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Anisotropy of favoured alpha transitions producing even-even deformed nuclei[*]

O.A.P. TAVARES

Conselho Nacional de Desenvolvimento Científico e Tecnológico-CNPq, Centro Brasileiro de Pesquisas Físicas-CBPF, Rua Dr. Xavier Sigaud 150, 22290-180 Rio de Janeiro-RJ, Brazil

Summary. – The anisotropy in favoured alpha transitions which produce even-even deformed nuclei is discussed. A simple, Gamow's-like model which takes into account the quadrupole deformation of the product nucleus has been formulated to calculate the alpha decay half-life. It is assumed that before tunneling into a purely Coulomb potential barrier the two-body system oscillates isotropically, thus giving rise to an equivalent, average preferential polar direction θ_0 (referrred to the symmetry axis of the ellipsoidal shape of the product nucleus) for alpha emission in favoured alpha transitions of even-even nuclei.

Key-words: Alpha decay; Anisotropic alpha emission; Alpha decay half-life

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[*]Dedicated to Professor Dr. H.G. de Carvalho on the occasion of his 80th birthday.

The present work reports on the anisotropy of ground-state to ground-state alpha transitions which produce even-even deformed nuclei. A simple, Gamow's-like model in which the quadrupole deformation of the product nucleus is taken into account has been formulated, yielding a formula with no adjustable parameter to calculate the half-life of favoured alpha decays of even-even parent nuclei.

In this approach the shape of the product nucleus is assumed to be the one of an ellipsoid of revolution with semi-axes $a = b \neq c$, the amount of deformation, d, being defined by the intrinsic electric quadrupole moment, Q_2, of the ground-state product nucleus.

The Q_α-value as well as the reduced mass of the disintegrating system, μ, are calculated from the nuclear (rather than the atomic) mass-values of the participating nuclides:

$$Q_\alpha = [m_P - (m_D + m_\alpha)]F \tag{1}$$

$$\frac{1}{\mu} = \frac{1}{m_D} + \frac{1}{m_\alpha} \ . \tag{2}$$

Here, m_P and m_D represent, respectively, the nuclear mass of the parent and daughter nucleus, m_α is the alpha-particle mass, and F is the mass-energy conversion constant. The nuclear mass is calculated by

$$m = M - Zm_e + B_{e,Z}, \tag{3}$$

where M is the atomic mass, $B_{e,Z}$ is the total binding energy of the Z electrons in the atom, and m_e is the electron rest mass. The $B_{e,Z}$-values are evaluated by

$$B_{e,Z} = 8.7 \cdot 10^{-6} \, \frac{Z^{2.517}}{F} \quad \mathrm{u}, \qquad Z \geq 60 \ , \tag{4}$$

which expression has been derived from data reported by HUANG *et al* [1]. Atomic mass values are those listed in the "1993 Atomic Mass Table" by AUDI and WAPSTRA[2], from which Table the values

$$\begin{aligned} m_e &= 548579903 \cdot 10^{-12} \ \mathrm{u} \\ m_\alpha &= 4.0015061747\mathrm{u} \\ F &= 931.494313 \ \frac{\mathrm{MeV}}{\mathrm{u}} \end{aligned}$$

have been taken and used throughly.

The frequency of oscillation for the relative two-body motion ($\lambda_0 \approx 10^{21} - 10^{22}\mathrm{s}^{-1}$) is calculated as

$$\lambda_0(\theta, d) = \frac{v}{2s_0(\theta, d)} \ , \tag{5}$$

where θ is the polar angle referred to the symmetry axis of the ellipsoid, $d = 250 \ Q_2/Z_2$ (d in fm^2 and Q_2 in barn) defines the degree of nuclear deformation, and Z_2 is the atomic number of the product nucleus; $v = (2Q_\alpha/\mu)^{1/2}$ is the relative velocity, and s_0 denotes the separation between the centres of the fragments at contact. This latter quantity is given by

$$s_0(\theta, d) = [A - (A - C)\cos^2\theta]^{-1/2} \tag{6}$$

with

$$A = (a + R_\alpha)^{-2}\ ,\ C = (c + R_\alpha)^{-2}\ , \tag{7}$$

where R_α represents the equivalent sharp charge radius of the alpha particle. The R_α-value is set equal to (1.62 ± 0.01)fm, as it comes from the charge density distribution resulting from data on elastic electron scattering from ^{4}He as obtained by SICK *et al.* [3].

