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The world is not enough

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Abstract

We show that the 5-dimensional model introduced by Randall and Sundrum is (half of) a wormhole, and that this is a general result in models of the RS type. We also **discuss the gravitational trapping of a scalar particle in 5-d spacetimes**. Finally, we present a simple model of brane-world cosmology in which the background is a static anti-de Sitter manifold, and the location of the two 3-branes is determined by the technique of "surgical grafting".

In recent years, high energy particle physics and gravity have been reconciled by means of the so–called brane worlds. [1]. From the phenomenological perspective these worlds provide an economic explanation of the hierarchy between the gravitational and electroweak mass scales. In particular, it has been suggested that the fundamental Plank scale M_* can be lowered all the way to TeV scale [2] by introducing extra dimensions [3]. In this framework the Standard Model (SM) is confined to a 3-brane [4] whereas gravity propagates freely through the extra dimensions. In such higher–dimensional models spacetime is usually taken to be the product of a 4–dimensional spacetime and a compact n-manifold. Consequently, the observed Plank scale is related to the higher dimensional scale gravity M_* through the relation $M_{\rm pl}^2 = M_*^{n+2} V_n$, being V_n the volume of the compact dimensional space. Unfortunately, the presence of large extra dimensions does not necessarily provide a satisfactory resolution of the hierarchy problem, which reappears in the large ratio between M_* and the compactification scale $\mu_c = V_n^{-1/n}$. Following the idea that SM fields may live in a 3-brane, Randall and Sundrum (RS) presented an alternative solution with the shape of a gravitational condenser: two branes of opposite tension (which gravitationally repel each other) stabilized by a slab of anti-de Sitter (AdS) space [5]. In this model the extra dimension is strongly curved, and the distance scales on the brane with negative tension are exponentially smaller than those on the positive tension brane.

Two different scenarios might be analyzed with this solution. We could assume that our world is the brane with negative tension, so as to obtain a correct ratio between the Plank and weak scales without the need to introduce a large hierarchy between M_* and μ_c [5]. Alternatively, if the visible brane is the one with positive tension, the configuration does not solve the hierarchy problem. However, the second brane can be moved to infinity and Newton's law is correctly reproduced on the brane–world in spite of the non-compact extra dimension [6]. In both cases, Poincaré invariance on the 3-brane was assumed [7]. Much of the recent work in this area has been devoted to lift this restriction, in order to see under what conditions it is possible to obtain the standard Friedmann-Robertson-Walker cosmological scenario on the visible brane [8]. In this Brief Report we shall show that the spacetime obtained in [5] has the structure of a wormhole. We shall also show here that any 5-dimensional non–factorizable geometry (i.e., one in which the components of the metric tensor depend on the coordinate of the compactified extra dimension) is accompanied by unavoidable violations of the null energy condition. Furthermore, we shall see that this is a direct consequence of the fact that within these models the visible brane is located at the throat of the wormhole. After studying these relations between wormholes and the phenomenology of RS spacetimes, **we discuss the mechanism of gravitational trapping for a scalar particle in a general static spacetime with diagonal 3-metric**. Afterwards we work out a new solution of Einstein's equation given by two slices of AdS spacetimes glued together at the location of the branes (one of which is at the wormhole's throat). Some features of the cosmology of this systems are presented. We close with a discussion.

Do we live in a wormhole throat? Morris and Thorne showed [9] that the basic feature of a wormhole is that the "flaring-out condition" must be satisfied at its throat. A throat is a closed spatial hypersurface such that one of the two future-directed null geodesic congruences orthogonal to it is just beginning to diverge on or near the surface. Stated mathematically, the expansion $\theta_{\pm}$ of one of the two orthogonal null congruences vanishes on the surface: $\theta_{+} = 0$ and/or $\theta_{-} = 0$, and the rate-of-change of the expansion along the same null direction ($u_{\pm}$) is positive-semi-definite at the surface: $d\theta_{\pm}/du_{\pm} \geq 0$ [10]. Before showing that the solution given by RS [5] can be interpreted as part of a 5–dimensional wormhole with its throat located on the negative tension brane, we recall the set up of the RS model.

