat Space-time

$$\begin{aligned}
\eta_{\mu\nu} &= \text{diag}(+, -, -, -) \\
x^\mu &\equiv (x^0, x^i) \doteq (t, \vec{x}), \quad x_\mu \equiv (x_0, x_i) \doteq (t, -\vec{x}) \\
\partial^\mu &\equiv (\partial^0, \partial^i) \doteq \left(\frac{\partial}{\partial t}, -\vec{\nabla}\right), \quad \partial_\mu \equiv (\partial_0, \partial_i) \doteq \left(\frac{\partial}{\partial t}, \vec{\nabla}\right) \\
A^\mu &\equiv (A^0, A^i) \doteq (\phi, \vec{A}), \quad A_\mu \equiv (A_0, A_i) \doteq (\phi, -\vec{A}) \\
F_{0i} &= \vec{E}_i, \quad F_{ij} = -\epsilon_{ijk} \vec{B}_k, \quad F^{0i} = -F_{0i} = -\vec{E}_i, \quad F^{ij} = F_{ij} = -\epsilon_{ijk} \vec{B}_k
\end{aligned}$$

2 The Lagrangian of the Dirac's Æther and Its Invariances

2.1 Equations of Motion

Let us consider a certain reference system moving with a constant 4-velocity relative to the æther reference frame. We take for action

$$\mathcal{S} = \int dx \left(-\frac{1}{4} F^2 + \tilde{J} \cdot A \right) \quad (1)$$

with

$$\tilde{J}^\mu \doteq \sigma v_\alpha F^{\alpha\mu} . \quad (2)$$

In these expressions we have $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and the field A is understood as a 1-form defined in a flat spacetime manifold. The (constant) parameter σ is associated to

the æther conductivity and \tilde{J} will be shown to be a conserved quantity. Here, the term $\tilde{J} \cdot A$ defines an interaction of the gauge field with itself.

The equation of motion for A_μ has the form

$$\partial_\nu F^{\nu\mu} + \sigma v^\mu \partial \cdot A - \sigma v_\nu \partial^\mu A^\nu = 0 \quad (3)$$

and it assumes the same form as the Maxwell equations in the presence of a source

$$\partial_\nu F^{\nu\mu} \equiv J^\mu \quad (4)$$

provided we identify

$$J^\mu = -\sigma v^\mu \partial \cdot A + \sigma v_\nu \partial^\mu A^\nu \quad (5)$$

with a conserved 4-current. This interpretation follows the same idea of Dirac's NE [1] in which the term $j^\mu \doteq \lambda A^\mu$ was interpreted as a 4-current. However, in Dirac's formalism the appearance of this 4-current originates from the introduction of a gauge fixing term $\frac{1}{2}\lambda(A^2 - k^2)$ in the action while in our approach J^μ , given in (5), arises from the presence of the interaction term $\tilde{J} \cdot A$.

Now, taking the divergence of (4) we obtain $0 = \partial_\mu J^\mu = \sigma v_\mu (\square A^\mu - \partial^\mu \partial \cdot A) = \sigma v_\mu \partial_\nu F^{\nu\mu} = \sigma v_\mu J^\mu = \sigma^2 (-v^2 \partial \cdot A + v_\alpha v_\beta \partial^\alpha A^\beta)$, i.e

$$\partial \cdot A = \frac{v_\alpha v_\beta}{v^2} \partial^\alpha A^\beta . \quad (6)$$

This constraint is a new feature of our model and since it involves the divergence of A it resembles a kind of gauge condition. Nonetheless, its origin is independent of any local symmetry of the action. In section 2.3 we will analyse how this condition combines with a local symmetry of the action imposing certain conditions to be satisfied by the gauge parameter.

