

The rotating detector and vacuum fluctuations*

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In this work we compare the quantization of a massless scalar field in an inertial frame with the quantization in a rotating frame. We used the Trocherries-Takeno mapping to relate measurements in the inertial and the rotating frames. An exact solution of the Klein-Gordon equation in the rotating coordinate system is found and the Bogolubov transformation between the inertial and rotating modes is calculated, showing that the rotating observer defines a vacuum state different from the Minkowski one. We also obtain the response function of an Unruh-De Witt detector coupled with the scalar field travelling in a uniformly rotating world-line. The response function is obtained for two different situations: when the quantum field is prepared in the usual Minkowski vacuum state and when it is prepared in the Trocherries-Takeno vacuum state. We also consider the case of an inertial detector interacting with the field in the rotating vacuum.

I Introduction

The key point of special relativity is that the Poincaré group is the symmetry group of all physical systems. The definitions of the Lorentz and Poincaré groups are based as groups of mappings that leave invariant the flat metric. The natural consequence is that an inertial observer (in an inertial reference frame) can assign a time and space location to any event occurring in space-time, using light clocks, etc. To obtain a “physical” interpretation of the Poincaré mapping we have to derive general relations between the space-time measurements made by different observers who are in different inertial frames.

So far, we have been considering only classes of measuring devices in inertial reference frames. However, suppose we are to make measurements with a device in non-inertial frames, as for example in a rotating disc. To discuss such measurements and to show how they can be incorporated into a space-time description, entails that distance and time measurements made with some arbitrary set of measuring devices can always be made to correspond to the coordinates of space-time by means of a suitable space-time mapping. In other words, in order to compare measurements made by inertial and non-inertial (e.g. rotating) observers, we

must present the mapping that relates the measurements made with the two different sets of devices.

For example, in a Galilean scenario it is possible to relate the space and time measurements made in a rotating frame to those in an inertial one by the mapping:

$$t = t', \quad (1)$$

$$r = r', \quad (2)$$

$$\theta = \theta' - \Omega t', \quad (3)$$

$$z = z', \quad (4)$$

where Ω is the constant angular velocity around the z axis of the inertial frame. In the above, the cylindrical coordinate system $x'^{\mu} = \{t', r', \theta', z'\}$ is adapted to an inertial observer and the rotating coordinate system $x^{\mu} = \{t, r, \theta, z\}$ is the one adapted to the rotating observer. Although some authors tried to construct the counterpart of this mapping incorporating special relativity [1], the final answer for this question is still open.

In two recent papers [2] it is assumed that the mapping which relates the inertial frame with the rotating one is given by:

$$t = t' \cosh \Omega r' - r' \theta' \sinh \Omega r', \quad (5)$$

$$r = r', \quad (6)$$

$$\theta = \theta' \cosh \Omega r' - \frac{t'}{r'} \sinh \Omega r', \quad (7)$$

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$$z = z'. \quad (8)$$

Such group of transformations was presented a long time ago by Trocherries and also Takeno [3]. In Takeno's derivation, three assumptions were made: (i) the transformation laws constitute a group; (ii) for small velocities we must recover the usual linear velocity law ($v = \Omega r$); and (iii) the velocity composition law is also in agreement with special relativity. In fact, the above transformation predicts that the velocity of a point at distance r from the axis is given by $v(r) = \tanh(\Omega r)$.

It is our purpose, in this work, to investigate how does an uniformly rotating observer see the Minkowski vacuum. In this direction we performed the quantization of a scalar field as an observer who rotates uniformly around some fixed point would do it. We assume the Trocherries-Takeno transformations (5-8) to compare measurements made in the inertial and in the rotating frames.

The canonical quantization of a scalar field in the rotating frame, related to the inertial one through coordinate transformations (1-4), was made by Denardo and Percacci and also by Letaw and Pfautsch [4]. To compare the quantizations performed in the inertial and rotating frames, the authors calculated the Bogolubov transformation [5] between the inertial modes $\psi_i(t', r', \theta', z')$ and the modes adapted to the rotating frame, $\bar{\psi}_j(t, r, \theta, z)$. Since they found that the Bogolubov coefficients β_{ij} are null, they conclude that the rotating vacuum or no-particle state, as defined by the rotating observer, is just the Minkowski vacuum $|0, M\rangle$.

