Massive Vectormesons and Gauge Theory

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Abstract

We show that the requirements of renormalizability and physical consistency imposed on perturbative interactions of massive vectormesons fix the interactions essentially uniquely i.e. the physical particle content together with renormalizability determines the theory. In particular physical consistency requires the presence of at least one additional (scalar) degree of freedom which was not part of the originally required physical particle content. In its simplest realization (probably the only one) this is a scalar field as envisaged by Higgs but without the Higgs condensate. The result agrees precisely with the usual quantization of a classical gauge theory by means of the Higgs mechanism. Hence the principles of local quantum physics via Bohr's correspondence explain the gauge principle as a selection principle among the many (semi)classical coupling possibilities. The statement that the renormalization and consistency requirements of spin=1 QFT lead to the gauge theory structure may be viewed as the inverse of 't Hooft's famous renormalizability proof in (quantized) gauge theories. We also comment on an alternative ghostfree formulation which avoids "field coordinates" alltogether and is analogous to a perturbative version of the d=1+1 formfactor program.

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1 Introduction

In the development of understanding of the renormalization aspects for spin=1 interactions, the classical concepts of gauge and fibre bundles have played a crucial role. Without the closely related Higgs mechanism it would be hard to imagine, how in the stage of QFT at the end of the 60^{ies}, the incorporation of the electro-weak interaction into the framework of renormalizable field theory could have been achieved. In the present article we will demonstrate that, although the Higgs mechanism via Higgs condensates within the setting of gauge theories is an efficient mnemotechnical device for the rapid construction of the physical result, it has no direct intrinsic physical content. Contrary to a widespread opinion within the physicists community, there is no "gauge principle" in the physical sense or an intrinsic physical meaning for "Higgs condensates".

We will show this by constructing the same physical results for interacting massive vectormesons in a quite different way which does not rely on the above concepts. In our approach based on the well-known real-time causal formulation of perturbative QFT, the renormalizability is the basic input requirement and the uniqueness and its gauge appearance in terms of (quasi)classical approximations are the results; with other words we would phrase the famous 't Hooft statement that gauge structure implies renormalizability the other way around. Since quantum theory is more fundamental than classical, this brings interacting vectormesons into harmony with Bohr's correspondence principle: it is the quantum theory which tells the classical which possibility among many couplings involving vectors and lower spin fields) it has to follow, namely the gauge invariant one.

We would not have gained much, and a cynic might claim that we have replaced one mystery (the gauge mystery) by another one (the renormalization mystery), but fortunately, we have some slightly more tangible results to offer. Our method brings into the open the long looming suspicion that the appearance of additional physical degrees of freedom (the alias Higgs particle but without vacuum condensates) is a necessity, following from perturbative consistency up to second order (no claim outside of perturbation theory is made!)[1]. In addition it suggests strongly that the physics of zero mass theory should be approached from massive vectormesons, the latter being conceptually (but not analytically) simpler. So as it happens often in physics, the new aspect does not so much lie in the physical results as such, but rather in the novel way in which they are obtained and in the interpretation associated with this derivation.

Ever since theories in which vectormesons or higher spin particles became physically relevant in the late 50^{ies}, there were two points of views to deal with such problem: to start from the Wigner particle picture and stay close to particles and scattering theory, or to quantize classical gauge field theory (canonically or by functional integrals) and to make contact with (infra)particles at a later stage. In fact Sakurai,

[1] Unfortunately this is not a structural theorem as e.g. the Goldstone theorem or the well-known necessary appearance of particle creation in scattering processes which have a nontrivial elastic part in d=3+1, but only a perturbative model dependent statement.

who introduced the term "Yang-Mills theory" [33], and who wanted to use quantized classical field theory for the description of massive vectormesons in strong interactions. encountered difficulties to reconcile the two points of view. In most of his contributions he therefore took a phenomenological non-geometric point of view. This was particularly advisable since, as a result of some criticism of Pauli, the use of the Yang-Mills model for the description of massive vectormesons was cast into question.

The discovery of the electro-weak theory and the renormalization of Yang-Mills theories [24] led to a drastic change of that picture. Whereas the first point of view, which tried to make the gauge principle for vectormesons pragmatically more palatable, found some protagonists (Lewellyn-Smith [30], Bell [4], Cornwall et al.[10]) mainly in the early days of gauge theories, the gauge or Yang-Mills point of view enjoyed general popularity and became the predominant one, irrespective of whether it could be derived from a more particle dominated on-shell approach or not.

The main reason in favor of the gauge point of view was not only its success in describing problems of quantum matter coupled to external electromagnetic fields as well as its esthetical mathematical appeal (which contributed largely to the acquirement of sophisticated knowledge which especially the younger generation of physicists have about fibre-bundles and to some degree also to the popularity it enjoys in mathematical circles), but rather the way in which, with some hindsight, it led physicists, like a guardian angel, into the allegedly safe harbor of renormalizable perturbation theories. It is probably not exaggerating to say that without its lead, Veltman and 't Hooft would not have been able to find the appropriate path into the problem of spin=1 renormalizability which culminated in 't Hooft's famous demonstration.

This created, at least for some physicist, the, as we will show in this paper, somewhat misleading impression that in addition to causality and spectral properties the general framework of QFT needs another principle, namely the gauge principle, for dealing with interactions. This way of thinking led eventually to the Higgs mechanism for the generation of vectormeson masses from spontaneously broken gauge invariance through "fattening via eating Goldstone bosons".

In this paper we propose a more intrinsic alternative framework which produces the same physical (gauge invariant) results without relying on the "guardian angle of gauge theory" but instead solely on the principles of (local quantum physics) (LQP). It uses a simplified free form of the BRS formalism [3] for a cohomological extension of the Wigner representation theory as a mathematically less formal operator substitute for the Faddeev-Popov formal arguments [21] on functional representations. This is motivated by renormalizability within the framework of causal perturbation (for all practical purposes equivalent to the Lagrangian approach); our contention is that with this proviso renormalizability of interacting massive vectormesons uniquely fixes the theory (including the necessity of containing additional Higgs-like matter content) in such a way that its (semi)classical and zero mass limit "explains" the quantum origin of the classical gauge concept. We put the word "explain" in quotation mark, because it basically reduces it to another not fully understood concept namely "renormalizability", with which we only feel more comfortable because it occurs on the more fundamental level of local quantum physics where there is still a future chance for a more profound understanding. It still possesses unsatisfactory (since nonintrinsic) formal unphysical features (ghosts) in intermediate steps. A throughout physical formulation requires to leave the framework of Feynman's perturbation in favor of a more on-shell formalism similar to that of (Kramers-Kronig) dispersion theory and is not yet available (section 5)

Our approach of course does not contradict the gauge approach if, together with the Higgs mechanism, one considers the gauge formalism as a mnemotechnical rule which facilitates the construction of that unique (fixed by the observable particle content) massive selfinteracting mesons within the renormalizable class (which would have been unique even without the rule). If the spin=1 theory is fixed already by renormalizability, it does not harm to add an additional computational rule which (at least for a majority of physicist) allows to speed up the calculation of the physical objects and creates additional confidence by making the relation with classical Maxwell theories more manifest. The gauge point of view becomes potentially harmful only if one takes it literally as a statement of intrinsic physical meaning and thus substitutes principles of local quantum physics by esthetical requirements from the theory of fibre bundles. We do not negate the crucial historical role which gauge theory via functional integrals and Faddeev-Poppov determinants played in the formulation of renormalization theory for spin=1 objects. However as mentioned already, today we know how to reformulate this directly as a cohomological extension of the spin=1 Wigner particle theory in order to lower the formal operator dimension of free fields from their physical value 2 to the formal value 1 without loosing their pointlike nature. As will be seen, this allows an alternative formulation for interacting massive vectormesons.

Most physicist have anyhow tacitly accepted the fact that there is only one renormalizable coupling of massive vectormesons and that consistency in second order requires the presence of other physical degrees of freedom. The present treatment puts this into evidence by dismissing the "gauge principle" as well as the Higgs mechanism as superfluous in favor of shifting more to the "renormalizability principle" since one uses the latter anyhow for the lower spins. The only truly intrinsic property of the massive case is the Schwinger charge screening as converted into a rigorous theorem by Swieca. We prefer to talk about charge liberation in the massless limit (taking the massive spin=1 theory as a reference) instead of charge screening of the massive theory (with the charged massless theory as a reference).

In more recent times a similar point of view has been taken up by Aste, Scharf and one of the authors (M.D.) [2]. We feel however that their "free perturbative operator gauge invariance", which is a pure quantum formulation of gauge invariance, is not really going sufficiently to the physical heart of the matter in pure massive theories. We will give preference to a perturbative approach of higher spin interactions using only the principles within local quantum physics. According to our viewpoint the above mentioned methods should be taken for what they are: technical shortcuts which allow to retain the standard perturbative formalism, but not as additions to the physical principles of local quantum physics. This may be somewhat surprising since the use of common sense of classical field theory would suggest that the number of possibilities of admissable polynomial couplings should *increase with spin and internal multiplicities.* This indeed applies up to spin=1/2, but beyond these low spins, the consistency requirements together with renormalizability takes over and works the other way around; namely to lower the number of possibilities; and for s>1 we do not even know if there are any renormalizable theories in the sense of this paper[2] .

To the extend that the reader considers those requirements on classical theories which follow from the more fundamental quantum field theory (imposing renormalizable perturbation) as more basic than those imposed by differential geometry, the gauge principle of classical theory is "explained" in terms of massive vectormeson renormalizability in QFT[3]. In the following we collect the arguments underlying this viewpoint. For pedagogical purpose and for reasons of brevity we exemplify our points in a particular class of theories which are the simplest about which both points of views can be expected to be applicable, namely selfinteracting models of massive vectormesons.

We want to stress that various aspects of this viewpoint are not new. As was already mentioned Schwinger, in a little noticed paper and some more extensive published lecture notes [40] thought about a massive phase in QED through the mechanism of charge screening but without (Higgs) vacuum condensates. In order to make his nonperturbative ideas of (Maxwellian interaction= renormalizable) massive vectormesons more palatable, he invented the 2-dim. Schwinger model[4]. Indeed as we know nowadays, the charge screening mechanism is intrinsic [45], whereas the Higgs-condensate mechanism is a mnemotechnical device going with a certain calculational approach which easily adapt to our often "classical" brains, but there is nothing physically intrinsic in those condensates, they rather depend totally depend on the use of the standard prescription. As already mentioned in more recent times a direct presentation of the massive case within the causal perturbation method without the spontaneous symmetry terminology has been given by the University of Zürich group [2] (Dütsch, Scharf and Aste). We should mention that the point of view advocated here (but perhaps not the detailed facts) is probably known to some people within the community of LQP. It also has been mentioned in the setting of Schwinger's work and of general quantum field theory by one of the authors (B. S) [35]. Some aspects of it appeared in the work of Grigore [22].

Our presentation is organized as follows. In section 2 we review the causal approach for massive vectormesons and its simplification as a result of the existence of a natural Fock reference space supplied by scattering theory. Our presentation uses a simplified quadratic BRS formalism. As a justification for the introduction of ghost fields we then describe the apparent clash between renormalizability and the operator dimension dim$A = 2$ of the free vectormeson operators in the usual causal setting and its

[2] In a recent paper Scharf and Wellmann [46] have shown that there exists no renormalizable theory for s=2 which satisfies (the free) perturbative (operator) gauge invariance.

[3] The minimal electromagnetic substitution law is a physical principle for external electromagnetic fields and has, outside of canonical quantization, no direct consequence for s=1 massive particle quantum fields.

[4] In fact in the Lowenstein-Swieca treatment of this model there is a chiral condensate (coming from the θ-degeneracy), but after the dust has settled, the physical content is described in terms of a massive free field only.

resolution via cohomological extension i.e. the ghost stuff (section 3). In the fourth section we show that the consistency requirements of spin=1 interactions are so strong that a list of the lowest dimension interpolating physical fields (the ones which we want to describe in our model as observable particles) not only fixes the form of these fields in terms of the auxiliary "classical" unphysical fields (which may be introduced already on the level of Wigner representation theory), but it also determines the form of the interaction density including the necessity of the perturbative presence of a physical degree of freedom, the alias Higgs particle, but now without its vacuum condensate. The argument is interesting in two aspects. On the one hand it shows that the particle content via the associated interpolating fields limits the possibilities of interaction for spin=1 particles much more severely than that of lower spin particles. Secondly it demonstrates the inconsistency of perturbation theory within the LQP setting without the appearance of an additional physical *quantum* object, which by minimality assumption is a scalar particle and agrees with the one described by the Higgs field.

In the fifth section we observe that general structural theorems of QFT in theories with asymptotic completeness assure the existence of LSZ-type power series in terms of the *physical Fock space operators* which in the previous BRS-like description were composites involving unphysical fields (ghosts). However we do not know an iterative law for the perturbative representation of the coefficient functions which parallels the standard iteration which uses time ordering or retarded products (for the fields). The general LSZ-like identification of coefficient functions of local fields involves multiple commutators of the local field [39] with the incoming free field (generalized formfactors). This scenario still holds, but its specialization to perturbation theory of physical vectormesons does not lead to the standard off-shell Feynman rules in terms of the physical incoming fields. The reason for this complication, which prevents the interchange of computations with the descend to the physical fields, is that the Wick-basis used for writing the latter in terms of linear combinations of composites (including ghosts) is not a natural basis for the physical fields (i.e. the fields which commute with the BRS charge Q).

Section 6 contains some remark of where one has to look for, if one wants to have a ghostfree formulation. In view of the fact that the previous sections have made clear that ghosts behave in some sense like catalyzers[5] in chemistry, this is not an academic problem but really goes to the root of understanding of renormalizability for higher spin where the standard causal approach breaks down. We are led to believe that such an approach must bypass the transition operator $S(g)$ and be on-shell i.e. directly deal with the on-shell S-matrix and multiparticle formfactors of physical fields. Since such a radical new formulation goes by far beyond the more modest goals set in this paper, we propose to take up these problems in a future publication.

Recently there has been an approach to understand the local observable *-algebras of nonabelian gauge theories without emphasizing the particle content, which in the LPQ framework is anyhow part

[5]First one introduces them in order to lower the operator dimension of W to four by decreasing the dimension of the vectorpotential (+ghosts) from two to one, and then in the cohomological descend the physical operator dimensions increase again while the ghosts are getting eliminated.

of the separate (in the algebraic approach) more difficult representation theory associated with states on the algebra [12]. This approach was specially aimed at the zero mass theories with infrared problems, because the method does not require the existence of an adiabatic limit. In that case one cannot use the scattering theory of the physical particles and the BRS operators cannot be written as bilinear operators in free fields but they receive interacting contributions in every order. Therefore one has to face the more difficult problem of a changing position of the physical cohomology space inside the extendend space depending on the perturbative order. Whereas in the present case the conceptual (but not necessarily analytical) simplicity of the massive case (existence of reference Fock space defined by scattering theory) plays an important role and the emphasis lies more on the particle side, the other approach relies on the dichotomy of algebras and states and the fact that the *local* nets of observable algebras do not require the understanding of difficult infrared problems; they are rather part of the difficult extraction of the particles characteristics from the local observable algebras. The present approach is in some sense inverse in that one starts with the observable particle content and grafts it on the existing BRS-extended framework of causal pertubations. The zero mass limit in our approach is conceptually complicated because it leads to charge liberation (the opposite of the Schwinger-Swieca charge screening) and the decoupling of the physical consistency (Higgs) degree of freedom. Those physical matter fields which will be charged in that limit cannot maintain their pointlike localization, rather it has to be semiinfinite spacelike (Mandelstam string-like). The linkage of these various phenomena generates the hope that by controlling those off-shell infrared problems which are necessary to implement this picture, one may actually get an insight into this (even in perturbation theory) notoriously difficult localization structure.

Since we only consider the present formalism as transitory on our way towards a completely ghostfree formulation, we did not try to invest much time in polishing our sometimes very messy pedestrian calculations.

2 Consistent perturbative construction of the S-matrix for massive gauge fields

The aim of this section is to construct the Stückelberg-Bogoliubov-Shirkov transition functional $S(g)$, which is the generating functional for the time-ordered products of Wick polynomials. As most functional quantities this object is not directly observable but it gives rise to fundamental physical observables as the S-matrix, formfactors and correlation functions of observable fields. The notation should not be misread as the S-matrix by which we always mean the scattering operator computed with the LSZ or Haag-Ruelle scattering theory. Our model is that of selfinteracting massive vectormesons. Our procedure is related to the one of Scharf, Aste and the first author [2], but similarly to a previous discussion by the other author [36] and to Grigore [22] as well as older articles as Lewellyn-Smith [30], we simply rely on physical consistency within the framework of local quantum physics and do not require such technical tools as "operator gauge invariance" although they tend to simplify calculations.

Using the Stückelberg-Bogoliubov-Shirkov-Epstein-Glaser method [6] [19] we make the following perturbative Ansatz for $S(\mathbf{g})$

$$S(\mathbf{g}) = \mathbf{1} + \sum_{n=1}^{\infty} \frac{i^n}{n!} \int d^4x_1 ... d^4x_n \, T_{j_1...j_n}(x_1, ..., x_n) g_{j_1}(x_1) ... g_{j_n}(x_n), \quad \mathbf{g} = (g_j)_{j=0}^{G}, \tag{1}$$

$g_j \in \mathcal{S}(\mathbf{R}^4)$, which is a formal power series in $\mathbf{g}$. The unknown $T_{j_1...j_n}$ are operator valued distributions [6]. They are constructed inductively by means of the following requirements (strongly influenced by the general Wightman-framework [44]):

(A) *Specification of the model in first order*: The first order expressions are the main input of the construction. They specify the model and must be local: $[T_j(x), T_k(y)] = 0$ for $(x-y)^2 < 0$. We assume that $T_0(x) \equiv W_0(x) \equiv W(x)$ is the physically relevant interaction density in Fock space (i.e. a Poincaré covariant scalar composite described by a Wick polynomial). $T_j \equiv W_j$, $j = 1, ..., G$ are auxiliary interactions. The interaction W_j is switched by the space-time dependent coupling "constant" $g_j \in \mathcal{S}(\mathbf{R}^4)$. The physically relevant S-matrix is obtained in the adiabatic limit: $g_0 \to$ const., $g_j \to 0$, $j = 1, ..., G$.

(B) *Permutation symmetry*: Due to the Ansatz (1) we may require permutation symmetry

$$T_{j_{\pi 1}...j_{\pi n}}(x_{\pi 1}, ..., x_{\pi n}) = T_{j_1...j_n}(x_1, ..., x_n), \qquad \forall \pi \in \mathcal{S}_n. \tag{2}$$

(C) *Causality*:

$$S(\mathbf{g}^{(1)} + \mathbf{g}^{(2)}) = S(\mathbf{g}^{(1)}) S(\mathbf{g}^{(2)}) \qquad \text{if} \qquad \cup_j \operatorname{supp} g_j^{(1)} \cap (\cup_j \operatorname{supp} g_j^{(2)} + \bar{V}_-) = \emptyset. \tag{3}$$

This requirement is equivalent to (see the appendix of [18]) [7]

$$T_{j_1...j_n}(x_1, ..., x_n) = T_{j_1...j_l}(x_1, ..., x_l) T_{j_{l+1}...j_n}(x_{l+1}, ..., x_n) \tag{4}$$

if

$$\{x_1, ..., x_l\} \cap (\{x_{l+1}, ..., x_n\} + \bar{V}_-) = \emptyset.$$

This means that $T_{j_1...j_n}(x_1, ..., x_n)$ is a (well-defined) time ordered product of $W_{j_1}(x_1)$, ..., $W_{j_n}(x_n)$. Hence we use the notation

$$T_{j_1...j_n}(x_1, ..., x_n) = T(W_{j_1}(x_1) ... W_{j_n}(x_n)) \tag{5}$$

Due to the induction with respect to the order n, the $T_{j_1...j_n}$ are uniquely fixed by causality up to the total diagonal $\Delta_n = \{(x_1, ..., x_n) | x_1 = x_2 = ... = x_n\}$. The extension of the $T_{j_1...j_n}$ to the total diagonal is nonunique. It is restricted by the following normalization conditions:

[6] For questions concerning domains we refer to [19]. For all operators which appear in sections 2 and 3 there exists a common dense invariant domain $\mathcal{D}$ and we restrict all operators to this subspace.