Let us start from the 5-dimensional action,

$$S = S_{\rm gravity} + S_{vis} + S_{hid}, \tag{1}$$

where

$$S_{\rm gravity} = \int d^4x \int_0^{\pi r_c} dy \sqrt{G}\, \{-\Lambda + 2M_*^3 R\}, \tag{2}$$

$$S_{\rm vis} = \int d^4x \sqrt{-g_{\rm vis}}\, \{\mathcal{L}_{\rm vis} - V_{\rm vis}\}, \tag{3}$$

and

$$S_{\rm hid} = \int d^4x \sqrt{-g_{\rm hid}}\, \{\mathcal{L}_{\rm hid} - V_{\rm hid}\}. \tag{4}$$

Here R stands for the 5-dimensional Ricci scalar in terms of the metric G_{AB}, and Λ is the 5-dimensional cosmological constant. The coordinate y is the extra dimension, and its range is $[0, \pi r_c]$, where r_c is the compactification radius. RS work on the space $S^1/\mathbf{Z_2}$. Eqs.(3) and (4) represent the action of 3-branes located at the orbifold fixed points at $y = 0, \pi r_c$. From the Lagrangians in (3) and (4) it has been separated out a constant vacuum energy which acts as a gravitational source even in the absence of particle excitations. In what follows we shall drop the $\mathcal{L}$ terms since they are not relevant in determining the classical 5-dimensional metric in the ground state. The Einstein equation for the above action reads,*

$$\begin{aligned}\sqrt{-G}\,(R_{AB} - \frac{1}{2}G_{AB}\,R) = -\frac{1}{4M_*^3}\,[\Lambda\,\sqrt{-G}\,G_{AB} \\ + V_{\rm vis}\,\sqrt{-g_{\rm vis}}\;g_{\mu\nu}^{\rm vis}\;\delta_A^{\mu}\,\delta_B^{\nu}\,\delta(y-y_0) \\ + V_{\rm hid}\,\sqrt{-g_{\rm hid}}\;g_{\mu\nu}^{\rm hid}\;\delta_A^{\mu}\,\delta_B^{\nu}\,\delta(y)].\end{aligned} \tag{5}$$

The line element

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^{\mu}dx^{\nu} + dy^2, \tag{6}$$

*Capital Latin indices run from 0 to 4 and refer to the entire 5-dimensional spacetime, Greek indices run from 0 to 3 and refer to the brane sub-spacetime; Latin indices from the middle of the alphabet $(i, j, k, \ldots)$ run from 1 to 4 and refer to constant t slices; Latin indices from the beginning of the alphabet $(a, b, c, \ldots)$ will run from 1 to 3 and will be used to refer to the 3-brane. As usual, $\eta_{\mu\nu}$ stands for the four dimensional Minkowski spacetime.

will be a solution of Eq.(5) provided that the boundary and bulk cosmological terms are related by a single scale k,

$$V_{\rm hid} = -V_{\rm vis} = 24\, M_*^3\, k, \quad \Lambda = -24\, M_*^3\, k^2. \tag{7}$$

We turn now to the analysis of the null geodesic congruences. We recall that in any static 5-dimensional spacetime every focusing (defocusing) of null geodesic congruences starts at a closed 3-dimensional hypersurface of maximal (minimal) area that, without loss of generality, can be located within a single constant-time spatial slice [11]. To describe the behavior of the brane-worlds, we conveniently adopt a Gaussian normal coordinate system in the neighborhood of each brane. We shall denote the 3-dimensional hypersurface swept out by each brane by Σ. Let us introduce a coordinate system $x_\perp^a$ on Σ. Next we consider all the geodesics which are orthogonal to Σ, and choose a neighborhood N around Σ so that any point $p \in N$ lies on one, and only one, geodesic. The first three coordinates of p are determined by the intersection of this geodesic with Σ. The full set of spatial coordinates is then given by $x^i = (x_\perp^a; \ell)$, wherein the hypersurfaces Σ under consideration are taken to be at $\ell = 0$, so that the spacetime metric reads

