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Electron-Electron Bound States in Maxwell-Chern-Simons-Proca QED_3

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We start from a parity-breaking MCS QED_3 model with spontaneous breaking of the gauge symmetry as a framework for evaluation of the electron-electron interaction potential and for attainment of numerical values for the e^-e^-- bound state. Three expressions ($V_{\text{eff}_{\uparrow\uparrow}}, V_{\text{eff}_{\uparrow\downarrow}}, V_{\text{eff}_{\downarrow\downarrow}}$) are obtained according to the polarization state of the scattered electrons. In an energy scale compatible with Condensed Matter electronic excitations, these three potentials become degenerated. The resulting potential is implemented in the Schrödinger equation and the variational method is applied to carry out the electronic binding energy. The resulting binding energies in the scale of $10-100$ meV and a correlation length in the scale of $10-30$Å are possible indications that the MCS-QED_3 model adopted may be suitable to address an eventual case of e^-e^- pairing in the presence of parity-symmetry breakdown. The data analyzed here suggest an energy scale of 10-100 meV to fix the breaking of the $U(1)$-symmetry.

Key-words: Electron-electron pairs; Bound states; Planar QED; Parity-breaking.

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I. INTRODUCTION

The advent of high-T_c superconductivity [1], in 1986, brought about a great excitation in both the theoretical and experimental physical panorama, drawing attention for the issue of formation of Cooper pairs in planar systems. In the late 90´s, there arose a field-theoretical approach to address the mechanism of electronic pairing: the evaluation of the electron-electron Möller scattering as a tool for the attainment of the e^-e^- interaction potential in the nonrelativistic approximation. This line of action searches for an attractive potential in such a way to induce the formation of correlated electron-electron pairs, (the charge carriers of the high-T_c superconductors). The present work shall follow this general procedure.

By direct application of the Gauss´s law in (1+2)-dimensions for the massless gauge field, the Coulombian interaction takes on the form of a confining potential ($\ln r$). The Kato condition [2] establishes the finiteness of the number of bound states, in $D = 1 + 2$, associated to a certain potential V, and can be used as a criterion for determining the character confining or condensating of the potential. The fact the logarithmic potential to be confining (according to the Kato criterion) indicates it does not lead to bound states, becoming clear the need of a finite range, screened interaction. The Chern-Simons (CS) term [3] is then introduced as the generator of (topological) mass for the photon, implying an intensive screening of the Coulombian interaction. The Maxwell-Chern-Simons (MCS) model, a particular case of Planar Quantum Electrodynamics - QED_3, then arose as a theoretical framework able for providing an attractive but not confining electron-electron interaction. This model was then used by some authors [4], [6], [8], [9] as basic tool for evaluation of the Möller scattering amplitude at tree-level, whose Fourier transform (in the Born approximation) yields the e^-e^- interaction potential. In a general way, these works have led to the same result: the electron-electron potential comes out attractive whenever the topological mass (ϑ) exceeds the electron mass (m_e). Georgelin and Wallet [10] started from two MCS-QED_3 Lagrangians, the first (second) with the gauge field nonminimally coupled to fermions (bosons), in such a way to consider the introduction of the anomalous momentum of the electron in the problem. Working in the perturbative regime ($1/k \ll 1$), these authors found an attractive potential for fermions ($V_{\psi\psi} < 0$), and also for scalar bosons ($V_{\varphi\varphi} < 0$), in the nonrelativistic approximation. The presence of the nonminimal coupling seems to be the key-factor for the attainment of the attractive potential between charges with the same sign. In this case, however, the potential remains negative even in the limit of a small topological mass ($\vartheta \ll m_e$), under a suitable choice of parameters. The nonrenormalizability of this model (due to the nonminimal coupling), however, implies a restriction to the validity of their results only at tree-level calculations.

All the MCS models, except the one exposed in Ref. [10], failed under the perspective of yielding a realistic electron-electron condensation into the domain of a Condensed Matter system due to the condition $\vartheta > m_e$, necessary for making the e^-e^- pairing feasible. One must believe to be unlikely the existence of a physical excitation with so large energy in a real solid state system (the superconductors usually are characterized by excitations in the meV scale). We will see that the introduction of the Higgs mechanism in the context of the MCS-Electrodynamics will bring out a negative contribution to the scattering potential that will allow a global attractive potential despite the condition $\vartheta > m_e$.

In a recent paper [14], we have derived an interaction potential associated to the scattering of two identically polarized electrons in the framework of a Maxwell-Chern-Simons QED_3 with spontaneous breaking of local-U(1) symmetry. Our result revealed the interesting possibility of an attractive electron-electron interaction whenever the contribution stemming from the Higgs sector overcomes the repulsive contribution from the gauge sector, which can be achieved by an appropriate fitting of the free parameters. In the present work, we generalize the results attained in Ref. [14], contemplating the existence of two fermionic families (ψ_+, ψ_-), and performing the numerical evaluation of the e^-e^- binding energies. The procedure here accomplished is analogous to the one enclosed in Ref. [14]: starting from a QED_3 Lagrangian (now built up by two spinor polarizations, ψ_+, ψ_-) with SSB, one evaluates the Möller scattering amplitudes (in the nonrelativistic approximation) having the Higgs and the massive photon as mediators and the corresponding interaction potential, that now emerges in three different expressions: $V_{\uparrow\uparrow}, V_{\uparrow\downarrow}, V_{\downarrow\downarrow}$ (depending on the spin polarization of the scattered electrons). The same theoretical possibility of attractiveness, pointed out in Ref. [14], is now manifested by these three potentials. A numerical procedure (variational method) is then implemented in order to carry out the binding energy of the Cooper pairs. Having in mind the nonrelativistic approximation, a reduced potential is implemented into the Schrödinger equation, whose numerical solution provides the data contained in Tables I, II, III. The achievement of binding energies in the meV scale and correlation length in the $10-30$Å scale is an indicative that the adopted MCS-QED_3 model may be suitable for addressing an eventual electronic pairing in a system endowed with parity-breaking.

