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Can Non Gravitational Black Holes Exist?

M. Novello, V. A. De Lorenci and E. Elbaz†

Centro Brasileiro de Pesquisas Físicas,
Rua Dr. Xavier Sigaud, 150, Urca
22290-180 – Rio de Janeiro, RJ – Brazil.

†*Institut de Physique Nucléaire de Lyon IN2P3-CNRS*
Université Claude Bernard
43 Bd du 11 Novembre 1918, F-69622 Villeurbanne Cedex, France.

Abstract

We claim that the existence of a mechanism such that photons may be trapped in a compact domain is not an exclusive property of gravitational forces. We show the case in which a non-linear electrodynamics allows such effect. In this latter case we should call this region an *Electromagnetic Black Hole* (EBH).

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1 Introduction

One of the most remarkable consequences of the attractive power of gravitational forces is probably the possibility of formation of black holes, regions of the spacetime in which photons are confined. However, from a fundamental point of view one should be tempted to ask: is this a typical and exclusive characteristic of gravity? Or, on the other hand, could it be possible that other interactions present analogous properties, e.g., to display a similar behavior like gravitational black holes (GBH), allowing the existence of a hidden part of the spacetime structure, unseen from the outside, without making appeal to gravity? In this paper we provide an affirmative answer to this question. Indeed, we will show that a model can be constructed such that purely electromagnetic forces can effectively lead to such a configuration. How is this possible?

First of all let us make some comments concerning the structure of the metric properties of spacetime in order to introduce our ideas. Since the advent of general relativity it has been widely accepted that the geometry of spacetime is driven uniquely by gravitational forces. Although this is a net consequence of the universality of such interaction, it is certainly not true that some effects of other interactions cannot be described in a similar framework, i.e., such that they can be interpreted as being nothing but a modification of the local metric properties. In order to provide an example – which will be used as the basis of the whole argumentation of the present paper – let us emphasize that it has been known for a long time that the wave propagation of nonlinear interactions could well be described in terms of an effective modification of the metric qualities of the underlying substractum. To be specific and antecipating our result, we shall see that electromagnetic disturbances generated in the framework of a nonlinear theory, do not propagate in an *a priori* Minkowskian background structure, but instead, propagate in a modified geometry that depends only on the character of the nonlinearity of the field. In the case of Born-Infeld theory, for instance, it has been shown [1] that it is the energy-momentum density of the non-linear field which is the essential cause by which the characteristic surfaces are in general not null cones of the background Minkowski geometry but instead null cones of another metric. Let us emphasize that the true responsible for the associated curvature of this effective geometry has nothing to do with gravitational process: it is a pure consequence of the assumed non-linearity of the electromagnetic field. We are not interested here in pointing out the obvious differences based on the distinct property that makes the geometrisation related to gravity to be a universal one[1], in comparison with that produced by electromagnetic forces. Notwithstanding its fundamental importance, for our purposes here such a distinction is not relevant.

Since all our argumentation in the present paper rests on electrodynamics and moreover on its nonlinearity, a few comments on it seems necessary. The modification of the metric properties of the underlying geometry through which the electromagnetic waves propagate is not an universal phenomenon. This means that the effective geometry is nothing but a convenient choice of representation of the field propagation in certain circumstances[2] (see, ref. [2] for more details).

[1]This is due to the validity of the equivalence principle.

[2]This is not a crucial distinction from general relativity as it can appears in the first sight, since, as it has been shown (see, for instance refs. [2], [3]), Einstein presentation of his gravity theory is

We are concerned here with the propagation of disturbances in a non-linear electromagnetic theory. This will be taken as a fundamental process. However, it seems worth to remark that such method of dealing with an equivalent geometry could well be applied to model processes occurring in the interior of a medium that contains interactions of charged particles and currents like, for instance, in a plasma. The reason for this is simple: it is the non-linear nature of the process that is the relevant condition for the application of the method of dealing with a modified geometry. Thus, it should not be a surprise that some of the consequences that we present here could well be tested in a terrestrial laboratory under special circumstances. As a major consequence of the equivalent geometrical interpretation an important property of the theory appears which can be synthesized as:

- **The discontinuities of nonlinear electromagnetic theories propagates in an effective non-Minkowskian geometry dependent only on the field properties.**

Let us show this and examine some of its consequences.