$$-e^\phi dt^2 + {}^{(4)}g_{ij}\, dx^i dx^j \;=\; -e^\phi dt^2 + {}^{(3)}g_{ab}\, dx^a dx^b + d\ell^2, \tag{8}$$

with ϕ the redshift function. The extrinsic curvature of each 3-surface is defined by

$$K_{ab} = \frac{1}{2}\,\frac{\partial g_{ab}}{\partial \ell}. \tag{9}$$

If we compute the variation in the area of Σ obtained by pushing the surface at $\ell = 0$ out to $\ell = \delta\ell(x)$ we get

$$\delta A(\Sigma) = \int \sqrt{{}^{(3)}g}\; {\rm tr}(K)\; \delta\ell(x)\; d^3x. \tag{10}$$

Since this expression must vanish for arbitrary $\delta\ell(x)$, the condition for the area to be extremal is simply ${\rm tr}(K) = 0$. In the case of the RS model described by Eq.(6) the extrinsic curvature is given by

$$K_{ab} = -k\; {\rm sg}(y)\; e^{-2k|y|}\; \eta_{ab} \tag{11}$$

Straightforward calculations show that the 3-sections located at $y = 0$ and $y = \pm\pi r_c$ are extremal. For the area to be minimal the additional constraint $\delta^2 A(\Sigma) \geq 0$ is required. Equivalently,

$$\frac{\partial\, {\rm tr}(K)}{\partial \ell} \geq 0. \tag{12}$$

For the RS spacetime we get

$$\frac{\partial {\rm tr}(K)}{\partial y} = -3\, k\, \delta(y), \tag{13}$$

then it is easily seen that the brane located at $y = 0$ represents a hypersurface of maximal area. The 3-surface at $y = \pm\pi r_c$ is instead a hypersurface of minimal area, *i.e.* a wormhole throat. Note that, due to the double orbifolding, the two different sides of the throat are actually topologically identified. Consequently, the RS model can be thought of as "half a wormhole".

It can be shown that any extension of the standard 4-dimensional world using a compactified non–factorizable dimension will posses hypersurfaces of maximal and minimal area. This assertion follows from the fact that the area of the 3-branes depends only on the extra coordinate. Then, as a function of the extra coordinate it must have an absolute maximum and an absolute minimum in the finite closed interval, and so forth must violate the null energy condition. Clearly, this is not the case with an infinite extra dimension [12].

Putting all this together, the huge hierarchy between the gravitational and weak scales seems to be the result of the change of the scales throughout the null geodesic congruence of a compactified dimension.

Gravitational trapping. As we mentioned above, in the RS framework nongravitational fields must be completely confined to the brane by some mechanism. On the other hand, the zero mode of gravity is localized on the brane and higher modes propagate freely in the extra dimension. Several confining mechanisms have been proposed in the past. For instance, in [13] it was shown that a potential well (originating in the nonlinearities of the eqs. of motion of a scalar field) can force scalar and spin 1/2 particles to live on the brane. Later, it was shown in [14] how massless gauge fields can be localized on a brane due to the dynamics of the gauge field outside the wall. Here we explore the possibility that particles can be trapped on the 3-brane by the sole action of gravity. This idea has been analyzed before in [15–18]. We will partially follow the work by Visser [15]. To see how the gravitational trapping works in our geometric approach, we shall briefly discuss the localization of a massless scalar field in 5-dimensional static spacetimes. The equation of motion is given by

