CBPF-NF-032/94

Algebraic Renormalization of Antisymmetric Tensor Matter Fields

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Abstract

The algebraic renormalization of a recently proposed abelian axial gauge model with antisymmetric tensor matter fields is presented.

Key-words: Renormalization; Gauge theories.

PACS: 11.15.Bt

1 Introduction

Antisymmetric tensor fields have been introduced since many years [1] and are object of continuous and renewed interests due to their connection with the topological field theories [2]. They are indeed the building blocks of a large class of Schwarz type topological theories [3]. These theories, also known as BF models, can be formulated in any space-time dimension and allow to compute topological invariants which generalize the three dimensional linking number [4]. Moreover they provide an example of ultraviolet finite field theories [5]. Let us point out that the antisymmetric tensor fields of the topological models are gauge fields, i.e. they possess zero modes which have to be gauge fixed in order to have a nondegenerate action.

More recently L. V. Avdeev and M. V. Chizhov [6] achieved the construction of a four dimensional abelian gauge model which includes antisymmetric second rank tensor fields as matter fields rather than gauge fields. The model contains also a coupling of the antisymmetric fields with chiral spinors and a quartic tensor self-interaction term. It exhibits several interesting features among which we underline the asymptotically free ultraviolet behaviour of the abelian gauge interaction. As shown by the authors [6] with an explicit one-loop computation, this is due to the fact that the contribution of the tensor fields to the gauge β-function is negative. Antisymmetric matter fields represent thus a possibility of introducing a new type of interaction in gauge theories.

The aim of this work is to give a regularization independent algebraic analysis of the model. The use of the algebraic method [7] in the present case is motivated by the fact that, besides the presence of γ_5 in the axial couplings of the classical action, the gauge transformations as well as the interaction vertices involving the antisymmetric matter fields explicitely contain the $\varepsilon_{\mu\nu\rho\sigma}$ tensor.

Moreover one has to be sure that the introduction of a new type of matter field does not lead to a new kind of anomaly. Indeed, as we shall see by exploiting the Wess-Zumino consistency condition [8], the tensor field gives rise to a nontrivial cocycle which is independent from the gauge field and which contains only matter fields. Its numerical coefficient turns out to be related to a set of one-loop Feynman diagrams which are built-up with the interaction vertex between the tensor and the spinor fields. However we will be able to show that the tensor-spinor vertex as well as the matter cocycle can be consistently eliminated by requiring that the model is invariant under an additional descrete symmetry. One is left thus with the usual gauge anomaly of the chiral QED. Furthermore the Adler-Bardeen theorem ensures that the latter is definitively absent if it is absent at the one-loop level, guaranteeing then the ultraviolet renormalizability of the model.

The paper is organized as follows. In Sect. 2 we introduce the classical action and we discuss its stability under radiative corrections. Sect. 3 is devoted to the analysis of the Wess-Zumino consistency condition. In Sect. 4 we present a geometrical derivation of the model in terms of covariant derivatives. Finally, Sect. 5 contains some useful relations and the conventions.

2 The model and its stability

The model is specified by the following classical action which, using the same notation of ref. [6], reads

$$
\begin{aligned}
S_{inv} = \int d^4x \biggl(\quad & -\frac{1}{4h^2}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu\partial_\mu\psi - \bar{\psi}\gamma^\mu\gamma_5 A_\mu\psi + \frac{1}{2}(\partial_\lambda T_{\mu\nu})^2 \\
& -2(\partial^\mu T_{\mu\nu})^2 + 4A_\mu\left(T^{\mu\nu}\partial^\lambda\widetilde{T}_{\lambda\nu} - \widetilde{T}^{\mu\nu}\partial^\lambda T_{\lambda\nu}\right) \\
& +4\left(\frac{1}{2}(A_\lambda T_{\mu\nu})^2 - 2(A^\mu T_{\mu\nu})^2\right) + y\bar{\psi}\sigma_{\mu\nu}T^{\mu\nu}\psi \\
& +\frac{q}{4}\left(\frac{1}{2}(T_{\mu\nu}T^{\mu\nu})^2 - 2T_{\mu\nu}T^{\nu\rho}T_{\rho\lambda}T^{\lambda\mu}\right) \biggr) \ ,
\end{aligned}
\tag{2.1}
$$