2.2 Global Gauge Invariance

The first invariance of the action we want to analyze is the one defined by a transformation parametrized by a global parameter λ ($\partial_\mu \lambda = 0$) and having the form

$$A_\mu \rightarrow A'_\mu = A_\mu + \lambda v_\mu . \quad (7)$$

Associated to this symmetry we have the following Noether current

$$\Theta^\mu = F^{\mu\nu} v_\nu - \sigma v^\mu A \cdot v + \sigma v^2 A^\mu \equiv \Theta^{\mu\nu} v_\nu \quad (8)$$

with

$$\Theta^{\mu\nu} \doteq F^{\mu\nu} - \sigma v^\mu A^\nu + \sigma v^\nu A^\mu . \quad (9)$$

In section **3** the quantity $\Theta^{\mu\nu}$ will be interpreted as the tensor $H^{\mu\nu}$, which in the æther frame becomes $H^{\mu\nu} = (\vec{D}, \vec{H})$. In this general case, it will be clear how $H^{\mu\nu}$ depends on the properties of the medium and on the 4-velocity v . In a system that is at rest relative to the æther we have

$$\Theta^\mu = (0, \vec{E} + \sigma \vec{A}) \quad (10)$$

and condition (6) gives $\vec{\nabla} \cdot \vec{A} = 0$. Therefore, the conservation equation of Θ^μ let us with

$$\vec{\nabla} \cdot \vec{E} = 0 . \quad (11)$$

In our model, there is another conserved current that has the form

$$\hat{J}^\mu = -\sigma v^\mu \partial \cdot A + \sigma v \cdot \partial A^\mu \quad (12)$$

and from which we obtain $\tilde{J} = \hat{J} - J$. Then, conservation of \tilde{J} follows immediately as the difference of two conserved currents. Equivalently, from (9) we can also think of \tilde{J}^ν as originating from the divergence of $\Theta^{\mu\nu}$,

$$\partial_\mu \Theta^{\mu\nu} = -\tilde{J}^\nu . \quad (13)$$

In the classical formulation of Electrodynamics in conducting media the non-homogeneous Maxwell equations are written covariantly as

$$\partial_\mu H^{\mu\nu} = -j_{ext}^\nu \quad (14)$$

with the tensor $H^{\mu\nu}$ having \vec{D} and \vec{H} as its components. In our model $\Theta^{\mu\nu}$ (9) generalizes the tensor $H^{\mu\nu}$ and equation (13) corresponds to (14). Then this allow us to interpret \tilde{J}^μ as the corresponding 4-current of our model, in much the same way as Dirac interpreted $j_\mu = \lambda A_\mu$ as a 4-current in [1]. The interaction term $\tilde{J} \cdot A$ in our action (1) parallels then the same interaction term of the usual electrodynamics.

The global symmetry (7) is a new feature of our model that has no similar counterpart in the usual Maxwell formulation. It is also possible to add a mass term to the action (1) that perserves this global symmetry. In fact, the action

$$\mathcal{S} = \int dx \left(-\frac{1}{4} F^2 + \tilde{J} \cdot A - \frac{1}{2} \sigma^2 A_\mu (v^2 g^{\mu\nu} - v^\mu v^\nu) A_\nu \right) \quad (15)$$

is invariant by (7). From (15) we obtain the following equation

$$\partial_\nu F^{\nu\mu} \equiv \bar{J}^\mu \doteq -\sigma v^\mu \partial.A + \sigma v_\nu \partial^\mu A^\nu + \sigma^2 (v^2 g^{\mu\nu} - v^\mu v^\nu) A_\nu \quad (16)$$

or equivalently

$$(g^{\mu\nu} (\square - \sigma^2 v^2) - \partial^\mu \partial^\nu + \sigma (v^\mu \partial^\nu - v^\nu \partial^\mu) + \sigma^2 v^\mu v^\nu) A_\nu = 0. \quad (17)$$

Here the conservation of the current \bar{J}^μ that follows from (16) doesn't produce any constraint over A . We also notice that associated to the global symmetry we have the same conserved current (8).