The configuration at contact is defined at the sharp surface of the neutron (rather than the charge) density distribution of the product nucleus. The semi-axes of the ellipsoidal shape of the product nucleus are determined by assuming that the neutron density distribution deforms in the same way as the charge density distribution does, where volume is preserved in both cases. Accordingly, the semi-axes are given by

$$a = b = \left(\frac{R_{n_2}^3}{c}\right)^{1/2}\ ,\ c = 2^{-1/3} R_{n_2}\left[(1+B)^{1/3} + (1-B)^{1/3}\right]\ , \tag{8}$$

where

$$B = \left[1 - 4\left(\frac{d}{3R_{ch_2}^2}\right)^3\right]^{1/2}\ , \tag{9}$$

and R_{n_2} and R_{ch_2} are, respectively, the equivalent sharp radius of the neutron and charge distribution of the product nucleus.

The R_{ch_2}-values are calculated from the average $\overline{\langle r_{ch}^2\rangle^{1/2}}$ of the nuclear root-mean-square charge radius values taken from a number of compilations and systematics of charge radii [4–13]. Here, the droplet model description for the radial moments (with contributions from the size, redistribution, and diffuseness) following MYERS and SCHMIDT [4] is used, thus obtaining

$$R_{ch_2} = \frac{1}{2}\left\{\left[4.41 \cdot 10^{-6} Z_2^2 + \frac{20}{3}\left(\overline{\langle r_{ch}^2\rangle^{1/2}}\right)^2 - 19.602\right]^{1/2} - 2.1 \cdot 10^{-3} Z_2\right\}\ . \tag{10}$$

This expression gives R_{ch_2}-values (expressed in fm) with uncertainty $\lesssim 1\%$.

The R_{n_2}-value is taken as the mean value $(R_{n_2}^M + R_{n_2}^D)/2$ of two equivalent sharp neutron radius evaluations. The first one (denoted by $R_{n_2}^M$) results from the droplet model description of atomic nuclei, the values of which (expressed in fm) are those listed in the table of equivalent sharp neutron radii by MYERS [14]. The second one (denoted by $R_{n_2}^D$) comes from the most recent systematics for neutron radii in even-even nuclei by DOBACZEWSKI *et al.* [13]. This systematic study predicts $\langle r_n^2\rangle^{1/2}$-values in excellent agreement with experimental neutron root-mean-square radii derived from the analysis of high-energy polarized proton scattering experiments on nuclei. According to [13] the root-mean-square neutron radii (expressed in fm) can be calculated by

$$\begin{aligned}(11)\qquad \langle r_n^2\rangle^{1/2} = r_n \;&=\; \left(\frac{3}{5}\right)^{1/2}\cdot 1.176A_2^{1/3}\left[1+\frac{3.264}{A_2}+0.1341\left(1-\frac{2Z_2}{A_2}\right)-\right.\\ &-\;\left.\frac{0.7121}{A_2^2}+\frac{4.828}{A_2}\left(1-\frac{2Z_2}{A_2}\right)\right]+\Delta r_n\ ,\end{aligned}$$

where A_2 is the mass number, and the Δr_n-values are taken from the color-code graph of figure 4 in Ref. [13]. By using the general relationship between $\langle r^2\rangle^{1/2}$ and R [15], and adopting the value 0.99 fm for the nuclear diffuseness, the $R^D_{n_2}$-values are obtained by

$$(12)\qquad R^D_{n_2} = \left(\frac{5}{12}\right)^{1/2} r_n\left\{1+\left[1-6\left(\frac{0.99}{r_n}\right)^2\right]^{1/2}\right\}\ .$$

The uncertainty associated with the R_{n_2}-values evaluated as described above results to be ~1–1.5%.