2 The General Framework

The nonlinear electrodynamic theory[3] is described by a Lagrangian L given uniquely in terms of the invariant $F \equiv F_{\mu\nu} F^{\mu\nu}$. We set[4]

$$L = L(F). \tag{1}$$

The corresponding equation of motion is given by

$$\{L_F F^{\mu\nu}\}_{,\nu} = 0 \tag{2}$$

in which L_F represents the functional derivative of the Lagrangian $(\delta L/\delta F)$ with respect to invariant F; L_{FF} is the second derivative.

This equation can be written in another, more appealing form, by just isolating the linear Maxwell term and taking all remaining non linear parts as an additional *internal* current to be added to the external one:

$$F^{\mu\nu}{}_{,\nu} = J^{\mu}_{int} + J^{\mu}_{ext} \tag{3}$$

in which the associated *internal* current, the self-term is given by

$$J^{\mu}_{int} \equiv -\frac{L_{FF}}{L_F} F_{,\nu} F^{\mu\nu}. \tag{4}$$

nothing but a choice of representation. Indeed, all contents of GR can be depicted without appeal to the geometrical description. However, due to its universality, such (geometric) representation acquired a status of uniqueness. It is precisely such universality that makes the most important difference between these processes. The equivalence principle guarantees that the geometry modification induced by gravity acts in all forms of existing matter and energy; on the other hand, only the *non-linear photon* experiences the modification of the geometry in the non-linear electrodynamics.

[3]We note that these remarks concern any spin-1 theory.

[4]We do not consider here the invariant constructed with the dual $F^{*}_{\mu\nu}$.

Written under this form it can be thought as nothing but a modelling of the response in a self-interacting way of some special plasma medium. Indeed, let us consider the quantity χ_ν,

$$\chi_\nu \equiv \frac{L_{FF}}{L_F} F_{,\nu}. \tag{5}$$

We define a normalized frame $n^\mu \equiv \chi^\mu/\chi^2$, which in the case χ^μ is time-like in the Minkowski background, could be identified to a real observer that co-moves with χ^μ. The extra term of the current assumes then the form

$$J^{int}{}_\mu = \sigma E_\mu \tag{6}$$

in which E_μ is the electric part of the field as seen in the frame n^μ and $\sigma(F)$ may depend on the field variables in a complicated way. Under the form of Eq. (6) the analogy with situations treated within Maxwell electrodynamics in material media is transparent.

¿From the definition of the energy-momentum tensor we obtain from the non-linear Lagrangian:

$$T_{\mu\nu} = -L\gamma_{\mu\nu} - 4L_F\, F_{\mu\alpha}\, F^\alpha{}_\nu. \tag{7}$$

Using the equation of motion and after some manipulation, one obtains the expression that contains all information of the balance of forces through the exchange of energy of the field and the currents independently of the particular form of the Lagrangian. Indeed, we obtain

$$T^{\mu\nu}{}_{,\nu} = -F^{\mu\nu}\, J^{ext}_\nu. \tag{8}$$

2.1 Propagation of the Discontinuities in Non-Linear Electrodynamics

Let Σ be a surface of discontinuity for the electromagnetic field. Following Hadamard's [4] condition let us assume that the field is continuous through Σ but its first derivative is discontinuous. We set

$$[F_{\mu\nu}]_\Sigma = 0, \tag{9}$$

and

$$[F_{\mu\nu,\lambda}]_\Sigma = f_{\mu\nu} k_\lambda, \tag{10}$$

in which the symbol $[J]_\Sigma$ represents the discontinuity of the function J through the surface Σ. Applying these conditions into the equation of motion (2) we obtain

$$L_F f^{\mu\nu}\, k_\nu + 2L_{FF}\, \xi F^{\mu\nu} k_\nu = 0, \tag{11}$$

where ξ is defined by

$$\xi \equiv F^{\alpha\beta}\, f_{\alpha\beta}. \tag{12}$$

The cyclic identity yields

$$f_{\mu\nu} k_\lambda + f_{\nu\lambda} k_\mu + f_{\lambda\mu} k_\nu = 0. \tag{13}$$