2.3 Local Gauge Invariance

The local gauge invariance depends on a parameter $\theta(x)$ and assumes the usual form

$$A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \theta. \quad (18)$$

Here, condition (6) adds new features to our analysis. In fact, let A' and A be two fields related by (18). Since both field configurations should obey (6) we must have

$$\partial.A' = \frac{1}{v^2} (v.\partial)(v.A') \Leftrightarrow \square\theta = \frac{v_\alpha v_\beta}{v^2} \partial^\alpha \partial^\beta \theta. \quad (19)$$

Consider that $\partial.A \neq 0$ and let us choose θ such that it ensures $\partial.A' = 0$. We should then have θ satisfying

$$\square\theta = -\partial.A. \quad (20)$$

This last condition together with the constraints (6,19) gives

$$\partial^\alpha \partial^\beta \theta = -\frac{1}{2} (\partial^\alpha A^\beta + \partial^\beta A^\alpha) \quad (21)$$

that represents a restriction stronger than that shown in (20). Equivalently, we can obtain equation (21) directly from $0 = \partial.A' = \frac{v_\alpha v_\beta}{v^2} \partial^\alpha (A^\beta + \partial^\beta \theta)$ by using (6).

2.4 Electrodynamics in the Æther's Reference Frame

Let us analyse our model in the æther reference frame. We have $v = (1, 0, 0, 0)$. We also suppose the æther as a medium without any given density of charge or current. From (4) we obtain the following equations

$$\vec{\nabla} \cdot \vec{E} = -\sigma \vec{\nabla} \cdot \vec{A} \quad (22)$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \sigma \frac{\partial \vec{A}}{\partial t} + \sigma \vec{E} \quad (23)$$

to which we add the homogeneous equations

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (24)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} . \quad (25)$$

Our equations involve just the fields (\vec{E}, \vec{B}) . The presence of additional terms depending on the potential vector in (22, 23) is understood as signaling the response of the medium to the presence of the fields (\vec{E}, \vec{B}) , a situation that resembles the phenomena of polarization and magnetization of a medium. Therefore, we rewrite (22) as

$$\vec{\nabla} \cdot \vec{D} = 0 \quad (26)$$

with $\vec{D} \doteq \vec{E} + \sigma \vec{A} + \vec{\nabla} \times \vec{K}$. At this point, \vec{K} is an arbitrary vector that can be thought of as playing a similar role of a gauge parameter. Now, we rewrite (23) as

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{D}}{\partial t} + \sigma \vec{E} - \frac{\partial}{\partial t} \vec{\nabla} \times \vec{K} \quad (27)$$

and we choose \vec{K} such that it satisfies

$$\frac{\partial}{\partial t} \vec{\nabla} \times \vec{K} = \sigma \vec{E} . \quad (28)$$

Then, using (25) we obtain

$$\frac{\partial}{\partial t} \vec{\nabla} \times (\sigma \vec{A} + \vec{\nabla} \times \vec{K}) = 0 . \quad (29)$$

The vector \vec{K} can be further restricted so that

$$\sigma \vec{A} + \vec{\nabla} \times \vec{K} = 0 . \quad (30)$$

This gives $\vec{E} = \vec{D}$ and $\vec{B} = \vec{H}$ and lead us to a set of equations

$$\vec{\nabla} \cdot \vec{E} = 0 \quad (31)$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} \quad (32)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (33)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (34)$$

that correspond to the Maxwell equations in free space. Conditions (28,30) can be interpreted as originating from the imposition of the temporal gauge. In fact, together they imply $\vec{\nabla} A_0 = 0$ that is naturally satisfied if we put $A_0 = 0$.

It is possible to give another description for our electrodynamics without using this vector \vec{K} . From (23) we can simply identify $\vec{D} = \vec{E} + \sigma\vec{A}$, $\vec{H} = \vec{B}$ that lead us to the following equations

$$\vec{\nabla} \cdot \vec{D} = 0 \quad (35)$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{D}}{\partial t} + \sigma \vec{E} \quad (36)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (37)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (38)$$

that coincides with the æther's equations obtained in [3, 4]. In the identification $\vec{D} = \vec{E} + \sigma\vec{A}$, $\vec{H} = \vec{B}$ the æther behaves like a medium that responds to the presence of the electric field by creating a polarization $\vec{P} = \sigma\vec{A}$. We also have a current $\vec{J} = \sigma\vec{E}$ which is in agreement with our supposition of the æther as being a medium with conductivity σ .