This paper is outlined as follows: in Section II, we present the QED_3 Lagrangian, its general features and one realizes the spontaneous breaking of U(1)-local symmetry that generates the Higgs boson and the Maxwell-Chern-Simons-Proca photon; in Section III, one evaluates the amplitudes for the Möller scattering; their Fourier transform will provide the e^-e^- interaction potentials $V_{\uparrow\uparrow}, V_{\uparrow\downarrow}, V_{\downarrow\downarrow}$ (despite the complex form of these potentials, they maintain the theoretical possibility of being attractive); in Section IV, one performs an analysis in order to obtain the e^-e^- binding energies by means of the numerical solution of the Schrödinger equation (by the variational method), whose results are disposed in Tables I, II, III. In Section V, we present our General Conclusions.

II. THE MCS QED$_3$ WITH SPONTANEOUS SYMMETRY BREAKING AND TWO SPINOR POLARIZATIONS

The action for a QED$_3$ model built up by two polarization fermionic fields (ψ_+, ψ_-), a gauge (A_μ) and a complex scalar field (φ), mutually coupled, and endowed with spontaneous breaking of a local U(1)-symmetry [12], [14], reads as

$$S_{QED-MCS} = \int d^3x\{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\overline{\psi}_+\gamma^\mu D_\mu\psi_+ + i\overline{\psi}_-\gamma^\mu D_\mu\psi_- + \frac{1}{2}\theta\epsilon^{\mu v\alpha}A_\mu\partial_v A_\alpha - m_e(\overline{\psi}_+\psi_+ - \overline{\psi}_-\psi_-) + \\ - y(\overline{\psi}_+\psi_+ - \overline{\psi}_-\psi_-)\varphi^*\varphi + D^\mu\varphi^* D_\mu\varphi - V(\varphi^*\varphi)\}, \tag{1}$$

where $V(\varphi^*\varphi)$ represents the sixth-power self-interaction potential,

$$V(\varphi^*\varphi) = \mu^2\varphi^*\varphi + \frac{\zeta}{2}(\varphi^*\varphi)^2 + \frac{\lambda}{3}(\varphi^*\varphi)^3, \tag{2}$$

which is responsible for the SSB; it is the most general one renormalizable in 1+2 dimensions [13]. The mass dimensions of the parameters μ, ζ, λ and y are respectively: 1,1,0 and 0. For the present purpose, we are interested only on stable vacuum, restriction satisfied by imposing some conditions on the potential parameters: $\lambda > 0, \zeta < 0$ and $\mu^2 \leq \frac{3\zeta^2}{16\lambda}$. The covariant derivatives are defined as: $D_\mu\psi_\pm = (\partial_\mu + ie_3A_\mu)\psi_\pm$ and $D_\mu\varphi = (\partial_\mu + ie_3A_\mu)\varphi$, where e_3 is the coupling constant of the $U(1)$-local gauge symmetry, here with dimension of (mass)$^{1/2}$, particularity that will be more explored in the numerical analysis section. In $(1+2)$ −dimensions, a fermionic field has its spin polarization fixed up by the mass sign [17]; however, in the action (1), it is manifest the presence of two spinor fields of opposite polarization. In this sense, it is necessary to stress that we have two positive-energy spinors (two spinor families), both solutions of the Dirac equation, each one with one polarization state according to the sign of the mass parameter, instead of the same spinor with two possibilities of spin-polarization.

Considering $\langle\varphi\rangle = v$, the vacuum expectation value for the scalar field product $\varphi^*\varphi$ is given by $\langle\varphi^*\varphi\rangle = v^2 = -\zeta/(2\lambda) + \left[(\zeta/(2\lambda))^2 - \mu^2/\lambda\right]^{1/2}$, while the condition for minimum reads as: $\mu^2 + \frac{\zeta}{2}v^2 + \lambda v^4 = 0$. After the spontaneous symmetry breaking, the scalar complex field can be parametrized by $\varphi = v + H + i\theta$, where H represents the Higgs scalar field and θ the would-be Goldstone boson; the SSB will be manifest when this parametrization is replaced in the action (1). Thereafter, in order to preserve the manifest renormalizability of the model, one adopts the ´t Hooft gauge by adding the fixing gauge term $\left(S^{gt}_{R_\xi} = \int d^3x[-\frac{1}{2\xi}(\partial^\mu A_\mu - \sqrt{2}\xi M_A\theta)^2]\right)$ to the broken action; finally, by retaining only the bilinear and the Yukawa interaction terms, one has,

$$S^{SSB}_{QED} = \int d^3x\Big\{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}M_A^2A^\mu A_\mu - \frac{1}{2\xi}(\partial^\mu A_\mu)^2 + \overline{\psi}_+(i\partial\!\!\!/ - m_{eff})\psi_+ + \overline{\psi}_-(i\partial\!\!\!/ + m_{eff})\psi_- + \frac{1}{2}\theta\epsilon^{\mu v\alpha}A_\mu\partial_v A_\alpha + \\ + \partial^\mu H\partial_\mu H - M_H^2H^2 + \partial^\mu\theta\partial_\mu\theta - M_\theta^2\theta^2 - 2yv(\overline{\psi}_+\psi_+ - \overline{\psi}_-\psi_-)H - e_3\left(\overline{\psi}_+A\!\!\!/\psi_+ + \overline{\psi}_-A\!\!\!/\psi_-\right)\Big\}, \tag{3}$$

whose mass parameters are:

$$M_A^2 = 2v^2e_3^2; \quad m_{eff} = m_e + yv^2; \quad M_H^2 = 2v^2(\zeta + 2\lambda v^2); \; M_\theta^2 = \xi M_A^2 \tag{4}$$

where ξ is an unphysical dimensionless gauge parameter.