The assumption is made out that before tunneling into the potential barrier the two-body system oscillates isotropically, giving rise to an average frequency of oscillation, $\bar{\lambda}_0(d)$, which does correspond to an equivalent, average preferential polar direction θ_0 for alpha emission. This assumption is expressed by

$$(13)\qquad \bar{\lambda}_0(d) = \frac{1}{4\pi}\int_0^\pi \lambda_0(\theta,d)2\pi\sin\theta d\theta = \lambda_0(d,\theta_0)\ ,$$

which leads to an anisotropic alpha emission at the average polar direction θ_0 in favoured alpha transitions of even-even nuclei.

The predicted anisotropy in alpha emission is expected to occur for cases of both prolate– and oblate-shaped product nuclei. Equations (5–7) and (13) are handled to give

(14)

$$\cos\theta_0 = \left\{\left(1-\frac{C}{A}\right)^{-1}-\left[\frac{1}{2}\left(\left(\frac{A}{C}-1\right)^{-1/2}+\left(1-\frac{C}{A}\right)^{-1}\cdot\text{arc sin}\left(1-\frac{C}{A}\right)^{1/2}\right)\right]^2\right\}^{1/2}\ ,$$

which is valid for prolate-shaped product nuclei, i.e., $Q_2>0$, $a<c$, and $C/A<1$, and

(15)

$$\cos\theta_0 = \left\{\left[\frac{1}{2}\left(\left(1-\frac{A}{C}\right)^{-1/2}+\left(\frac{C}{A}-1\right)^{-1}\cdot\text{arc sinh}\left(\frac{C}{A}-1\right)^{1/2}\right)\right]^2-\left(\frac{C}{A}-1\right)^{-1}\right\}^{1/2}\ ,$$

which is valid for oblate-shaped product nuclei, i.e., $Q_2<0$, $a>c$, and $C/A>1$.

The equivalent, average preferential polar direction for alpha emission is found to vary very weakly within the range of deformation of known even-even nuclei [16, 17]. In fact, from equations (14) and (15) it results that the θ_0-values are found in the range $\sim 53.4^0 - 54.7^0$ for all prolate cases, and in the range $\sim 54.8^0 - 56.0^0$ for the oblate ones.

The decay constant,

$$\lambda(d) = \bar{\lambda}_0(d) P(d, \theta_0) \ , \tag{16}$$

is therefore calculated at such average preferential θ_0 direction, where

$$P(d, \theta_0) = \exp[-G(d, \theta_0)] \tag{17}$$

is the penetrability factor through a purely Coulomb potential barrier, $V(d, \theta_0, s)$, at separation $s \geq s_0(d, \theta_0)$, and $G(d, \theta_0)$ is Gamow's factor for decay given by the classical WKB-integral approximation

$$G(d, \theta_0) = \frac{2}{\hbar} \int_{s_0(d,\theta_0)}^{s'(d,\theta_0)} \{2\mu[V(d, \theta_0, s) - Q_\alpha]\}^{1/2} \, \mathrm{d}s \ , \tag{18}$$

in which the outer turning point s' is defined by

$$V(d, \theta_0, s') = Q_\alpha \ . \tag{19}$$

The Coulomb potential energy for the interaction between the alpha particle ($Z_1 = 2$) and the product nucleus (supposed to have only quadrupole deformation) at separations $s \geq s_0$ has been deduced as [18]:

$$V(d, \theta_0, s) = \frac{Z_1 Z_2 e^2}{s} \, g(d, \theta_0, s) \ , \tag{20}$$

where, for prolate deformations ($Q_2 > 0, \ \ d > 0, \quad x = d/s^2 > 0$)

