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SPIN GLASS D-VECTOR MODEL ON THE BETHE LATTICE

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ABSTRACT

The stability of the spin-glass phase of the disordered D-vector model on the Bethe lattice is studied in the zero field limit. We found that no breaking of replica symmetry occurs and for continuous symmetry spin system ($D > 1$) a low moments spin glass solution near T_c exist only when the spin dimensionality is less than the branching number of the Bethe lattice. The non-zero field limit is also discussed.

Key words: Short range spin glass; Replica symmetry; D-Vector Model.

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In this work we study a short range D-component spin-glass model on the Bethe lattice. The Ising case (D=1) has been studied by Thouless [1] and it was found that in a presence of a field breaking of replica symmetry occurs in the same fashion as in the Sherrington-Kirkpatrick (SK) infinite range model [2,3]. Unlike the SK-model the present study might give some informations about the behaviour of short ranged systems. Earlier studies of vector spin-glasses with long-range interactions [4] also present a replica symmetry breaking transition and in the presence of an external magnetic field H show a critical line $T_c(H)$ at which the order parameters associated with transverse degrees of freedom go continuously to zero and irreversibility sets in.

We consider a model described by the reduced Hamiltonian

$$- \beta H = \sum_{<ij>} K_{ij} \vec{S}_i . \vec{S}_j + \sum_i \vec{B} . \vec{S}_i \qquad (1)$$

where the spins $\vec{S} = (S^1, S^2, \ldots S^D)$, $|\vec{S}| = D^{\frac{1}{2}}$, occupy the sites of a Cayley tree of coordination number $z=(q+1)$, $\vec{B}=\beta\vec{H}$ and the K_{ij} are the random nearest neighbour exchange interaction with a probability distribution characterized by $<K_{ij}> = 0$ and $<K_{ij}^2> = K^2$. The normalized partition function of the system can be written following [5] as

$$Z = \int d\vec{S}_0 \delta(D-|\vec{S}_0|^2) \exp\{\vec{B}.\vec{S}_0 + \sum_{i=1}^{z} \vec{B}_{0i}\vec{S}_0\} / \int d\vec{S}_0 \delta(D-|\vec{S}_0|^2) \qquad (2)$$

where $\vec{B}_{0i}$, the effective local field which a given neighbouring spin $\vec{S}_i$ exerts on the central spin $\vec{S}_0$, is defined by

$$\lambda_i \exp\{\vec{B}_{ij}.\vec{S}_i\} = \int d\vec{S}_j\ \delta(D-|\vec{S}_j|^2)\ \exp\{K_{ij}\vec{S}_i.\vec{S}_j + \vec{B}.\vec{S}_j\}\ \times$$
$$\times\ \exp\{\sum_{\ell=1}^{q} \vec{B}_{j\ell}.\vec{S}_j\} \quad . \tag{3}$$

In (3) (ij) label a given pair of spins belonging to two successive generation of the Cayley tree. Since we are interested only in the local properties deep inside the tree (Bethe lattice), one have to find the "fixed function" solution the functional recursion equation (3). One expect that in the paramagnetic phase the local field deep inside the tree should have component only along the external applied field while in the condensed phase transverse ordering might also occurs. Here we are primarily studing the behaviour of the system very close to the transition line $T_c(H)$ and we assume for the moment that only the longitudinal component of the local field is relevant, i.e.

$$\vec{B}_{ij}.\vec{S}_i \cong f_{ij}S_i^1 \tag{4}$$

where we have chosen $\vec{H} = (H,0,0,\ldots)$. Eq.(4) is certainly exact in the whole paramagnetic phase. Integrating (3) over the spins variables with help of (4) and expanding up to fifth order in the fields we get

$$f_{ij} = t(B+F_j) - \frac{(1-Dt^2)}{D+2}\ (B+F_j)^3 + a(B+F_j)^5 \tag{5}$$

where

$$a = \{\frac{D(D-1)}{2K_{ij}}\left[t^2 + (\frac{2}{DK_{ij}^2} - \frac{(D-2)}{2D})(1 - \frac{t}{K_{ij}} - \frac{(D^2+4)}{2D(D+2)}\right] +$$
$$+ \frac{D^2(D+8)}{(D+2)} t^5 - \frac{3D(D+4)}{(D+2)} t^3 - \frac{2(D^2-2D-4)}{(D+4)} t\}/(D+2)(D+4) \qquad (6)$$