[7] The non-trivial part of this equivalence is that in the n-th order expression of (3) only special testfunctions appear, whereas (4) holds on $\mathcal{S}(\mathbf{R}^{4n})$.

(D) *Poincaré covariance*;

(E) *Unitarity*: $S(\mathbf{g})^{-1} = S(\mathbf{g})^*$ for $\mathbf{g} = (g_0, 0, ..., 0)$, g_0 real valued;

(F) *Scaling degree*: The degree of the singularity at the diagonal, measured in terms of Steinmann' scaling degree [42][8][8], may not be increased by the extension. This ensures *renormalizability by power counting* if the scaling degree (or 'mass dimension') of all W_j is ≤ 4. This degree is a tool which is related to Weinberg's power counting.

Additional normalization conditions must be imposed, if one wants to maintain further symmetries or relations [9], e.g. discrete symmetries (P,C,T), 'operator gauge invariance' (29-30) or the field equations of the interacting fields ((**N4**) in [12]), which can be obtained from the functional $S(\mathbf{g})$ (1) by Bogoliubovs formula (see sect.4).

The existence of the adiabatic limit restricts the extension additionally: for *pure massive* theories Epstein and Glaser [20] proved that, with correct mass and wave function (re)normalization the adiabatic limit of the functional $S(\mathbf{g})$ exists in the strong operator sense and it is this limit which we call 'S-matrix'. More precisely setting $\mathbf{g}_\epsilon(x) := (g_0(\epsilon x), 0, ...0)$ the limit

$$S_n\psi \equiv \lim_{\epsilon \longrightarrow 0} S_n(\mathbf{g}_\epsilon)\psi \tag{6}$$

exists $\forall\psi \in \mathcal{D}$, where $g_0 \in \mathcal{S}(\mathbf{R}^4)$, $g := g_0(0) > 0$ is the coupling constant and $S_n(\mathbf{g})$ (S_n resp.) denotes the n-th order of the functional $S(\mathbf{g})$ (S-matrix resp.). It follows that the S-matrix is unitary as an operator valued formal power series in Fock space [20]: $S = \sum_n S_n$, $S_n \sim g^n$, $S^*S = \mathbf{1} = SS^*$ on $\mathcal{D}$.

Due to this fact we solely consider models in which all fields are massive. In order that it makes physically sense to consider the S-matrix, we assume that there are no unstable physical particles as e.g. the W- and Z-bosons in the electroweak theory.

In *gauge theories* the crucial problem is the elimination of the unphysical degrees of freedom. In the S-matrix framework this problem turns into the requirement that the S-matrix induces a well-defined unitary operator on the space of physical states (this is discussed in detail below). We will see that this condition is very restrictive: it determines the possible interactions to a large extent.

Let us first consider the free incoming fields. We quantize the free gauge fields $(A_a^\mu)_a = 1, ..., M$ in Feynman gauge

$$(\Box + m_a^2)A_a^\mu = 0, \quad [A_a^\mu(x), A_b^\nu(y)] = ig^{\mu\nu}\delta_{ab}\Delta_{m_a}(x-y), \quad A_a^{\mu\,*} = A_a^\mu, \tag{7}$$

where Δ_m is the Pauli-Jordan distribution to the mass m. The representation of this *-algebra requires an indefinite inner product space. We, therefore, work in a Krein Fock space $\mathcal{F}$. We denote the scalar

[8] We adopt here the notion 'scaling degree' to operator valued distributions by using the strong operator topology. Note that the scaling degree of a Wick monomial agrees with its mass dimension.

[9] We consider symmetries and relations which are satisfied away from the total diagonal, due to the causal factorization (5) and the inductive assumption. Poincaré covariance (D) and unitarity (E) are of this type.

product by $(.,.)$ and A^+ is the adjoint of A w.r.t. $(.,.)$. Let J be the Krein operator: $J^2 = 1$, $J^+ = J$. Then the indefinite inner product $< ..., >$ is defined by

$$< a, b > \equiv (a, Jb), \qquad a, b \in \mathcal{F} \tag{8}$$

and $*$ denotes the adjoint with respect to $< .,. >$:

$$O^* \equiv JO^+J, \qquad < Oa, b > = < a, O^*b > \tag{9}$$

Let Q be an (unbounded) *-symmetrical nilpotent operator in $\mathcal{F}$

$$Q = Q^* \quad \text{(on the dense invariant domain } \mathcal{D}\text{)}, \qquad Q^2 = 0 \tag{10}$$

By means of $Q^2 = 0$ one easily finds that $\mathcal{D}$ is the direct sum of three, pairwise orthogonal (w.r.t. $(.,.)$) subspaces [25][27]

$$\mathcal{D} = \text{ran}\, Q \oplus (\ker Q \cap \ker Q^+) \oplus \text{ran}\, Q^+ \tag{11}$$

$$\ker Q = \text{ran}\, Q \oplus (\ker Q \cap \ker Q^+), \qquad \ker Q^+ = \text{ran}\, Q^+ \oplus (\ker Q \cap \ker Q^+) \tag{12}$$

In addition we assume

$$J|_{\ker Q \cap \ker Q^+} = \mathbf{1} \qquad \text{(positivity assumption)} \tag{13}$$

Then the $< ..., >$-product is positive definite on

$$\mathcal{H}_{\text{phys}} \equiv \ker Q \cap \ker Q^+ \tag{14}$$

and $\mathcal{H}_{\text{phys}}$ is interpreted as the *physical subspace* of $\mathcal{F}$. We denote the projectors on ran$Q$ ($\mathcal{H}_{\text{phys}}$, ran$Q^+$ rsp.) by P_- (P_0, P_+ rsp.)

$$\mathbf{1} = P_- + P_0 + P_+ \qquad (\text{on } \mathcal{D}) \tag{15}$$

Note $Q = P_-QP_+$. The positivity (13) and $Q = Q^*$ imply [27]

$$P_0 = P_0JP_0, \qquad J = P_0JP_0 + P_-JP_+ + P_+JP_- \tag{16}$$

and hence

$$P_0^* = P_0, \qquad P_-^* = P_+ \tag{17}$$

Let $S : \mathcal{D} \to \mathcal{D}$ be the (strong) adiabatic limit (6) of $S(g)$. We define

$$S_{ab} \equiv P_aSP_b, \qquad a, b \in \{-, 0, +\} \tag{18}$$

and obtain

$$P_aS^*P_b = (S_{(-b)\,(-a)})^* \tag{19}$$

For pedagogical reasons we introduce the matrix notation according to the decomposition (11)

$$J = \begin{pmatrix} 0 & 0 & P_- J P_+ \\ 0 & P_0 & 0 \\ P_+ J P_- & 0 & 0 \end{pmatrix} \tag{20}$$

$$S = \begin{pmatrix} S_{--} & S_{-0} & S_{-+} \\ S_{0-} & S_{00} & S_{0+} \\ S_{+-} & S_{+0} & S_{++} \end{pmatrix} \tag{21}$$

and

$$S^* = \begin{pmatrix} (S_{++})^* & (S_{0+})^* & (S_{-+})^* \\ (S_{+0})^* & (S_{00})^* & (S_{-0})^* \\ (S_{+-})^* & (S_{0-})^* & (S_{--})^* \end{pmatrix}$$

by means of (19).

An alternative definition of the physical states is

$$\mathcal{H}'_{\text{phys}} \equiv \frac{\ker Q}{\operatorname{ran} Q} \tag{22}$$

where the scalar product in $\mathcal{H}'_{\text{phys}}$ is defined such that the map

$$\mathcal{H}_{\text{phys}} \longrightarrow \mathcal{H}'_{\text{phys}} : \phi \longrightarrow [\phi] \tag{23}$$

is a pre Hilbert space isomorphism. ($[\phi]$ denotes the equivalence class of ϕ.) The first definition (i.e. $\mathcal{H}_{\text{phys}}$) has the advantage that the set of physical states is a *sub*space of the Krein Fock space $\mathcal{F}$, which has a clear particle interpretation. But $\mathcal{H}_{\text{phys}}$ is not Lorentz invariant (in contrast to $\mathcal{H}'_{\text{phys}}$). The change of its position inside the total space under L-transformations is of course a result of the lack of L-invariance of J.

Let $S^*S = \mathbf{1} = SS^*$ (on $\mathcal{D}$). We now discuss two different formulations of the physical consistency of the S-matrix:

(i) A consistent S-matrix theory requires

$$P_0 S^* P_0 S P_0 = P_0 = P_0 S P_0 S^* P_0 \quad \Longleftrightarrow \quad (S_{00})^* = (S_{00})^{-1} \quad \text{on} \quad \mathcal{H}_{\text{phys}}.$$

(ii) In the framework of the definition (22) of the physical states consistency means that S and $S^{-1} = S^*$ induce well-defined operators on the factor space $\mathcal{H}'_{\text{phys}}$ by the definition

$$[\mathcal{O}][\phi] \equiv [\mathcal{O}\phi], \qquad \mathcal{O} = S, S^* \tag{24}$$

This holds true iff

$$\mathcal{O} \ker Q \subset \ker Q \qquad \wedge \qquad \mathcal{O} \operatorname{ran} Q \subset \operatorname{ran} Q, \qquad \mathcal{O} = S, S^* \tag{25}$$

Due to $[\mathcal{O}]^* = [\mathcal{O}^*]$ the physical S-matrix $[S]$ is then unitary.

The following Lemma states that (i) is a truly weaker condition than (ii) and gives equivalent formulations of (ii).

Lemma 1: The following statements (a)-(g) are equivalent and they imply (h). But (h) does not imply the other statements if $Q \neq 0$.

(a) $SS^* = \mathbf{1} = S^*S$ and $S\ker Q \subset \ker Q$ (i.e. $S_{+-} = 0 = S_{+0}$).

(b) $SS^* = \mathbf{1} = S^*S$ and $[Q, S]|_{\ker Q} = 0$.

(c) $SS^* = \mathbf{1} = S^*S$ and $S\mathrm{ran}\, Q \subset \mathrm{ran}\, Q$ (i.e. $S_{+-} = 0 = S_{0-}$).

(d) The matrix S (21) has the form

$$S = \begin{pmatrix} S_{--} & S_{-0} & S_{-+} \\ 0 & S_{00} & -S_{00}(S_{-0})^*(S_{--})^{*-1} \\ 0 & 0 & (S_{--})^{*-1} \end{pmatrix} \tag{26}$$

where S_{--} and S_{00} are invertible (on $\mathrm{ran}\, Q$, $\mathcal{H}_{\mathrm{phys}}$ rsp.) and S_{00}. S_{--}, S_{-0}, S_{-+} satisfy

$$(S_{00})^* = (S_{00})^{-1}, \qquad S_{-+}(S_{--})^* + S_{-0}(S_{-0})^* + S_{--}(S_{-+})^* = 0.$$

(e) $SS^* = \mathbf{1} = S^*S$ and $S^*\ker Q \subset \ker Q$ (i.e. $(S_{+-})^* = 0 = (S_{0-})^*$).

(f) $SS^* = \mathbf{1} = S^*S$ and $[Q, S^*]|_{\ker Q} = 0$.

(g) $SS^* = \mathbf{1} = S^*S$ and $S^*\mathrm{ran}\, Q \subset \mathrm{ran}\, Q$ (i.e. $(S_{+-})^* = 0 = (S_{+0})^*$).

(h) $SS^* = \mathbf{1} = S^*S$ and $(S_{00})^* S_{00} = P_0 = S_{00}(S_{00})^*$.

Proof: (b) $\Leftrightarrow$ (a) $\Leftrightarrow$ (g) and (f) $\Leftrightarrow$ (e) $\Leftrightarrow$ (c) hold trivially true.

(a) $\Leftrightarrow$ (e): (a) implies $S_k \ker Q \subset \ker Q$ for each order S_k of S. Therefore,

$$(S^*)_n = (S^{-1})_n = \sum_{r=1}^{n} (-1)^r \sum_{n_1,\dots,n_r \geq 1,\; n_1+\dots+n_r = n} S_{n_1} \dots S_{n_r}$$

maps $\ker Q$ in $\ker Q$ and, hence, this holds also true for $S^* = \sum_n (S^*)_n$. (e) $\Rightarrow$ (a) follows analogously.

(a),(c) $\Leftrightarrow$ (d): By a straightforward calculation one verifies that the equations

$$S_{+-} = 0,\; S_{+0} = 0,\; S_{0-} = 0 \quad \text{and} \quad \sum_c (S_{(-c)(-a)})^* S_{cb} = \delta_{ab} P_a = \sum_c S_{ac} (S_{(-b)(-c)})^* \tag{27}$$

are equivalent to (d).

(a),(c) $\Rightarrow$ (h): Choosing $a = 0 = b$ in (27) we obtain $(S_{00})^* S_{00} = P_0 = S_{00}(S_{00})^*$.

To show that (h) does not imply the other statements for $Q \neq 0$ ($\Leftrightarrow \mathrm{ran}\, Q \neq 0 \Leftrightarrow \mathrm{ran}\, Q^+ \neq 0$), we give two examples for S which satisfy (h) but not (d):

- The condition (h) is invariant under an exchange of Q and Q^+. Therefore, there exists a solution S of (h) which maps $\ker Q^+$ in $\ker Q^+$ (and/or $\mathrm{ran}\, Q^+$ in $\mathrm{ran}\, Q^+$), i.e. S is a lower triangular matrix.

- The following S-matrix fulfills (h) and $S_{+-} \neq 0 \neq S_{-+}$ (if $cd \neq 0$):

$$S = \begin{pmatrix} ae^{i\alpha}P_- & 0 & ice^{i\alpha}P_- JP_+ \\ 0 & e^{i\phi}P_0 & 0 \\ ide^{i\alpha}P_+ JP_- & 0 & be^{i\alpha}P_+ \end{pmatrix} \tag{28}$$

with $a, b, c, d \in \mathbf{R}$, $\alpha, \phi \in \mathbf{R}$ and $ab + cd = 1$. $\square$

In references [2] physical consistency of the S-matrix is satisfied by requiring a perturbative condition which implies $[Q, S] = 0$, namely the

'free perturbative operator gauge invariance' [10]: Let $W \equiv T_0$ be the interaction Lagrangian. Then there exists W_1^ν with

$$[Q, W(x)] = i\partial_\nu W_1^\nu(x) \tag{29}$$

and the time ordered products of W and W_1^ν fulfil

$$[Q, T(W(x_1)...W(x_n))] = i\sum_{l=1}^{n} \partial_\mu^{x_l} T(W(x_1)...W_1^\mu(x_l)...W(x_n)) \tag{30}$$

Let us assume that (30) holds true to all orders $\leq (n-1)$. Due to the causal factorization (4) the requirement (30) is then satisfied away from the total diagonal, i.e. on $\mathcal{S}(\mathbf{R}^{4n} \setminus \Delta_n)$. Hence, (30) is an additional normalization condition for $T(W...W)$ and $T(W...W_1^\mu...W)$. But it is a highly non-trivial task to prove that there exists an extension to the diagonal which satisfies (30) and the other normalization conditions (D), (E) and (F).

In contrast to $[Q, S] = 0$ or $[Q, S]|_{\ker Q} = 0$, the Q-divergence condition (29-30) makes sense also in models in which the adiabatic limit does not exist, e.g. in massless nonabelian gauge theories. For massless $SU(N)$-Yang-Mills theories it has been proved that the Q-divergence condition (29-30) (more precisely the corresponding C-number identities which imply (29-30)) can be satisfied to all orders [14],[15], and that these C-number identities imply the usual Slavnov-Taylor identities [16]. In addition, (29-30) determines to a large extent the possible structure of the model (see below). We emphasize that this is a *pure quantum formulation of gauge invariance*, without reference to classical physics.

In the present case of pure massive gauge theories (or more generally in gauge theories in which the strong adiabatic limit (6) of the transition functional $S(g)$ exists) we proceed in an alternative way. *We do not require the Q-divergence condition (29-30) as a new physical principle, instead we simply require physical consistency of the S-matrix*, which means

$$[Q, S]|_{\ker Q} = 0 \tag{31}$$

[10] The reason for this name stems from the fact that Q is the generator of the BRST-transformation of the free incoming fields. For the present purpose we will simply refer to it as the "Q-divergence condition" for time-ordered products since it has nothing to do with the classical notion of gauge invariance in the differential geometric setting of fibre bundles, but secures that the true S-matrix is physical. Roughly speaking it is the off shell version of physicality of the on-shell S-matrix.

by Lemma 1. This is a weaker condition than (29-30). But we will see that it determines the theory to the same extent. Our procedure is similar to Grigore [22].[11]

First we construct the operator Q (10) which defines the physical states. For massless gauge theories the procedure is well-known [29], [13]. The nilpotency of Q gives reason to introduce an anticommuting pair of ghost fields $u_a, \tilde{u}_a$ for each gauge field A_a. Then we define[12]

$$Q \equiv \int d^3x \sum_a \partial_\nu A_a^\nu(x) \overleftrightarrow{\partial}_0 u_a(x) \tag{32}$$

Turning to massive gauge fields A_a (7) we give the ghost fields the same masses m_a (otherwise the current $\sum_a \partial_\nu A_a^\nu \overleftrightarrow{\partial}_0 u_a$ or $\sum_a (\partial_\nu A_a^\nu + m_a\phi_a) \overleftrightarrow{\partial}_0 u_a$ (see below) would not be conserved)

$$(\Box + m_a^2)u_a = 0, \quad (\Box + m_a^2)\tilde{u}_a = 0, \quad \{u_a(x), u_b(y)\} = 0, \quad \{\tilde{u}_a(x), \tilde{u}_b(y)\} = 0,$$

$$\{u_a(x), \tilde{u}_b(y)\} = -i\delta_{ab}\Delta_{m_a}(x-y), \quad u_a^* = u_a, \quad \tilde{u}_a^* = -\tilde{u}_a \tag{33}$$

If we insert the massive A_a and u_a fields into the formula (32) for Q the nilpotency is lost

$$2Q^2_{\text{naive}} = \{Q_{\text{naive}}, Q_{\text{naive}}\} = \int d^3x \int d^3y \, [\partial_\mu A_a^\mu(x), \partial_\nu A_b^\nu(y)] \overleftrightarrow{\partial}_{x_0} \overleftrightarrow{\partial}_{y^0} u_a(x) u_b(y) \not= 0 \tag{34}$$

To restore the nilpotency we proceed as follows: to each gauge field A_a we consider a scalar field ϕ_a with the same mass m_a

$$(\Box + m_a^2)\phi_a = 0, \quad [\phi_a(x), \phi_b(y)] = -i\delta_{ab}\Delta_{m_a}(x-y), \quad \phi_a^* = \phi_a \tag{35}$$

Then, due to $[\partial_\mu A_a^\mu + m_a\phi_a, \partial_\nu A_b^\nu(y) + m_b\phi_b] = 0$, the gauge charge

$$Q \equiv \int d^3x \sum_a (\partial_\nu A_a^\nu(x) + m_a\phi_a(x)) \overleftrightarrow{\partial}_0 u_a(x) \tag{36}$$

is nilpotent and symmetrical. Later we shall see that an additional scalar field H with arbitrary mass $m_H \geq 0$ is needed

$$(\Box + m_H^2)H = 0, \quad [H(x), H(y)] = -i\Delta_{m_H}(x-y), \quad H^* = H \tag{37}$$

The notation H is reminiscent of the *Higgs* field, but one with no "vacuum condensate" (leaving aside the academic point of whether Higgs idea would allow for our terminology): $< \Omega|H(x)|\Omega >= 0$, where Ω

[11] However, there are two mistakes in these papers. It is overlooked that the trilinear terms in $W_{(0)}$ vanish in the adiabatic limit due to energy-momentum conservation and, hence, in first order the condition (31) yields no information about the trilinear terms in $W_{(0)}$ (see appendix A).