III. THE ELECTRON-ELECTRON SCATTERING POTENTIAL IN THE NONRELATIVISTIC LIMIT

In the low-energy limit (Born approximation), the two-particle interaction potential is given by the Fourier transform of the two-particle scattering amplitude [18]. It is important to stress that, in the case of the nonrelativistic Möller scattering, one should consider only the t-channel (direct scattering) [18] even for indistinguishable electrons, since in this limit they recover the classical notion of trajectory. The Möller scattering will be mediated by two particles: the Higgs scalar and the massive gauge field. From the action (3), one reads off the propagators associated to the Higgs scalar and Maxwell-Chern-Simons-Proca field:

$$\langle H(k)H(-k)\rangle = \frac{i}{2}\frac{1}{k^2-M_H^2}; \quad \langle A_\mu(k)A_\nu(-k)\rangle = -i\left\{\frac{k^2-M_A^2}{(k^2-M_A^2)^2-k^2\theta^2}\left(\eta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)+ \right.$$
$$\left. +\frac{\xi}{(k^2-\xi M_A^2)}\frac{k_\mu k_\nu}{k^2}+\frac{\theta}{(k^2-M_A^2)^2-k^2\theta^2}i\epsilon_{\mu\alpha\nu}k^\alpha\right\}. \tag{5}$$

The photon propagator can be split in the following form,

$$\langle A_\mu A_\nu\rangle = -i\left[\frac{C_+}{k^2-M_+^2}+\frac{C_-}{k^2-M_-^2}\right](\eta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2})+\frac{-i\xi k_\mu k_\nu}{k^2(k^2-\xi M_A^2)}+i\left[\frac{C}{k^2-M_+^2}-\frac{C}{k^2-M_-^2}\right]\epsilon_{\mu\alpha\nu}k^\alpha,$$

with the positive definite constants C_+, C_-, C and the quadratic masses poles M_+^2 and M_-^2 given by:

$$C_\pm = \frac{1}{2}\left[1\pm\frac{\theta}{\sqrt{4M_A^2+\theta^2}}\right]; \quad C=\frac{1}{\sqrt{4M_A^2+\theta^2}}; \quad M_\pm^2=\frac{1}{2}\left[(2M_A^2+\theta^2)\pm|\theta|\sqrt{4M_A^2+\theta^2}\right]. \tag{6}$$

From the action (3), it is easy to extract the vertex Feynman rules: $V_{\psi_\pm H\psi_\pm} = \pm 2ivy; V_{\psi A\psi} = ie_3\gamma^\mu$. Since in the low-energy limit only the t-channel must be considered, the whole scattering amplitudes are written in the form:

$$-i\mathcal{M}_{\pm H\pm} = \overline{u}_\pm(p_1)(\pm 2ivy)u_\pm(p_1^{'})\left[\langle H(k)H(-k)\rangle\right]\overline{u}_\pm(p_2)(\pm 2ivy)u_\pm(p_2^{'}), \tag{7}$$
$$-i\mathcal{M}_{\pm H\mp} = \overline{u}_\pm(p_1)(\pm 2ivy)u_\pm(p_1^{'})\left[\langle H(k)H(-k)\rangle\right]\overline{u}_\mp(p_2)(\mp 2ivy)u_\mp(p_2^{'}), \tag{8}$$
$$-i\mathcal{M}_{\pm A\pm} = \overline{u}_\pm(p_1)(ie_3\gamma^\mu)u_\pm(p_1^{'})\left[\langle A_\mu(k)A_\nu(-k)\rangle\right]\overline{u}_\pm(p_2)(ie_3\gamma^\nu)u_\pm(p_2^{'}), \tag{9}$$
$$-i\mathcal{M}_{\pm A\mp} = \overline{u}_\pm(p_1)(ie_3\gamma^\mu)u_\pm(p_1^{'})\left[\langle A_\mu(k)A_\nu(-k)\rangle\right]\overline{u}_\mp(p_2)(ie_3\gamma^\nu)u_\mp(p_2^{'}). \tag{10}$$