$$\partial_A(\sqrt{G}\, G^{AB}\partial_B\Psi) = 0. \tag{14}$$

In order to solve Eq.(14), we assume that the redshift function depends only the extra coordinate, and that the metric on the brane-world is diagonal. This assumption implies that $\mathrm{tr}(K) \equiv \partial_\ell g^{^{(3)}}/2\, g^{^{(3)}}$, where ${}^{(3)}g$ is the determinant of the 3-metric. Moreover, we adopt for Ψ the following *Ansatz*:

$$\Psi = e^{-ip_\mu x^\mu}\, \psi(\ell), \tag{15}$$

(with $p_\mu p^\mu = -m^2$) which represents a travelling wave in the four dimensional submanifold. Using the change of variables $\psi(\ell) = u(\ell)\exp\left(-1/2\int^\ell\{\phi'(z) + \mathrm{tr}[K(z)]\}\, dz\right)$, one gets an equation for $u(\ell)$

$$u''(\ell) + \left\{e^{-2\phi}m^2 - \frac{1}{4}[\phi' + \mathrm{tr}(K)]^2 - \frac{1}{2}\phi'' - \frac{1}{2}[\mathrm{tr}(K)]'\right\} u(\ell) = 0. \tag{16}$$

By means of the transformations $z = z(\ell)$ and $u(z) = \gamma(z)\sigma(z)$, this equation can be re-written as a one-dimensional Schrödinger-like equation

$$\frac{d^2\sigma}{dz^2} + V(z)\ \sigma = -m^2\sigma, \tag{17}$$

with

$$V(z) = \frac{1}{\gamma}\frac{d^2\gamma}{dz^2} + \frac{d(\ln\gamma)}{dz}\frac{z''}{(z')^2} - \frac{[\phi' + \mathrm{tr}(K)]^2 + 2\phi'' + 2[\mathrm{tr}(K)]'}{4\,(z')^2}, \tag{18}$$

and $z(l) = \int e^{-\phi} dl$, $\ln\gamma = 1/2 \int \phi' e^{\phi} z' dl$ (the prime denotes derivative w.r.t ℓ). The potential $V(z)$ will determine whether the scalar particle is localized on the brane, and will also yield the 4-dimensional mass spectrum. Let us remark that this equation is valid for any static model in which the 3-metric is diagonal (e.g. the "Visser gauge" can be obtained by fixing $K_{ab} = 0$ [15]). In the specific case of the RS model we have

$$z(\ell) = \frac{1}{k}\mathrm{sg}(\ell)(e^{k|\ell|} - 1), \qquad \gamma(z) = (1 + k|z|)^{-1/2}, \tag{19}$$

and

$$V(z) = \frac{3}{2}k\,\delta(z) - \frac{15}{4}\frac{k^2}{(1 + k|z|)^2}. \tag{20}$$

The zero-mass solution takes the form $\sigma_0(z) = k^{-1}(1 + k|z|)^{-3/2}$. As in the case of gravitons Eq. (17) gives rise to a tower of continuum states with $m^2 > 0$ [6]. A general discussion on the mass spectrum can be found in [19]. The analysis carried out here in terms of the extrinsic curvature and the redshift function can be straightforwardly generalized to nonzero spin fields. The case of particles with spin 1, 1/2 and 3/2 in the RS model was studied in [19].

The cosmological solution. We shall now construct a solution of Eq.(5) by connecting "incomplete" AdS spaces according to the technique introduced by Visser [20], we shall take two copies of 5-dimensional AdS spacetimes and join them with the junction condition formalism [21]. Namely, first remove from each AdS spacetime identical regions of the form $\mathcal{R} \times \Omega$, where Ω is a 4-dimensional compact spacelike hypersurface and $\mathcal{R}$ is a timelike straight line. Identify now these two incomplete spacetimes along the timelike boundaries $\mathcal{R} \times \partial\Omega$. The resulting spacetime $\mathcal{M}$ is geodesically complete and posses two AdS asymptotic regions connected by a wormhole. The throat of the wormhole is just the junction $\partial\Omega$ at which the two original AdS spacetimes are identified.