where (h, y, q) are coupling constants and $T_{\mu\nu} = -T_{\mu\nu}$ is a second rank antisymmetric tensor field with

$$
\begin{aligned}
\widetilde{T}_{\mu\nu} &= \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}T^{\rho\sigma} \ , \\
\frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\widetilde{T}^{\rho\sigma} &= -T_{\mu\nu} \ ,
\end{aligned}
\tag{2.2}
$$

the $\varepsilon_{\mu\nu\rho\sigma}$ tensor being normalized as

$$
\varepsilon_{\mu_1\mu_2\mu_3\mu_4}\varepsilon^{\nu_1\nu_2\nu_3\nu_4} = -\delta^{[\nu_1}_{\mu_1}...\delta^{\nu_4]}_{\mu_4} \ . \tag{2.3}
$$

We use the Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(+---)$ and we denote $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ (see Sect. 5 for the conventions).

Notice that the quadratic term in the antisymmetric field of expression (2.1) is nondegenerate, i.e. $T_{\mu\nu}$ is a matter field. This feature allows also for the introduction of a spinor-tensor interaction (y-term) as well as for a quartic tensor self-interaction (q-term). It is easy to check that the action (2.1) is left invariant by the following gauge transformations [6]:

$$
\begin{aligned}
\delta A_\mu &= \partial_\mu\omega \ , \\
\delta\psi &= -i\omega\gamma_5\psi \ , \\
\delta\bar{\psi} &= -i\omega\bar{\psi}\gamma_5 \ , \\
\delta T_{\mu\nu} &= -2\omega\widetilde{T}_{\mu\nu} \ .
\end{aligned}
\tag{2.4}
$$

Introducing now a covariant Feynman gauge

$$
S_{gf} = -\frac{1}{2\alpha}\int d^4x \, (\partial A)^2 \ , \tag{2.5}
$$

the gauge-fixed action

$$\Sigma = S_{inv} + S_{gf} \ , \tag{2.6}$$

obeys the Ward identity

$$\mathcal{W}(x)\Sigma = -\frac{1}{\alpha}\partial^2\partial A \ , \tag{2.7}$$

where $\mathcal{W}(x)$ denotes the local Ward operator[1]

$$\mathcal{W}(x) = -\partial_\mu \frac{\delta}{\delta A_\mu} - i\frac{\delta}{\delta\psi}\gamma_5\psi - i\bar{\psi}\gamma_5\frac{\delta}{\delta\bar{\psi}} - \widetilde{T}_{\mu\nu}\frac{\delta}{\delta T_{\mu\nu}} \ . \tag{2.8}$$

The classical action Σ is known to be constrained, besides the Ward identity (2.7), by a set of descrete symmetries: i.e. parity $\mathcal{P}$ and charge conjugation $\mathcal{C}$ [9, 10]. They act as

i) Parity $\mathcal{P}$

$$\begin{aligned}
& x \to x_p = (x^0, -x^i) \ , \qquad i = 1,2,3 \\
& \psi(x_p) = \gamma^0\psi(x) \ , \\
& A_0(x_p) = -A_0(x) \ , \qquad A_i(x_p) = A_i(x) \ , \\
& T_{0i}(x_p) = -T_{0i}(x) \ , \qquad T_{ij}(x_p) = T_{ij}(x) \ .
\end{aligned} \tag{2.9}$$

ii) Charge conjugation $\mathcal{C}$

$$\begin{aligned}
& \psi \to \psi^c = C\bar{\psi}^T \ , \qquad C = i\gamma^0\gamma^2 \ , \\
& A_\mu \to A^c_\mu = A_\mu \ , \\
& T_{\mu\nu} \to T^c_{\mu\nu} = -T_{\mu\nu} \ .
\end{aligned} \tag{2.10}$$

The fields (A, ψ, T) have respectively dimension $(1, 3/2, 1)$.