Another way to compare different quantizations is to study the vacuum activity of a quantum field, and this is performed introducing a measuring device which couples with the quantum field via an interaction Lagrangian. (In the following discussion we will take into account the detector model due to Unruh and De Witt [6, 7].) Using first-order perturbation theory, it is possible to calculate the probability of excitation per unit proper time (excitation rate) of such a detector, that is, the probability per unit time that the detector, traveling in a given world-line and initially prepared in its ground state, to wind up in an excited state when it interacts with the field in a given state [8]. As an example, consider that the detector is in an inertial frame and the field is prepared in the Minkowski vacuum state. In this situation the detector will remain in the ground state (null excitation rate). This is easily understood, because there are no inertial particles in the Minkowski vacuum state. On the other hand, if the detector is put in a world-line of an observer with constant proper acceleration and the field is prepared again in the Minkowski vacuum, the detector has a non-null probability to suffer a transition to an excited state. This is the so-called Unruh-Davies effect and the quantitative result is in agreement with the fact that the Minkowski vacuum state is seen as a thermal state

by the accelerated observer, with temperature proportional to its proper acceleration [6]. The construction of a quantum field theory with the implementation of the Fock space in Rindler's manifold leads to define the Rindler vacuum state $|0, R\rangle$. For completeness, in the situation when the field is prepared in such a state and the detector is uniformly accelerated, the detector remains inert. Again one is tempted to conclude that this is so because there are no Rindler particles in $|0, R\rangle$ to be detected in the uniformly accelerated frame.

The agreement between the response of a detector and canonical quantum field theory seems not to occur for more general situations. Indeed, as was shown by Letaw and Pfautsch, if the detector is put in a uniformly rotating world-line and the field is prepared in the Minkowski vacuum state $|0, M\rangle$, it is found a non-null excitation rate, in spite of the fact that $|0, M\rangle$ is considered as the vacuum state for a rotating observer, as discussed above. The rotating detector is excited even though there are no particles as an orbiting observer would define them (see also [9]).

Recently Davies *et al* [10] (see also [11]) solved this paradox, still assuming the Galilean coordinate transformations (1-4) between the inertial and rotating frames. First of all note that the world-line of an observer in the rotating frame is an integral curve of the Killing vector $\xi = (1 - \Omega^2 r^2)^{-\frac{1}{2}} \frac{\partial}{\partial t}$, which is time-like only for $\Omega r < 1$. Therefore, for a given angular velocity Ω there will be a maximum value of the radial coordinate $r_{max} = \frac{1}{\Omega}$ (the light cylinder) for which an observer a distance $r > r_{max}$ will be moving faster than light. The Bogolubov coefficient β is a scalar product over a spacelike hypersurface, where the radial coordinate ranges over $0 \leq r < \infty$ and the notion of Bogolubov transformations outside the light cylinder becomes obscure. In order to circumvent this problem, Davies *et al* introduced a perfectly conducting cylinder with radius $a < r_{max}$ and they prove that the response of the detector vanishes when it is put a distance $r < a$ from the rotation axis. They conclude that "a rotating particle detector corotating with a rotating vacuum state registers the absence of quanta", although they continue to regard the Minkowski vacuum as the rotating vacuum state.

The aim of this paper is two-fold. The first one is to present an exact solution of the Klein-Gordon equation in the Trocherries-Takeno coordinate system and to show that the Bogolubov coefficients β between inertial and rotating modes are not zero. This fact proves that there is a Trocherries-Takeno vacuum state adapted to rotating observers. The second one is to analyse the behavior of an apparatus device, a detector which is coupled with the scalar field travelling in inertial or rotating world-lines, interacting with the field in the Minkowski or the Trocherries-Takeno vacuum states.

We organize this paper as follows. In Section II we second quantize a massless scalar field in Takeno's ro-

tating coordinate system, and also compare this quantization with the usual one in the inertial frame via the calculation of the Bogolubov coefficients. In Section III we introduce the measuring apparatus – the Unruh-DeWitt detector [6]. We calculate its response function when it is rotating and the field is prepared in two different states: the Trocherries-Takeno vacuum state and the Minkowski vacuum state. We also consider the case of an inertial detector interacting with the field in the rotating vacuum. Conclusions are given in Section IV. In this paper we use $\hbar = c = k_B = 1$.