For definiteness, the coordinates on $\partial\Omega$ can be taken to be the angular variables $(\chi,\ \theta,\ \phi)$ which are always well defined, up to an overall rotation, for a spherically symmetric configuration. We shall refer again to a Gaussian normal coordinate system in the neighborhood of the throat,

$$ds^2 = -(1 - \lambda r^2)\,dt^2 + dy^2 + r^2\,d\Omega_3^2, \tag{21}$$

where

$$\frac{dr}{dy} = \sqrt{1 - \lambda r^2}, \tag{22}$$

$\lambda = \Lambda/24M_*^3$, r and t denote the values of the corresponding AdS coordinates, and $d\Omega_3^2$ stands for the line element of the three sphere.

Associating positive values of G_{AB} to one side of the throat and negative values to the other side, without loss of generality the metric in the neighborhood of the brane can be written as,

$$G_{AB} = \Theta(y - y_0)\ G^{+}_{AB} + \ \Theta(-y + y_0)\ G^{-}_{AB}. \tag{23}$$

At the boundary of the two regions, the energy momentum tensor can be expressed in terms of the jump of the extrinsic curvature $\mathcal{K}^A_B = K^{A\,+}_B - K^{A\,-}_B$,

$$K^{\pm}_{AB} = \frac{1}{2}\frac{\partial G_{AB}}{\partial y}\bigg|_{y=y_0^{\pm}} = \frac{1}{2}\frac{\partial G^{\pm}_{AB}}{\partial y}. \tag{24}$$

At this stage, it is convenient to introduce the coordinate which denotes the proper time as measured along the surface of the wormhole throat τ. Then, for the dynamic case, the geometry is uniquely specified by a single degree of freedom, the radius of the throat $a(\tau)$. Across the boundary, the jumps of the extrinsic curvature are given by [22] (dots indicate derivative with respect to τ)

$$\mathcal{K}^{\tau}_{\tau} = \frac{2\,(\ddot{a} - \lambda a)}{\sqrt{1 - \lambda a^2 + \dot{a}^2}}, \tag{25}$$

and

$$\mathcal{K}^{\chi}_{\chi} = \mathcal{K}^{\theta}_{\theta} = \mathcal{K}^{\phi}_{\phi} = \frac{2}{a}\sqrt{1 - \lambda a^2 + \dot{a}^2}. \tag{26}$$

Replacing in Eq.(5) we obtain,

$$V_{\rm vis}\, g^{\rm vis}_{\mu\nu}\ \delta^{\mu}_{A}\,\delta^{\nu}_{B} \ = 4M_*^3\,[\mathcal{K}_{AB} \ - {\rm tr}(\mathcal{K})\, g^{\rm vis}_{\mu\nu}\ \delta^{\mu}_{A}\,\delta^{\nu}_{B}\,], \tag{27}$$

or equivalently,

$$V_{\rm vis} \ = -24M_*^3\ \frac{\sqrt{1 - \lambda a^2 + \dot{a}^2}}{a}, \tag{28}$$

$$V_{\rm vis} \ = -8M_*^3\ \frac{2 - 3\lambda a^2 + 2\dot{a}^2 + \ddot{a}a}{a\sqrt{1 - \lambda a^2 + \dot{a}^2}}. \tag{29}$$

Now, Eq.(5) together with the covariant conservation of the stress energy determine the classical dynamics of the system [23],

$$\dot{a}^2 - \left[\frac{\Lambda}{24M_*^3} + \left(\frac{V_{\rm vis}}{24\,M_*^3}\right)^2\right]\ a^2 = -1. \tag{30}$$