The first two expressions represent the scattering amplitude mediated by the Higgs particles for equal and opposite electron polarizations, while in the last two ones the mediator is the massive Chern-Simons-Proca photon. The spinors $u_+(p)$, $u_-(p)$ stand for the positive-energy solution of the Dirac equation, satisfying the normalization conditions: $\overline{u}_\pm(p)u_\pm(p)=\pm 1$. Working in the center-of-mass frame, the momenta of the interacting particles and the momentum transfer take a simpler form, useful for writing the spinors $u_+(p)$, $u_-(p)$, as it is properly shown in the Appendix. With these definitions, one carries out the fermionic current elements, also displayed in the Appendix, so that the evaluation of the scattering amplitudes (for low momenta approximation), at tree-level, associated to the Higgs and the gauge particle become:

$$\mathcal{M}_{Higgs} = -2v^2y^2\left(\frac{1}{\overrightarrow{k}^2+M_H^2}\right), \tag{11}$$

$$\mathcal{M}_{\uparrow A\uparrow} = \mathcal{M}_1+\mathcal{M}_2+\mathcal{M}_3, \quad \mathcal{M}_{\downarrow A\downarrow} = \mathcal{M}_1-\mathcal{M}_2+\mathcal{M}_3, \quad \mathcal{M}_{\uparrow A\downarrow} = \mathcal{M}_{\downarrow A\uparrow} = \mathcal{M}_1+\mathcal{M}_3,$$

with:

$$\mathcal{M}_1 = e_3^2\left[\frac{C_+}{\overrightarrow{k}^2+M_+^2}+\frac{C_-}{\overrightarrow{k}^2+M_-^2}\right], \ \mathcal{M}_2=\frac{e_3^2\overrightarrow{k}^2}{m_{\text{eff}}}\left[\frac{C}{\overrightarrow{k}^2+M_+^2}-\frac{C}{\overrightarrow{k}^2+M_-^2}\right], \ \mathcal{M}_3=\frac{-i\sin\phi}{(1-\cos\phi)}\mathcal{M}_2, \tag{12}$$

where it was used $\overrightarrow{k}^2 = 2p^2(1-\cos\phi)$. Furthermore, it is clear that the Higgs amplitude is independent of the electron polarization, while the gauge amplitude splits into three different expressions, depending on the polarization of the scattered electrons. The terms $\mathcal{M}_1$,$\mathcal{M}_2$ correspond to the real part of the Möller scattering amplitude, while $\mathcal{M}_3$

describes the Aharonov-Bohm amplitude for fermions [4], [8], [10]. The interaction potentials are obtained through the Fourier transform of the scattering amplitude (inside the Born approximation limit): $V(\overrightarrow{r}) = \int \frac{d^2k}{(2\pi)^2}\mathcal{M}e^{i\overrightarrow{k}.\overrightarrow{r}}$. According to this approximation, Eq.(11) yields an attractive Higgs potential,

$$V_{Higgs}(r) = -\frac{1}{2\pi}2v^2y^2K_o(M_Hr), \tag{13}$$

while in the gauge sector there appear three different potentials (depending on the polarization state):

$$V_{gauge\ \uparrow\uparrow}(r) = V_1(r) + V_2(r) + V_3(r), \quad V_{gauge\ \uparrow\downarrow}(r) = V_1(r) + V_3(r), \quad V_{gauge\ \downarrow\downarrow}(r) = V_1(r) - V_2(r) + V_3(r),$$

$V_1(r)$, $V_2(r)$, $V_3(r)$ being respectively the Fourier transforms of the amplitudes $\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3$, given explicitly by:

$$V_1(r) = \frac{e_3^2}{2\pi}\Big[C_+K_o(M_+r) + C_-K_o(M_-r)\Big], \tag{14}$$

$$V_2(r) = -\frac{e_3^2}{2\pi}\frac{C}{m_{\rm eff}}\Big[M_+^2K_o(M_+r) - M_-^2K_o(M_-r)\Big], \tag{15}$$

$$V_3(r) = 2\frac{e_3^2}{2\pi}\frac{Cl}{m_{\rm eff}r}\Big[M_+K_1(M_+r) - M_-K_1(M_-r)\Big]. \tag{16}$$

The complete potential expressions are obtained joining the Higgs and gauge contributions: $V(r) = V_{Higgs} + V_{gauge}$:

$$V(r)_{\uparrow\uparrow} = -\frac{1}{2\pi}2v^2y^2K_o(M_hr) + \frac{e_3^2}{2\pi}\Big\{(C_+ - \frac{C}{m}M_+^2)K_o(M_+r) + (C_- + \frac{C}{m_{\rm eff}}M_-^2)K_o(M_-r) + \\ + 2\frac{Cl}{m_{\rm eff}r}(M_+K_1(M_+r) - M_-K_1(M_-r)\Big\}, \tag{17}$$

$$V(r)_{\uparrow\downarrow} = -\frac{1}{2\pi}2v^2y^2K_o(M_hr) + \frac{e_3^2}{2\pi}\Big\{C_+K_o(M_+r) + C_-K_o(M_-r) + 2\frac{Cl}{m_{\rm eff}r}[M_+K_1(M_+r) + \\ - M_-K_1(M_-r)\Big\}, \tag{18}$$

$$V(r)_{\downarrow\downarrow} = -\frac{1}{2\pi}2v^2y^2K_o(M_hr) + \frac{e_3^2}{2\pi}\Big\{(C_+ + \frac{C}{m_{\rm eff}}M_+^2)K_o(M_+r) + (C_- - \frac{C}{m_{\rm eff}}M_-^2)K_o(M_-r) \\ + 2\frac{Cl}{m_{\rm eff}r}(M_+K_1(M_+r) - M_-K_1(M_-r)\Big\}. \tag{19}$$

Here, $K_o(x)$ and $K_1(x)$ are the modified Bessel functions and l is the angular momentum. The last three equations represent the tree-level potentials evaluated at the Born approximation. Now, it is convenient to define the limit of validity of the potentials (17), (18), (19). They have been derived in the low-energy limit, consequently they must be valid in the perturbative regime, where the loop corrections are negligible before the semi-classical terms. For a typical MCS model, the perturbative limit is given by $\frac{e^2}{\theta} \ll 1$; in the case of the present model, nevertheless, there are four dimensionless parameters e_3^2/m, e_3^2/M_H, e_3^2/M_+, e_3^2/M_-. According to the discussion realized in Ref. [14], the pertubative regime is valid whenever $e_3^2/M_+ \ll 1$ and $y \ll 1$ (the first condition obviously implies $e_3^2/m \ll 1$).