By using the terminology introduced below the second mistake can be described as follows: the Higgs field(s) is/are treated as scalar partner(s) (with arbitrary mass $m_H \geq 0$) of the massless gauge field(s), which does/do not appear in Q and, hence, is/are physical. By chance this works for the electroweak theory (there is one massless gauge field and one Higgs field is needed). But e.g. in the present case of pure massive gauge theories, there would be no Higgs field and such a model is physically inconsistent (31) to second order. This mistake is not made in [2].

[12] The convergence of this integral (and also of the corresponding expression (36) in the massive theory) can be shown by using a method of Requardt [32],[12].

denotes the vacuum of the free fields. For the representation of A_a (7), $u_a, \tilde{u}_a$ (33), ϕ_a (35) and H (37) in a Krein Fock space $\mathcal{F}$ and especially the definition of J we refer to [27][13]. From the commutation relations

$$[Q, A_a^\mu] = i\partial^\mu u_a, \quad [Q, \phi_a] = im_a u_a, \quad \{Q, u_a\} = 0,$$

$$\{Q, \tilde{u}_a\} = -i(\partial_\mu A_a^\mu + m_a \phi_a), \quad [Q, H] = 0 \tag{38}$$

we conclude that $\mathcal{H}_{\rm phys}$ (14) is the linear span of the set of states $B_1 ... B_l|0>$, $l \in \mathbf{N}_0$ ($|0> \in \mathcal{F}$ is the vaccuum), where $B_1, ..., B_l$ are transversal gauge boson fields (three polarizations) or H-fields. Using this explicit result the positivity assumption (13) can be verified [27]. We emphasize that H is physical, in contrast to the fields ϕ_a, which are sometimes called 'scalar ghosts'. The latter correspond to the Stückelberg fields.

We are now looking for the possible interactions $W \equiv W_0$ which satisfy the following requirements:

(a) W is a Wick polynomial in the free incoming fields. Each monomial in W has at least three factors,

(b) W is invariant with respect to Poincaré transformations,

(c) the number of u-fields agrees with the number of $\tilde{u}$-fields in each monomial of W (i.e. the 'ghost number' is zero),

(d) the scaling degree (or mass dimension) of W is ≤ 4 (this is necessary for renormalizability by power counting),

(e) $W = W^*$ (unitarity of the S-matrix in first order),

(f) physical consistency (31).

Ignoring consistency (31) to orders $n \geq 2$, there remains the freedom to add divergence and coboundary couplings to W

$$W^{(\beta,\gamma)} = W + \sum_l \beta_l \partial_\nu D_l^\nu + \sum_j \gamma_j \{Q, K_j\}, \qquad \beta_l, \gamma_j \in \mathbf{R} \tag{39}$$

where D_l^ν, K_j are restricted by (a)-(e). Taking additionally consistency (31) to orders $n \geq 2$ into account, it seems that this freedom can be maintained, due to the following result. It is shown in [17] that the Q-divergence condition (30) (which implies (31)) can be satisfied to all orders (by choosing suitable normalizations) for any $(\boldsymbol{\beta}, \boldsymbol{\gamma})$, if the 'generalized (free perturbative operator) gauge invariance' [14] holds true for $(\boldsymbol{\beta}, \boldsymbol{\gamma}) = (\mathbf{0}, \mathbf{0})$. Moreover, under this assumption, one can prove

$$P_0 T(W^{(\beta,\gamma)}(x_1) ... W^{(\beta,\gamma)}(x_n)) P_0 = P_0 T(W^{(0,0)}(x_1) ... W^{(0,0)}(x_n)) P_0 + \text{divergences} \tag{40}$$

[13] In [27] it was not realized that the Higgs field is needed and, hence, it is missing there. The H-field is represented similarly to the other scalar fields ϕ_a (with $J = \mathbf{1}$ in the H-Fock space).

[14] The 'generalized (free perturbative operator) gauge invariance' is the following statement. To a given $W \equiv W_0$ there exist W_1^ν and $W_2^{\mu\nu}$ with

$$[Q, W_0] = i\partial_\nu W_1^\nu, \quad \{Q, W_1^\nu\} = i\partial_\mu W_2^{\mu\nu}, \quad [Q, W_2^{\mu\nu}] = 0$$

(see [17]) and, hence, the physical S-matrix S_{00} is independent from (β,γ).

Now we make the most general Ansatz for W (up to divergence and coboundary terms) which satisfies (a)-(e)

$$\begin{aligned} W &= f_{abc} : A_{a\,\mu} A_{b\,\nu} \partial^\nu A_c^\mu : + f^1_{abc} : u_a \partial^\mu \tilde{u}_b A_{c\,\mu} : \\ &+ d_{abc}(: A_a^\mu \phi_b \partial_\mu \phi_c : - : A_a^\mu \partial_\mu \phi_b \phi_c :) + e_{abc} : A_a^\mu A_{b\,\mu} \phi_c : \\ &+ h_{abc} : \tilde{u}_a u_b \phi_c : + j_{abc} : \phi_a \phi_b \phi_c : + k_{ab}(: H A_a^\mu \partial_\mu \phi_b : - : \partial_\mu H A_a^\mu \phi_b :) \\ &+ l_{ab} : A_a^\mu A_{b\,\mu} H : + p_{ab} : H \tilde{u}_a u_b : + q_{ab} : H \phi_a \phi_b : + r_a : H^2 \phi_a : + s : H^3 : \\ &+ (\text{quadrilinear terms, e.g. } \sim : AAAA :, : AAu\tilde{u} :, : AA\phi\phi :, : H^4 :) \end{aligned} \tag{41}$$

where f_{abc}, f^1_{abc}, d_{abc}, e_{abc}, h_{abc}, j_{abc}, k_{ab}, l_{ab}, p_{ab}, q_{ab}, r_a, $s \in \mathbf{R}$ are arbitrary constants (which are introduced without any knowledge about a gauge group). From now on we specialize to the simplest non-trivial model: we consider three selfinteracting massive gauge fields: $m_a > 0$, $a = 1, 2, 3$ [15].

Our aim is to determined the parameters in W (41) and the tree normalization terms (see (49) below) by the consistency condition (31). The hope is that one can conclude from

$$0 = [Q, S_n]|_{\ker Q} = \frac{i^n}{n!} \int dx_1 ... dx_n \, [Q, T(W(x_1)...W(x_n))]|_{\ker Q} \tag{42}$$

that $[Q, T(W(x_1)...W(x_n))]$ must be a sum of divergences of local operators and that then one can follow the calculations in [2].

But there is a difficulty at first order, which is explained in detail in appendix A. The adiabatic limit of the trilinear terms in W vanishes (except possibly the H-couplings) due to energy-momentum conservation and in agreement with the fact that for stable particles there are no S-matrix elements with three particle legs; the lowest tree contributions involve ≥ 4 legs. Hence, we get no information about these terms from $[Q, S_1]|_{\ker Q} = 0$. For the quadrilinear terms $W^{(4)}$ in W the procedure works (see also appendix A): we obtain that $[Q, W^{(4)}]$ must be the divergence of a Wick polynomial. In [22] it is shown that this implies $W^{(4)} = 0$ except for a term $\sim : H^4 :$. But the latter term will be excluded by consistency (31) to second order. (The well-known $: H^4 :$-coupling is of second order in g, it appears in the framework of causal perturbation theory as a tree normalization term of $T(W(x_1)W(x_2))$, see below.)

To determined the parameters in the trilinear terms $W^{(3)}$ of W (41) we compute the tree diagrams

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and the time ordered products of W_0, W_1^ν and $W_2^{\mu\nu}$ fulfil

$$[Q, T(W_{j_1}(x_1)...W_{j_n}(x_n))]_\mp = i \sum_{l=1}^{n} \partial^{x_l} T(W_{j_1}(x_1)...W_{j_l+1}(x_l)...W_{j_n}(x_n)), \quad j_1, ..., j_n \in \{0, 1, 2\},$$

where we have the anticommutator on the l.h.s. iff $(j_1 + ... + j_n)$ is odd, and where $T(...W_3(x_l)...) \equiv 0$ by definition. Similarly to (30), the second equation is a normalization condition on the time ordered products. A proof that it can be satisfied to all orders is still missing in any nonabelian model. Nevertheless we strongly presume that this statement holds true for all models which are BRST-invariant, especially for the pure massive $SU(2)$-model studied below.

[15] The resulting model is usually called '$SU(2)$ Higgs-Kibble model'. It is obtained from the electroweak theory, which is studied in detail in [2], by setting the Weinberg angle $\Theta_W = 0$ and omitting the photon field, which decouples in this case.

in $T(W^{(3)}(x_1)W^{(3)}(x_2))$ (which depend on these parameters) and require

$$\int d^4x_1\, d^4x_2\, [Q, T(W^{(3)}(x_1)W^{(3)}(x_2))|_{\rm tree}]|_{\ker Q} = 0. \tag{43}$$

We only give a *heuristic* description of the calculation (a rigorous formulation is still missing):

- For the terms with $x_1 \neq x_2$ the T-product factorizes, e.g. for $x_1 \notin x_2 + \bar{V}^-$ we have

$$[Q, T(W^{(3)}(x_1)W^{(3)}(x_2))|_{\rm tree}] = [Q, W^{(3)}(x_1)]W^{(3)}(x_2)|_{\rm tree} + W^{(3)}(x_1)[Q, W^{(3)}(x_2))]|_{\rm tree}. \tag{44}$$

This makes it plausible that the cancellation of these terms yields the same restrictions as the Q-divergence condition to first order (29): $[Q, W^{(3)}] = i\partial_\nu W_1^{(3)\,\nu}$ for some Wick monomial $W_1^{(3)\,\nu}$.[16] In this way we obtain

$$\begin{aligned} f_{abc} &= -f^1_{abc} \quad \text{and} \quad f_{abc} \quad \text{is totally antisymmetric,} \\ d_{abc} &= f_{abc}\frac{m_b^2 + m_c^2 - m_a^2}{4m_b m_c}, \quad e_{abc} = f_{abc}\frac{m_b^2 - m_a^2}{2m_c}, \\ h_{abc} &= f_{abc}\frac{m_a^2 + m_c^2 - m_b^2}{2m_c}, \quad j_{abc} = 0, \quad r_a = 0 \end{aligned} \tag{45}$$

and some relations for k_{ab}, l_{ab}, p_{ab} and q_{ab} [2]. There results no restriction on s. One easily verifies that these values of the parameters are not only necessary for (29), they are also sufficient. By absorbing a constant factor in g we obtain

$$f_{abc} = \epsilon_{abc}. \tag{46}$$

The latter are the structure constants of $su(2)$. So the gauge group structure is not put in, it comes out as a consequence of physical consistency and the Ansatz (41) for W. For more complicated models (i.e. more than three gauge fields) this conclusion is impossible at this stage, because one does not know that the $f'_{abc}s$ satisfy the Jacobi identity. But then the latter is obtained in the next step (see below). So far it is possible that all couplings involving the H-field vanish, i.e. the Higgs field is not yet needed.

- But the cancellation of the terms with $x_1 = x_2$ in (43) cannot be achieved without H-couplings. This condition yields important results:[17]

  - The f_{abc}'s must fulfil the Jacobi identity. (In our simple model this is already known (46).) [18]
  - The masses must agree

$$m \equiv m_1 = m_2 = m_3. \tag{47}$$

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[16] It is obvious that the Q-divergence condition to first order implies the cancellation of the terms $x_1 \neq x_2$ in (43), but here we proceed in the opposite direction.

[17] These results agree precisely with the ones derived from the Q-divergence condition (for second order tree diagrams) in [2].

[18] Stora [43] found (for an arbitrary number of massless selfinteracting gauge fields) that the Q-divergence condition to first order implies that the coupling parameters are totally antisymmetric and that the Q-divergence condition for second order tree diagrams yields the Jacobi identity.

- The H-coupling parameters take the values

$$k_{ab}=\frac{\kappa}{2}\delta_{ab},\quad l_{ab}=-\frac{\kappa m}{2}\delta_{ab},\quad p_{ab}=\frac{\kappa m}{2}\delta_{ab},\quad q_{ab}=\frac{\kappa m_H^2}{4m}\delta_{ab} \tag{48}$$

where $\kappa\in\{-1,1\}$. The parameter s is still free.

- There is no term $\sim: H^4:$ in W (i.e. in first order in g).

- In $T(W(x_1)W(x_2))|_{\rm tree}$ the C-number distributions are Feynman propagators with derivatives: $\Delta^F(x_1-x_2)$, $\partial_\mu\Delta^F(x_1-x_2)$ and $\partial_\nu\partial_\mu\Delta^F(x_1-x_2)$. The first two extend uniquely to the diagonal $x_1=x_2$ and the last one has a distinguished extension, namely $\partial_\nu\partial_\mu\Delta^F(x_1-x_2)$ [19]. We denote this extension by $T(W(x_1)W(x_2))|^0_{\rm tree}$. So-called 'tree normalization terms'

$$N(x_1,x_2)=C\delta(x_1-x_2):B_1(x_1)B_2(x_2)B_3(x_3)B_4(x_4):,\qquad C\in\mathbf{R}\,\text{or}\,i\mathbf{R} \tag{49}$$

($B_1,...,B_4\in\{A^\mu,\,u,\,\tilde u,\,\phi,\,H\}$) can be added to $T(W(x_1)W(x_2))|^0_{\rm tree}$, if they satisfy the properties (b) (Poincaré covariance), (c) (ghost number), (d) (scaling degree) and (e) (unitarity) which are required above for W (here they restrict $N(x_1,x_2)$). These tree normalization terms (49) correspond to the quadrilinear terms of order g^2 (g denotes the coupling constant) in the interaction Lagrangian of the conventional theory, they have the same influence on the perturbation series of the S-matrix. *The cancellation of the terms $x_1=x_2$ in (43) fixes the possible tree normalization terms* (i.e. the constants C in (49)) *uniquely* (in terms of s), except for [20]

$$N_{H^4}(x_1,x_2)=\lambda\delta(x_1-x_2):H^4(x_1):,\qquad \lambda\in\mathbf{R}. \tag{50}$$

• The parameters s and λ, which are still free, are determined by physical consistency (42) for the tree diagrams to *third* order (analogously to [2]):

$$s=\frac{m_H^2}{4m},\qquad \lambda=-\frac{m_H^2}{16m^2}. \tag{51}$$

So W and the tree diagram normalizations to second order are completely determined (up to the sign κ, which is conventional) and these terms agree precisely with the interaction Lagrangian obtained by the Higgs mechanism. The Higgs potential is not put in here, it is derived from physical consistency. Spontaneous symmetry breaking plays no role in this approach, because we *start with the massive free incoming fields.* A proof that physical consistency can be satisfied to all orders (by choosing suitable normalizations) is missing up to now, but we are convinced that this holds true.

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[19] The general extension which is Poincaré covariant (D) and does not increase the scaling degree (F) reads: $\partial_\nu\partial_\mu\Delta^F(x_1-x_2)+Cg_{\nu\mu}\delta(x_1-x_2)$, $C\in\mathbf{C}$ arbitrary.

[20] For the expert we mention that tree normalization terms with $B_1,...,B_4$ exclusively scalar fields (e.g. N_{H^4}) are required. In contrast to the other tree normalization terms (i.e. the tree normalization terms with gauge field factors) they violate the normalization condition **(N3)** in [12] (or (43) in [19]). But this is no harm.

3 Renormalizability and Ghosts

The formulation of massive selfinteracting vectormesons of the previous section will now serve as a point of departure for a more fundamental conceptual discussion. Similar to Weinberg (Weinbergs old work on Feynman rules for higher spin) we start with Wigner's theory of particle representations; in our case because we want to avoid any parallelism to (quasi)classical systems (quantization) and (for reasons which will become gradually clear to the reader) develop local quantum physics from an intrinsic point of view as much as possible. In particular we would like to understand the curious phenomenon that, contrary to the classical situation, the possibilities of perturbative renormalizable QFT are the more restrictive, the higher the spin. Whereas the classical theory is in need of an additional selection principle (the gauge principle in case of zero mass), local quantum physics for spin≥ 1 is more restrictive: *the particle content and renormalizability fix the vectormeson theory* (where the possible triviality for spin> 1 is a special case). It is this result red backward into (quasi)classical field theory, which, in the spirit of Bohr's correspondence principle gives a fundamental physical support for the *classical gauge selection principle* (which then gives the strong link with the mathematical-esthetical appeal to fibre bundles). As a side result we will learn that the massive theory fulfils the Schwinger-Swieca [40] [45] screening mechanism, and that (similar to the gauge interpretation in terms of a Higgs field) the theory has more physical degrees of freedom than the massive vectormesons from which we started in zeroth order, namely consistent perturbation theory requires the introduction of a scalar H-field (the Higgs field without Higgs condensate[21]) of the previous section.

It is well-known that the step from the Wigner representation theory of particles (positive energy irreducible representations of the Poincaré group with finite spin/helicity) to local free fields is described in terms of intertwiners u, v[22]

$$\psi^{[n_+,n_-]}(x) = \int \sum_{s_3=-s}^{s} \left\{ e^{-ipx} u(\vec{p}, s_3) a(\vec{p}, s_3) + e^{ipx} v(\vec{p}, s_3) b^*(\vec{p}, s_3) \right\} \frac{d^3p}{2\omega} \tag{52}$$

which intertwine the Wigner representation matrices $D^{(s)}(R(\Lambda, p))$ with matrices of the covariant Lorentz group representation $D^{[n_+,n_-]}(\Lambda)$

$$D^{[n_+,n_-]}(\Lambda) u(p) = u(\Lambda p) D^{(s)}(R(\Lambda, \Lambda p)) \tag{53}$$

where for convenience we have collected the $(2s+1)$ $u^{(n_+,n_-)}(p)$ mixed (un)dotted $(2n_+ + 1)(2n_- + 1)$ component u-spinors into a rectangular $(2n_+ + 1)(2n_- + 1) \times (2s+1)$ matrix $u(p)$ and similar for the $v's$.