Multiplying this equation by $k^\lambda\, F^{\mu\nu}$ yields

$$\xi k_\nu\, k_\mu \eta^{\mu\nu} + 2\, F^{\mu\nu} f_{\nu\lambda} k^\lambda\, k_\mu = 0. \tag{14}$$

From the Eq. (11) it results:

$$f_{\mu\nu}\, k^{\nu} = -\, 2\, \frac{L_{FF}}{L_F}\, \xi F_{\mu\nu}\, k^{\nu}, \tag{15}$$

and, after some algebraic manipulations the equation of propagation of the disturbances is obtained:

$$\{\gamma^{\mu\nu} + \Lambda^{\mu\nu}\}\, k_{\mu} k_{\nu} = 0. \tag{16}$$

The new quantity, $\Lambda^{\mu\nu}$, is defined by

$$\Lambda^{\mu\nu} \equiv -4\, \frac{L_{FF}}{L_F}\, F^{\mu\alpha}\, F_{\alpha}{}^{\nu}. \tag{17}$$

The main lesson we learn from this is that in the non-linear electrodynamics the disturbances propagate not in the Minkowskian background but in an effective geometry which depends on the energy distribution of the field. The net effect of the nonlinearity can thus be summarized in the following property.

- The disturbances of nonlinear electrodynamics are null geodesics that propagate in the modified effective geometry:

$$g^{\mu\nu} = \gamma^{\mu\nu} - 4\, \frac{L_{FF}}{L_F}\, F^{\mu\alpha}\, F_{\alpha}{}^{\nu}. \tag{18}$$

In these formulas $\gamma_{\mu\nu}$ is the Minkowski metric written in an arbitrary system of coordinates. A simple inspection on this formula shows that only in the particular linear case of Maxwell electrodynamics does the discontinuity of the electromagnetic field propagate in a Minkowski background. ¿From equation (18), we obtain the specific form of the components of the metric tensor:

$$g^{00} \;=\; 1 - 4\, \frac{L_{FF}}{L_F} E^2, \tag{19}$$

$$g^{ij} \;=\; \gamma^{ij} + 4\, \frac{L_{FF}}{L_F}\, \left(E^i\, E^j + B^i\, B^j - \gamma^{ij}\, B^k\, B_k\right), \tag{20}$$

$$g^{ol} \;=\; -4\, \frac{L_{FF}}{L_F} \gamma^{li} \epsilon_{ijk}\, E^j\, B^k, \tag{21}$$

in which we have set $E^2 = -\, E_{\alpha}\, E^{\alpha}$.

A direct inspection on the above formulas of the associated geometry allows us to envisage the possibility of generating a null surface exclusively in terms of electromagnetic processes. This situation happens to occur when the properties of the non linearity is such that it induces the equality

$$E^2 = L_F / 4 L_{FF}. \tag{22}$$

Before going into a specific model, let us make here a comment. Linear photons propagate in a Minkowskian underlying background. Non linear photons propagate in an effective geometry given by Eq. (18). Note, however, that this situation is not competitive to gravity processes. The reason for this is easy to understand: the above modified geometry (in case of non-linear electrodynamics) is not a universal one. Indeed, other kinds of particles and radiations behave as if the background metric is that dealt with in special relativity: the charged particles, e. g., electrons follow time-like paths with respect to Minkowski metric.

In the appendix we present a simple example of the application of such procedure in the case of electrodynamics in a dielectric medium.