A remarkable point to be highlighted concerns the attainment of three different potentials: $V(r)_{\uparrow\uparrow}, V(r)_{\uparrow\downarrow}, V(r)_{\downarrow\downarrow}$. Our results put in explicit evidence the dependence of the potential on the spin state. Were parity preserved, this would not be the result; however, by virtue of the explicit breaking of parity, as induced by the Chern-Simons term, expressions (17), (18), (19) differ from one another as it can be understood on the basis of parity transformation arguments. Another signal of parity-breaking is the linear dependence of V on l: $l \rightarrow -l$ is not a symmetry of the potential.

Although the gauge invariance is broken by the appearance of a Proca mass during the SSB, one expects that the interaction potential associated to the system comes to preserve the characteristics of the original Lagrangian (before the SSB). This fact leads us to study a way to assure the gauge invariance of the effective interaction potential. Analysis of the Galilean limit of the field theories in (1+2) dimensions, carried out by Hagen [7], have shown that the 2-body scattering problem, as mediated by a gauge particle, must lead to an effective potential that preserves the structure of a perfect square form $(l-\alpha^2)^2$, and can be identified with the Aharonov-Bohm scattering potential. The quartic order term (α^4) is related to the presence of 2-photon diagrams induced by the seagull vertex $(\varphi^*\varphi A_\mu A^\mu)$, and thus associated to the gauge invariance of the resulting potential. In this way, the potential structure $(l-\alpha^2)^2$ must be also pursued in more complex electron-electron scatterings panoramas, in order to ensure gauge invariance. Actually, this is just the signal of a more general result. Electron-electron scatterings, in general, no matter the complexity of the interactions, must exhibit the combination $(l-\alpha^2)^2$ for the sake of gauge invariance of the final result. This kind of procedure is found in Ref. [8], where a nonrelativistic interaction potential was derived in the context of a MCS-QED$_3$ (without scalar sector), in the perturbative regime, $1/k \ll 1$, with k being the statistic parameter (in our present case $k \equiv 4\pi\theta/e_3^2$). In this reference, in order to ensure the gauge invariance, at the low-energy approximation, one takes into account the two-photons diagrams, which amounts to adding up to the tree-level potential the quartic order term $\left\{\frac{e^2}{2\pi\theta}[1-\theta r K_1(\theta r)]\right\}^2$, turning out into the following gauge-invariant effective potential form [4], [8]:

$$V_{MCS}(r) = \frac{e^2}{2\pi}\left[1-\frac{\theta}{m_e}\right]K_0(\theta r) + \frac{1}{m_e r^2}\left\{l - \frac{e^2}{2\pi\theta}[1-\theta r K_1(\theta r)]\right\}^2 . \tag{20}$$

In the expression above, the first term corresponds to the electromagnetic potential, whereas the last one incorporates the centrifugal barrier (l/mr^2), the Aharonov-Bohm term and the 2-photon exchange term. One observes that this procedure becomes necessary when the model is analyzed or defined out of the pertubative limit. In Ref. [10], for instance, one accomplishes an evaluation of the scattering potential, in the Born approximation, whose final result is not supplemented by the term $\left\{\frac{e^2}{2\pi\theta}[1-\theta r K_1(\theta r)]\right\}^2$, under the justification that derivation has been done in the pertubative regime $(1/k \ll 1)$. In such a regime, the 2-photon term becomes negligible (for it is proportional to $1/k^2$) and shows itself unable to jeopardize the gauge invariance of the model.

In a scenery where one searches for applications to Condensed Matter Physics, one must require $\theta \ll m_e$, and the scattering potential given by Eq.(20) then comes out positive. This implication prevents a possible application of this kind of model to superconductivity, where the characteristic energies are of meV order. Since the effective electron mass $(m_{\rm eff} = m_e + yv^2)$ is $\sim 10^5 eV$, energy scale much greater than that corresponding to the condensed matter interactions (meV), one must impose the following condition on the physical excitations of the model:

$$m_{\rm eff} \gg \vartheta, M_A, M_\pm . \tag{21}$$

In the limit $M_A \to 0$, one has: $M_+ \sim \vartheta$; in this situation, the dimensionless parameter e_3^2/M_+ reduces to e_3^2/ϑ, that now lies outside the pertubative regime, since ϑ is now small $(\sim meV)$. Therefore, in this energy scale, our results may not be restricted to the pertubative limit; the consideration of the 2-photon term to Eqs.(17, 18, 19) becomes then relevant in order to assure the gauge invariance of these potentials. As a final result, one rewrites the three expressions for the effective-gauge-invariant scattering potentials:

$$\begin{aligned} V_{\rm eff_{\uparrow\uparrow}}(r) = &-\frac{1}{2\pi}2v^2y^2K_0(M_H r) + \frac{e_3^2}{2\pi}\left\{\left[(C_+ - \frac{C}{m_{\rm eff}}M_+^2\right]K_0(M_+ r) + \left[C_- + \frac{C}{m_{\rm eff}}M_-^2\right]K_0(M_- r)\right\} \\ &+\frac{1}{m_{\rm eff}r^2}\left\{l + \frac{e_3^2}{2\pi}Cr[M_+K_1(M_+ r) - M_-K_1(M_- r)]\right\}^2 , \end{aligned} \tag{22}$$