Let us first note that the covariantized inner product in the *one-particle Wigner space* for $s \geq 1$ contains necessarily first or higher powers of momenta. Associated with this is the fact that the intertwiners u, v have a dimension ≥ 1 which immediately translates into an operator dimension of the field

[21] We use the notation H-field only in order to avoid any association with Higgs condensates.

[22] The letter u, which was used in the previous section for a ghost field, means here an intertwiner. The ghost fields in the Wigner one-particle space will be denoted by $(\omega, \tilde{\omega}, \varphi)$. The corresponding Fock space fields are the fields $(u, \tilde{u}, \phi)$ of the previous section.

dim$\psi \geq 2$. Since interaction densities $W_0 \equiv W$ are at least trilinear in free fields and since the smallest possible operator dimension is 1 (for scalar fields), it is impossible to satisfy the renormalizability condition dim$W \leq 4$ within the Stückelberg-Bogoliubov-Epstein-Glaser operator approach. Inspired by the idea of a cohomological representation of the physical space and the physical observables in the previous section one looks for a cohomological extension of the Wigner space for massive vectormesons in order that the associated two-point function (or propagation kernel) has a milder (renormalizable) high momentum behavior. The results of the previous section also suggest the simplest possibility to achieve that, namely to use three additional indefinite metric scalar wave functions (where on the Wigner level the "statistics" is yet undetermined). More precisely we form an extended Hilbert space H_{ext} by the multicomponent wave functions $(A_\mu, \omega, \bar{\omega}, \varphi)$ defined on the mass shell. H_{ext} has in addition to a positiv definite inner product another one which does not have this property and correponds to the Krein structure of the previous section. The latter is relevant in order to have at least some pseudo-unitary Lorentz covariance and a definition of modular localized subspaces. On H_{ext} we define a BRS-like operator by

$$s_W \begin{pmatrix} A_\mu^a(p) \\ \omega_a(p) \\ \bar{\omega}_a(p) \\ \varphi_a(p) \end{pmatrix} = \begin{pmatrix} p_\mu \omega_a(p) \\ 0 \\ -p^\mu A_\mu^a(p) - im_a\varphi_a(p) \\ im_a\omega_a(p) \end{pmatrix}. \tag{54}$$

The so defined s_W-operation defines a differential space since the definition easily leads to $s_W^2 = 0$. One then uses this s_W in order to write the following cohomological representation for the physical (Wigner) Hilbert space H_W in terms of the above extended space H_{ext}

$$\begin{aligned} H_W &= \frac{\ker s_W}{\operatorname{ran} s_W} \\ &= cl.\left\{ A_\mu(p) \mid p^\mu A_\mu(p) = 0, -\int A_\mu(p)A^\mu(p)\frac{d^3p}{2\omega} < \infty \right\} \\ &= cl.\left\{ A_\mu(p) \mid -\int A_\mu(p)(g^{\mu\nu} - \frac{p^\mu p^\nu}{m^2})A_\nu(p)\frac{d^3p}{2\omega} < \infty \right\} \end{aligned} \tag{55}$$

i.e. we obtain the L^2-closure of the space of transversal vector wave functions which in terms of the associated fields (52) had the high dimension dim$A = 2$ which was responsible for the lack of renormalizable interactions within the original (nonextended) formulation. (The transversality condition does not lower the dimension of the vector wave functions or the corresponding fields.) On the other hand the extended Hilbert space has no transversality condition and obeys the classical assignment of dimensions (i.e. dim$A = 1$ in H_{ext}). This is due to the fact that ghost contributions damp the high momentum behavior of the associated two-point function.[23]

[23] In the Lagrangean framework there is an alternative method to lower the dimension of the gauge field A from 2 to 1. In contrast to massless gauge fields, the Proca field ($\mathcal{L} = -\frac{1}{4}F^2 + \frac{1}{2}m^2A^2$, $F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$) has a well-defined propagator without introducing a gauge fixing term, but it has dimension dim$A = 2$. Stückelbergs trick is to add a "gauge fixing term" $\mathcal{L}_1 = -\frac{1}{2}\lambda(\partial \cdot A)^2$, $\lambda > 0$ (this misleading terminology is chosen because because $\mathcal{L}_1$ has the same form as

The extension has however an influence on the modular localization theory in Wigner space where the latter is (via the CCR/CAR functor) the preempted locality in Fock space. For example the wedge (Rindler or Bisognano-Wichmann) localized subspace involves instead of the simple complex-conjugation in addition the Krein operator η i.e. one has for the action of the pre-Tomita operator[24]

$$(S\psi)(p) = (\eta\overline{\psi})(p). \tag{56}$$

Where ψ is the multi-component wave function involving the ghosts in addition to the vectorpotential. The standard modular localization theory can be found in [35] and the adaptation to the present extended "pseudo-modular" case will be treated in a separate paper. The physical Wigner subspace (more precisely it is a cohomologically defined factorspace) is precisely characterized by the validity of the "correct" modular localization associated with the Tomita theory. Although our ghost extension of the (m,s=1) Wigner representation is not uniquely fixed (we chose a "minimal" extension) we believe that any other cohomological extension which also lowers the $\dim A = 2$ down to its classical value $\dim A = 1$ will contain the minimal and possible additional pieces which do not change the physical content[25].

The next step from particles to fields is the answer to the question of what is the action of s_W on the *multiparticle tensor space* [11]. From the usual Fock space formalism we are used to the following action of derivations δ on tensor products

$$\delta(\psi_1 \otimes \psi_2) = \delta\psi_1 \otimes \psi_2 + \psi_1 \otimes \delta\psi_2$$

It is easy to see that the tensorproduct action of s_W must include a grading in order to maintain the nilpotency

$$s(\psi_1 \otimes \psi_2) = s\psi_1 \otimes \psi_2 + \psi_1(-)^{\text{degree}\,\psi_1} \otimes s\psi_2, \tag{57}$$

where s denotes the Fock space version of s_W.[26] So, two of the three scalar ghost fields $(u, \tilde{u}, \phi)$[27], which are companions of each massive vector meson field, are required to be graded fermionic fields and the third one must be bosonic.

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the gauge fixing Lagrangean in the massless case), which damps the high momentum behavior of the propagator such that $\dim A = 1$. In this Stückelberg formalism, the necessity to introduce ghosts shows up in the fact that without ghosts it is impossible to define a *stable* physical subspace or factor space. For $\lambda = 1$ (Feynman gauge) (7) one obtains the same free gauge fields as in the quantization of the cohomological extension of the Wigner one-particle space. We prefer the latter method, because we do not want to rely on a Lagrangean framework, instead we want to be close to the particle picture.

[24] The pre-Tomita operator S has nothing to do with the above nilpotent operators s_W (54) (in the one particle Wigner space) or s (57) in Fock space.

[25] The geometrical Faddeev-Poppov method can also be considered as a minimal extension of the functional measure. It owes its unique appearance more to geometric than quantum physical reasoning.

[26] From the context it should be clear whether we mean by the letter s the present nilpotent Fock space operator s or the spin.

[27] We recall that $(u, \tilde{u}, \phi)$ are the Fock space fields of the previous section and correspond to $(\omega, \tilde{\omega}, \varphi)$ in the Wigner one-particle space.

In this way we obtain the Fock space formalism of the previous section with $[Q,\cdot]$ being the implementation of s in Fock space. Whereas the ghost formalism can be pursued back into the Wigner one-particle theory, the necessity to choose trilinear couplings at first order in g with coefficients fulfilling group theoretical symmetry, as well as the necessity of enlargement of the vectormeson setup by additional physical degrees of freedom (whose simplest and perhaps only realization are the scalar H-fields) only shows up as a consistency requirement above the zeroth order.

4 Determination by Field Content

In the second section the physical consistency was formulated in terms of the S-matrix (31). We are now looking for a corresponding condition on the *interacting physical fields*. General fields (including composites) are defined in terms of the Bogoliubov transition functional $S(\mathbf{g})$ (1) in Fock space as formal power series in g_0. The interacting field $W_{j\,\mathrm{int}}(x;g_0W)$ to the interaction $W\equiv W_0$ and corresponding to the Wick polynomial W_j, $j=1,...,G$ of free fields, is defined by

$$W_{j\,\mathrm{int}}(x;g_0W)\equiv\frac{\delta}{i\delta g_j(x)}S(g_0,0,...,0)^{-1}S(g_0,0,...,g_j,0,...)|_{g_j=0}. \tag{58}$$

By inserting (1) one obtains the perturbative expansion of the interacting fields

$$W_{j\,\mathrm{int}}(x;g_0W)=W_j(x)+\sum_{n=1}^{\infty}\frac{i^n}{n!}\int d^4x_1...d^4x_n\,g_0(x_1)...g_0(x_n)R(W(x_1)...W(x_n);W_j(x)), \tag{59}$$

with the 'totally retarded products'

$$R(A_1(x_1)...A_n(x_n);A(x))\equiv\sum_{I\subset\{1,...,n\}}(-1)^{|I|}\bar{T}(A_i(x_i),\,i\in I)T(A_k(x_k),\,k\in I^c,A(x)), \tag{60}$$

where $A_1,...,A_n,A$ are Wick polynomials, $I^c\equiv\{1,...,n\}\setminus I$ and $\bar{T}$ denotes the 'antichronological product'. The corresponding generating functional is $S(\mathbf{g})^{-1}$. The antichronological products can be obtained uniquely from the time ordered products by the usual inversion of a formal power series

$$\bar{T}(A_1(x_1)...A_n(x_n))=\sum_{P\in\mathrm{Part}\,\{1,...,n\}}(-1)^{|P|+n}\prod_{p\in P}T(A_i(x_i),\,i\in p). \tag{61}$$

By means of causality (4) one easily finds that the R-products (60) have totally retarded support with respect to the distinguished coordinate x.

$$\mathrm{supp}\,R(A_1(x_1)...A_n(x_n);A(x))\subset\{(x_1,...x_n;x)\mid x_i\in x+\bar{V}_-,\ \forall i=1,...n\}. \tag{62}$$

In a pure massive theory the strong adiabatic limit of the interacting fields exists as a formal power series in $g:=g_0(0)$:

$$W_{j\,\mathrm{int}}(x)\psi\equiv W_{j\,\mathrm{int}}(x;W)\psi\equiv\lim_{\epsilon\to 0}W_{j\,\mathrm{int}}(x;g_{0\,\epsilon}W)\psi,\quad\text{where}\quad g_{0\,\epsilon}(x)\equiv g_0(\epsilon x),\ \psi\in\mathcal{D}, \tag{63}$$

because this holds true for $S(\mathbf{g})$ (6) [20].

A $*$-symmetrical interacting field in the adiabatic limit (i.e. $\phi_{\rm int}(x) \equiv \phi_{\rm int}(x;W) = \lim_{\epsilon\to 0}\phi_{\rm int}(x;g_{0\,\epsilon}W)$, $\phi^*_{\rm int} = \phi_{\rm int}$) is called *physical* (or an *observable*) if it induces a well-defined operator on the factor space $\mathcal{H}'_{\rm phys}$ (22). This holds true iff

$$\phi_{\rm int}(f;W)\ker Q \subset \ker Q \quad \wedge \quad \phi_{\rm int}(f;W)\,{\rm ran}\, Q \subset {\rm ran}\, Q, \quad \forall f \in \mathcal{S}(\mathbf{R}^4) \tag{64}$$

(cf.(25)). This is equivalent to

$$[Q,\phi_{\rm int}(f;W)]|_{\ker Q} = 0, \qquad \forall f \in \mathcal{S}(\mathbf{R}^4). \tag{65}$$

(The nontrivial part of this statement is that (65) implies $\phi_{\rm int}(f)\,{\rm ran}\,Q \subset {\rm ran}\,Q$. However, using the notations of sect. 2, the condition (65) is equivalent to $\phi_{\rm int}(f)_{+-} = 0 = \phi_{\rm int}(f)_{+0}$. Hence $\phi_{\rm int}(\bar f)_{0-} = (\phi_{\rm int}(f)_{+0})^* = 0$. Together we obtain $\phi_{\rm int}(f)\,{\rm ran}\,Q \subset {\rm ran}\,Q$.)

Let $F^{\mu\nu}_a \equiv \partial^\mu A^\nu_a - \partial^\nu A^\mu_a$. We now *require that (for each $a = 1,...,M$) there exists a physical field* $\mathcal{F}^{\mu\nu}_{a\,\rm int}$, i.e.

$$[Q,\mathcal{F}^{\mu\nu}_{a\,\rm int}(x;W)]|_{\ker Q} = 0, \tag{66}$$

with the additional properties:

(i) $\mathcal{F}^{\mu\nu}_{a\,\rm int} = F^{\mu\nu}_{a\,\rm int} + \sum_k c_k\psi^{\mu\nu}_{ak\,\rm int}$, where the c_k are formal power series of (constant) complex numbers and $\mathcal{F}^{\mu\nu}_{a\,\rm int}$ agrees in zeroth order with the free $F^{\mu\nu}_a$, i.e. the c_k vanish to zeroth order: $c_k^{(0)} = 0$;

(ii) the zeroth order $\psi^{\mu\nu}_{ak}$ of $\psi^{\mu\nu}_{ak\,\rm int}$ is a Wick monomial which has precisely one gauge field factor F or A, the other factors are ghost or scalar fields, and $\psi^{\mu\nu}_{ak} \neq F^{\mu\nu}_b$, $\forall a,b \in \{1,...,M\}$;

(iii) the scaling degree (or mass dimension) of ψ_{ak} is ≤ 4;

(iv) $\mathcal{F}^{\mu\nu}_{a\,\rm int}$ is a Lorentz tensor of second rank and is antisymmetrical in (μ,ν);

(v) the ghost number of $\mathcal{F}^{\mu\nu}_{a\,\rm int}$ is zero;

(vi) $\mathcal{F}^{\mu\nu}_{a\,\rm int}(x;W)^* = \mathcal{F}^{\mu\nu}_{a\,\rm int}(x;W)$.

The requirement (ii) is badly motivated (except the demand that ψ_{ak} must be a Wick monomial), its main purpose is to shorten the calculations. However, the condition $\psi^{\mu\nu}_k \neq F^{\mu\nu}$ in (ii) is necessary for the uniqueness of $\mathcal{F}_{\rm int}$: the fields

$$(1+\sum_{k=1}^{\infty} b_k g^k)\mathcal{F}^{\mu\nu}_{a\,\rm int}, \qquad b_k = {\rm const.} \in \mathbf{R}$$

satisfy all other requirements if $\mathcal{F}_{\rm int}$ does so. It is an interesting question how far (ii) can be weakened such that the uniqueness of $\mathcal{F}_{\rm int}$ does not get lost.

Due to the normalization condition (F) (scaling degree) for the time ordered products, the property (iii) implies ${\rm dim}\mathcal{F}^{\mu\nu}_{a\,\rm int} \leq 4$. To specify (iv), (v) and (vi) note that they must be fulfilled in particular by the Wick monomials ψ_{ak} (in the case of (vi) the coefficients c_k are also involved: $c_k^*\psi^*_{ak} = c_k\psi_{ak}$). However, these three requirements also restrict the normalization of the higher orders of $\psi^{\mu\nu}_{ak\,\rm int}$ and $F^{\mu\nu}_{a\,\rm int}$. It is easy to see that these additional normalization conditions can be fulfilled (e.g. by antisymmetrization in

(μ,ν) of an arbitrary Poincaré covariant extension) and we assume that the normalizations are always done in such a way.

The most general Ansatz which is compatible with (i)-(vi) reads

$$\begin{aligned}
\mathcal{F}^{\mu\nu}_{d\,\mathrm{int}} &= F^{\mu\nu}_{d\,\mathrm{int}} + t_{dab}(F^{\mu\nu}_a\phi_b)_{\mathrm{int}} + \tilde{t}_{dab}(A^\mu_a\partial^\nu\phi_b - A^\nu_a\partial^\mu\phi_b)_{\mathrm{int}} \\
+v_{dabc}(&: F^{\mu\nu}_a\phi_b\phi_c :)_{\mathrm{int}} + \tilde{v}_{dabc}(: A^\mu_a\partial^\nu\phi_b\phi_c : - : A^\nu_a\partial^\mu\phi_b\phi_c :)_{\mathrm{int}} \\
+w_{dabc}(&: F^{\mu\nu}_a u_b\tilde{u}_c :)_{\mathrm{int}} + \tilde{w}_{dabc}(: A^\mu_a u_b\partial^\nu\tilde{u}_c : - : A^\nu_a u_b\partial^\mu\tilde{u}_c :)_{\mathrm{int}} \\
+w'_{dabc}(&: A^\mu_a\partial^\nu u_b\tilde{u}_c : - : A^\nu_a\partial^\mu u_b\tilde{u}_c :)_{\mathrm{int}} \\
& +x_{da}(F^{\mu\nu}_a H)_{\mathrm{int}} + \tilde{x}_{da}(A^\mu_a\partial^\nu H - A^\nu_a\partial^\mu H)_{\mathrm{int}} \\
& +y_{dab}(F^{\mu\nu}_a\phi_b H)_{\mathrm{int}} + \tilde{y}_{dab}(A^\mu_a\partial^\nu\phi_b H - A^\nu_a\partial^\mu\phi_b H)_{\mathrm{int}} \\
& +y'_{dab}(A^\mu_a\phi_b\partial^\nu H - A^\nu_a\phi_b\partial^\mu H)_{\mathrm{int}} \\
+z_{da}(&: F^{\mu\nu}_a H^2 :)_{\mathrm{int}} + \tilde{z}_{da}(: A^\mu_a\partial^\nu HH : - : A^\nu_a\partial^\mu HH :)_{\mathrm{int}},
\end{aligned} \tag{67}$$

where $t_{dab},\dots,w'_{dabc},x_{da},\dots,\tilde{z}_{da}$ are (arbitrary) constants which are formal power series in $\mathbf{R}$

$$t_{dab} = \sum_{k=1}^\infty t^{(k)}_{dab}g^k, \qquad x_{da} = \sum_{k=1}^\infty x^{(k)}_{da}g^k, \qquad \text{etc..}$$

We define

$$\mathcal{F}^{\mu\nu\,(k)}_d \equiv t^{(k)}_{dab}F^{\mu\nu}_a\phi_b + \dots + x^{(k)}_{da}F^{\mu\nu}_a H + \dots \qquad \forall k\geq 1,$$

hence[28]

$$\mathcal{F}^{\mu\nu}_{d\,\mathrm{int}} = F^{\mu\nu}_{d\,\mathrm{int}} + \sum_{k=1}^\infty \mathcal{F}^{\mu\nu\,(k)}_{d\,\mathrm{int}}g^k. \tag{68}$$

By definition we may assume $v_{dabc} = v_{dacb}$.

We require that the interaction density W satisfies the properties (a)-(f) listed in section 2, especially $\dim W \leq 4$ which ensures renormalizability. Additionally we assume that W contains no quadrilinear terms. Most probably this assumption is not necessary, i.e. the other requirements (including the physicality (66)) exclude the quadrilinear terms. But this still needs to be checked and will be left open here.