According to Schwinger's idea of structureless vacuum [7], an electromagnetic field disturbs the vacuum affecting its properties of homogeneity and isotropy. This is exactly the situation we have obtained in our model, where the presence of an electromagnetic field in a vacuum with conductivity σ produces a response of the medium ($\vec{D} \neq \epsilon\vec{E}$) that signalizes its non-isotropy.

3 Material Relations Involving $H^{\mu\nu}$ and $F_{\mu\nu}$

In section 2.4, we have obtained the following material relations

$$\vec{D} = \vec{E} + \sigma\vec{A} \quad (39)$$

$$\vec{H} = \vec{B} \quad (40)$$

for a flat spacetime and for the case of a reference frame at rest relative to the æther. In the case of a curved spacetime and for a non-inertial reference frame moving relative to the æther, we are supposed to have a more complicated relation between H and F . Indeed, the general form for the material relations between the tensors $H^{\mu\nu}$ and $F_{\mu\nu}$ in a medium that is at rest in *any* reference frame with a metric $g_{\alpha\beta}$ has been established in [8] as:

$$\sqrt{-g}H^{\alpha\beta} = \sqrt{-g}g^{\alpha\gamma}g^{\beta\kappa}S_{\gamma}^{\mu}S_{\kappa}^{\nu}F_{\mu\nu} \quad (41)$$

where the tensor S_β^α characterizes the electromagnetic properties of the medium. For our later purpose, it is convenient to rewrite (41) as

$$\sqrt{-g}H^{\alpha\beta} = \sqrt{-g}g^{\alpha\gamma}g^{\beta\kappa}S_{\gamma\kappa}^\mu A_\mu \quad (42)$$

where the third rank mixed tensor $S_{\gamma\kappa}^\mu$ relates to S_β^α by

$$S_{\gamma\kappa}^\mu = (S_\gamma^\nu S_\kappa^\mu - S_\gamma^\mu S_\kappa^\nu)\partial_\nu. \quad (43)$$

As an application of (41) it was shown in [8] that for the vacuum (considered from an inertial reference frame) the tensor S_β^α assumes the form $S_\beta^\alpha = \delta_\beta^\alpha$ and the material equations become $\sqrt{-g}H^{\alpha\beta} = \sqrt{-g}g^{\alpha\mu}g^{\beta\nu}F_{\mu\nu}$ which reproduces the usual equations of free electrodynamics in a curved background [9, 10]. Also, in the case of a linear isotropic medium that is at rest in an inertial reference frame the tensor S_β^α is given by $S_0^0 = \epsilon\sqrt{\mu}$, $S_1^1 = S_2^2 = S_3^3 = \frac{1}{\sqrt{\mu}}$ and we obtain the usual relations $\vec{D} = \epsilon\vec{E}$ and $\vec{H} = \frac{1}{\mu}\vec{B}$ for the electrodynamics in a medium.

In our model, in order to define the tensor $H^{\alpha\beta}$ for a generic reference frame in a curved background and to find a suitable material relation of the type (42) we should first follow the procedure of section 2.4 that allow us to define $H^{\mu\nu}$ directly from the equation of motion for A_μ . Explicitly, let us take the equation of motion in a curved background,

$$\partial_\nu(\sqrt{-g}F^{\nu\mu} - \sigma\sqrt{-g}v^\nu A^\mu + \sigma\sqrt{-g}v^\mu A^\nu) = J^\mu \equiv -\sigma\sqrt{-g}v_\nu F^{\nu\mu} \quad (44)$$

then we define

$$\sqrt{-g}H^{\alpha\beta} \doteq \sqrt{-g}(F^{\alpha\beta} - \sigma v^\alpha A^\beta + \sigma v^\beta A^\alpha). \quad (45)$$