II Canonical quantization in the inertial and rotating frames

In this section we will make a comparison between the quantizations of the massless Klein-Gordon field performed in the inertial frame and in the rotating one

(Trocherries-Takeno), when the two coordinate systems are related by the mapping (5-8). Such a comparison will be made by calculating the Bogolubov transformation [5] between the inertial and rotating modes, solutions of the respective Klein-Gordon equations. For the quantization in the inertial frame one chooses cylindrical coordinates on $t' = \text{constant}$ hypersurfaces and writes the Klein-Gordon equation in terms of them. We just quote the results of refs. [4]. Positive-frequency modes (with respect to inertial time t'), solutions of the Klein-Gordon equation, are found to be:

$$v_{q'm'k'}(t', r', \theta', z') = N_1 e^{ik'z' + im'\theta'} e^{-i\omega't'} J_{m'}(q'r'), \quad (9)$$

where $\omega'^2 = q'^2 + k'^2$, $J_{m'}(q'r')$ are Bessel functions well-behaved at the origin and $N_1 = [2\pi(2\omega')^{1/2}]^{-1}$ is a normalization factor. In the above, $m' = 0, \pm 1, \pm 2, \pm 3, \dots$, $0 \leq q' < \infty$ and $-\infty < k' < \infty$. In this way the field is expanded as:

$$\phi(t', r', \theta', z') = \sum_{m'} \int dq' q' dk' \left[b_{q'm'k'} v_{q'm'k'}(t', r', \theta', z') + b_{q'm'k'}^\dagger v_{q'm'k'}^*(t', r', \theta', z') \right], \quad (10)$$

where the coefficients $b_{q'm'k'}$ and $b_{q'm'k'}^\dagger$ are, respectively, the annihilation and creation operators of the inertial quanta of the field and satisfy the usual commutation rule $[b_i, b_j^\dagger] = \delta_{ij}$. In the above, the modes v_i and v_i^* are called, respectively, positive and negative frequency modes with respect to the Killing vector $\partial/\partial t'$. It is important to stress that in stationary geometries, such as the Minkowski space-time, the definition of positive and negative frequency modes has no ambiguities. The Minkowski vacuum state is then defined by

$$b_{q'm'k'} |0, M\rangle = 0, \quad \forall q', m', k'. \quad (11)$$

We now consider the quantization in the rotating frame.

Assuming the mapping (5-8) to connect measurements made in the rotating frame and those made in the inertial one, the line element in the rotating coordinates assumes the non-stationary form [2]:

$$ds^2 = dt^2 - (1+P)dr^2 - r^2 d\theta^2 - dz^2 + 2Qdrd\theta + 2Sdt dr, \quad (12)$$

where P, Q and S are given by:

$$P = \left(\frac{Y}{r^2} + 4\Omega\theta t\right) \sinh^2 \Omega r - \frac{\Omega}{r}(t^2 + r^2\theta^2) \sinh 2\Omega r + \Omega^2 Y, \quad (13)$$

$$Q = r\theta \sinh^2 \Omega r - \frac{1}{2}t \sinh 2\Omega r + \Omega r t, \quad (14)$$

$$S = \frac{t}{r} \sinh^2 \Omega r - \frac{1}{2}\theta \sinh 2\Omega r - \Omega r \theta, \quad (15)$$

where $Y = (t^2 - r^2\theta^2)$. Note that this metric presents no event horizons. In order to implement the canonical quantization first we have to solve the Klein-Gordon equation in the Trocherries-Takeno coordinate system:

$$\square\phi(t, r, \theta, z) = 0. \quad (16)$$

It is possible to show that a complete set, basis in the space of solutions of the Klein-Gordon equation is given by $\{u_{qmk}, u_{qmk}^*\}$, where

$$u_{qmk}(t, r, \theta, z) = N_2 e^{ikz} \exp \left[i (m \cosh \Omega r + \omega r \sinh \Omega r) \theta - i \left(\frac{m}{r} \sinh \Omega r + \omega \cosh \Omega r \right) t \right] J_m(qr), \quad (17)$$

where $\omega^2 = q^2 + k^2$ and N_2 is a normalization factor. Again, $m = 0, \pm 1, \pm 2, \pm 3, \dots$, $0 \leq q < \infty$ and $-\infty < k < \infty$. One sees that these modes are well-behaved throughout the whole manifold. Making use of the transformations (5-8) one can show that these modes are of positive frequency by using the criterium

$$\phi(t, r, \theta, z) = \sum_m \int dq q dk \left(a_{qmk} u_{qmk}(t, r, \theta, z) + a_{qmk}^\dagger u_{qmk}^*(t, r, \theta, z) \right), \quad (18)$$

where the coefficients a_{qmk} and a_{qmk}^\dagger are, respectively, the annihilation and creation operators of the Trocherries-Takeno quanta of the field. The vacuum state defined by the rotating observer is thus the Trocherries-Takeno vacuum state $|0, T\rangle$ and it is given by

$$a_{qmk} |0, T\rangle = 0, \quad \forall q, m, k. \quad (19)$$

The many-particle states of the theory can be obtained through successive applications of the creation operators on the vacuum state.