We now replace the requirement (66) by

$$[Q,\mathcal{F}^{\mu\nu}_{a\,\mathrm{int}}(x;W)] = 0. \tag{69}$$

This may be justified in the following way[29]. Since we are working in the adiabatic limit our Q (36) (constructed in terms of incoming free fields) agrees with the Kugo-Ojima operator Q_{int} [29], [12] which

[28] By definition the map $\phi\rightarrow\phi_{\mathrm{int}}$ (ϕ a Wick monomial) is linear with C^∞-functions as coefficients (here the coefficients are constants).

[29] We thank Klaus Fredenhagen for bringing this argument to our attention.

implements the BRST-transformation of the interacting fields. Hence $[Q, \mathcal{F}^{\mu\nu}_{a\,\mathrm{int}}(x;W)]$ is again a local operator. But by the Reeh-Schlieder theorem [31] such an operator is zero if it vanishes on the vacuum, which is an element of the kernel of Q. Thus the requirements (66) and (69) are equivalent.[30]

The main (completely new) result of this section is that the *physicality (69) fixes the parameters* $t_{dab}, \ldots, \tilde{z}_{da}$ *in the Ansatz (67) and the interaction* W (including the tree normalization terms to second order (49)) up to the same non-uniqueness as in section 2, namely the addition of divergence- and coboundary couplings to W (39). Especially we will see that *an additional physical degree of freedom is required: the Higgs field* H *is needed in the interaction* W *as well as in the field* $\mathcal{F}^{\mu\nu}_{d\,\mathrm{int}}$.

We verify this statement in appendix B by explicit calculation of the tree diagrams to lowest orders. We do this only for the simplest non-trivial model, namely three selfinteracting massive gauge fields ($m_a > 0$, $a = 1, 2, 3$) as in section 2. We make the same Ansatz for W as in section 2 , but without the quadrilinear terms. Up to divergence- and coboundary couplings (39) this is the most general trilinear Ansatz which fulfills the requirements (a)-(e) of section 2.

The parameters in the Ansatz for W and $\mathcal{F}^{\mu\nu}_{d\,\mathrm{int}}$ (67) are determined by inserting these expressions into the physicality requirement (69). Here we only state the results:

- for W we obtain precisely the same expression as in section 2,
- we have computed the parameters in $\mathcal{F}_{\mathrm{int}}$ (which are formal power series) up to second order in g

$$\begin{aligned}\mathcal{F}^{\mu\nu}_{d\,\mathrm{int}} = F^{\mu\nu}_{d\,\mathrm{int}} - \frac{g}{m}\epsilon_{dbc}(F^{\mu\nu}_b\phi_c)_{\mathrm{int}} + &\quad \frac{g}{m}(F^{\mu\nu}_d H)_{\mathrm{int}} \\ -\frac{g^2}{4m^2}[\delta_{da}\delta_{bc} - (\delta_{dc}\delta_{ba} + \delta_{db}\delta_{ca})] &\quad (:\ F^{\mu\nu}_a\phi_b\phi_c :)_{\mathrm{int}} \\ -\frac{g^2}{2m^2}\epsilon_{dbc}(F^{\mu\nu}_b\phi_c H)_{\mathrm{int}} + \frac{g^2}{4m^2} &\quad (:\ F^{\mu\nu}_d H^2 :)_{\mathrm{int}} + \mathcal{O}(g^3).\end{aligned} \tag{70}$$

To get a better understanding of the latter result we identify these physical fields $\mathcal{F}^{\mu\nu}_{d\,\mathrm{int}}$ (70) as gauge invariant fields in the framework of spontaneous symmetry breaking of the $SU(2)$ gauge symmetry. In this semiclassical picture the scalar fields ϕ_a (35) and H (37) form two $SU(2)$ doublets

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_2 + i\phi_1 \\ v + H - i\phi_3 \end{pmatrix} \tag{71}$$

and

$$\tilde{\Phi} = \frac{1}{\sqrt{2}}\begin{pmatrix} v + H + i\phi_3 \\ -\phi_2 + i\phi_1 \end{pmatrix}, \tag{72}$$

where v is the vacuum expectation value of the original (i.e. non-shifted) field $\tilde{H} = v + H$. v is proportional to the gauge boson mass

$$m = \frac{gv}{2}. \tag{73}$$

[30] This argument cannot be applied to $[Q, S]|_{\ker Q} = 0$ (31), because the S-matrix is non-local.

The composite fields

$$\mathcal{G}_3^{\mu\nu} := \tilde{\Phi}^* F^{\mu\nu} \tilde{\Phi} = -\Phi^* F^{\mu\nu} \Phi = -\sum_{a=1,2,3} F_a^{\mu\nu} \Phi^* \sigma_a \Phi \tag{74}$$

(σ_a are the Pauli matrices) and

$$\mathcal{G}_1^{\mu\nu} := \frac{1}{2}(\tilde{\Phi}^* F^{\mu\nu} \Phi + \Phi^* F^{\mu\nu} \tilde{\Phi}), \qquad \mathcal{G}_2^{\mu\nu} := \frac{i}{2}(\tilde{\Phi}^* F^{\mu\nu} \Phi - \Phi^* F^{\mu\nu} \tilde{\Phi}) \tag{75}$$

are $SU(2)$ gauge invariant, or equivalently they are invariant with respect to the classical[31] BRST-transformation

$$s(F_a^{\mu\nu}) = ig\epsilon_{abc} F_b^{\mu\nu} u_c, \quad s(\phi_a) = imu_a + \frac{ig}{2}(u_a H + \epsilon_{abc}\phi_b u_c), \quad s(H) = -\frac{ig}{2} u_a \phi_a. \tag{76}$$

By multiplying out the matrices in $\mathcal{G}_d^{\mu\nu}$ (74-75) we find that (with a suitable normalization) the corresponding quantum fields are proportional to $\mathcal{F}_{d\,\mathrm{int}}^{\mu\nu}$, so far as we have determined the constant power series $t_{dab}, ..., \check{z}_{da}$ (67), i.e.

$$\mathcal{G}_{d\,\mathrm{int}}^{\mu\nu}(x; W) = \frac{v^2}{2} \mathcal{F}_{d\,\mathrm{int}}^{\mu\nu}(x; W) + \mathcal{O}(g^3). \tag{77}$$

Finally some more remarks on the validity of the Schwinger-Swieca screening property are in order. Since the physical field strength is an operator in Hilbert space (positive definite metric) which fulfills a Maxwell type equation and since the particle spectrum has an isolated mass hyperboloid for the vectormeson, the theorem of Swieca [45] is applicable and hence the charge defined in terms of the large surface integral over the physical field strength vanishes. This charge srceening is more interesting if additional spinor matter is present.

5 Renormalizability for s$\geq$ 1 without Ghosts?

We have seen that the cohomological representation achieves the magic trick of rescuing renormalizability, whereas the naive application of the standard causal perturbation theory based on interactions W in terms of local Wick polynomials in physical (Wigner) field coordinates (i.e. the naive implementation of Weinberg's program of deriving Feynman rules from Wigner's particle theory for higher spin particles), fails on the count of renormalizability. The trick which rescues the situation, employs ghosts in intermediate steps, but it does so in a way unaccustomed in basic physics which is reminiscent of a catalyzer in chemistry. This is to say a physical problem of perturbativly coupled massive spin =1 free fields interactions, which a priori has nothing to do with ghosts, had to be cohomologically extended, because that was apparently the only way to reconcile the standard perturbative machinery (of deformation of free theories by $W's$) with the short distance renormalizability requirement. But at the end, after

[31] To avoid problems of defining products of interacting fields (this can be done by means of (58)) and the BRST-transformation thereof, we only consider *classical* fields here.

the cohomological descend, one obtains local physical vectormeson fields and a physical S-matrix (both renormalizable) within a physical Fock space of massive spin one particles, together with new physical degrees of freedom. This is certainly a result which totally conceals the intermediate presence of ghosts. Although the final result agrees formally with (the gauge invariant part of) renormalized gauge theory, the underlying physical idea and the words used to describe it are quite different. Instead of the Higgs mechanism, which generates the s=1 mass through an additionally introduced (by hand) scalar field with nonvanishing vacuum expectation value, and which has no visible intrinsic (gauge-invariant) meaning, the vectormeson mass in the present approach is directly linked with the afore-mentioned Schwinger-Swieca screening mechanism. The latter, together with the renormalizability requirement, demands the presence of additional degrees of freedom which we realized as scalar particles and identified with the Higgs field without its vacuum condensate.

Although we have neither demonstrated uniqueness of the ghosts nor of the new physical degree of freedom, we believe, that as already mentioned in the third section, our minimal solution is unique in the sense that any other solution involving higher spin ghost and induced physical objects always contains our minimal solution (plus possible additional couplings with coupling parameters which may be set to zero). Although the (interacting) physical fields have the same locality and spectral properties as in a renormalizable s<1 coupling, the operator dimension of the lowest dimensional field which interpolates the s=0 and s=1 particles do not have the usual form of logarithmic corrections on the canonical (free field) dimensions. Rather they involve higher Wick-polynomials up to operator dimension 4. This is the consequence of the fact that the cohomological descend has rendered the standard formulas in terms of R-products, which were responsible for the canonical dimension + logarithmic corrections, invalid (as a result of the necessity of constructing Q-invariant linear combinations). This rearrangement of the original Wick-basis of all fields in favor of a physical basis (for which the Wick-basis becomes unnatural) is the cause of this problem of mismatch which leads to complicated formulas for the Q-invariant physical basis in terms of the original Wick-basis. Whereas symmetries of the theory under quite general conditions can be naturally worked into the renormalization scheme (so that the renormalized expression in terms of Wick-products maintains its classical appearance), this is apparently not possible for the physical subalgebra. The reason is that the cohomological descend is a procedure somewhat removed from (quasi)classical computations. It also indicates that even though the final physical degrees of freedom are local and can be described in terms of covariant pointlike fields, the formalism deviates in "some way" from the standard local behavior. On a very formal level (ignoring regularisations of line integrals) we could in fact also have obtained the lowering of the operator dimension dim$A$ from two to one without any cohomological extension by allowing nonpointlike and noncovariant vectorpotentials which are interpolating the same particles as the covariant ones. Although on-shell objects as the S-matrix of the final QFT are unaffected, as long as those fields remain almost local in the sense of [23], it is not known how to deal with such objects in a causal approach based on interaction polynomials W and transition operators $S(g)$.

Since the quantization arguments in favor of a gauge theoretical formulation are usually based on the massless case, let us look at arguments pointing towards non-pointlike vectorpotentials for ($m = 0$, $h = 1$) (m is the mass and h denotes the helicity) i.e. photons. It is interesting to note that the local quantum theory of free photons does indeed not allow the introduction of any local covariant vector potential, thus amplifying the suspicion that spin one interacting theory are somehow "less local" than a renormalizable lower spin theory. In order to see this, it is helpful to briefly revisit Wigner's analysis of semiinteger helicity zero mass irreducible positive energy representations and their relation to covariant fields where this phenomenon is visible even before renormalization.

For zero mass the problem with vectorpotentials comes from the noncompact structure of the little group (stability group) of a lightlike vector say $p_R = (1, 0, 0, 1)$, which is the twofold covering of the euclidean group $E(2)$ (and is denoted by $\tilde{E}(2)$). Whereas the rotation has the interpretation as a helicity rotation (rotation around the third axis), the "translations" are Lorentz-transformations which tilt the t-z wedge, leaving its upper light like vector unchanged. As far as the two transversal coordinates are concerned they behave like 2-parametric Galilei velocity transformations (i.e. "light cone translations" without the energy positivity property) with the two longitudinal light cone translations playing the role of the Hamiltonian resp. the central mass in the quantum mechanical representation theory of the Galilei group. The embedding of $\tilde{E}(2)$ into $SL(2, \mathbf{C})$ for the above choice of reference vector is

$$\alpha(\rho, \theta) = \begin{pmatrix} e^{i\frac{1}{2}\theta} & \rho \\ 0 & e^{-i\frac{1}{2}\theta} \end{pmatrix}, \quad p_R \sim \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}, \tag{78}$$

where θ is the angle for the rotations around the 3-axis and $\rho = \rho_1 + i\rho_2$ parameterizes the euclidean translations by the vector (ρ_1, ρ_2). The unitary representation theory of this noncompact group is somewhat more complicated than that of $SU(2)$. But it is obvious that the representations fall into two classes; the "neutrino- photon" class with $U(\alpha(\rho, 0)) = 1$ i.e. trivial representation of the euclidean translations, and the remaining faithful "continuous spin" (infinite dimensional) representations with $U(\alpha(\rho, 0)) \neq 1$. A more detailed analysis shows that the latter lacks the strong localization requirements which one must impose on those positive energy representations which are used for the description of particles.[32] Hence the Wigner theory does not allow to describe photon operators in terms of covariant vector fields. On the other hand a covariant field strength $F_{\mu\nu}$ has the following intertwiner representation in terms of the Wigner annihilation/creation operators for circular polarized photons $a_{\pm}^{\#}(k)$

$$F_{\mu\nu}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \left\{ e^{-ikx} \sum_{\pm} u_{\mu\nu}^{(\pm)}(k) a_{\pm}(k) + h.c. \right\} \frac{d^3k}{2|k|} \tag{79}$$

$$u_{\mu\nu}^{(\pm)}(k) \simeq k_\mu e_\nu^{(\pm)}(k) - k_\nu e_\mu^{(\pm)}(k) \tag{80}$$

Here $e_\mu^{(\pm)}(k)$ are the polarization vectors which are obtained by application of the following Lorentz transformation to the standard reference vectors $\frac{1}{\sqrt{2}}(0, \pm 1, i, 0)$ (which are $\perp$ $(1, 0, 0, 1)$): a rotation of

[32] For the nonfaithful representation ($m = 0, h =$ semiinteger) ($U(\alpha(\rho, 0)) \neq 1$) one finds that the infinite set of intertwiners in (52) is restricted to $u^{(n_+, n_-)}$ with $n_- = n_+ \pm h$ [47].

the z-axis into the momentum direction $\vec{n} \equiv \frac{\vec{k}}{\omega}$ (fixed uniquely by the standard prescription in terms of two Euler angles) and a subsequent Lorentz-boost along this direction which transforms $(1, \vec{n})$ into $k = \omega(1, \vec{n})$. It is these vectors that do not behave covariant under those Lorentz-transformations which involve the above "little group translations" but rather produce an affine transformation law

$$G(\rho)e^{(\lambda)}(p_R) = e^{(\lambda)}(p_R) + \begin{cases} -\frac{1}{2}(\bar{\rho}, 0, 0, \bar{\rho}), & \lambda = + \\ +\frac{1}{2}(\rho, 0, 0, \rho), & \lambda = - \end{cases} \qquad \rho = \rho_1 + i\rho_2, \tag{81}$$

($G(\rho)$ is the Minkowski space representation of the euclidean translations by (ρ_1, ρ_2)) whereas under x-y rotations the $e^{(\lambda)}$ picks up the standard Wigner phase factor. The polarization vectors do not behave as 4-vectors since they are not invariant under the euclidean translations in $\tilde{E}(2)$, as one would have expected for a (nonexisting!) bona fide intertwiner from the $(0, h = 1)$ Wigner representation to the $D^{[\frac{1}{2}, \frac{1}{2}]}$ covariant representation. Rather the intertwiner only has L-covariance up to additive gauge transformations i.e. up to affine longitudinal terms. For general Lorentz transformations this affine law reads:

$$(U(\Lambda)e)_\mu(k) = \Lambda_\mu^\nu e_\nu(\Lambda^{-1}k) + k_\mu H(\Lambda, k) \tag{82}$$

This peculiar manifestation of the $(0, h = 1)$ little group $\tilde{E}(2)$ is the cause for the appearance of the local gauge issue in local quantum physics. Unfortunately this quantum origin is somewhat hidden in the quantization approach, where it remains invisible behind the geometrical interpretation in terms of fibre bundles. In terms of the potential in the physical Fock space we have

$$U(\Lambda)A_\mu(x)U^*(\Lambda) = \Lambda_\mu^{-1\nu} A_\nu(\Lambda x) + \partial_\mu H \tag{83}$$

where H is a concrete operator involving the $a_\pm^\#$ and $e_\pm$ from (80). Formally the string-like localized vectorpotential (with localization chosen in the spacelike n-direction) may be written as a line integral over the field strength

$$\begin{aligned} \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) &= F_{\mu\nu}(x) \\ A_\nu &= \frac{1}{n \cdot \partial} n^\mu F_{\mu\nu} \curvearrowright n \cdot A = 0 \end{aligned} \tag{84}$$

Where the regularization needed in order to make mathematical sense out of these stringlike objects and related technical problems will not be discussed in this qualitative presentation.

Comparing with the massive case, one notes that the problematic aspects of the use of vectorpotentials in the local description of s=1 show up already in the quantum aspects of free Wigner photons before the renormalization procedure. What is in common to both cases is the fact that no existing formulation of renormalized perturbation theory is capable to treat $\dim A_\mu = 1$ nonlocal vectorpotentials in the framework of Wick-polynomials W, both cases can be dealt with in terms of the same remedy namely cohomological extension by ghosts within a BRS-like formalism. As a result of the availability of arguments based on scattering theory the BRS formalism can be simplified in the massive case whereas in the massless theory one must use the full BRS-formalism which suffers contribution from interactions.

In order to get rid of the "ghost-catalyzers" in favor of a ghostfree formulation, one has to go significantly outside the present perturbative framework. A hint where to look comes from the observation that the ghosts have been introduced to lower the operator dimension of A_μ from two to one, while still maintaining the (formal) locality and the covariance of the operators. This is clearly an off-shell short distance argument. So this observation suggests to look for an on-shell formulation. on-shell quantities are the true S-matrix and formfactors of physical operators A between multiparticle scattering states:

$$S \qquad |p_1,...p_n\rangle^{out} = |p_1,...p_n\rangle^{in} \tag{85}$$
$$^{out}\langle p_1'...p_m'|\, A\, |p_1,...p_n\rangle^{in}$$

Note that we defined the formfactors in such a way that the matrix elements of S are equal to the formfactor of the identity operator. So the first question is: can one compute the perturbative expansion of S while staying on-shell all the time. The causal approach used before does not fulfill this requirement, since the Bogoliubov transition operator $S(g)$ is off shell (in the sense of this paper) and only approaches the mass shell in the asymptotic limit[33]. In fact it is not possible to formulate causality which involves products of fields directly on-shell, its indirect formulation goes through the notoriously elusive crossing symmetry together with some inexorably linked subtle analytic continuation properties. So the question can be reformulated as: does unitarity, crossing symmetry and a lowest order (tree-structure) input for S allow an on-shell iterative approach ala Epstein and Glaser?

By an on-shell construction of S we mean its direct construction without the (analytically simpler but conceptually more complicated way) detour through $S(g)$ with g off-shell. Up to one loop order such construction already exists in the literature and was recently used for calculations in thermal QFT [7]. By contour deformation of the standard one loop integral one obtains a manifestly crossing-symmetric sum over (forward and backward) mass shell integrations. In this way the one-loop integration will be reduced to mass shell integrations (one of the loop particles is on its mass shell) over tree diagrams which have been closed at two ends. Such representations were used in particular in d=1+1 [5][34], and the techniques probably date back to the 60^{ies} [26]. Note that the basic input are the tree diagrams and not, as in the causal approach, the interaction vertices (which for the trilinear case generically vanish on-shell). Therefore the one-loop approximation of S for vectormesons will not need ghosts whereas the

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[33] The difference between on- and off-shell looks quite innocuous in momentum space, however the spacetime aspects could be different. In order to illustrate this point, just look at the massive Thirring model. The $S(g)$ resp. the field correlation functions show the full field theoretic vacuum- and one-particle polarization structure (virtual particle structure) whereas the S-matrix and the closely related PFG wedge generators are "quantum mechanical" i.e. obey particle number conservation.