3 The Model

In order to show a specific situation that represents a configuration of an effective geometry for electromagnetic forces, let us concentrate here in a simple model. We set

$$L = -\frac{F}{4}\left(1 - \frac{F}{b}\right)^{-1}, \tag{23}$$

where the constant b has dimensionality of density of energy (we use units in which $c = 1$). We should note that this theory is a very close approximation of Maxwell electrodynamics in the case the constant b is large.

For our purposes, it is convenient to seek for a spherically symmetric and static solution of this theory. A direct computation shows that in the spherical coordinate system (t, r, θ, ϕ) a particular solution can be found uniquely in terms of a radial electric component given by:

$$F_{01} = f(r), \tag{24}$$

where the function $f(r)$ obeys the relation:

$$\frac{f(r)}{\left[b + 2f(r)^2\right]^2} = \frac{C_0}{r^2} \tag{25}$$

and the parameter C_0 is related to the electric charge located at the origin.

¿From the previous section, we conclude that the effect of the nonlinearity on the propagation of the discontinuities is to induce the electromagnetic waves to follow a null cone of the modified geometry given by

$$g^{\mu\nu} = \gamma^{\mu\nu} - \frac{8}{b - F}\, F^{\mu\beta} F_{\beta}{}^{\nu}, \tag{26}$$

in which the Minkowski metric $\gamma_{\mu\nu}$ has the form

$$ds^2 = dt^2 - dr^2 - r^2\, d\theta^2 - r^2 \sin^2\theta\, d\phi^2. \tag{27}$$

It then follows that the non-null components of the metric tensor of the effective geometry, as seen by the photon disturbances, are:

$$g^{00} = \frac{b - 6f(r)^2}{b + 2f(r)^2}, \tag{28}$$

$$g^{11} = -\,g^{00}, \tag{29}$$

$$g^{22} = -\,\frac{1}{r^2}, \tag{30}$$

$$g^{33} = \frac{1}{r^2 \sin^2\theta}. \tag{31}$$

Just for completeness, let us exhibit the non-vanishing components of the energy-momentum tensor of the field:

$$T^0{}_0 = \frac{bf(r)^2}{2}\left\{\frac{b-2f(r)^2}{[b+2f(r)^2]^2}\right\}, \tag{32}$$

$$T^1{}_1 = T^0{}_0, \tag{33}$$

$$T^2{}_2 = -\frac{bf(r)^2}{2b+4f(r)^2}, \tag{34}$$

$$T^3{}_3 = T^2{}_2. \tag{35}$$

In linear Maxwell theory the energy-momentum tensor is traceless. This property is no longer true in the non-linear case. A direct inspection on this expression shows that the field energy is well behaved throughout all space except at the origin.

3.1 Electromagnetic Black Holes (EBH)

Electromagnetic disturbances in a non-linear theory follow null cones (geodesics) in an effective geometry. The best way to analyze the properties of their paths is then to examine the equations of motion of the geodesics in the effective metric $g_{\mu\nu}$ given by

$$ds^2 = \left[\frac{b+2f(r)^2}{b-6f(r)^2}\right] dt^2 - \left[\frac{b+2f(r)^2}{b-6f(r)^2}\right] dr^2 - r^2\, d\theta^2 - r^2 \sin^2\theta\, d\phi^2. \tag{36}$$

Solving the variational problem

$$\delta \int ds = 0, \tag{37}$$

the solutions of the Euler-Lagrange equations that follow are:

$$\left[\frac{b+2f(r)^2}{b-6f(r)^2}\right] \dot{t} = l_0, \tag{38}$$

$$r^2 \dot{\phi} = h_0, \tag{39}$$

where l_0 and h_0 are constants of motion related to the photon energy. The system was reduced to a planar orbit by the choice of the initial conditions:

$$\dot{\theta} = 0, \qquad \theta = \frac{\pi}{2}. \tag{40}$$

The null property of the geodesics allows to obtain in a direct way the equation of the radial component. Using Eqs. (36) - (40), results:

$$\dot{r}^2 + \frac{b-6f(r)^2}{[b+2f(r)^2]^2}\left\{\frac{h_0^2 f(r)}{C_0} - l_0^2\left[b-6f(r)^2\right]\right\} = 0. \tag{41}$$