In order to study the stability [7] of the model under radiative corrections we look at the most general solution of the equation

$$\mathcal{W}(x)\widetilde{\Sigma} = 0 \ , \tag{2.11}$$

where $\widetilde{\Sigma}$ is an integrated local polynomial in the fields and their derivatives with dimension four, invariant under parity $\mathcal{P}$ and charge conjugation $\mathcal{C}$. $\widetilde{\Sigma}$ represents the most general local gauge invariant counterterm which one can freely add at each order of perturbation theory. It can be parametrized as

$$\widetilde{\Sigma} = \widetilde{\Sigma}(A, \psi) + \widetilde{\Sigma}(A, \psi, T) \ , \tag{2.12}$$

[1]As usual the spinor derivative $\frac{\delta}{\delta\psi}$ acts from the right to the left.

where $\widetilde{\Sigma}(A,\psi)$ depends only on the fields $(A,\psi,\bar{\psi})$ and $\widetilde{\Sigma}(A,\psi,T)$ collects the dependence on the new tensor matter field $T_{\mu\nu}$. The stability condition (2.11), due to the fact that the Ward operator $\mathcal{W}(x)$ is linear, splits into the two conditions

$$\mathcal{W}(x)\widetilde{\Sigma}(A,\psi)=0\ , \tag{2.13}$$

and

$$\mathcal{W}(x)\widetilde{\Sigma}(A,\psi,T)=0\ . \tag{2.14}$$

The first equation (2.13) yields the well known local invariant counterterm of the chiral QED [9]

$$\widetilde{\Sigma}(A,\psi)=\int d^4x\left(-\frac{\rho}{4}F_{\mu\nu}F^{\mu\nu}+\sigma\left(i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi-\bar{\psi}\gamma^{\mu}\gamma_5A_{\mu}\psi\right)\right)\ , \tag{2.15}$$

with (ρ,σ) arbitrary parameters. Turning now to the second term $\widetilde{\Sigma}(A,\psi,T)$ it follows that, using the algebraic identity valid in four dimensions

$$\varepsilon^{\alpha\beta\mu\nu}\partial^{\sigma}+\varepsilon^{\sigma\alpha\beta\mu}\partial^{\nu}+\varepsilon^{\nu\sigma\alpha\beta}\partial^{\mu}+\varepsilon^{\mu\nu\sigma\alpha}\partial^{\beta}+\varepsilon^{\beta\mu\nu\sigma}\partial^{\alpha}=0\ , \tag{2.16}$$

it can be parametrized as

$$\begin{aligned}\widetilde{\Sigma}(A,\psi,T)=\int d^4x\Big(\ & a(T_{\mu\nu}T^{\mu\nu})^2+bT_{\mu\nu}T^{\nu\rho}T_{\rho\lambda}T^{\lambda\mu}+c(\partial_{\lambda}T_{\mu\nu})^2\\ &+d(\partial^{\mu}T_{\mu\nu})^2+e\partial AT_{\mu\nu}\widetilde{T}^{\mu\nu}+mA_{\mu}T^{\mu\nu}\partial^{\lambda}\widetilde{T}_{\lambda\nu}\\ &+nA_{\mu}\widetilde{T}^{\mu\nu}\partial^{\lambda}T_{\lambda\nu}+p(A^{\mu}T_{\mu\nu})^2\\ &+u(A_{\mu}T_{\lambda\nu})^2+v\bar{\psi}\sigma_{\mu\nu}T^{\mu\nu}\psi\Big)\ ,\end{aligned} \tag{2.17}$$

with (a,b,c,d,e,m,n,p,u,v) constant parameters. Condition (2.14) implies that $\widetilde{\Sigma}(A,\psi,T)$ depends only on three parameters, i.e.

$$\begin{aligned}\widetilde{\Sigma}(A,\psi,T)=\ &\int d^4x\Bigg(c\bigg(\frac{1}{2}(\partial_{\lambda}T_{\mu\nu})^2-2(\partial^{\mu}T_{\mu\nu})^2+4A_{\mu}T^{\mu\nu}\partial^{\lambda}\widetilde{T}_{\lambda\nu}\\ &-4A_{\mu}\widetilde{T}^{\mu\nu}\partial^{\lambda}T_{\lambda\nu}+4\Big(\frac{1}{2}(A_{\mu}T_{\lambda\nu})^2-2(A^{\mu}T_{\mu\nu})^2\Big)\bigg)\\ &+a\Big(\frac{1}{2}(T_{\mu\nu}T^{\mu\nu})^2-2T_{\mu\nu}T^{\nu\rho}T_{\rho\lambda}T^{\lambda\mu}\Big)+v\bar{\psi}\sigma_{\mu\nu}T^{\mu\nu}\psi\Bigg)\ .\end{aligned} \tag{2.18}$$

One sees thus that the most general local gauge invariant counterterm contains five free independent parameters (ρ,σ,c,a,v). They are easily seen to correspond to a renormalization of the coupling constants (h,y,q) and to field amplitude redefinitions[2]. This proves the stability of the classical action under radiative corrections.