$$\begin{aligned} V_{\rm eff_{\uparrow\downarrow}}(r) = &-\frac{1}{2\pi}2v^2y^2K_0(M_H r) + \frac{e_3^2}{2\pi}\ [C_+K_o(M_+ r) + C_-K_o(M_- r)] + \frac{1}{m_{\rm eff}r^2}\left\{l + \frac{e_3^2}{2\pi}Cr[M_+K_1(M_+ r) + \right. \\ &\left. -M_-K_1(M_- r)]\right\}^2 , \end{aligned} \tag{23}$$

$$\begin{aligned} V_{\rm eff_{\downarrow\downarrow}}(r) = &-\frac{1}{2\pi}2v^2y^2K_0(M_H r) + \frac{e_3^2}{2\pi}\left\{\left[C_+ + \frac{C}{m_{\rm eff}}M_+^2\right]K_o(M_+ r) + \left[C_- - \frac{C}{m_{\rm eff}}M_-^2\right]K_o(M_- r)\right\} \\ &+\frac{1}{m_{\rm eff}r^2}\left\{l + \frac{e_3^2}{2\pi}Cr[M_+K_1(M_+ r) - M_-K_1(M_- r)]\right\}^2 , \end{aligned} \tag{24}$$

where $\frac{l^2}{mr^2}$ represents the centrifugal barrier, and the term proportional to C^2 comes from the 2-photon exchange.

In the energy scale given by condition (21), the proportionality coefficients of $V_2(r)$ become negligible:

$$m_{\rm eff} \gg \vartheta, M_A, M_\pm \qquad \Longrightarrow \quad \frac{C}{m_{\rm eff}} M_+^2 \ll 1,\ \frac{C}{m_{\rm eff}} M_-^2 \ll 1. \tag{25}$$

As a consequence of these considerations, one can observe that only the first term of the expressions (22, 23, 24) is attractive, which corresponds to the Higgs interaction. At the same time, the potential $V_2(r)$ reveals itself small before $V_1(r)$ and $V_3(r)$, leading to a simplification in the expressions (22), (23), (24), that degenerate to a single form:

$$\begin{aligned} V_{\rm eff}(r) = &-\frac{1}{2\pi} 2v^2 y^2 K_0(M_H r) + \frac{e_3^2}{2\pi}\left[C_+ K_o(M_+ r) + C_- K_o(M_- r)\right] + \frac{1}{m_{\rm eff} r^2}\Bigg\{ l + \\ &+\frac{e_3^2}{2\pi} C r [M_+ K_1(M_+ r) - M_- K_1(M_- r)]\Bigg\}^2, \end{aligned} \tag{26}$$

The fact that $C_\pm > 0$, $\forall\ \vartheta, M_A$ makes the second term (proportional to $e^2/2\pi$) of the equation above to be positive, revealing the repulsive nature of gauge sector. This trivial analysis shows that the potentials (22), (23), (24) will be attractive only when the contribution originated from the Yukawa interaction overcomes the one coming from the gauge sector, which can be achieved by accomplishing a suitable fitting on the model parameters. The fulfillment of this condition can render the formation of $e^- e^-$ bound states feasible , once the above potentials are "weak" in the sense of Kato criterion, analyzed by Chadan *et al.* [2] in the context of the low-energy scattering theory in $(1+2)$ dimensions.

Finally, it is instructive to show how the gauge sectors of the potentials (22), (23),(24) behave in the limit of a vanishing Proca mass: $M_A \longrightarrow 0$. In this case, the propagator of the gauge field reduces to the Maxwell-Chern-Simons one, leading to the following limits:

$$M_+ \longrightarrow \theta; M_- \longrightarrow 0; C_+ \longrightarrow 1; C_- \longrightarrow 0; K_1(M_- r) \longrightarrow \frac{1}{M_- r}; C \longrightarrow \frac{1}{\theta}; \tag{27}$$

$$\lim_{M_A \longrightarrow 0} V_{\uparrow\uparrow}(r) = \frac{e_3^2}{2\pi}\left(1 - \frac{\theta}{m_{\rm eff}}\right) K_o(\theta r) + \frac{1}{m_{\rm eff} r^2}\left[l - \frac{e_3^2}{2\pi\theta}(1 - \theta r K_1(\theta r))\right]^2, \tag{28}$$

$$\lim_{M_A \longrightarrow 0} V_{\uparrow\downarrow}(r) = \frac{e_3^2}{2\pi} K_o(\theta r) + \frac{1}{m_{\rm eff} r^2}\left[l - \frac{e_3^2}{2\pi\theta}(1 - \theta r K_1(\theta r))\right]^2, \tag{29}$$

$$\lim_{M_A \longrightarrow 0} V_{\downarrow\downarrow}(r) = \frac{e_3^2}{2\pi}\left(1 + \frac{\theta}{m_{\rm eff}}\right) K_o(\theta r) + \frac{1}{m_{\rm eff} r^2}\left[l - \frac{e_3^2}{2\pi\theta}(1 - \theta r K_1(\theta r))\right]^2. \tag{30}$$

One remarks that Eq. (28) encloses exactly the same result achieved by Dobrolibov [8] *et al.* and others [4], [9] for the scattering of two up-polarization electrons, which enforces the generalization realized in this paper.