We can re-write this equation in the more convenient form:

$$\dot{r}^2 + V_{eff} = l_0^2 \tag{42}$$

in which the potential V_{eff} is defined by:

$$V_{eff} = \frac{b - 6f(r)^2}{[b + 2f(r)^2]^2} \left\{ \frac{h_0^2 f(r)}{C_0} - l_0^2 \left[b - 6f(r)^2 \right] \right\} + l_0^2. \qquad (43)$$

Thus, the motion of the photon in such non-linear theory can be described as a particle dotted with energy l_0^2 immersed in a central field of forces characterized by the potential V_{eff}.

4 Conclusion

The important fact to be noticed here is related to the behavior of the metric (36) in the critical point $f(r_c)^2 = b/6$. Indeed coordinates t and r interchange their corresponding roles when crossing the $r = r_c$ null surface[5]. Note however that the potential V_{eff} is well behaved at r_c.

We can thus conclude from this remark and from what we have learned in this paper, that configurations of structures containing hidden regions, like black holes, are not restricted to gravitational forces. Non linear electromagnetic interaction can also produce similar objects. In this paper we presented a simple purely electromagnetic model in which we have neglected the gravitational effects.

We should like to point out that in order to compare the above EBH (Electromagnetic Black Hole) to standard GBH (Gravitational Black Hole) one must couple this non linear electromagnetic theory to gravity. We postpone this analysis for a forthcoming paper.

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Appendix

A – Dielectric Constant and the Effective Geometry

In order to show the non-familiar reader the treatment that involves dealing with the propagation of the non-linear theory as a modification of the background geometry, we will present here the simplest possible case of the standard Maxwell theory in a dielectric medium. We will show how it is possible to present the wave propagation of linear electrodynamics in a medium in terms of a modified geometry of the spacetime.

In this section we take the Maxwell theory in a medium such that the electromagnetic field is represented by two anti-symmetric tensors $F_{\mu\nu}$ and $P_{\mu\nu}$ given in terms of the electric

[5] See appendix B for more details.

and magnetic vectors, as seen by an arbitrary observer endowed with a four-velocity v^{μ}, by the standard expressions:

$$F_{\mu\nu} = E_{\mu}\, v_{\nu} - E_{\nu}\, v_{\mu} + \eta_{\mu\nu}{}^{\rho\sigma}\, v_{\rho}\, H_{\sigma} \tag{A.1}$$

and

$$P_{\mu\nu} = D_{\mu}\, v_{\nu} - D_{\nu}\, v_{\mu} + \eta_{\mu\nu}{}^{\rho\sigma}\, v_{\rho}\, B_{\sigma}. \tag{A.2}$$

The Maxwell equations are:

$$\partial^{\nu}\, F^{*}_{\mu\nu} = 0, \tag{A.3}$$

$$\partial^{\nu}\, P_{\mu\nu} = 0. \tag{A.4}$$

Following Hadamard, we consider the discontinuities on the fields as given by

$$\begin{aligned} [\partial_{\lambda}\, E_{\mu}]_{\Sigma} &= k_{\lambda}\, e_{\mu} \\ [\partial_{\lambda}\, D_{\mu}]_{\Sigma} &= k_{\lambda}\, d_{\mu} \\ [\partial_{\lambda}\, H_{\mu}]_{\Sigma} &= k_{\lambda}\, h_{\mu} \\ [\partial_{\lambda}\, B_{\mu}]_{\Sigma} &= k_{\lambda}\, b_{\mu}. \end{aligned} \tag{A.5}$$