This procedure is equivalent to the introduction of the antisymmetric polarization tensor of the medium $M^{\alpha\beta}$ [8]

$$\sqrt{-g}H^{\alpha\beta} = \sqrt{-g}(F^{\alpha\beta} + M^{\alpha\beta}) \Leftrightarrow \sqrt{-g}H^{\alpha\beta} = \sqrt{-g}g^{\alpha\nu}g^{\beta\mu}(F_{\nu\mu} + M_{\nu\mu}) \quad (46)$$

provided we identify from (45,46)

$$M^{\alpha\beta} = -\sigma v^\alpha A^\beta + \sigma v^\beta A^\alpha \Leftrightarrow M_{\alpha\beta} = -\sigma v_\alpha A_\beta + \sigma v_\beta A_\alpha \quad (47)$$

with $F^{\mu\nu} = g^{\mu\alpha}g^{\nu\beta}F_{\alpha\beta}$, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, $M^{\mu\nu} = g^{\mu\alpha}g^{\nu\beta}M_{\alpha\beta}$. Finally, in order to obtain the material equations for our model we extend definition (42) by allowing $S_{\alpha\beta}^\mu$ to be a generic operator not restricted by (43) but given by

$$S_{\alpha\beta}^\mu = \delta_\beta^\mu(\partial_\alpha - \sigma v_\alpha) - \delta_\alpha^\mu(\partial_\beta - \sigma v_\beta). \quad (48)$$

Here, the tensor $S_{\alpha\beta}^{\mu}$ may contain not only electromagnetic properties of the medium (as it is the case of (43)) but also information about the reference frame (implicit in the four-velocity v_{α}). Adopting the convention $\vec{D}_i = \sqrt{-g}H^{i0}$, $\vec{H}_i = -\frac{1}{2}\epsilon_{ijk}\sqrt{-g}H^{jk}$ [8] and considering a flat spacetime we obtain (in the æther frame) from (42,48) the relations given in (39,40).

4 The Dirac's Æther in a Curved Background

4.1 Some Questions on the Cosmic Magnetic Induction

The origin of the cosmic magnetic induction (CMI) is still unknown. In fact, until now no theory has completely succeeded on explaining the evolution of the CMI, from its generation in the early universe to our present time in galaxies, stars and probably in the extragalactic medium. The standard dynamo action, for instance, has left many questions without answer [11, 12]. Recently, one special type of mechanism for the amplification of the CMI has been studied in [5] where the amplification was understood as being caused by the expansion of the cosmological background. This new kind of theoretical preview was called *geometric amplification* (GA) because the only agent responsible for this effect was the scale geometric factor $R(t)$. One of the advantages of this approach is that the amplification can be an alternative for the standard mechanism recently contested in [11].

The radio emission of very distant cosmic objects determines an upper limit for the intensity of the CMI. According to the literature, in the extra-galactic medium the CMI is supposed to have an intensity less than 10^{-13}T ⁴.

4.2 The Model in the Friedmann's Geometries

Considering these ideas, we will now apply our model to a curved background. We will be particularly interested on finding solutions for the CMI that may provide an explanation for the amplification of this field from the initial value of $\approx 10^{-19}\text{T}$ to the suggested actual value of $\approx 10^{-13}\text{T}$. Here, we will assume that the conductivity of Dirac's Æther is $\approx 10^{-19}/\text{s}$.

Let us take as coordinate system the cylindrical coordinates of Schrödinger $x^{\mu} = (t; \rho, \phi, \zeta)$ in a Friedmann cosmological background minimally coupled with the electro-

⁴Other models have considered a generation of a primordial CMI of magnitude $\approx 10^{-21}\text{T}$ [11].

magnetic field. The metric tensor in all three Friedmann geometries is written as

$$g_{\mu\nu} = \text{diag} \left[R^2(t)(1; -1, -u^2, -w^2) \right], \quad (49)$$

where the functions $u(\rho)$ and $w(\rho)$ define the type of three-geometry (with constant curvature k_c) under consideration. These functions and the different scale factors are shown in table 1. We assume here the same time dependent 4-potential with cylindrical symmetry

Table 1: The functions u , w and $R(t)$ where $\alpha = 10^{26}\text{m}$.

u and w	k_c	$R(t)$
$u = \rho, w = 1$	0	$(\alpha/2)t^2$
$u = \sin \rho, w = \cos \rho$	+1	$\alpha(1 - \cos t)$
$u = \sinh \rho, w = \cosh \rho$	-1	$\alpha(\cosh t - 1)$