We are now ready to compare both quantizations by using the Bogolubov transformations [5]. Since both sets of modes are complete, one can expand modes (9) in terms of modes (17) and vice-versa, the coefficients of this expansion being called Bogolubov coefficients. For instance:

$$u_i(x) = \sum_j \alpha_{ij} v_j(x) + \beta_{ij} v_j^*(x) \quad (20)$$

of di Sessa [12], which states that a given mode is of positive frequency if it vanishes in the limit $(t') \rightarrow -i\infty$, where t' is the inertial time coordinate, while u_j^* are modes of negative frequency. In this way, the field operator is expanded in terms of these modes as:

and conversely:

$$v_j(x) = \sum_i \alpha_{ij}^* u_i(x) - \beta_{ij} u_i^*(x), \quad (21)$$

where $\alpha_{jj'} = (u_j, v_{j'})$ and $\beta_{jj'} = -(u_j, v_{j'}^*)$, where the scalar product is defined by:

$$(\phi, \chi) = -i \int_{\Sigma} d\Sigma^\mu \sqrt{-g} [\phi \partial_\mu \chi^* - \chi^* \partial_\mu \phi], \quad (22)$$

with $d\Sigma^\mu = n^\mu d\Sigma$, where n^μ is a future-oriented unit vector orthogonal to the spacelike hypersurface Σ . As Σ we will choose the hypersurface $t' = 0$, t' being the inertial time. The relevant coefficient for our present analysis is the β coefficient since it is this coefficient which gives the content of rotating particles in the Minkowski vacuum [5], and for such a calculation we need to express the rotating modes $u_j(x)$ in terms of the inertial coordinates, using transformations (5-8). In this way:

$$\begin{aligned} u_{qmk}(t', r', \theta', z') &= N_2 e^{ikz'} \exp(i\theta' [m \cosh 2\Omega r' + \omega r' \sinh 2\Omega r']) \\ &\times \exp\left(-it' \left[\omega \cosh 2\Omega r' + \frac{m}{r'} \sinh 2\Omega r'\right]\right) J_m(qr') \end{aligned} \quad (23)$$

and the Bogolubov coefficient $\beta_{jj'}$ is written as:

$$\begin{aligned} \beta_{jj'} &= +i \int_0^\infty r' dr' \int_{-\infty}^\infty dz' \int_0^{2\pi} d\theta' \left[u_j(x') \frac{\partial v_{j'}(x')}{\partial t'} - v_{j'}(x') \frac{\partial u_j(x')}{\partial t'} \right] \\ &= N_1 N_2 \int_{-\infty}^\infty dz' e^{i(k+k')z'} \int_0^\infty r' dr' \left[\omega' - \omega \cosh 2\Omega r' - \frac{m}{r'} \sinh 2\Omega r' \right] J_{m'}(q'r') J_m(qr') \\ &\times \int_0^{2\pi} d\theta' \exp(i\theta' [m' + m \cosh 2\Omega r' + \omega r' \sinh 2\Omega r']). \end{aligned} \quad (24)$$

The first integral is easily evaluated to a delta func-

$$\int_0^{2\pi} d\theta' \exp(i\theta' A_{m,m'}(r, \omega)) = (iA_{m,m'}(r, \omega))^{-1} [\exp(2\pi i A_{m,m'}(r, \omega)) - 1],$$

where

$$A_{m,m'}(r, \omega) = m' + m \cosh 2\Omega r' + \omega r' \sinh 2\Omega r'. \quad (25)$$

$$\begin{aligned} \beta_{jj'} &= 2\pi N_1 N_2 \delta(k + k') \int_0^\infty r' dr' \left(\omega' - \omega \cosh 2\Omega r' - \frac{m}{r'} \sinh 2\Omega r' \right) J_{m'}(q' r') J_m(q r') \\ &\times (iA_{m,m'}(r, \omega))^{-1} [\exp(2\pi i A_{m,m'}(r, \omega)) - 1]. \end{aligned} \quad (26)$$

The resulting expression is difficult to evaluate, but nonetheless it is non-zero. (In the appendix we give an indirect proof that it is non-zero.) This means that the two vacua considered are non-equivalent, i.e., $|0, M\rangle \neq |0, T\rangle$, which means that the Minkowski vacuum $|0, M\rangle$ contains rotating quanta, i.e., Trocheries-Takeno particles [5].