Using the constitutive relations[6]

$$d_{\mu} = \epsilon\, e_{\mu} \tag{A.6}$$

$$b_{\mu} = \frac{h_{\mu}}{\mu} \tag{A.7}$$

one obtains after a straightforward calculation

$$k_{\mu}\, k_{\nu}\, [\eta^{\mu\nu} + (\epsilon\, \mu - 1) v^{\mu}\, v^{\nu}] = 0. \tag{A.8}$$

This shows that even the simple case of the evolution of the wave front in standard Maxwell equation in a medium can be interpreted in terms of an effective geometry $g^{\mu\nu}$ that depends not only on the medium properties ϵ and μ, but also on the observer's velocity, given by:

$$g^{\mu\nu} \equiv \eta^{\mu\nu} + (\epsilon\, \mu - 1) v^{\mu}\, v^{\nu}. \tag{A.9}$$

This ends our proof.

B – Null Surface

Let us consider the surface $\psi = r = const$ in the case of the solution examined in the previous section. We are interested here in the analysis of the characteristics of the equation of motion of the non linear electromagnetic field in the neighborhood of the critical radius defined by relation

$$f(r = r_c) = \sqrt{\frac{b}{6}} \tag{B.1}$$

[6]We deal here with the simplest case of linear isotropic relations, just for didactic reasons.

that result in the value for this radial coordinate

$$r_c = \frac{4b}{3}\sqrt{C_0\sqrt{\frac{6}{b}}}. \tag{B.2}$$

Using the metric $g^{\mu\nu}$, we have

$$\psi_\mu \psi_\nu g^{\mu\nu} = \psi_1 \psi_1 g^{11} = -(\psi_1)^2 \left[\frac{b - 6f(r)^2}{b + 2f(r)^2}\right] \tag{B.3}$$

where we have set $\psi_\mu = \partial_\mu \psi$. At the value $r = r_c$, this relation vanishes showing that the surface ψ is a null surface at the critical radius, for the non-linear photon.

C – Born-Infeld Model

The result that we have shown here, concerning the existence of EBH depends not only on the nonlinearity but also on the specific form of theory. The most popular non-linear model, the Born-Infeld [5] theory, does not admit such structure (EBH) in its corresponding *static* configuration. Indeed, the simplest way to show this is by a direct inspection on the properties of the corresponding static solution for this case. The Born-Infeld Lagrangian is

$$L = -\frac{b^2}{4}\left\{\sqrt{1 + \frac{2\,F}{b^2}} - 1\right\}. \tag{C.1}$$

From the previous formuli, the g^{00} component of the effective geometry takes the form

$$g^{00} = 1 + \frac{4E^2}{b^2 - 4\,E^2} \tag{C.2}$$

that results in the inverse — since in this solution we have the metric diagonal — given by

$$g_{00} = \frac{b^2 - 4\,E^2}{b^2} \tag{C.3}$$

which vanishes for the following value of the electric field:

$$E = \frac{b}{2}. \tag{C.4}$$

Therefore, this value of the electric field is just the upper limit of validity of this function, as it can be noticed by a direct analysis of the above Lagrangian.

References

[1] J. Plebansky, in *Lectures on Non-Linear Electrodynamics*, Ed. Nordita (Copenhagen, 1968).

[2] M. Novello, V. A. De Lorenci and L. R. de Freitas, Annals of Physics, 254, 1, 83 (1997).

[3] R. P. Feynman, in *Lectures on Gravitation*, California Institute of Technology (1962), unpublished.

[4] J. Hadamard, in *Leçons sur la propagation des ondes et les équations de l'hydrodynamique.* Paris, Ed. Hermann, (1903); V. D. Zakharov, in *Gravitational Waves in Einstein's Theory.* New York, John Wiley & Sons, Inc. (1973).

[5] M. Born, Nature, **132**, 282 (1933); Proc. Roy. Soc. **A 143**, 410 (1934); M. Born and L. Infeld, Nature **133**, 63 (1934).