[2] As it happens in the ordinary QED, the gauge fixing term (2.5) is not renormalized.

3 The Wess-Zumino consistency condition

At the quantum level the action Σ is replaced by a vertex functional

$$\Gamma = \Sigma + O(\hbar) \ , \tag{3.1}$$

which obeys the broken Ward identity

$$\mathcal{W}(x)\Gamma = -\frac{1}{\alpha}\partial^2\partial A + \mathcal{A}(x) \cdot \Gamma \ , \tag{3.2}$$

where the insertion $\mathcal{A} \cdot \Gamma$ represents the possible breaking induced by the radiative corrections. According to the Quantum Action Principle [11] the lowest order nonvanishing contribution to the breaking - of order $\hbar$ at least -

$$\mathcal{A} \cdot \Gamma = \mathcal{A} + O(\hbar\mathcal{A}) \ , \tag{3.3}$$

is a local functional with dimension four, even under charge conjugation $\mathcal{C}$, odd under parity $\mathcal{P}$[3] and constrained by the Wess-Zumino consistency condition [8]

$$\mathcal{W}(x)\mathcal{A}(y) \ - \ \mathcal{W}(y)\mathcal{A}(x) = 0 \ . \tag{3.4}$$

The latter stems from the algebraic relation

$$\mathcal{W}(x)\mathcal{W}(y) \ - \ \mathcal{W}(y)\mathcal{W}(x) = 0 \ . \tag{3.5}$$

As it is well known the theory will be anomaly free if condition (3.4) admits only the trivial solution, i.e.

$$\mathcal{A}^{tr}(x) = \mathcal{W}(x)\Delta \ , \tag{3.6}$$

with Δ an integrated local polynomial with dimension four and even under parity $\mathcal{P}$ and charge conjugation $\mathcal{C}$. On the other hand nontrivial cocycles of (3.4)

$$\mathcal{A}^{nontr}(x) \neq \mathcal{W}(x)\Delta \ , \tag{3.7}$$

cannot be reabsorbed as local counterterms and represent real obstructions in order to have an invariant quantum vertex functional.

To study the Wess-Zumino consistency condition we proceed as before and we write

$$\mathcal{A}(x) = \mathcal{A}_1(A,\psi) + \mathcal{A}_2(A,\psi,T) \ , \tag{3.8}$$

where $\mathcal{A}_1(A,\psi)$ depends only on $(A,\psi,\bar{\psi})$ and $\mathcal{A}_2$ contains the field $T_{\mu\nu}$. Equation (3.4) splits thus into the two conditions

$$\mathcal{W}(x)\mathcal{A}_1(y) \ - \ \mathcal{W}(y)\mathcal{A}_1(x) = 0 \tag{3.9}$$

[3]This property follows from the fact that the Ward operator $\mathcal{W}(x)$ (2.8) is odd under parity transformations and even under charge conjugation.

and

$$\mathcal{W}(x)\mathcal{A}_2(y) \;-\; \mathcal{W}(y)\mathcal{A}_2(x) = 0 \ . \tag{3.10}$$

The first equation (3.9) yields, modulo trivial cocycles, the usual abelian gauge anomaly [12]

$$\mathcal{A}_1 = r\varepsilon_{\mu\nu\rho\sigma}\partial^\mu A^\nu \partial^\rho A^\sigma \ . \tag{3.11}$$