$$g(d, \theta_0, s) = \frac{3}{2} \left\{ \frac{n}{x^{1/2}} \text{arc sinh} \left[x^{1/2} \left(\frac{2}{m - x} \right)^{1/2} \right] + p \right\} \ , \tag{21}$$

and, for oblate deformations ($Q_2 < 0, \ \ d < 0, \quad x = d/s^2 < 0$)

$$g(d, \theta_0, s) = \frac{3}{2} \left\{ \frac{n}{(-x)^{1/2}} \text{arc sin} \left[(-x)^{1/2} \left(\frac{2}{m - x} \right)^{1/2} \right] + p \right\} \ , \tag{22}$$

with

$$\begin{cases} m = 1 + (1 - 2x \ \cos 2\theta_0 + x^2)^{1/2} \\[2ex] n = 1 + \dfrac{1 - 3\cos^2\theta_0}{2x} \\[2ex] p = \dfrac{2^{1/2} \cos^2\theta_0}{x(m + x)^{1/2}} - \dfrac{(m + x)^{1/2} \sin^2\theta_0}{2^{1/2} x(m - x)} \ , \end{cases} \tag{23}$$

and $e^2 = 1.4399652$ MeV·fm is the square of the electronic charge.

Finally, by expressing masses in u, energies in MeV, lengths in fm, and time in yr, the following formula can be used as routine to calculate the alpha decay half-life:

$$T_{1/2} = 3.16 \cdot 10^{-30} \left(\frac{\mu}{Q_\alpha}\right)^{1/2} s_0(d,\theta_0) \exp\left[0.5249578932\ (Z_1 Z_2 \mu)^{1/2} I\right] \ , \tag{24}$$

where

$$I = \int_{s_0}^{s'} \left[\frac{g(d,\theta_0,s)}{s} - q\right]^{1/2} \mathrm{d}s \qquad , \qquad q = \frac{Q_\alpha}{Z_1 Z_2 e^2} \ , \tag{25}$$

and s' is the solution of the equation $g(d,\theta_0,s') = qs'$. Equation (24) gives absolute values for the half-life in the sense that it does not contain any adjustable parameter.

The preferred alpha decay of ^{252}Cf isotope has been selected to test the present model. Since the deformation parameters for ^{248}Cm product nucleus have been calculated as $\beta_2 = 0.235$, $\beta_3 = 0$, $\beta_4 = 0.04$, and $\beta_6 = -0.036$ [17], it follows that ^{248}Cm exhibits essentially a quadrupole deformation. The measured intrinsic electric quadrupole moment for ^{248}Cm is reported as $Q_2 = 12$ b [10], for which case it yields $\theta_0 = 54.135^0$. The corresponding predicted half-life results to be (2.7 ± 1.0)yr, in quite good agreement with the measured value of (3.18 ± 0.09)yr [19]. If the assumption of spherical-shaped product nucleus was made the predicted half-life by the present Gamow's-like model in the spherical approximation, i.e., $Q_2 = 0$, $d = 0$, $a = b = c = R_{n_2}$, $s_0 = R_{n_2} + R_\alpha$, $V(s) = Z_1 Z_2 e^2/s$, and $s' = Z_1 Z_2 e^2/Q_\alpha$, would result (1.94 ± 0.58)yr. This latter value is clearly more distant from the experimental one than is the calculated half-life when the quadrupole deformation of ^{248}Cm product nucleus is taken into account. Figure 1 shows the effect of deformation of the product nucleus (assumed to be an ellipsoid of revolution) on potential Coulomb barrier for the favoured decay $^{252}\mathrm{Cf} \rightarrow {}^{248}\mathrm{Cm} + {}^4\mathrm{He}$. The potential energy, $V(s)$, calculated in the separation region $s_0 \leq s \leq s'$ at the extreme emission directions $\theta = 0$ (pole) and $\theta = \pi/2$ (equator) is compared with the potential barrier obtained in the spherical approximation. It is seen that not only the $V(s)$-curves differ from each other, but large differences are noted mainly among the separation-values at the contact configuration. The combined effect from such differences leads therefore to different predicted half-life values.