[34] The main motivation of that paper was to point out that in d=1+1 all one-loop integrals can be described in terms of one universal master function times the tree expression. In this way the factorization property of the tree approximation in theories without real particle creation was shown to remain stable in passing to the one-loop terms. There is no such master representation for $d \geq 2+1$.

off-shell $S(g)$ does. Unfortunately the authors only express their belief in a generalization to multi-loops [48] but indicated no proof. We hope to return to this important point in the near future.

Believing for a moment that the answer is positive, the next question is: can we compute the perturbation expansion of formfactors for a known perturbative expansion of S? At this point this program would merge and become a special application of a much broader construction of a kind of perturbation theory of local algebras which avoids the use of individual fields. Such an approach has been recently formulated by one of the present authors (B. S.) [37][34] but up to date it only has been applied to the special case of d=1+1 factorizing models. Its main new ingredients are modular theory and the existence of **polarization free** one-particle generators of wedge algebras (PFG's). These PFG's are operators which are wedge-localized and whose one time application to the vacuum vector creates a clean one particle state, without those particle/antiparticle polarization clouds which are inexorably associated with smaller than wedge localizations at least in the presence of interactions. These PFG's are so to say the best compromise between particles and fields because the (von Neumann) algebras generated by them contain on the one hand no annihilators of the vacuum (which is a typical property of fields restricted to regions which permit a nontrivial spacelike complement), and on the other hand they have the mentioned PFG property (related to the fact that a wedge is transformed into itself by an appropriate Lorentz-boost). Their correlation functions are uniquely fixed in terms of products of "nonoverlapping" S-matrices and their thermal KMS-property which expresses their modular wedge localization [38] is equivalent to the crossing symmetry of the S-matrix. Their mixed correlation functions with one additional operator $A \in \mathcal{A}(W)$ are closely related to the above generalized formfactors. For the mentioned case of a d=1+1 factorizing S-matrix, the Fouriertransforms of the PFG's turned out to coalesce with the Zamolodchikov-Faddeev operators, whch shows that PFG's are too noncommutative in order to be visible in a quantization or euclidean approach. In this case one can go beyond perturbation theory and obtain the conceptual basis [37][34] as well as a computational formalism for the Karowski-Weisz-Smirnov [28][41]Bootstrap-Formfactor programm.

It should be clear that such a perturbative modular construction program for non-factorizing higher dimensional theories, if feasible, would not need any ghosts. This is so since their only role was to allow the incorporation of vectormesons into the standard renormalization approach based on the off-shell time-ordered (or retarded) vacuum expectations, i.e. they are short-distance behavior-improving objects (by lowering the operator dimension of the interaction density). A perturbative approach based on modular localization and PFG's however does not use such concepts at all. In fact the wedge localized PFG's and their correlation functions have a kind of natural cutoff as a result of their extremely large localization and their only role is to create the wedge algebras. In this context the formfactors are not associated with individual operators in this algebra, but rather one is dealing with spaces of formfactors formed by all elements in that algebra sandwiched between in and out vectors. Instead of the short distance behavior of special (Lagrangian) field coordinates, the existence problem of models now depends on the nontriviality of double cone algebras obtained via intersecting wedge algebras [37][34] [38]. Even if there would be already sufficiently many concrete results in this new approach, it would be advisable to put this into a separate paper because the very concepts and methods will be very different from those used in this paper.

6 Future Perspectives

The history of gauge theory and its connection with higher spin renormalizable QFT is one of the most revealing and fascinating in the development of concepts in QFT. Contrary to a widespread opinion, we believe that this is still an unfinished story with possible future surprises in store.

The usefulness of the role of the "gauge principle" as a selection principle in the classical setting is beyond any controversy. This is even strengthened if we move to the realm of the semiclassical setting for quantum matter coupled to external fields (minimal electromagnetic-coupling), in fact it is only this setting which has the most direct relation to the differential geometric concepts of fibre bundles. The full QFT involving perturbatively interacting massive vectormesons however does not need such a principle if one demands (as one does for any other theory) renormalizability. To the contrary, the spin one situation has a very rigid connection of the assumed observable particle content with the admissable interaction, as we saw in section 4. In some way this explains the (quasi)classical gauge principle in the Bohr correspondence limit. We also argued that the occurrence of additional *physical* degrees of freedom in interactions is a consequence of the required renormalizability of spin=1 situations. The simplest and (under reasonable assumptions) perhaps the only perturbative consistent possibility is a scalar particle: the ubiquitous Higgs, but now stripped of its vacuum expectation hallmark and delegated to its consisteny role in the perturbative version of the Schwinger-Swieca screening. Again we expect that this important consistency feature will come out much more clearly in a ghostfree approach.

It is interesting to look back and ask if and where this alternative viewpoint on massive vectormesons has occurred before. Naturally in all approaches leaning heavily on quantization and differential geometry, one does not find it. This includes the famous Yang-Mills paper and presumably also its older predecessor, a 1938/39 conference report by Oscar Klein (which, as a consequence of the second world war, did not lead to a theoretical discussion on vectormesons). In fact the mentioned particle physics criticism by Pauli. concerning the limitation to zero mass and the ensuing infrared problems, was well taken and it was only through the incorporation of the Higgs mechanism into standard model that Weinberg and Salam were able to make that theory work and to overcome Pauli's criticism. Within a gauge-based approach this was also the only way to accomplish this.

However already before the contribution of Higgs there were attempts to counteract Pauli's gauge dictum against massive vectormesons by slightly different ideas. As already mentioned, one was Schwinger's screening mechanism.

The point of view which we have been favoring (at the end of section 4) is inverse to Schwinger's. Instead of screening the zero mass situation via the Schwinger-Higgs mechanism, we would like to start from the conceptually simpler massive theory and talk about liberation of charges. The advantage would be that the necessary decoupling of the physical (the Higgs analogue) degree of freedom, which was required by perturbative renormalization in that limit, will be inexorably linked with the emergence of the infraparticle aspects (including their semiinfinite stringlike localization) of the liberated charge carriers.

Question about the possible nonintrinsic nature of the Higgs mechanism are actually quite old but remained dormant for a long time, probably as a result of the unexpected enduring success of the standard model. But sometimes people did worry about it. For example P. W. Anderson [1] investigated an interesting physical analogy to the plasmon problem and made the following comment about the mechanism behind massive photons: "How, then, if we were confined to a plasma as we are to the vacuum and could only measure renormalized quantities, might we try to determine whether, before turning on the effects of electromagnetic interaction, A (the vectorpotential) had been a massless gauge field and... . As far as we can see this is not possible." But even the well-known Elitzur's theorem can be interpreted as pointing into this direction.

It is perhaps not even the significant phenomenological success of the standard model which chained most physicist to the interpretation of the vectormeson mass in terms of the Higgs mechanism within gauge theory , but rather the desire to subjugate all physics to the (classical) "elegance" of differential geometry than to open up some new and difficult conceptual problems about real time local quantum physics.

Our viewpoint suggests that it may be physically very fruitful to look for a ghostfree formulation in form of a on-shell perturbation theory of the S-Matrix and the closely related wedge algebra by using some features of Tomita's modular theory as adapted to the wedge situation. This would be the safest and most systematic way to decide whether there exist other higher spin s>1 theories which lead to renormalizable physical theories (outside the standard power counting for the operator dimension of the interaction polynomial W) without knowing any cohomological trick. It may be also the safest way to understand whether perturbative consistency arguments requiring the presence of more physical degree of freedoms (the true story behind the Higgs particle) are part of a bigger story.

If we succeed to direct some of the attention, which has been given during the last 25 years to geometric/mathematical aspects back towards the the many open and interesting conceptual local quantum problems, the time it took for writing this paper was well spent.

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7 Appendix A: The S-matrix to first order

First we prove the following statement (which is of general interest):

Lemma 2: Let $V(x)$ be an arbitrary Wick polynomial with derivatives (i.e. an arbitrary element of the Borchers class of the free fields which are present) and let all free fields be massive. Then the relation

$$\lim_{\epsilon\to 0}\int d^4x\, V(x)g(\epsilon x)=0, \qquad (g\in\mathcal{D}(\mathbf{R}^4),\quad g(0)=1) \tag{86}$$

implies that there exists another Wick polynomial (with derivatives) W_1^μ such that

$$V=\partial_\mu W_1^\mu+V_0. \tag{87}$$

Thereby V_0 is the sum of all Wick monomials $C:\partial^{a_1}A_{j_1}(x)...\partial^{a_n}A_{j_n}(x):$ ($C=$const., A_j is a free field with mass m_j) in V for which the masses $m_{j_1},...,m_{j_n}$ are such that the solution for $(k_1,...,k_n)\in\mathbf{R}^{4n}$ of

$$k_1+...+k_n=0\quad\bigwedge\quad k_l^2=m_{j_l}^2\quad\forall l=1,...,n \tag{88}$$

contains no non-empty open subset of the manifold $k_l^2=m_{j_l}^2\quad\forall l=1,...,n$.

A simple example for a term in V_0 is $:\varphi(x)^3:$, where $(\Box+m^2)\varphi=0$, $m>0$.

Proof:[35] To simplify the notations we assume that the free fields are bosonic scalars, the generalization to other types of free fields is obvious. Global factors 2π are omitted in the whole proof. We write V in the form

$$V(x)=\sum_n\sum_{j_1,...,j_n}P_{j_1...j_n}(\partial^{x_1},...,\partial^{x_n}):A_{j_1}(x_1)...A_{j_n}(x_n):|_{x_1=...=x_n=x}, \tag{89}$$

where the sum over n is finite, the A_j's are free fields and $P_{j_1...j_n}(\partial^{x_1},...,\partial^{x_n})$ is a polynomial in the partial derivatives $\partial_\mu^{x_l}$, $1\le l\le n$. Obviously we may replace $P_{j_1...j_n}(\partial^{x_1},...,\partial^{x_n})$ by

$$\bar{P}_{j_1...j_n}(\partial^{x_1},...,\partial^{x_n})=\frac{1}{n!}\sum_{\pi\in S_n}P_{j_{\pi(1)}...j_{\pi(n)}}(\partial^{x_{\pi(1)}},...,\partial^{x_{\pi(n)}}) \tag{90}$$

in (89). In terms of annihilation and creation operators the free fields read

$$A_l(y)=\int d^4p\,\delta(p^2-m_l^2)\tilde{A}_l(p)e^{ipy},\quad \tilde{A}_l(p):=[\Theta(-p^0)a_l(-\vec{p})+\Theta(p^0)b_l^+(\vec{p})] \tag{91}$$

($a_l=b_l$ is possible), where $[b_l(\vec{p}),b_j^+(\vec{k})]=\delta_{lj}2\sqrt{\vec{p}^2+m_l^2}\delta^3(\vec{p}-\vec{k})$ and similar for $[a_l,a_j^+]$. Now we insert (89-91) into (86)

$$\begin{aligned}0&=\lim_{\epsilon\to 0}\sum_n\int dx\,g(\epsilon x)\int dx_1...dx_n\,\delta(x_1-x,...,x_n-x)\cdot\\&\quad\cdot\sum_{j_1,...,j_n}\bar{P}_{j_1...j_n}(\partial^{x_1},...,\partial^{x_n}):A_{j_1}(x_1)...A_{j_n}(x_n):\\&=\sum_n\int d^4p_1...d^4p_n\sum_{j_1,...,j_n}\bar{P}_{j_1...j_n}(ip_1,...,ip_n)\delta^4(p_1+...+p_n)\cdot\\&\quad\cdot\prod_{l=1}^n\delta(p_l^2-m_{j_l}^2)[\Theta(-p_l^0)a_{j_l}(-\vec{p}_l)+\Theta(p_l^0)b_{j_l}^+(\vec{p}_l)].\end{aligned} \tag{92}$$

[35] Klaus Fredenhagen told us the main idea of proof. The method is strongly influenced by the proof of lemma 2 (including appendix b)) in [9].

(Here we have exchanged the order of integrations. This can be justified by considering matrix elements between wave packets.) Taking inproper matrix elements $< b^+_{r_1}(\vec{k}_1)...b^+_{r_s}(\vec{k}_s)\Omega|...a^+_{r_{s+1}}(-\vec{k}_{s+1})...a^+_{r_t}(-\vec{k}_t)\Omega >$ of (92), where Ω is the vacuum of the free fields, we obtain[36]

$$0 = \bar{P}_{r_1...r_t}(ik_1, ..., ik_t)\delta^4(k_1 + ... + k_t) \quad \text{where} \quad k_l = (\pm\sqrt{\vec{k}_l^2 + m^2_{r_l}}, \vec{k}_l)\ \forall l, \tag{93}$$

more precisely $k^0_l > 0$ for $l = 1, ..., s$ and $k^0_l < 0$ for $l = s + 1, ..., t$. If $r_1, ..., r_t$ are such that the corresponding term belongs to $(V - V_0)$, then the condition (93) restricts the polynomial $\bar{P}_{r_1...r_t}$ in the following way. We make a Taylor expansion with respect to $k = (k_1 + ... + k_t)$ at $k = 0$:

$$\bar{P}_{r_1...r_t}(ik_1, ..., ik_t) = \bar{P}_{r_1...r_t}(ik_1, ..., ik_{t-1}, -i(k_1 + ...k_{t-1})) + ik_\mu \dot{Q}^\mu_{r_1...r_t}(k_1, ..., k_t) \tag{94}$$

where $\dot{Q}^\mu_{r_1...r_t}$ is also a polynomial. From (93) we conclude (for all $(r_1, ..., r_t)$ which appear in $(V - V_0)$) that $\bar{P}_{r_1...r_t}(ik_1, ..., ik_{t-1}, -i(k_1 + ... + k_{t-1}))$ vanishes if $k^0_l = \pm\sqrt{\vec{k}_l^2 + m^2_{r_l}}$, $\forall l = 1, ..., t-1$ and $-(k_1 + ... + k_{t-1})^0 = \pm\sqrt{(\vec{k}_1 + ... + \vec{k}_{t-1})^2 + m^2_{r_t}}$ (the sign of k^0_l obeys the above prescription). Buchholz and Fredenhagen proved in appendix b) of [9] [37] that this implies that there exist other polynomials $P^{(l)}(k_1, ..., k_{t-1})$ $(l = 1, ..., t)$ (for simplicity we omit the indices $r_1, ..., r_t$) such that

$$\begin{aligned} \bar{P}(ik_1, ..., ik_{t-1}, -i(k_1 + ...k_{t-1})) &= \sum_{l=1}^{t-1}(k_l^2 - m^2_{r_l})P^{(l)}(k_1, ..., k_{t-1}) \\ &+ ((k_1 + ... + k_{t-1})^2 - m^2_{r_t})P^{(t)}(k_1, ..., k_{t-1}). \end{aligned} \tag{95}$$

We conclude

$$\begin{aligned} \bar{P}(ik_1, ..., ik_t)\prod_{l=1}^t \delta(k_l^2 - m^2_{r_l}) &= [((k_1 + ... + k_{t-1})^2 - k_t^2)P^{(t)}(k_1, ..., k_{t-1}) \\ &\quad + ik_\mu\dot{Q}^\mu(k_1, ..., k_t)]\prod_{l=1}^t \delta(k_l^2 - m^2_{k_l}) \\ &= i(k_1 + ... + k_t)_\mu \quad Q^\mu(ik_1, ..., ik_t) \quad \prod_{l=1}^t \delta(k_l^2 - m^2_{k_l}), \end{aligned} \tag{96}$$

where $Q^\mu(ik_1, ..., ik_t) := -i(k_1 + ... + k_{t-1} - k_t)^\mu P^{(t)}(k_1, ..., k_{t-1}) + \dot{Q}^\mu(k_1, ..., k_t)$. Next we symmetrize Q^μ according to (90) and denote the result by $\bar{Q}^\mu$. The equation (96) holds still true if we replace Q^μ by $\bar{Q}^\mu$. Summing up we obtain

$$\begin{aligned} (V - V_0)(x) &= \sum_n \sum_{j_1, ..., j_n} \int d^4p_1...d^4p_n\, i(p_1 + ... + p_n)_\mu \bar{Q}^\mu_{j_1...j_n}(ip_1, ..., ip_n) \cdot \\ &\quad \cdot e^{i(p_1 + ... + p_n)x} : \prod_{l=1}^n \delta(p_l^2 - m^2_{j_l})[\Theta(-p^0_l)a_{j_l}(-\vec{p}_l) + \Theta(p^0_l)b^+_{j_l}(\vec{p}_l)] : \\ &= \partial^x_\mu \sum_n \sum_{j_1, ..., j_n} \bar{Q}^\mu_{j_1...j_n}(\partial^{x_1}, ..., \partial^{x_n}) : A_{j_1}(x_1)...A_{j_n}(x_n) : |_{x_1 = ... = x_n = x} \end{aligned}$$

[36] Usually there are several possibilities to contract the creation and annihilation operators. All give the same contribution due to the permutation symmetry of $\bar{P}_{r_1...r_t}$ (90).

[37] This proof is non-elementary, it relies on results of the theory of polynomial rings. The proof is given for vanishing masses. It is quite obvious that the statement holds also in the massive case, but we have not worked out a proof.

(we set $\bar{Q}^{\mu}_{r_1...r_t} := 0$ if $(r_1, ..., r_t)$ belongs to V_0) which is the assertion (87). □

Physical consistency (31) to first order means

$$\lim_{\epsilon\to 0} \int d^4x\, g(\epsilon x)[Q, W(x)]|_{\ker Q} = 0. \tag{97}$$

By an obvious modification of the above Lemma we conclude from this condition that there exists a Wick polynomial W_1^{μ} such that

$$[Q, W(x)]|_{\ker Q} = (i\partial_\mu W_1^{\mu}(x) + [Q, W_{(0)}(x)])|_{\ker Q}, \tag{98}$$

where $W_{(0)}$ is that part of W which satisfies the kinematical equations (88) for a discrete set of momenta only, similarly to V_0 in the Lemma. (For a free field φ note that $[Q, \varphi(x)]$ satisfies the same field equation as φ and hence has the same mass.) Since $([Q, W(x)] - i\partial_\mu W_1^{\mu}(x) - [Q, W_{(0)}(x)])$ is a local operator, we can apply the Reeh-Schlieder theorem and obtain that the latter expression vanishes everywhere (cf. (69)), i.e. we may omit the restriction to $\ker Q$ on both sides in (98). In addition W must satisfy the requirements (a)-(e) listed in section 2.

Quadrilinear terms in $W(x)$ (i.e. terms $\sim: AAAA :$, $\sim: AAu\tilde{u} :$, $\sim: u\tilde{u}u\tilde{u} :$, $\sim: \phi\phi\phi\phi :$, $\sim: \phi\phi AA :$, $\sim: \phi\phi u\tilde{u} :$, $\sim: \phi\phi H^2 :$, $\sim: AAH^2 :$, $\sim: u\tilde{u}H^2 :$, $\sim: H^4 :$) belong to $(W - W_{(0)})$, hence they must satisfy the Q-divergence condition to first order (29). The only solution is $\sim: H^4 :$ [22], but such a term will be excluded by physical consistency (31) to second order.