$F_j = \sum_{\ell=1}^{q} f_{j\ell}$ and $t = I_{D/2}(DK_{ij})/I_{D/2-1}(DK_{ij})$ is the generalized hyperbolic tangent with $I_n(DK_{ij})$ being the modified Bessel function of first kind of order n. By Squaring eq.(5) and averaging over the K_{ij} the critical temperature at zero field is immediately obtained from the first order coefficient of $\langle f^2\rangle$ (assumed non zero) and given by

$$1 = q \langle t^2\rangle \qquad (7)$$

This result, which has been already obtained by [6] for D=3 is exact and independent of (4) since, on the average, the system is isotropic in the spin space. However the stability study of the system will decide in what conditions the condensed phase sets in below T_c. Therefore we now turn to a discussion of breaking of replica symmetry and its stability analysis at zero field following [1]. Introducing another set of effective fields g_i for the same sample $\{K_{ij}\}$ (second replica) and multipling eqs.(5) for both local fields we obtain up to the fourth moments that

$$\langle fg\rangle_i = t_1\langle fg\rangle_j - \frac{2(t_1-Dt_2)}{(D+2)}\left[\langle f^3g\rangle_j + 3(q-1)\langle f^2\rangle_j\langle fg\rangle_j\right]$$

$$<f^2>_i = t_1<f^2>_j - \frac{2(t_1-Dt_2)}{(D+2)} \left[<f^4>_j + 3(q-1)<f^2>_j^2\right]$$

$$<f^3g>_i = t_2\left[<f^3g>_i + 3(q-1)<f^2>_j<fg>_j\right] \tag{8}$$

$$<f^4>_i = t_2 \left[<f^4>_j + 3(q-1)<f^2>_j^2\right]$$

where $t_n=q<t^{2n}>$. From eqs. (8) one finds two possible solutions: a paramagnetic one where $<f^2>=<fg>=0$ and a spin-glass one with

$$<f^2> = \frac{(D+2)}{6(q-1)} \frac{(t_1-1)(1-t_2)}{(t_1-Dt_2)} \tag{9}$$

and $<fg>=0$ (i.e. no breaking of replica symmetry at zero field). The stability of these solutions are given by the behavior of the two relevant eigenvalues of eq. (8). For the paramagnetic fixed point solution we have $\lambda_1=\lambda_2=t_1$ which are obviously less (equal) (great) than unit above (at) (below) the critical temperature. For the spin-glass solution we get $\lambda_1= 1$ and

$$\lambda_2= x + (x^2-t_1t_2)^{1/2} \tag{10}$$

where $x = \frac{1}{2} [2(1+t_1t_2)- t_1-t_2]$. λ_1 which is associated with the fluctuations of $<fg>$ is always unity and λ_2 associated with the fluctuations of $<f^2>$ is less (equal) (great) than unity below (at) (above) the critical temperature. For the Ising case (D=1) one can easily see that $<f^2>$ given by (10) is always positive below T_c. However for continuous symmetry spin systems (D>1) the spin glass solution can only exist when the spin dimensionality is less than the branching number q=z-1 of the Bethe lattice, i.e. D< q. If D>q, at zero field, no second order transition is found in agreement with the spherical model studied by Derrida et al [7].

The set of eq. (5) is the one appropriate for investigating the system in the presence of a field. From (5) we can generalize eq. (8) and up to second order in the fields and sixth order in the local fields one is led to a 9x9 stability matrix. This study is presently underway and the results will be published elsewhere. This work was partially supported by CNPq, CAPES and FINEP (Brazilian Agencies).

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