[5] given by

$$A^\mu = (0; 0, 1, 0)f(t)/R^2(t), \quad (50)$$

where $f(t)$ is a function to be determined by the field equations. The non-zero components of the field strength $F_{\mu\nu}$ are F_{02} and F_{12} . Therefore, for the orthonormal basis we have chosen, the non-null components of the fields \mathbf{E} and $c\mathbf{B}$ are $E_\phi = -\dot{f}u^2/R^2$, $cB_\zeta = 2fu u'/R^2$ ⁵. To study how the expansion of the Universe influences the time evolution of the electromagnetic field, it is appropriate to define the time dependent quantities $\mathcal{E}(t) \equiv |\mathbf{E}/u| = |\dot{f}|/R^2$ and $\mathcal{B}(t) \equiv |\mathbf{B}/u'| = 2|f|/(cR^2)$ that for simplicity we will also refer as the electric field and magnetic induction.

In a curved space-time with metric minimally coupled to the electromagnetic field we obtain from \mathcal{S} the following equations

$$F^{\mu\nu}_{;\mu} + \frac{\sigma}{c}(A^\mu v^\nu - A^\nu v^\mu)_{;\mu} = J^\nu \quad (51)$$

where the semicolon denotes the covariant derivative, $J^\nu = (-\sigma/c)v_\mu F^{\mu\nu}$ and $v_\mu = R(t)\delta_\mu^0$ the four-velocity of the æther.

Here we notice that our new equations (51) differ from the ones of our previous model [5] by the presence of a skew-symmetric term $(A^\mu v^\nu - A^\nu v^\mu)_{;\mu}$ instead of the term $(1/\lambda^2)A^\nu$

⁵The dot means d/dt , the prime means $d/d\rho$, and c , in this section, is written explicitly as the light velocity.

(that comes from the Proca term). In both cases, the qualitative behaviour of the new term will be very similar to the one obtained in the Proca model as we can see after comparing Table 3 below and the graphics of our preceding paper.

For $\nu = 2$ we have

$$\ddot{f} + \left[4 k_c - \frac{\sigma}{c} \dot{R} \right] f = 0 \quad (52)$$

and according to the geometry, we have to solve the equations below:

$$\begin{aligned} \ddot{f} - \alpha t \frac{\sigma}{c} f &= 0 \quad \text{for } k_c = 0 ; \\ \ddot{f} + \left(4 - \frac{\sigma}{c} \alpha \sin t \right) f &= 0 \quad \text{for } k_c = +1 ; \\ \ddot{f} - \left(4 + \frac{\sigma}{c} \alpha \sinh t \right) f &= 0 \quad \text{for } k_c = -1 . \end{aligned} \quad (53)$$

We assume an initial field of magnitude $\mathcal{B}(t_i) \approx 10^{-19}\text{T}$. Since our model should also contain a weak initial electric field to be dissipated during the evolution of Universe, we shall assume the existence of an electric field of magnitude $\mathcal{E}(t_i) \approx 10^{-4}\text{V/m}$. These limits are fixed in order to give, as a final result, a realistic value for the modulus of $\mathcal{B}(t_f)$ that agrees with the one established by the usual theory of the cosmic fields. These initial values we have assumed don't perturb the gravitational field, as it is evident from simple calculations which show the energy-momentum tensor of the electromagnetic and gravitational field related by a factor above 10^{10} . These results then justify the minimal coupling of these fields as a suitable framework to study the influence of the very strong geometry of the Universe on the very weak electromagnetic phenomenon.

Using the same method employed in [5], we will integrate numerically equations (53) from the initial cosmic conformal time $t_i = 0.0890$ to the final time $t_f = 1.6100$. In standard cosmology this range corresponds respectively to the final stage of the matter-radiation coupling and our current epoch. We have obtained around 20,000 points in the numerical integration of which, in Table 2, we displayed only a small ensemble of these points that can give us a qualitative view of the amplification phenomenon. These results are very similar to the ones of our latter work [5]. In Table 3 we compared the initial and the final value of the fields for each geometry. The data show the important amplification of the \mathcal{B} field, which is very welcomed by astrophysicists, and the overall reduction of the electric field. It should be noticed that these results are determined not only by the evolution of the function $f(t)$, (which constrains the field equations) but also by the direct contribution of the geometry as given by the scale factor, $R(t)$, that is present in both