A trilinear term in $W(x)$ (with masses m_1, m_2 and m_3) does not belong to $W_{(0)}$ iff there exists a permutation $\pi \in \mathcal{S}_3$ such that

$$m_{\pi(3)} > m_{\pi(1)} + m_{\pi(2)}. \tag{99}$$

But physical consistency to higher orders will require $m_1 = m_2 = m_3 =: m$ apart from the H-couplings ($m_H > 0$ will not be restricted; see sect. 2). Neglecting this exception we conclude

$$S_1^{(3)} = \lim_{\epsilon\to 0} \int d^4x\, g(\epsilon x) W^{(3)}(x) = 0 \tag{100}$$

and $[Q, S_1^{(3)}] = 0$ (31) is trivially satisfied. (The upper index (3) refers to the trilinear terms.) To get restrictions on $W^{(3)}$ we must consider physical consistency (31) to orders $n \geq 2$, especially for second order tree diagrams.

8 Appendix B: Determination of the parameters in W and $\mathcal{F}_{\text{int}}$ by requiring $[Q, \mathcal{F}_{\text{int}}(x; W)] = 0$

The calculation is lengthy, hence we concentrate on the essential points. We determind the parameters in the Ansatz for W (41) and $\mathcal{F}_{\text{int}}$ (67) by inserting these expressions into the physicality requirement (69) to lowest perturbative orders. Once a parameter is fixed, its value will be used in the following calculations without mentioning it.

- To zeroth order the condition is trivially fulfilled due to[38]

$$[Q, F_d^{\mu\nu}] = 0. \tag{101}$$

- To first order the equation

$$[Q, \mathcal{F}_d^{(1)}(x)] + i \int dx_1 \, [Q, R(W(x_1); F_d(x))] = 0 \tag{102}$$

is required. The second term contains tree diagrams only. By means of (60) they can be written in the form

$$\begin{aligned} R(W(x_1); F_d^{\mu\nu}(x)) &= \Theta(x^0 - x_1^0)[F_d^{\mu\nu}(x), W(x_1)] \\ +\delta(x - x_1) i C \frac{1}{2}(f_{bcd} - f_{cbd}) &: A_b^\mu(x_1) A_c^\nu(x_1) :, \end{aligned} \tag{103}$$

where C is an arbitrary constant. The terms $\sim \Theta(x^0 - x_1^0)$ have propagators $\Delta^{\text{ret}}(x - x_1)$, $\partial\Delta^{\text{ret}}(x - x_1)$, $\partial\partial\Delta^{\text{ret}}(x - x_1)$, where Δ^{ret} is the retarded fundamental solution of the Klein Gordon equation, $\text{supp}\,\Delta^{\text{ret}} \subset \bar{V}^+$. The term $\sim C\delta(x - x_1)$ is due to the non-uniqueness of the extension of $T(W(x_1); F_d^{\mu\nu}(x))$ to the diagonal $x_1 = x_2$. This is similar to the tree normalization terms (49). The tensor f_{bcd} which is the same as in the Ansatz for W[39] is antisymmetrized according to the normalization condition $\mathcal{F}_{d\,\text{int}}^{\mu\nu} = -\mathcal{F}_{d\,\text{int}}^{\nu\mu}$ (iv). Now let ψ_1, ψ_2 be arbitrary free fields and $y_1, y_2 \in \mathbf{R}^4$, $(y_j - x)^2 < 0$ $(j = 1, 2)$. By commuting (102) with $\psi_1(y_1)$ and $\psi_2(y_2)$ we obtain

$$0 = \int dx_1 \, [[[Q, [F_d(x), W(x_1)]], \psi_1(y_1)], \psi_2(y_2)]\Theta(x^0 - x_1^0). \tag{104}$$

We conclude that for $x_1 \in (x + \bar{V}^-) \setminus \{x\}$ the expression $[Q, [F_d(x), W(x_1)]]$ must be a divergence of a local operator with respect to x_1,[40] i.e. there must exist a two-point distribution $R_{1d}^\tau(x_1, x)$ with

$$\Theta(x^0 - x_1^0)[Q, [F_d(x), W(x_1)]] = \partial_\tau^{x_1} R_{1d}^\tau(x_1, x) + \delta(x - x_1) P_d(x), \tag{105}$$

where $P_d(x)$ is a Wick polynomial (with constant coefficients) in free fields (possibly with derivative). In contrast to the adiabatic limit of W (86) there is no kinematical restriction (in the sense of (88)) here, because one of the free field operators in $W(x_1)$ is contracted with $F(x)$ and, hence, the corresponding momentum must not be on-shell. We have not worked out a general proof (in the style of appendix

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[38] If we would not have made the assumption $c_k^{(0)} = 0$ (i), we would find here that most of the zeroth order coefficients must vanish. It would remain $\mathcal{F}_d^{\mu\nu\,(0)} = F_d^{\mu\nu} + x_{da}^{(0)} F_a^{\mu\nu} H + z_{da}^{(0)} : F_a^{\mu\nu} H^2 :$ only. $x^{(0)}$ and $z^{(0)}$ are then forced to vanish by higher orders of (69). But this complicates the calculation a lot and the assumption $c_k^{(0)} = 0$ is reasonable. So we do not do this effort here.

[39] We use here an additional normalization condition, namely that the term $\sim C\delta(x - x_1) : AA :$ has the same colour tensor as the corresponding non-local term (i.e. the term in $\Theta(x^0 - x_1^0)[F^{\mu\nu}(x), W(x_1)]$ with the same external legs).

[40] Note that every operator valued distribution $G(x_1, ..., x_n)$, $(x_j \in \mathbf{R}^4)$ can be written as divergence of a *non-local* operator, e.g. $G(x_1, ..., x_n) = \partial_\nu^{x_1} \int dy \, \partial^\nu D^{\cdots}(x_1 - y) G(y, x_2, ..., x_n)$, where $D^{\cdots}$ is a fundamental solution of the wave equation.

A) of the step from (104) to (105). But the explicit calculations show that this conclusion is correct.[41] Inserting (105) and (103) into (102) it results

$$[Q,\mathcal{F}_d^{\mu\nu\,(1)}(x)] + iP_d^{\mu\nu}(x) - C\frac{1}{2}(f_{bcd} - f_{cbd})[Q, :A_b^\mu(x)A_c^\nu(x):] = 0, \tag{106}$$

which means that the "local terms"[42] must satisfy the condition (102) separately. Later when we shall know more about W (which determines P_d by (105)), we will compute most of the coefficients in $\mathcal{F}_d^{(1)}$ (67) and the normalization constant C from (106). Inserting (101) into (105) we find that W must fulfill

$$[F_d(x),[Q,W(x_1)]] = \partial_\tau^{x_1} R_{1d}^\tau(x_1,x) \quad \text{for} \quad x_1 \in (x+\bar{V}^-)\setminus\{x\}. \tag{107}$$

Obviously this condition is truly weaker than $[Q,W]$ =(divergence of a Wick polynomial) (29). But the higher orders of the physicality requirement (69) yield more information about W.

- To second order (69) reads

$$\begin{aligned}[Q,\mathcal{F}_d^{(2)}(x)] + i\int dx_1\,[Q,R(W(x_1);\mathcal{F}_d^{(1)}(x))]&\\ +\frac{i^2}{2}\int dx_1dx_2\,[Q,R(W(x_1)W(x_2);F_d(x))] &= 0\end{aligned} \tag{108}$$

The tree diagrams in the third term have the structure[43]

$$\begin{aligned}&R(W(x_1)W(x_2);F_d(x))|_{\text{tree}}\\ =&\sum \mathcal{D}_1^{\text{ret}}(x-x_1)\mathcal{D}_2^{\text{ret}}(x_1-x_2) : B_1(x_1)B_2(x_2)B_3(x_2): + (x_1 \leftrightarrow x_2)\end{aligned} \tag{109}$$

where $\mathcal{D}_j^{\text{ret}} = \Delta^{\text{ret}}, \partial\Delta^{\text{ret}}, (\partial\partial\Delta^{\text{ret}} + C_j\delta)$ (C_j are arbitrary constants) and with free fields B_k (possibly with derivative), i.e. $B_k \in \{A^\mu, \partial A, \partial\partial A, (\partial)u, (\partial)\tilde{u}, (\partial)\phi, (\partial)H\}$. All other diagrams in (108) have less than three legs at the vertex/vertices $\neq x$. Now let $y_l \in \mathbf{R}^4$, $l = 1,2,3$, with $(y_l - x)^2 < 0$ and let ψ_l, $l = 1,2,3$, be arbitrary free fields. In the triple commutator of (108) with $\psi_1(y_1)$, $\psi_2(y_2)$ and $\psi_3(y_3)$ only the terms (109) survive (the terms $\sim C_1\delta(x-x_1)$ in (109) do not contribute either)

$$\begin{aligned}0 &= \int dx_1dx_2\,[[[[Q,R(W(x_1)W(x_2);F_d(x))|_{\text{tree}}],\psi_1(y_1)],\psi_2(y_2)],\psi_3(y_3)]\\ &= \int dx_1dx_2\,[[[[Q,\{[F_d(x),T(W(x_1)W(x_2))] - W(x_1)[F_d(x),W(x_2)]\\ &\quad -W(x_2)[F_d(x),W(x_1)]\}|_{\text{tree}}],\psi_1(y_1)],\psi_2(y_2)],\psi_3(y_3)],\end{aligned} \tag{110}$$

where we have inserted (60), (61) and the causal factorization (4) of the time-ordered products due to $x \notin \{x_1,x_2\}+\bar{V}^-$. Similarly to the step from (104) to (105) we conclude that $[Q,R(W(x_1)W(x_2);F_d(x))|_{\text{tree}}]$ must be a sum of divergences (with respect to x_1 or x_2) of local operators for $x_1,x_2 \in (x+\bar{V}^-)\setminus\{x\}$.

[41] This remark concerns also the analogous (more complicated) steps from (110) to (113) and from (129) to (130) in second and third order.

[42] We set quotation marks because the splitting of a distribution in a local and a non-local part is non-unique.

[43] Note that totally retarded products (60) contain connected diagrams only.

Now we transform (110) by means of identities of the kind

$$[Q,(M(x)N(y))|_{\text{tree}}] = [Q,M(x)N(y)]|_{\text{tree}} \tag{111}$$

$$= ([Q,M(x)]N(y))|_{\text{tree}} + (M(x)[Q,N(y)])|_{\text{tree}}, \tag{112}$$

where M and N are arbitrary Wick polynomials. Then using (101) and (107) we find the condition

$$[F_d(x),[Q,T(W(x_1)W(x_2))]]|_{\text{tree}} - [Q,W(x_1)][F_d(x),W(x_2)]|_{\text{tree}}$$
$$-[Q,W(x_2)][F_d(x),W(x_1)]|_{\text{tree}} = \text{div}_{x_1} + (x_1 \leftrightarrow x_2) \text{ for } x_1,x_2 \in (x+\bar{V}^-)\setminus\{x\}. \tag{113}$$

where div_y means some divergence of a local operator with respect to y. Next we specialize to the subregion $x_1^0 > x_2^0$, where $T(W(x_1)W(x_2))$ factorizes. By means of again (107) we see that W must satisfy

$$[[F_d(x),W(x_1)],[Q,W(x_2)]]|_{\text{tree}} = \text{div}_{x_1} + \text{div}_{x_2} \text{ for } x_1,x_2 \in (x+\bar{V}^-)\setminus\{x\},\ x_1^0 > x_2^0. \tag{114}$$

Inserting the Ansatz for W into this condition one finds that (114) yields the same restrictions on the parameters in W as the Q-divergence condition to first order (29):

$$[Q,W] = \text{divergence of a Wick polynomial}. \tag{115}$$

In other words we obtain the same results for the parameters in W as in (45), especially $f_{abc} = \epsilon_{abc}$. We recall that these values of the parameters are not only necessary for (115), they are also sufficient. Hence (107) is also fulfilled. By inserting (115) into (113) we find the condition

$$[F_d(x),[Q,T(W(x_1)W(x_2))|_{\text{tree}}]] = \text{div}_{x_1} + (x_1 \leftrightarrow x_2) \quad \text{for} \quad x_1,x_2 \in (x+\bar{V}^-)\setminus\{x\}. \tag{116}$$

This is a necessary but not sufficient condition for $[Q,T(W(x_1)W(x_2))|_{\text{tree}}] = \text{div}_{x_1} + (x_1 \leftrightarrow x_2)$. To get the full information of this latter condition we need to go to third order.

- However, first we return to the "local terms" to first order (106), because we shall need the validity of this equation. We explicitly calculate the terms on the l.h.s. of (105), transform them into divergence form (as far as possible) and obtain[14]

$$P_d^{\mu\nu} = \epsilon_{bcd}[F_b^{\mu\nu}u_c - A_b^\mu\partial^\nu u_c + A_b^\nu\partial^\mu u_c]. \tag{117}$$

Inserting this into (106) we find that (106) is fulfilled iff the normalization

$$C = -1 \tag{118}$$

is chosen and the first order coefficients in $\mathcal{F}_{d\,\text{int}}^{\mu\nu}$ take the values

$$t_{dbc}^{(1)} = -\frac{1}{m_c}\epsilon_{bcd},\quad \tilde{t}^{(1)} = 0,\quad v^{(1)} = 0,\quad \tilde{v}^{(1)} = 0,\quad w^{(1)} = 0,\quad \tilde{w}^{(1)} = 0,$$
$$w'^{(1)} = 0,\quad \tilde{x}^{(1)} = 0,\quad y^{(1)} = 0,\quad \tilde{y}^{(1)} = 0,\quad y'^{(1)} = 0,\quad \tilde{z}^{(1)} = 0. \tag{119}$$

[14] By the experience of [2] we know that all contributions to P_d come from the contraction of $F_d(x)$ with $\partial A(x_1)$ in the first term of $W(x_1)$ (41).

$x^{(1)}$ and $z^{(1)}$ are still arbitrary. Note that (107), (118) and (119) are not only necessary for (102), together they are also sufficient.

- We now consider the condition (69) to third order, i.e. the analogous equation to (102), (108). By commuting this equation with arbitrary free fields $\psi_1(y_1)$, $\psi_2(y_2)$, $\psi_3(y_3)$, $\psi_4(y_4)$, $(y_l - x)^2 < 0\ \forall l$, we obtain

$$0 = \frac{i^2}{2!}\int dx_2 dx_3\, [[[[[Q, R(W(x_2)W(x_3); \mathcal{F}^{(1)}(x))], \psi_1(y_1)], \psi_2(y_2)], \psi_3(y_3)], \psi_4(y_4)]$$
$$+\frac{i^3}{3!}\int dx_1 dx_2 dx_3\, [[[[[Q, R(W(x_1)W(x_2)W(x_3); F(x))], \psi_1(y_1)], ...\psi_4(y_4)]. \tag{120}$$

Only tree diagrams of the following types contribute (we use the same notations as in (109)): in the first term

$$\sum \mathcal{D}_1^{\rm ret}(x - x_2)\mathcal{D}_2^{\rm ret}(x - x_3) : B_0(x)B_1(x_2)B_2(x_2)B_3(x_3)B_4(x_3) : +(x_2 \leftrightarrow x_3) \tag{121}$$

(where $B_0(x)$ is a free field or $\equiv 1$), in the second term

$$\sum \mathcal{D}_3^{\rm ret}(x - x_1)\mathcal{D}_4^{\rm ret}(x_1 - x_2)\mathcal{D}_5^{\rm ret}(x_1 - x_3) : B_5(x_2)B_6(x_2)B_7(x_3)B_8(x_3) : +... \tag{122}$$

and

$$\sum \mathcal{D}_6^{\rm ret}(x - x_1)\mathcal{D}_7^{\rm ret}(x_1 - x_2)\mathcal{D}_8^{\rm ret}(x_2 - x_3) : B_9(x_1)B_{10}(x_2)B_{11}(x_3)B_{12}(x_3) : +... \tag{123}$$

where the dots mean terms obtained by cyclic permutations. In the first term in (120) we take (60) and the causal factorization basing on $x \notin (\{x_2, x_3\} + \bar{V}^-)$ into account. By means of (115) it results

$$\frac{i^2}{2!}\int dx_2 dx_3\, [[[[\{[[Q, \mathcal{F}^{(1)}(x)], T(W(x_2)W(x_3))] - W(x_2)[[Q, \mathcal{F}^{(1)}(x)], W(x_3)]$$
$$-W(x_3)[[Q, \mathcal{F}^{(1)}(x)], W(x_2)]\}|_{\rm tree}, \psi_1(y_1)]..., \psi_4(y_4)]. \tag{124}$$

Additionally we have used that due to (121) only the disconnected diagram of $T(W(x_2)$ $W(x_3))$ contributes, hence $[...[\mathcal{F}^{(1)}(x), [Q, T(W(x_2)W(x_3))]]..., \psi_4(y_4)] = {\rm div}_{x_2} + {\rm div}_{x_3}$ by (115).