The same happens to $^{190}\mathrm{Pt} \rightarrow {}^{186}\mathrm{Os} + {}^4\mathrm{He}$ decay ($Q_2 = 5.4$ b, $\theta_0 = 54.314^0$), for which case the predicted half-life of $(2.6 \pm 1.0) \cdot 10^{11}$yr agrees quite well with the most recent measured value of $(3.2 \pm 0.1) \cdot 10^{11}$yr [20]. In this case, the value $(2.3 \pm 0.7) \cdot 10^{11}$yr results if one assumes the spherical shape for ^{186}Os product nucleus.

An example of even-even oblate-shaped daughter nucleus is found in $^{184}\mathrm{Pb} \rightarrow {}^{180}\mathrm{Hg} + {}^4\mathrm{He}$ decay, with experimental half-life of $1.7 \cdot 10^{-8}$yr. The quadrupole deformation parameter for ^{180}Hg is reported as $\beta_2 = -0.122$ [17], which corresponds to an intrinsic electric quadrupole moment of $Q_2 = -3.12$ b. The equivalent, average preferential direction for alpha emission is calculated as $\theta_0 = 55.0^0$, and the predicted half-life results to be $1.4 \cdot 10^{-8}$yr, in quite good agreement with the measured value. Since the amount of oblate deformation for ^{180}Hg product nucleus is small, the predicted half-life does not differ appreciably from $1.2 \cdot 10^{-8}$yr as obtained under the spherical-shaped approximation for ^{180}Hg.

The examples reported above show that little changes from the spherical to either prolate or oblate shape of the product nucleus may explain the expected anisotropy in favoured alpha decays of even-even nuclei. The experimental half-lives of preferred alpha transitions producing deformed nuclei should be better reproduced when deformation is taken into account in Gamow's-like model than in its spherical approximation. A systematic half-life prediction study on these lines should be worked out.

To conclude, it is worthwhile to mention that a microscopic description of the alpha decay in axially deformed nuclei has been presented recently by DELION *et al.* [21-23] and STEWART *et al.* [24], and experimental observations of remarkably pronounced preferential alpha emission in odd-A alpha emitters have been reported by SOINSKI and SHIRLEY [25], WOUTERS *et al.* [26], and very recently by SCHUURMANS *et al.* [27].