III Detector excitation rate

We now pass to consider the probability of excitation of a detector which is moving in a circular path at constant angular velocity Ω , interacting with the scalar field. The initial state of the detector is its ground state and for the initial state of the field we will consider the two vacuum states: the usual Minkowski vacuum state and also the Trocheries-Takeno vacuum state. The interaction with the field may cause transitions between the energy levels of the detector and if it is found, after the interaction, in an excited state, one can say that it has detected a vacuum fluctuation of the field [11].

As a detector we shall be considering mainly the detector model of Unruh-De Witt [6, 7], which is a system with two internal energy eigenstates with monopole matrix element between these two states different from zero. According to standard theory [8], the probability of excitation per unit proper time of such a system (normalized by the selectivity of the detector), or simply, its excitation rate, is given by:

$$R(E) = \int_{-\infty}^{\infty} d\Delta t e^{-iE\Delta t} G^+(x(t), x(t')), \quad (27)$$

tion $2\pi\delta(k + k')$, while the third one gives us:

Thus, we obtain

where $\Delta t = t - t'$, $E > 0$ is the difference between the excited and ground state energies of the detector and $G^+(x(t), x(t'))$ is the positive-frequency Wightman function calculated along the detector's trajectory. Let us note that the positive-frequency Wightman function is given by

$$G^+(x(t), x(t')) = \langle 0 | \phi(x(t)) \phi(x(t')) | 0 \rangle, \quad (28)$$

where $|0\rangle$ is the vacuum state of the field, which can either be $|0, M\rangle$ or $|0, T\rangle$. Let us consider first the second possibility.

If one splits the field operator in its positive and negative-frequency parts with respect to the Trocheries-Takeno time coordinate t , as $\phi(x) = \phi^+(x) + \phi^-(x)$ (we wrote $\phi(x)$ for $\phi(x(t))$), where $\phi^+(x)$ contains only annihilation operators and $\phi^-(x)$ contains only creation operators (see Eq.(18)), and also considers $|0\rangle$ as the Trocheries-Takeno vacuum state, i.e., $|0\rangle = |0, T\rangle$ then, using Eq.(19), one finds that:

$$G_T^+(x(t), x(t')) = \sum_i u_i(x) u_i^*(x'), \quad (29)$$

where the subscript T stands for the Wightman function calculated in the Trocheries-Takeno vacuum state. Considering now the modes given by Eq.(17) and that we are interested in the situation where the detector is at rest in the Trocheries-Takeno frame, i.e., $\theta = \text{constant}$, $z = \text{constant}$ and $r = R_0 = \text{constant}$, one finds:

$$G_T^\pm(x(t), x(t')) = \sum_{m=-\infty}^{\infty} \int_0^\infty dq q \int_{-\infty}^{\infty} dk N_2^2 e^{-i[\frac{m}{R_0} \sinh \Omega R_0 + \omega \cosh \Omega R_0] \Delta t} J_m^2(q R_0). \quad (30)$$

Putting the above expression in Eq.(27), we find:

$$R_T^{(r)}(E, R_0) = \sum_{m=-\infty}^{\infty} \int_0^\infty dq q \int_{-\infty}^{\infty} dk N_2^2 J_m^2(q R_0) \int_{-\infty}^{\infty} d\Delta t e^{-i[E + \frac{m}{R_0} \sinh \Omega R_0 + \omega \cosh \Omega R_0] \Delta t}. \quad (31)$$

(In the above, the subscript T stands for the Takeno vacuum and the superscript (r) stands for the rotating world-line followed by the detector.) The last inte-

gral gives us $2\pi\delta\left(E + \frac{m}{R_0} \sinh \Omega R_0 + \omega \cosh \Omega R_0\right)$, for which the argument is non-null only if $m < 0$; we can take the summation index to run for $m = 1, 2, 3, \dots$, leaving us with

$$R_T^{(r)}(E, R_0) = 2\pi \sum_{m=1}^{\infty} \int_0^\infty dq q \int_{-\infty}^{\infty} dk N_2^2 J_m^2(q R_0) \delta\left(E - \frac{m}{R_0} \sinh \Omega R_0 + \omega \cosh \Omega R_0\right). \quad (32)$$

The above expression predicts excitation for the detector, and depends in a non-trivial way on the position R_0 where it is put. So we once again arrive at the same confrontation between canonical quantum field theory and the detector formalism, which was settled by Letaw and Pfautsch and Padmanabhan: how is it possible for the *orbiting* detector to be excited in the *rotating* vacuum? However a crucial distinction exists between our present analysis and the above-mentioned works: we state, as proved in the last section, that the rotating vacuum **is not** the Minkowski vacuum. We now analyse the two independent origins of the non-null excitation rate, Eq.(32).