Table 2: Some Numerical Data (F=Flat, E=Elliptic, H=Hyperbolic)

t	0.0890	0.1000	0.2352	0.5779	1.5793	1.6100
(F) $\log \mathcal{E} $	-3.0004	-3.1804	-4.7392	-6.2559	-8.0335	-8.0147
$\log c\mathcal{B} $	-11.6993	-4.7811	-5.2158	-6.2569	-7.5110	-7.5378
(E) $\log \mathcal{E} $	-2.9997	-3.2035	-4.7034	-6.4784	-7.8183	-7.8333
$\log c\mathcal{B} $	-11.6988	-4.8575	-5.2248	-6.3072	-8.6010	-8.7447
(H) $\log \mathcal{E} $	-3.0009	-3.2052	-4.6926	-6.2743	-8.1737	-8.2008
$\log c\mathcal{B} $	-11.6999	-4.8591	-5.2266	-6.2841	-7.6994	-7.7205

mathematical expressions for \mathcal{E} and \mathcal{B} . It is the interchange between the gravitational and electromagnetic fields that imposes, as the Universe evolves, the decreasing of the electric field and the amplification of the \mathcal{B} field.

Table 3: The Reduction of \mathcal{E} and the Amplification of \mathcal{B}

Geometry	\mathcal{E}	\mathcal{B}
Flat ($k_c = 0$)	$\times 10^{-5}$	$\times 10^{+4}$
Elliptic, ($k_c = +1$)	$\times 10^{-5}$	$\times 10^{+3}$
Hyperbolic, ($k_c = -1$)	$\times 10^{-5}$	$\times 10^{+4}$

5 Conclusion

Our model for the \mathcal{A} ether of Dirac allow us to adapt our description to any observer, be it inertial or not. In the case of an observer at rest relative to the \mathcal{a} ether, in a flat spacetime, we have obtained the same description of previous models [3, 4] but with different material relations involving the fields $\vec{D}, \vec{H}, \vec{E}, \vec{B}$. This accounts for the fact that although the \mathcal{a} ether is a medium with conductivity, the presence of an electromagnetic field disturbs its isotropy. The equations we have obtained in this case had the same form as the Maxwell equations in a conductive medium provided we adopted a certain “gauge” choice for the vector \vec{K} (28,30). Here we notice the same role of the gauge condition as in the original work of Dirac’s NE [1], e.g. as a condition that determines a certain physics. In the case

of a curved background, an observer with $v_\alpha = R(t)\delta_\alpha^0$ will see the same phenomenon of amplification of the magnetic induction and reduction of the electric intensity that had already been observed in [5] in the context of a Proca electromagnetic field in a Dirac æther.

The material relations between the polarization tensor and the gauge potential given in (41,48) shows that the tensor $H^{\mu\nu}$ is related to the electromagnetic properties of the medium (encoded into the tensor $S_{\alpha\beta}^\mu$) and the metric tensor. This raises an interesting picture of a mutual effect between the electromagnetic field and the geometry of the spacetime. In fact, if the GA describes how the expansion of the universe influences the electromagnetic fields as a kind of geometric background effect that should be added to well-established non-geometric effects, there is also the possibility for the electromagnetic field to influence the expansion of the universe [13].

The geometric relations between the electromagnetic field and the metric tensor involves the scale factor $R(t)$. The amplification of the field \mathcal{B} in this case is determined by the minimal coupling we adopted. In other couplings between the electromagnetic and the gravitational fields the interchange could be more rapid and/or more intense, as we can see in [14]. Our results confirm once more the strict relations between the electromagnetic and gravitational phenomena. Furthermore, the model we elaborated for the Dirac's Æther in a curved space-time suggests it can also be applied to the study of the electromagnetism in the large scale structure of the Universe.

The investigation of quantum phenomena brings us to the realm of quantum electrodynamics. Any attempt to quantize our model should first address the question of a reinterpretation of the 4-velocity v_μ at the quantum level so that we can associate to it a quantum field $v(x)$ depending of the spacetime. This investigation will be presented in another work.

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