IV. NUMERICAL ANALYSIS

The numerical procedure adopted here consists on the implementation of the variational method for the Schrödinger equation supplemented by the interaction potential (26). In this sense, it is necessary to expose some properties of the wavefunction representing the $e^- e^-$ and of the two-dimensional Schrödinger equation.

A. The composite wave-function and the Schrödinger equation

The Pauli exclusion principle states the antisymmetric character of the total two-electron wavefunction (Ψ) with respect to an electron-electron permutation: $\Psi(\rho_1, s_1, \rho_2, s_2) = -\Psi(\rho_2, s_2, \rho_1, s_1)$. Assuming that no significant spin-orbit interaction takes place, the function Ψ can be split into three independent functions: $\Psi(\rho_1, s_1, \rho_2, s_2) = \psi(R)\varphi(r)\chi(s_1, s_2)$, which represent, respectively, the center-of-mass wave function, the relative one, and the spin wave function (R and s being the center-of-mass and spin coordinates respectively, while r is the relative coordinate of the electrons). Taking into account the Pauli principle, the total wavefunction Ψ in the center-of-mass frame reads as

$$\Psi^{S=1} = \varphi_{odd}(r)\chi^{S=1}_{even}(s_1, s_2)\ , \quad \Psi^{S=0} = \varphi_{even}(r)\chi^{S=0}_{odd}(s_1, s_2)\ , \tag{31}$$

where $\chi^{S=0}$, $\chi^{S=1}$, $\varphi_{even}(r)$, $\varphi_{odd}(r)$ stand respectively for the (antisymmetric) singlet spin-function, the (symmetric) spin triplet, the even space-function ($l = 0$: s-wave, $l = 2$: d-wave), and the odd space-function ($l = 1$: p-wave: , $l = 3$: f-wave).

Within the nonrelativistic approximation, the binding energy associated to an e^-e^- pair is given by planar Schrödinger equation for the relative space-function $\varphi(r)$,

$$\frac{\partial^2\varphi(r)}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi(r)}{\partial r} - \frac{l^2}{r^2}\varphi(r) + 2\mu_{eff}[E - V(r)]\varphi(r) = 0\ , \tag{32}$$

where $V(r)$ represents the interaction potential given by Eq. (26), and $\mu_{eff} = \frac{1}{2}m_{\rm eff}$, is the effective reduced mass of the system. By means of the following transformation $\varphi(r) = \frac{1}{\sqrt{r}}\ g(r)$, one has

$$\frac{\partial^2 g(r)}{\partial r^2} - \frac{l^2 - \frac{1}{4}}{r^2}g(r) + 2\mu_{eff}[E - V(r)]g(r) = 0\ . \tag{33}$$

B. The Variational Method and the Choice of the trial function

To work out the variational method, one must take as starting point the choice of the trial function that represents the generic features of the e^-e^- pair. The definition of a trial function must observe some conditions, such as the asymptotic behavior at infinity, the analysis of its free version and its behavior at the origin. For a zero angular momentum ($l = 0$) state, the Eq.(33) becomes

$$\left\{\frac{\partial^2}{\partial r^2} + \frac{1}{4r^2} + 2\mu_{eff}[E + C_s K_0(M_H r)]\right\}g(r) = 0, \tag{34}$$

whose free version ($V(r) = 0$), for $l = 0$ state, $\left[\frac{\partial^2}{\partial r^2} + \frac{1}{4r^2} + k^2\right]u(r) = 0$, has as solution $u(r) = B_1\sqrt{r}J_0(kr) + B_2\sqrt{r}Y_0(kr)$, with B_1 and B_2 being arbitrary constants and $k = \sqrt{2\mu_{eff}E}$. In the limit $r \to 0$, this solution goes simply as $u(r) \longrightarrow \sqrt{r} + \lambda\sqrt{r}\ln(r)$. Since the second term in the last equation behaves like an attractive potential, $-1/4r^2$, this implies the possibility of obtaining a bound state ($E < 0$) even for $V(r) = 0$ [2]. This is not physically acceptable, leading to a restriction on the needed self-adjoint extension of the differential operator $-d^2/dr^2 - 1/4r^2$. Among the infinite number of self-adjoint extensions of this operator, the only physical choice corresponds to the Friedrichs extension ($B_2 = 0$), which behaves like $\sqrt{r}$ at the origin, indicating this same behavior for $u(r)$. In this way the behavior of the trial function at the origin is determined. The complete equation, $V(r) \neq 0$, will preserve the self-adjointness of free Hamiltonian, if the potential is "weak" in the sense of the Kato condition: $\int_0^\infty r(1 + |\ln(r)|)|V(r)|dr < \infty$. Provided the interaction potential, given by Eq. (26), satisfies the Kato condition, the self-adjointness of the total Hamiltonian is assured and the existence of bound states is allowed. On the other hand, at infinity, the trial function must vanish asymptotically in order to fulfill square integrability. Therefore, a good choice can then be given by $g(r) = f(r)\exp(-\beta r)$, where $f(r)$ is a well-behaved function that satisfies the limit condition: $\lim_{r\to 0} f(r) = \sqrt{r}$. By simplicity, the trial function (for zero angular momentum) read as

$$g(r) = \sqrt{r}\exp(-\beta r)\ , \tag{35}$$

where β is a free parameter whose variation approximately determines an energy minimum.