Note that the present situation of a Unruh-De Witt detector being excited when put in an orbiting world-line interacting with the field in the rotating (Trocherries-Takeno) vacuum is to be contrasted with the two following situations. In fact, this same detector *is not excited* whether it is in an inertial world-line and interacting with the field in the iner-

tial (Minkowski) vacuum [5, 8] or when it is uniformly accelerated and interacting with the field in the accelerated (Rindler) vacuum [13]. However note that both Minkowski and Rindler space-times are static, differently of the Trocherries-Takeno metric, and differently also of the rotating metric obtained using the galilean transformation, which are examples of non-static metrics. Recall that using instead the galilean transformation to a rotating frame it was also found by Letaw and Pfautsch a non-null excitation rate for the orbiting detector in the rotating vacuum, considered by them as $|0, M\rangle$. Therefore the excitation in the present case may be attributed to the non-staticity of the Trocherries-Takeno metric.

The other origin of the excitation found above for the detector can be attributed to the detector model we adopted [14]. Indeed, note that splitting the field operator in its positive and negative-frequency parts with respect to rotating time t in Eq.(28), one can express the Wightman function as:

$$\begin{aligned} G^+(x, x') &= \langle 0 | \phi(x) \phi(x') | 0 \rangle \\ &= \langle 0 | \phi^+(x) \phi^+(x') | 0 \rangle + \langle 0 | \phi^+(x) \phi^-(x') | 0 \rangle \\ &+ \langle 0 | \phi^-(x) \phi^+(x') | 0 \rangle + \langle 0 | \phi^-(x) \phi^-(x') | 0 \rangle. \end{aligned} \quad (33)$$

In the case where $|0\rangle = |0, T\rangle$, because of Eq.(19) only the second term above is non-vanishing, corresponding to the emission (creation) of a Trocherries-Takeno quantum with simultaneous excitation of the detector and this is the term responsible for the non-vanishing excitation rate, Eq.(32). In the context of quantum optics photodetection is regarded as photo*absorption* processes only [15, 16], a detector being able to be excited only when it absorbs (annihilates) a quantum of the field. In this way terms like the second one above are discarded and only terms like the third one are taken into account, such a procedure being called the rotating-wave approximation [16]. Therefore purely absorptive detectors (Glauber model) always give vanishing excitation rate in the vacuum state of the field. From this discussion, it is clear that the Glauber detector model will not be excited when put in the orbiting world-line and the field is in the Trocherries-Takeno vacuum state. Another context in which the inclusion of the antiresonant term (second one above) plays a crucial role is that of accelerated observers, where the thermal character of the Minkowski vacuum as seen by a Rindler observer is not revealed if one uses the Glauber correlation function, but only if

one uses the Wightman one. In effect, the Wightman correlation function includes the vacuum fluctuations that are omitted in the Glauber function [14], and to these very vacuum fluctuations can be attributed the non-vanishing excitation rate Eq.(32). Because of this feature the model of Unruh-De Witt is also called a fluctuometer [17].

We now discuss the other case of putting the detector in an orbiting trajectory and preparing the scalar field in the usual inertial vacuum $|0, M\rangle$. Writing $|0, M\rangle$ for $|0\rangle$ in Eq.(28), it is easy to show that the positive-frequency Wightman function is given by:

$$G_M^+(x(t), x(t')) = \sum_j v_j(x) v_j^*(x'), \quad (34)$$

where M stands for the Minkowski vacuum state. As the rate of excitation Eq.(27) is given in terms of the detector's proper time, we shall express Eq.(34) in terms of the rotating coordinates, using the inverse of Takeno's transformations. Let us begin with $G_M^+(x(t'_1), x(t'_2))$, in inertial coordinates, with identifications $r'_1 = r'_2 = R_0$ and $z'_1 = z'_2$, as demanded for this case:

$$G_M^+(x(t'_1), x(t'_2)) = \sum_{m=-\infty}^{\infty} \int_0^{\infty} dq q \int_{-\infty}^{\infty} dk N_1^2 e^{-i\omega(t'_1-t'_2)+im(\theta'_1-\theta'_2)} J_m^2(qR_0). \quad (35)$$

The inverse of Takeno's transformations read

$$t' = t \cosh \Omega r + r \theta \sinh \Omega r, \quad (36)$$

$$r' = r, \quad (37)$$

$$\theta' = \theta \cosh \Omega r + \frac{t}{r} \sinh \Omega r, \quad (38)$$

$$z' = z. \quad (39)$$

Using the above in Eq.(35) and taking note of the fact that the detector is at rest in the rotating frame, i.e., $\theta_1 = \theta_2$, we see that in this manner the Minkowski Wightman function is a function of the difference in proper time $\Delta t = t_1 - t_2$, which allows us to calculate the rate of excitation of the orbiting detector when the field is in the Minkowski vacuum:

$$R_M^{(r)}(E, R_0) = 2\pi \sum_{m=1}^{\infty} \int_0^{\infty} dq q \int_{-\infty}^{\infty} dk N_1^2 J_m^2(qR_0) \delta\left(E - \frac{m}{R_0} \sinh \Omega R_0 + \omega \cosh \Omega R_0\right). \quad (40)$$

The result above is very much like Eq.(32), with the exception that in the above it appears the normalization of the inertial modes N_1 instead of N_2 .