In the second term in (120) the situation is more complicated. The diagrams of the type (123) obey the causal factorization basing on $x \notin (\{x_1, x_2, x_3\} + \bar{V}^-)$. But for the diagrams of the type (122) we only know $x \notin (\{x_2, x_3\} + \bar{V}^-)$ (or promulgated configurations). To fix the position of the third vertex we consider two smooth functions h_1, h_2 with

$$1 = h_1(y) + h_2(y)\ \forall y \in x + \bar{V}^-, \quad h_1 \in \mathcal{D}(\mathbf{R}^4), \quad x \in {\rm supp}\, h_1,$$
$${\rm supp}\, h_2 \cap x + \bar{V}^+ = \emptyset, \quad (y_l - z)^2 < 0\ \forall z \in {\rm supp}\, h_1,\ l = 1, 2, 3, 4. \tag{125}$$

With that the second term in (120) can be written in the form

$$\frac{i^3}{3!}\int dx_1 dx_2 dx_3\, (h_1(x_1)h_2(x_2)h_2(x_3) + h_2(x_1)h_1(x_2)h_2(x_3) + h_2(x_1)h_2(x_2)h_1(x_3)$$
$$+h_2(x_1)h_2(x_2)h_2(x_3))[...[Q, R(W(x_1)...; F(x))|_{\rm tree}]..., \psi_4(y_4)]. \tag{126}$$

In the terms with a factor $h_1(x_l)$ the diagrams of the type (122) contribute only, and we may insert the causal factorization due to $\{x, x_l\} \cap (\{x_j, x_k\} + \bar{V}^-) = \emptyset$ (where $\{l, j, k\} = \{1, 2, 3\}$). So the contribution of $(h_1(x_1)h_2(x_2)h_2(x_3) + h_2(x_1)h_1(x_2)h_2(x_3) + h_2(x_1)h_2(x_2)h_1(x_3))$ in (126) reads

$$\frac{i^3}{2!}\int dx_1 dx_2 dx_3\, h_1(x_1)h_2(x_2)h_2(x_3)[...\{[[Q, R(W(x_1); F(x))], T(W(x_2)W(x_3))] \\ -W(x_2)[[Q, R(W(x_1); F(x))], W(x_3)] \\ -W(x_3)[[Q, R(W(x_1); F(x))], W(x_2)]\}|_{\text{tree}} ..., \psi_4(y_4)], \tag{127}$$

where we have used $[Q, T(W(x_2)W(x_3))] = \text{div}_{x_2} + \text{div}_{x_3}$, which holds true here for the same reason as in (124). >From (102) and (103) we know

$$\begin{aligned} & [Q, \mathcal{F}^{(1)}(x)] + i\int dx_1\, h_1(x_1)[Q, R(W(x_1); F(x))] \\ = & -i\int dx_1\, h_2(x_1)[Q, R(W(x_1); F(x))] = -i\int dx_1\, h_2(x_1)\partial_\tau^{x_1}[F(x), W_1^\tau(x_1)], \end{aligned} \tag{128}$$

where we have used $[Q, W] = \partial_\tau W_1^\tau$ (115). We now insert (127), (126) and (124) into (120). By means of (128) we obtain

$$\begin{aligned} 0 = & \frac{i^3}{3!}\int dx_1 dx_2 dx_3\, h_2(x_1)h_2(x_2)h_2(x_3)[...\{[Q, R(W(x_1)W(x_2)W(x_3); F(x))|_{\text{tree}}] \\ & -\partial_\tau^{x_1}([[F(x), W_1^\tau(x_1)], T(W(x_2)W(x_3))] - W(x_2)[[F(x), W_1^\tau(x_1)], W(x_3)] \\ & -W(x_3)[[F(x), W_1^\tau(x_1)], W(x_2)])|_{\text{tree}} - \partial^{x_2}(...) - \partial^{x_3}(...)\}..., \psi_4(y_4)], \end{aligned} \tag{129}$$

where $\partial^{x_2}(...)$ and $\partial^{x_3}(...)$ are obtained from $\partial^{x_1}(...)$ by cyclic permutation. The next step is analogous to the step from (104) to (105), but more complicated because one has to care about the cancellation of the boundary terms. Taking the freedom in the choice of h_2 into account, (129) is equivalent to the existence of a local operator $R_3^\tau(x_1; x_2, x_3; x)$ with

$$\begin{aligned} [Q, R(W(x_1)W(x_2)W(x_3); F(x))]|_{\text{tree}} &= \partial_\tau^{x_1}R_3^\tau(x_1; x_2, x_3; x) + \text{cyclic permutations}, \\ \forall x_1, x_2, x_3 &\in (x + \bar{V}^-) \setminus \{x\}, \end{aligned} \tag{130}$$

because the contribution from e.g. $\partial^{x_1}(...)$ in (129) is equal to

$$-\frac{i^3}{3!}\int dx_1 dx_2 dx_3\, (\partial_\tau h_2)(x_1)h_2(x_2)h_2(x_3)[...R_3^\tau(x_1; x_2, x_3; x)..., \psi_4(y_4)]. \tag{131}$$

Let us explain this latter statement. First note that due to (62) and (125) there is only a contribution in (131) for $x_1, x_2, x_3 \in (x + \bar{V}^-) \setminus \{x\} \wedge x_1 \notin (\{x_2, x_3\} + \bar{V}^-)$. In this region we have

$$\begin{aligned} R(W(x_1)W(x_2)W(x_3); F(x)) = & [[F(x), W(x_1)], T(W(x_2)W(x_3))] \\ & -W(x_2)[[F(x), W(x_1)], W(x_3)] \\ & -W(x_3)[[F(x), W(x_1)], W(x_2)] \end{aligned} \tag{132}$$

due to causal factorization. Now we take into account that (in this region) $\partial_\tau^{x_1}R_3^\tau(x_1; x_2, x_3; x)$ comes from the $[Q, W(x_1)]$-terms in $[Q, R(WWW; F)]$ (where $R(WWW; F)$ is given by (132)). However, by replacing $W(x_1)$ by $\partial_\tau W_1^\tau(x_1)$ in (132) we obtain exactly the $\partial^{x_1}(...)$-term in (129).

By means of causal factorization and (101), (115), (116) the condition (130) can be written in the form

$$\begin{aligned}
&\{[F(x),[Q,T(W(x_1)W(x_2)W(x_3))]]+[Q,T(W(x_2)W(x_3))][W(x_1),F(x)]\\
&+[Q,T(W(x_1)W(x_2))][W(x_3),F(x)]+[Q,T(W(x_1)W(x_3))][W(x_2),F(x)]\}|_{\rm tree}\\
=\;&{\rm div}_{x_1}+\text{cyclic permutations} \qquad (133)
\end{aligned}$$

for $x_1,x_2,x_3\in(x+\bar{V}^-)\setminus\{x\}$. Next we specialize to the subregion $x_1\notin(\{x_2,x_3\}+\bar{V}^-)$. Again by causal factorization and (116) we find that $T(W(x_2)W(x_3))$ must fulfill

$$[[F(x),W(x_1)],[Q,T(W(x_2)W(x_3))]]|_{\rm tree}={\rm div}_{x_1}+{\rm div}_{x_2}+{\rm div}_{x_3} \qquad (134)$$

for $x_1,x_2,x_3\in(x+\bar{V}^-)\setminus\{x\}\wedge x_1\notin(\{x_2,x_3\}+\bar{V}^-)$. Inserting the explicit expression for W (with the values of the parameters obtained so far) one sees that this condition is equivalent to

$$[Q,T(W(x_2)W(x_3))|_{\rm tree}]={\rm div}_{x_2}+(x_2\leftrightarrow x_3). \qquad (135)$$

As we mentioned in section 2 the latter requirement is satisfied iff the masses agree $m\equiv m_1=m_2=m_3$ (47), the parameters of the H-coupling take the values (48)[45] and if the normalization constants in the tree normalization terms (49) are suitably chosen (up to λ (50) they are uniquely fixed by (135)). So far there remain two free parameters in W and $T(WW)|_{\rm tree}$: s (41) (cf. (48)) and λ (50). In section 2 they have been determined by

$$[Q,T(W(x_1)W(x_2)W(x_3))|_{\rm tree}]={\rm div}_{x_1}+\text{cyclic permutations}. \qquad (136)$$

By inserting (135) into (133) we find the weaker requirement

$$[F(x),[Q,T(W(x_1)W(x_2)W(x_3))|_{\rm tree}]]={\rm div}_{x_1}+\text{cyclic permutations} \qquad (137)$$

for $x_1,x_2,x_3\in(x+\bar{V}^-)\setminus\{x\}$. But the latter condition does not determined s and λ. One needs to consider the physicality condition (69) to fourth order. There the values (51) for s and λ are obtained, as can be seen by an analogous procedure. So we obtain exactly the same results for the parameters in W and the tree normalizations in $T(WW)$ as in section 2. We recall that they agree completely with the interaction Lagrangian obtained by the Higgs mechanism.

- Now we have explained how to handle the essential difficulties which appear in the determination of the parameters in W (41) and in $\mathcal{F}_{\rm int}$ (67) by the physicality (69). We do not give further details. Instead we give a heuristic summary and the remaining results. To n-th order the requirement (69) reads

$$0=[Q,\mathcal{F}_d^{(n)}(x)]+\sum_{l=1}^{n}\frac{i^l}{l!}\int dx_1...dx_l\,[Q,R(W(x_1)...W(x_l);\mathcal{F}_d^{(n-l)}(x))] \qquad (138)$$

where $\mathcal{F}_d^{(0)}\equiv F_d$.

[45] In the following calculations the undetermined sign κ in (48) is chosen to be $\kappa=1$.

(I) In the "non-local terms" ($x_j \neq x$ for at least one j) we can apply the causal factorization (4) of the time-ordered products. Then the non-local terms in (138) cancel due to

$$[Q, T(W(x_1)...W(x_k))] = \text{sum of divergences of local operators}, \quad 1 \leq k \leq n, \tag{139}$$

and the physicality (138) to lower orders $< n$ (see e.g. (128-129)). We have seen that for the tree diagrams to lowest orders the condition (139) is not only sufficient for the cancellation, it is also necessary. But the lowest orders tree diagrams of (139) fix the parameters in W and $T(WW)|_{\text{tree}}$, as was shown in [2].

For the cancellation of the "non-local terms" in (138) it is not important which kind of physical fields we require to exist, the properties (i)-(vi) of $\mathcal{F}_{\text{int}}$ (67) are essentially not needed. Hence we conjecture that already the existence of *any* non-trivial observable fixes the interaction.

(II) The cancellation of the remaining terms in (138), which are "local" (i.e. their support is the total diagonal $x_j = x, \forall j$), requires a suitable normalization of the time-ordered products $T(W(x_1)...W(x_l); \mathcal{F}_d^{(n-l)}(x))$ (see e.g. (118)) and a suitable choice of the parameters in $\mathcal{F}_d^{(n-l)}$, $l = 0, 1, ..., n-1$. [46] In this way the latter parameters are *uniquely* determined (at least to lowest orders).

So the cancellation of the "local terms" to second order requires

$$\begin{aligned} t^{(2)} = 0, \quad \tilde{t}^{(2)} = 0, \quad v^{(2)}_{dabc} = -\frac{1}{2m^2}[(\delta_{da} - \frac{m}{2}x^{(1)}_{da})\delta_{bc} - \frac{1}{2}(\delta_{dc}\delta_{ba} + \delta_{db}\delta_{ca})], \\ \tilde{v}^{(2)} = 0, \quad w^{(2)} = 0, \quad \tilde{w}^{(2)} = 0, \quad w'^{(2)} = 0, \quad \tilde{x}^{(2)} = 0, \\ y^{(2)}_{dbc} = \frac{1}{2m^2}\epsilon_{dbc} - \frac{1}{m}x^{(1)}_{da}\epsilon_{abc}, \quad \tilde{y}^{(2)} = 0, \quad y'^{(2)} = 0, \quad \tilde{z}^{(2)} = 0, \quad z^{(1)} = 0. \end{aligned} \tag{140}$$

The very last equation results from the terms $\sim g^2 : F(x)u(x)H^2(x) :$. In this case the cancellation is of a special kind as explained below. The other equations are obtained analogously to (106), (119). They come from bilinear and trilinear terms. $x^{(1)}$, $x^{(2)}$ and $z^{(2)}$ are still arbitrary.

To third order the terms $\sim g^3 : F(x)u(x)\phi(x)\phi(x) :$, ($\sim g^3 : F(x)u(x)H^2(x) :$ respectively) cancel iff

$$x^{(1)}_{da} = \frac{1}{m}\delta_{da}, \qquad (z^{(2)}_{da} = \frac{1}{4m^2}\delta_{da} \text{ respectively}). \tag{141}$$

Similarly one finds to fourth order that $x^{(2)}$ must vanish.

Summing up (119), (140) and (141) we obtain the result (70). But we have not checked that the cancellation of the "local terms" in (138) can be achieved for loop diagrams and for tree diagrams to higher orders. For the tree diagrams with more than three external legs (e.g. the very last equation in (140) and the terms considered in (141)) there is no contribution from $\mathcal{F}_d^{(n)}$ in (138) due to our Ansatz (67). Hence the parameters in $\mathcal{F}_d^{(n-l)}$, $l \geq 1$ are available only (besides normalization constants). However, most of the latter parameters have already been fixed (or restricted) by (138) to lower orders. (For example to achieve the cancellation of the terms considered in (141), it is important that $x^{(1)}$ ($z^{(2)}$ resp.) has not been fixed previously.) So we have shown the existence of observables $\mathcal{F}^{\mu\nu}_{d\,\text{int}}$, $d = 1, 2, 3$, only

[46] Mainly parameters in $\mathcal{F}_d^{(n)}$ are determined; most of the parameters in $\mathcal{F}_d^{(n-l)}$, $1 \leq l \leq n-1$, have already been fixed in earlier steps of the induction.

partially. But there is a strong hint that this holds true for all terms to all orders from the identification (given in the main text) of our physical fields $\mathcal{F}^{\mu\nu}_{d\,\mathrm{int}}$ (70) as gauge invariant fields in the framework of spontaneous symmetry breaking of the $SU(2)$ gauge symmetry.

References

[1] P. W Andersen, Phys. Rev. **130**, (1963) 439

[2] A. Aste, M. Dütsch and G. Scharf: 'On gauge invariance and spontaneous symmetry breaking', J. Phys. A, 30 (1997) 5785

M. Dütsch and G. Scharf: "Perturbative gauge invariance: the electroweak theory", hep-th/9612091, to appear in Ann. Phys. (Leipzig)

A. Aste, M. Dütsch and G. Scharf: "Perturbative gauge invariance: the electroweak theory II", hep-th/9702053, to appear in Ann. Phys. (Leipzig)

[3] C. Becchi, A. Rouet and R. Stora: "Renormalization of the abelian Higgs-Kibble model", *Commun. Math. Phys.* **42** (1975), 127

C. Becchi, A. Rouet and R. Stora: "Renormalization of gauge theories", *Annals of Physics (N.Y.)* **98** (1976), 287

[4] J. S. Bell: "High energy behaviour of tree diagrams in gauge theories", Nucl. Phys. B60 (1973) 427-436

[5] B. Berg: "The Massive Thirring Model: Particle Scattering in Perturbation Theory", *N. Cimento A* **41** (1977) 58

[6] N. N. Bogoliubov and D. V. Shirkov: "Introduction to the Theory of Quantized Fields", New York (1959)

[7] F. T. Brandt, Ashok Das, J. Frenkel, A. J. da Silva: "Retarded Greens Functions and Forward Scattering Amplitudes in Thermal Field Theory", hep-th/9809177

[8] R. Brunetti and K. Fredenhagen: "Interacting quantum fields in curved space: Renormalization of φ^4", gr-qc/9701048, *Proceedings of the Conference 'Operator Algebras and Quantum Field Theory', held at Accademia Nazionale dei Lincei, Rome, July 1996*

R. Brunetti and K. Fredenhagen: "Microlocal analysis and interacting quantum field theories: Renormalization on physical backgrounds", math-ph/9903028

[9] D. Buchholz and K. Fredenhagen: "A Note on the Inverse Scattering Problem in Quantum Field Theory", *Commun. Math. Phys.* **56** (1977), 91

[10] J.M. Cornwall, D. N. Levin and G. Tiktopoulos: "Derivation of gauge invariance from high-energy unitarity bounds on the S matrix", Phys. Rev. D10 (1974) 1145

[11] M. Dubois-Violette, private communication

[12] M. Dütsch and K. Fredenhagen: "A local (perturbative) construction of observables in gauge theories: the example of QED", hep-th/9807078, *Commun. Math. Phys.* **203** (1999) 71

[13] M. Dütsch, T. Hurth, F. Krahe and G. Scharf: "Causal construction of Yang-Mills theories. I." *N. Cimento A* **106** (1993), 1029

[14] M. Dütsch, T. Hurth and G. Scharf: "Causal construction of Yang-Mills theories. IV. Unitarity", *N. Cimento A* **108** (1995), 737

[15] M. Dütsch: "On gauge invariance of Yang-Mills theories with matter fields", *N. Cimento A* **109** (1996), 1145

[16] M. Dütsch: "Slavnov-Taylor identities from the causal point of view", *Int. J. Mod. Phys. A* **12** (1997) 3205

[17] M. Dütsch: "Non-Uniqueness of Quantized Yang-Mills Theories", J. Phys. A, 29 (1996) 7597

[18] M. Dütsch: "Causal Functional Integral for the S-Matrix of QED", N. Cim. A102 (1989) 1469

[19] H. Epstein and V. Glaser: "The role of locality in perturbation theory", *Ann. Inst. H. Poincaré A* **19** (1973), 211

[20] H. Epstein and V. Glaser: "Adiabatic limit in perturbation theory" in G.Velo and A.S.Wightman (eds.): Renormalization theory (1976) 193-254

[21] L. D. Faddeev and V. N. Popov: "Feynman diagrams for the Yang-Mills field", *Phys. Lett. B* **25** (1967), 29

[22] D. R. Grigore: 'On the uniqueness of the non-abelian gauge theories in Epstein-Glaser approach to renormalisation theory', hep-th/9806244

D. R. Grigore: The standard model and its generalisations in Epstein-Glaser approach to renormalisation theory', hep-th/9810078

D. R. Grigore: The standard model and its generalisations in Epstein-Glaser approach to renormalisation theory II: the fermion sector and the axial anomaly'. hep-th/9903206

[23] R. Haag, "Local Quantum Physics", Springer Verlag 1992

[24] G. 't Hooft, "The glorious days of physics, Renormalization of Gauge Theories" hep-th/9812203

[25] S.S. Horuzhy, A.V. Voronin: "Remarks on mathematical structure of BRST theories",*Commun. Math. Phys.* **123** (1989), 677

[26] G. Källén and J.S. Toll: "Special Class of Feynman Integrals in Two-Dimensional Space-Time", *Journ. Math. Phys.* **6** (1965) 299

[27] F. Krahe: "A causal approach to massive Yang-Mills theories". *Acta Phys. Polonica B* **27** (1996), 2453

[28] M. Karowski and P. Weisz, Nucl. Phys. **B 139**, (1978) 445

[29] T. Kugo and I. Ojima: "Local covariant operator formalism of non-abelian gauge theories and quark confinement problem", *Suppl. Progr. Theor. Phys.* **66** (1979), 1

[30] C. H. Lewellyn Smith, in Proc. 5th Hawaii Topical Conf. Part. Phys., ed. P. N. Dobson et al., Univ. of Hawaii Press, Honolulu, Hawaii (1973)

[31] H. Reeh and S. Schlieder: "Bemerkungen zur Unitäräquivalenz von Lorentzinvarianten Feldern", *N. Cimento* **22** (1961), 1051

[32] M. Requardt: "Symmetry conservation and integrals over local charge densities in quantum field theory", *Commun. Math. Phys.* **50** (1976), 259

[33] J. J. Sakurai, Ann. Phys. (N.Y.) **11**, (1961) 1

[34] B. Schroer and H. -W. Wiesbrock, hep-th/9812251, to be published in Rev. in Math. Phys.

[35] B. Schroer, Ann. Phys. **255**, No 3, (1997) 270

[36] B. Schroer, "QFT at the Turn of the Century: old principles with new concepts" (an essay on local quantum physics), FU-SFB288 preprint, 1998

[37] B. Schroer, *Modular Wedge Localzation and the d=1+1 Formfactor Program*, in print, scheduled to appear in Annals of Physics **275**, (1999)

[38] B. Schroer: "Modular Theory and Eyvind Wichmann's Contributions to modern Particle Physics Theory", hep-th/9906071

[39] Schweber, "An Introduction to Relativistic Quantum Field Theory", Part Four, Row, Peterson and Company 1961

[40] J. Schwinger, Phys. Rev. **125**, (1962) 397

[41] F. A. Smirnov, Int.J. Mod. Phys. **A4**, (1989) 156

[42] O. Steinmann, "Perturbation expansions in axiomatic field theory", Lecture Notes in Physics **11**, Springer-Verlag (1971)

[43] R. Stora: "Local gauge groups in quantum field theory: perturbative gauge theories", talk given at the workshop "Local Quantum Physics" at the Erwin-Schroedinger-Institute, Vienna (1997)

[44] R. F. Streater and A.S. Wightman, "TCP, Spin & Statistics and All That"

[45] J. A. Swieca, Phys Rev. D **13**, (1976) 312

[46] G. Scharf and M. Wellmann, hep-th/9903055

[47] S. Weinberg: "The Theory of Quantized Fields", Vol.I

[48] P. Weisz: "Perturbation theory check of a proposed exact Thirring model S-matrix", *Nucl. Phys. B* **122**, (1977) 1