After integration we get,

$$a = a_0\ \cosh\{a_0^{-1}\,\tau\}, \tag{31}$$

where $a_0 = (\lambda + (V_{\rm vis}/24M_*^3)^2)^{-1/2}$ is the minimum radius of the brane. Contrary to what happens in the RS model, if $\lambda < 0$ and $|\lambda| \geq (V^2_{\rm vis}/24M_*^3)$, a has no real solution. However, if either $\lambda > 0$, or $\lambda < 0$ and $|\lambda| < (V^2_{\rm vis}/24M_*^3)$, the wormhole does not collapse but it is bounded by a minimum size a_0 [24]. The expression for the Hubble constant in this solution is

$$H^2 = -\frac{1}{a^2} + \frac{\Lambda}{24M_*^3} + \left(\frac{V_{\rm vis}}{24M_*^3}\right)^2 . \quad (32)$$

Its behavior is similar to the one obtained for the RS models, i.e., $H \propto V_{\rm vis}$ rather than $V_{\rm vis}^{1/2}$ as in conventional cosmology. However, as the world approaches to the minimum size the expansion tends to zero. Furthermore, unlike the standard spherical Robertson-Walker case this world experience an everlasting expansion.

To find out the behavior of the surface of maximal area one must redefine the jump in the extrinsic curvature $\mathcal{K}_B^A = K_B^{A\,-} - K_B^{A\,+}$, and afterwards repeat *mutatis mutandis* the entire computation. It is easily seen that except for the sign of the vacuum energy which results flipped ($V_{\rm hid} > 0$), one can just replace the sub-index vis by hid in every expression. The expansion rate of the hidden brane is then given by,

$$\left(\frac{\dot{b}}{b}\right)^2 = -\frac{1}{b^2} + \frac{\Lambda}{24M_*^3} + \left(\frac{V_{\rm hid}}{24M_*^3}\right)^2 , \quad (33)$$

where b is the radius of the hidden brane.†

Discussion. Let us suggest a possible way to understand the hierarchy problem in the scenario presented here. To preserve the stability of the system, we need to impose a constraint on the relation between δa and δb, the increments in the visible and hidden radii during an interval of time as measured by synchronized clocks on each brane. If this constraint is given, the strategy for solving the hierarchy problem is as follows. First one has to select δa and δb so as to obtain the correct relation between the mass scales, then Eq.(22) will fix λ to obtain that hierarchy. The value of the present Hubble parameter together with Eq. (32) set $V_{\rm vis}$. Finally, from Eq. (33) one can determine the value of $V_{\rm hid}$.

Our solution can be regarded as a 5-dimensional spacetime (with two brane worlds located at fixed coordinates in the internal dimension) propagating through a static AdS background manifold. A more general case would be given by lifting the static restriction on the background. This may have some important consequences. Consider for instance an expanding 3-dimensional spacelike hypersurface embedded in a 4+1-dimensional spacetime. At each point on the hypersurface there are two null vectors orthogonal to the hypersurface, and associated to these two null vectors there exist two null geodesic congruences that are well defined on an open neighborhood of the hypersurface. While the surface propagates along the null geodesic congruence of the extra dimension the mass scale changes. Hence, if the speed of the surface (along the internal dimension) is comparable to its expansion rate, the signals that

†Let us mention that the dynamics of a domain wall in the RS-AdS_5 bulk was analyzed in (Kraus Ref. [8]). With the help of the same formalism the author found a rather similar solution where the bulk cosmological constant is not exactly cancelled by the one on the brane.

travel into the world may display an additional redshift or blueshift. In the former case the observers on the brane would interpret its motion as an inflationary scenario.

We have proved above that the geometrical structure of the manifold given by RS corresponds to half of a wormhole geometry, and its mirror image. We also showed that any geometry with a compactified non-factorizable dimension will inevitably posses hypersurfaces of maximal and minimal area. Now, the dependence of the extrinsic curvature of the RS solution with the extra coordinate, given by Eq.(11), coincides with that of the warp factor in the RS metric. In turn, this factor is responsible for the renormalization of the mass that solves the hierarchy problem in the RS model. This coincidence may be suggesting a relation between the extrinsic curvature and the change in the mass scale. We will pursue this line of research somewhere else.

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