Finally, let us suppose that it is possible to prepare the field in the rotating vacuum and the detector is in an inertial world-line and let us calculate the excitation

rate in this situation:

$$R_T^{(i)}(E, R_0) = \int_{-\infty}^{\infty} d\Delta t' e^{-iE\Delta t'} G_T^+(x(t_1), x(t_2)), \quad (41)$$

where the superscript (i) stands for the inertial world-line followed by the detector, $\Delta t'$ is the difference in

proper time in the inertial frame, and $G_T^+(x(t), x(t'))$ is given by Eq.(29). It is not difficult to write $G_T^+(x(t), x(t'))$ in terms of the inertial coordinates, re-

calling that now the detector is not at rest in the rotating frame. We have therefore the result that:

$$R_T^{(i)}(E, R_0) = 2\pi \sum_{m=-\infty}^{\infty} \int_0^{\infty} dq q \int_{-\infty}^{\infty} dk N_2^2 J_m^2(qR_0) \times \delta \left(E - \left(\omega \Omega R_0 - \frac{m}{R_0} \right) \sinh(2\Omega R_0) - (m\Omega - \omega) \cosh(2\Omega R_0) \right). \quad (42)$$

In order to study the activity of the Trocherries-Takeno vacuum, we calculated the rate of excitation of an Unruh-De Witt detector in two different situations: when it is put in the orbiting and in an inertial world-lines. Since in the first case we found a non-null rate, contrary to the idea that the orbiting detector co-rotating with the rotating vacuum should not perceive anything, it can be considered as a noise of the rotat-

ing vacuum, being it perceived regardless of the state of motion of the detector. This amounts to say that the inertial detector will also measure this noise, and we normalize the rate in this situation by subtracting from it the value Eq.(32), resulting in a normalized excitation rate for the inertial detector in interaction with the field in the rotating vacuum:

$$\mathcal{R}_T^{(i)}(E, R_0) = 2\pi \sum_{m=-\infty}^{\infty} \int_0^{\infty} dq q \int_{-\infty}^{\infty} dk N_2^2 J_m^2(qR_0) \times \left[\delta \left(E - \left(\omega \Omega R_0 - \frac{m}{R_0} \right) \sinh(2\Omega R_0) - (m\Omega - \omega) \cosh(2\Omega R_0) \right) - \delta \left(E - \frac{m}{R_0} \sinh \Omega R_0 + \omega \cosh \Omega R_0 \right) \right] \quad (43)$$

IV Summary and discussions

In this work we quantize a massless scalar field in a uniformly rotating frame and compare this quantization with the usual one in an inertial frame. As a difference with regard to previous works, we assume a coordinate transformation between both frames that takes into account the finite velocity of light and is valid in the whole manifold. In doing so, a material point orbiting around the axis of rotation never exceeds the speed of light, no matter how far it is from the axis. The metric, when written in rotating coordinates, presents no event horizons, although it is non-static and non-stationary. We recourse to a criterium of di Sessa to define positive and negative frequency modes in the rotating frame and the field is quantized along these lines. Such a quantization entails a vacuum state and by using the Bogolubov transformations we were able to show that this vacuum state is inequivalent to the Minkowski one. This is the main result of the paper. This results in that the Minkowski vacuum is seen as a many-rotating-particle state by a uniformly rotating

observer, although it can not be seen as a thermal state (of Takeno particles), as in the case of a uniformly accelerated observer (for Rindler particles) and thus we cannot assign a temperature to it.

We obtain the response function of an Unruh-De Witt detector in three different situations: the orbiting detector is interacting with the field prepared in the rotating and in the Minkowski vacuum states, and finally the detector is travelling in an inertial world-line and the field is prepared in the rotating vacuum. In the first case it is found that the detector gets excited and we attributed this excitation to two different causes: Firstly, we are using the Unruh-De Witt detector model instead of a purely absorptive detector as the Glauber's model, and secondly the Trocherries-Takeno metric is non-static. Because the rotating vacuum excites even a rotating detector, we consider this as a noise which will be measured by any other state of motion of the detector. In this way, when calculating the response of the inertial detector in the presence of the rotating vacuum we subtract from it this noise.