Turning to the second equation (3.10) it is not difficult to show the existence of a nontrivial $T_{\mu\nu}$-dependent cocycle. It reads

$$\mathcal{A}_2 = \eta\bar{\psi}\sigma_{\mu\nu}\gamma_5\psi T^{\mu\nu} \ , \tag{3.12}$$

with η a numerical coefficient. Notice that expression (3.12) depends only on the matter fields and does not contain the gauge field A. Acting now on the Ward identity (3.2) with the test operator

$$\frac{\delta}{\delta\psi(y)}\frac{\delta}{\delta\bar{\psi}(z)}\frac{\delta}{\delta T_{\mu\nu}(u)} \ , \tag{3.13}$$

and setting all the fields equal to zero, one easily checks that the numerical coefficient η, at its lowest order, is related to two kinds of one-loop $1PI$ diagrams $\Gamma^{(1)}_{A\psi\bar{\psi}T}$ and $\Gamma^{(1)}_{\psi\bar{\psi}T}$, respectively a set of three box diagrams with four amputated external legs of the type $(A, \psi, \bar{\psi}, T)$ and three triangle diagrams with $(\psi, \bar{\psi}, T)$ as external amputated legs. The appearence of the matter cocycle $\bar{\psi}\sigma_{\mu\nu}\gamma_5\psi T^{\mu\nu}$ could jeopardize the Adler-Bardeen theorem [13, 14] of the gauge anomaly and spoil the renormalizability of the theory.

One is forced then to impose further constraints on the model in order to avoid the presence of the cocycle (3.12). To this purpose let us remark that all the diagrams which contribute to the coefficient η are built-up by making use of the tensor-spinor three vertex $(\psi\bar{\psi}T)$ of (2.1). Therefore the simplest mechanism which one may impose in order to exclude the matter cocycle is to require from the very beginning that the classical action (2.1) is invariant under an additional descrete symmetry

$$T_{\mu\nu} \rightarrow -T_{\mu\nu} \ . \tag{3.14}$$

It is apparent that this new invariance forbids the presence in (2.1) of the three vertex $(\psi\bar{\psi}T)$. The latter, as it follows by combining (3.14) with the stability analysis of Sect. 2, will be not reintroduced by the radiative corrections. In addition, requirement (3.14) excludes the appearence of (3.12) as an anomaly in the Ward identity (3.2). We are left then with the gauge anomaly (3.11). However, the Adler-Bardeen theorem [14] ensures that if the coefficient r vanishes at one-loop order it will vanish at all orders of perturbation theory, guaranteing thus the perturbative renormalizability of the model. As it is well known [9] (see also the detailed discussion of the authors [6]), the vanishing of the coefficient r at the one-loop order in the present abelian case is achieved by introducing a partner for every charged particle with opposite axial gauge charge.

Of course, the descrete symmetry (3.14) will extend to the partner of the tensor matter field $T_{\mu\nu}$.

4 A geometrical construction

In this section we present a simple geometrical derivation of the model. Let us begin by introducing the *covariant derivative*

$$\nabla_\mu T_{\rho\sigma} := (\partial_\mu \widetilde{T}_{\rho\sigma} - 2A_\mu T_{\rho\sigma}) \ , \tag{4.1}$$

and

$$\nabla_\mu \widetilde{T}_{\rho\sigma} = \frac{1}{2}\varepsilon_{\rho\sigma\alpha\beta}\nabla_\mu T^{\alpha\beta} = -(\partial_\mu T_{\rho\sigma} + 2A_\mu \widetilde{T}_{\rho\sigma}) \ . \tag{4.2}$$

The name *covariant* is justified by the fact that (∇T) and $(\nabla \widetilde{T})$ transform under the gauge transformations (2.4) as the fields T and $\widetilde{T}$, i.e.

$$\begin{aligned} \delta(\nabla_\mu T_{\rho\sigma}) &= -2\omega(\nabla_\mu \widetilde{T}_{\rho\sigma}) \ , \\ \delta(\nabla_\mu \widetilde{T}_{\rho\sigma}) &= 2\omega(\nabla_\mu T_{\rho\sigma}) \ . \end{aligned} \tag{4.3}$$

This important property allows to construct gauge invariant quantities in a very simple way. It is almost immediate indeed to verify that the term

$$\tilde{S} = \int d^4x \left((\nabla^\mu T_{\mu\nu})(\nabla^\sigma T_\sigma{}^\nu) + (\nabla^\mu \widetilde{T}_{\mu\nu})(\nabla^\sigma \widetilde{T}_\sigma{}^\nu) \right) \ , \tag{4.4}$$

is gauge invariant.