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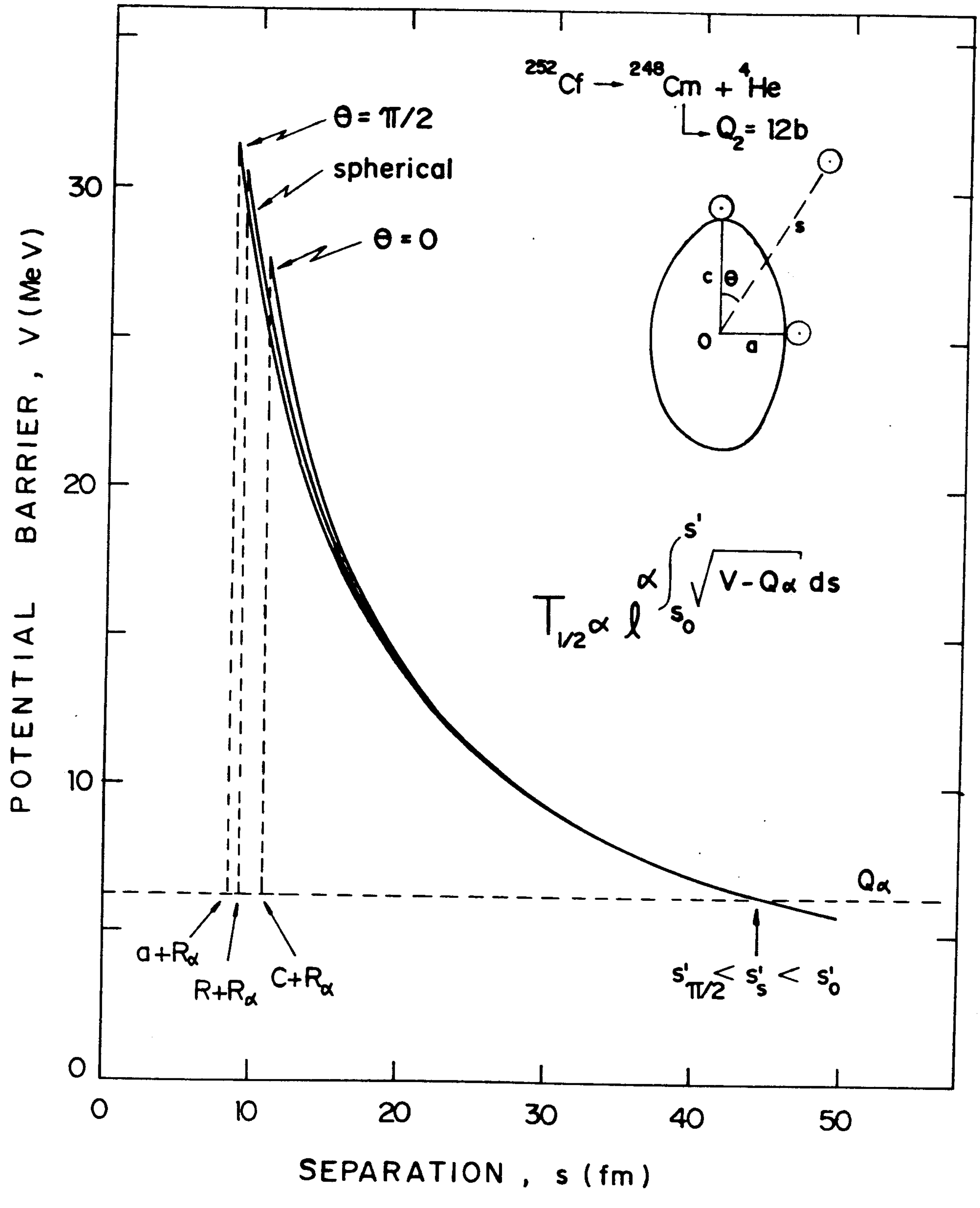
Figure Caption

Figure 1 Coulomb potential barrier, $V(s)$, plotted against separation, s, for the favoured alpha decay of ^{252}Cf isotope. The ^{248}Cm product nucleus is supposed to be an ellipsoid of revolution (semi-axes $a = b < c$) of total charge uniformly distributed in the volume. θ is the polar emission angle referred to the symmetry axis of the ellipsoid. The Q_α-value for decay is represented by the horizontal dashed line. The curves represent the calculated potential barrier for $\theta = 0$ and $\theta = \pi/2$ (ellipsoidal shape), and for the spherical approximation of the product nucleus as indicated. Also shown are the inner (s_0) and outer (s') turning points in each case.



$^{252}Cf \rightarrow {}^{248}Cm + {}^{4}He$
$Q_2 = 12b$
$\theta = \pi/2$
spherical
$\theta = 0$
s
c
θ
O
a
$T_{1/2} \propto \ell^{\alpha \int_{s_0}^{s'} \sqrt{V - Q_\alpha}\, ds}$
Q_α
$a+R_\alpha$
$R+R_\alpha$
$C+R_\alpha$
$s'_{\pi/2} < s'_s < s'_0$
POTENTIAL BARRIER, V (MeV)
SEPARATION, s (fm)
0
10
20
30
40
50