V Appendix

show that:

We now give indirect proofs that the Bogolubov coefficient β is in fact non-zero. The calculations in the text

$$\begin{aligned} \beta_{jj'} &= 2\pi N_1 N_2 \delta(k+k') \int_0^\infty r' dr' \left(\omega' - \omega \cosh 2\Omega r' - \frac{m}{r'} \sinh 2\Omega r' \right) J_{m'}(q'r') J_m(qr') \\ &\times (iA_{m,m'}(r, \omega))^{-1} [\exp(2\pi i A_{m,m'}(r, \omega)) - 1], \end{aligned} \quad (44)$$

where

$$A_{m,m'}(r, \omega) = m' + m \cosh 2\Omega r' + \omega r' \sinh 2\Omega r'. \quad (45)$$

As for $m = m' = 0$ the integrands go like $1/\sinh(2\Omega r)$, and $1/\sinh(2\Omega r)$ diverges for $r \rightarrow 0$, let us calculate $\beta_{0,1}$, which is for $m = 0$ and $m' = 1$.

$$\begin{aligned} \beta_{01} &= -2\pi i N_1(m'=1) N_2(m=0) \delta(k+k') \int_0^\infty r dr \frac{(\omega' - \omega \cosh 2\Omega r)}{(1 + r\omega \sinh 2\Omega r)} J_0(qr) J_1(q'r) \\ &\times [\exp(2\pi i(1 + r\omega \sinh 2\Omega r)) - 1]. \end{aligned} \quad (46)$$

So we call I the integral above:

$$I = \int_0^\infty r dr \frac{(\omega' - \omega \cosh 2\Omega r)}{(1 + r\omega \sinh 2\Omega r)} J_0(qr) J_1(q'r) [\exp(2\pi i(1 + r\omega \sinh 2\Omega r)) - 1]. \quad (47)$$

The first integral is the only one which is ω' -dependent, i.e., it is a function of ω' . So, if we prove that it is different from zero, we prove that

$\beta_{01} \neq 0$, since the second integral is not sufficient to make it zero. Let us call it $I(\omega')$:

$$I(\omega') = \omega' \int_0^\infty r dr \frac{J_0(qr) J_1(q'r)}{(1 + r\omega \sinh 2\Omega r)} [\exp(2\pi i(1 + r\omega \sinh 2\Omega r)) - 1]. \quad (48)$$

Let us pick the second integral above, since again the first one is not capable to make $I(\omega') \neq 0$:

$$I_2(\omega') = -\omega' \int_0^\infty r dr \frac{J_0(qr) J_1(q'r)}{(1 + r\omega \sinh 2\Omega r)}. \quad (49)$$

There is not an analytical expression for $I_2(\omega')$, but using the Maple one can calculate particular values, such

as:

$$\int_0^\infty r dr \frac{J_0(r) J_1(r)}{(1 + r \sinh r)} = 0.1830959596, \quad (50)$$

so we proved, very indirectly, that $\beta \neq 0$.

A different way to prove that $\beta \neq 0$ is to study the high-velocity limit of β_{00} :

$$\beta_{00} = -2\pi i N_1(m'=0) N_2(m=0) \delta(k+k') \int_0^\infty r dr \frac{\omega' - \omega \cosh(2\Omega r)}{\omega r \sinh(2\Omega r)} J_0(qr) J_0(q'r) [e^{2\pi i \omega r \sinh(2\Omega r)} - 1]. \quad (51)$$

As $\frac{1}{\sinh x} \rightarrow 0$ as $x \rightarrow \infty$, and $\frac{\cosh x}{\sinh x} \rightarrow 1$ as $x \rightarrow \infty$,

we find:

$$\lim_{\Omega \rightarrow \infty} \beta_{00} = -2\pi i N_1(m' = 0) N_2(m = 0) \delta(k + k') \int_0^\infty dr J_0(qr) J_0(q'r) \left[1 - e^{2\pi i \omega r \sinh(2\Omega r)} \right]. \quad (52)$$

The first of these two integrals can be found in Gradshteyn, in terms of hypergeometric functions:

$$\int_0^\infty dr J_0(qr) J_0(q'r) = \frac{1}{q + q'} F \left[\frac{1}{2}; \frac{1}{2}; 1; \frac{4qq'}{(q + q')^2} \right], \quad (53)$$

and it is non-zero.

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