Observe that a different contraction of the Lorentz indices

$$\int d^4x \left((\nabla_\mu T_{\rho\nu})(\nabla^\mu T^{\rho\nu}) + (\nabla_\mu \widetilde{T}_{\rho\nu})(\nabla^\mu \widetilde{T}^{\rho\nu}) \right) \ , \tag{4.5}$$

identically vanishes due to the property

$$\widetilde{T}_{\rho\sigma}\widetilde{T}^{\rho\sigma} = -T_{\rho\sigma}T^{\rho\sigma} \ . \tag{4.6}$$

Expression (4.4) is easily computed to be

$$\begin{aligned} \tilde{S} = -\int d^4x \Big(\ & \frac{1}{2}(\partial_\lambda T_{\mu\nu})^2 - 2(\partial^\mu T_{\mu\nu})^2 + 4A_\mu T^{\mu\nu}\partial^\lambda \widetilde{T}_{\lambda\nu} \\ & -4A_\mu \widetilde{T}^{\mu\nu}\partial^\lambda T_{\lambda\nu} + 4\Big(\frac{1}{2}(A_\mu T_{\lambda\nu})^2 - 2(A^\mu T_{\mu\nu})^2\Big)\Big) \ , \end{aligned} \tag{4.7}$$

i.e. $\tilde{S}$ is nothingh but the $(A-T)$ dependent part of the expression (2.1). We have thus recovered in an elegant and more geometrical way the initial invariant classical action S_{inv}. The generalization of the above construction for a nonabelian version of (2.1) is under investigation.

5 Conventions

We give here some useful relations involving the $\varepsilon_{\mu\nu\rho\sigma}$ and the Dirac γ-matrices.

We have

$$\begin{aligned} \widetilde{T}_{\mu\lambda}\widetilde{T}^{\lambda\nu} &= T_{\mu\lambda}T^{\lambda\nu} + \frac{1}{2}\delta^{\nu}_{\mu}T_{\alpha\beta}T^{\alpha\beta} \ , \\ T_{\mu\lambda}\widetilde{T}^{\lambda\nu} &= \frac{1}{4}\delta^{\nu}_{\mu}T_{\alpha\beta}\widetilde{T}_{\alpha\beta} \ . \end{aligned} \tag{5.1}$$

From equation (2.16) one gets

$$\begin{aligned} (\partial_\sigma \widetilde{T}_{\mu\rho})\widetilde{T}^{\sigma\rho} &= -\frac{1}{2}T^{\alpha\beta}\partial_\mu T_{\alpha\beta} - T_{\mu\rho}\partial^\sigma T^{\rho}_{\sigma} \ , \\ \widetilde{T}_{\mu\rho}\partial_\sigma \widetilde{T}^{\sigma\rho} &= -\frac{1}{2}T^{\alpha\beta}\partial_\mu T_{\alpha\beta} - (\partial_\sigma T_{\lambda\mu})T^{\sigma\lambda} \ , \end{aligned} \tag{5.2}$$

as well as

$$\widetilde{T}_{\alpha\beta}\partial^2 T^{\alpha\beta} = -2T_{\alpha\beta}\partial_\nu\partial^\beta \widetilde{T}^{\nu\alpha} - 2\widetilde{T}_{\alpha\beta}\partial_\nu\partial^\beta T^{\nu\alpha} \ . \tag{5.3}$$

Concerning the γ-matrices, here taken in the Dirac representation, we use [6, 10]:

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \ , \qquad {\gamma_5}^2 = 1 \ , \tag{5.4}$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] \ , \qquad \frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta} = \gamma_5\sigma^{\mu\nu} \ . \tag{5.5}$$

For the charge conjugation matrix C

$$C = i\gamma^0\gamma^2 \ , \qquad C^2 = -1 \ , \qquad C^{-1} = -C \ , \tag{5.6}$$

and

$$C\gamma_\mu C = \gamma_\mu^T \ , \qquad C\gamma_5 C = -\gamma_5 = -\gamma_5^T \ . \tag{5.7}$$

Acknowledgements

We wish to thank José Helayel-Neto for a carefully reading of the manuscript. The *Conselho Nacional de Desenvolvimento Cientifico e Tecnologico*, *CNPq*-Brazil is gratefully acknowledged for the financial support.

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