An analogous procedure can be undertaken to determine the behavior of the trial function when the angular momentum is different from zero ($l \neq 0$). In this case, and in the limit $r \longrightarrow 0$, Eq.(33) reduces to $\left[\frac{\partial^2}{\partial r^2} - \frac{l^2-\frac{1}{4}}{r^2} + k^2\right] u(r) = 0$, whose general solution reads as $u(r) = B_1 r^{(l+1/2)} + B_2 r^{(-l+1/2)}$. For $l > 0$, the choice $r^{(l+1/2)}$ entails a trial function that is well-behaved at the origin. Since the Schrödinger equation depends only on l^2, any of the choices, $l > 0$ or $l < 0$, is enough to provide the energy values of the physical states and one gets

$$g(r) = r^{1/2+l} \exp(-\beta r) \ , \tag{36}$$

where β is again a spanning free parameter to be numerically fixed in order to maximize the binding energy. Though this last result is mathematically correct, we should point out that the discussion regarding non-zero angular momentum states here is merely for the sake of completeness. The true wave-function in this case should include the angular components which remain precluded in this approach.

C. The Analysis of the Potential and the Numerical Data

The numerical analysis of the potentials $V_{\mathrm{eff}_{\uparrow\uparrow}}, V_{\mathrm{eff}_{\uparrow\downarrow}}, V_{\mathrm{eff}_{\downarrow\downarrow}}$ is totally dependent on the parameters of the field-theoretical model. As a first step, it is convenient to realize an analysis on the relevant parameters and thereafter to initiate a numerical procedure. The central purpose of this section is to demonstrate that the potentials obtained are attractive and lead to the formation of bound states e^-e^-, whose energy is situated into a range relevant to some Condensed Matter systems, like the high-T$_c$ superconductors.

As well-known, to parallel-spin states (spin triplet) there must be a p-wave (spin triplet and $l = 1$) associated, whereas the antiparallel-spin states (spin singlet) are linked to an s-wave (spin singlet and $l = 0$). Here, despite the parity-breakdown associated to the state $l = 1$, the s-wave can also appear as solution, since it is not necessarily manifested in all states. Given the degeneracy of the potentials $V_{\mathrm{eff}_{\uparrow\uparrow}}, V_{\mathrm{eff}_{\uparrow\downarrow}}, V_{\mathrm{eff}_{\downarrow\downarrow}}$ on the reduced potential (26), the issue concerning the wavefunction symmetry looses some of its status: both the s- and p-wave appear as solutions for the system. According to Eqs. (35), (36), the implementation of the variational method requires a trial-function with $r^{1/2}-$behaviour at the origin in the case of an s-wave and a $r^{3/2}-$behaviour for a p-wave.

Before starting the numerical calculations, it is instructive to show the relevant parameters:

$$e_3^2 = \frac{e^2}{l_\perp} = \frac{1}{137,04}\frac{1973,26}{l_\perp} = \frac{14,399}{l_\perp}, \tag{37}$$

$$\alpha = \frac{\vartheta}{M_A}, \tag{38}$$

$$\zeta < 0, \ \lambda \geq \frac{3}{4}\frac{|\zeta|}{\upsilon^2}, \tag{39}$$

$$\lambda = \frac{3}{4}\frac{|\zeta|}{\nu^2} \Longrightarrow M_H^2 = \nu^2|\zeta|, \tag{40}$$

$$\lambda = \frac{|\zeta|}{\nu^2} \Longrightarrow M_H^2 = 2\nu^2|\zeta|. \tag{41}$$

Specifically, in $D = 1+2$, the electromagnetic coupling constant squared, e_3^2, has dimension of mass, rather than the dimensionless character of the usual four-dimensional QED$_4$ coupling constant. This fact might be understood as a memory of the third dimension that appears (into the coupling constant) when one tries to work with a theory intrinsically defined in three space-time dimensions. This dimensional peculiarity could be better implemented through the definition of a new coupling constant in three space-time dimensions [4], [5]: $e \longrightarrow e_3 = e/\sqrt{l_\perp}$, where $l_\perp$ represents a length orthogonal to the planar dimension. The smaller is $l_\perp$, the smaller is the remnant of the frozen dimension, the larger is the planar character of the model and the coupling constant e_3, what reveals its effective nature. In this sense, it is instructive to notice that the effective value of e_3^2 is larger than $e^2 = 1/137$ whenever $l_\perp < 1973.26$ Å, since 1 (Å)$^{-1} = 1973.26\, eV$. This particularity broadens the repulsive interaction for small $l_\perp$ and requires an even stronger Higgs contribution to account for a total attractive interaction. Finally, this parameter must be evaluated inside a range appropriated to not jeopardize the planar nature of the system, so that one requires that: $2 < l_\perp < 15$Å. The parameter α is defined as the ratio between the Proca mass and the Chern-Simons mass, while ζ, λ are parameters of $V-$potential and are important to assure a stable vacuum, condition given by Eq. (39). The imposition of some relations between ζ, λ, ν^2, like Eqs.(40) e (41), imply a kind of expression for the Higgs mass that exhibit dependence only on ν^2 and $|\zeta|$. This set of conditions impose a lower bound for the Higgs mass: $M^2_{H\,\mathrm{min}} = 3|\zeta|/4\lambda$.

Besides the factors above, the entire determination of the potential (26) also depends on v^2, the vacuum expectation value (v.e.v.), and on y, the parameter that measures the coupling between the fermions and the Higgs scalar. Being a free parameter, v^2 indicates the energy scale of the spontaneous breakdown of the $U(1)-$local symmetry, usually determined by some experimental data associated to the phenomenology of the model under investigation, as it occurs in the electroweak Weinberg-Salam model, for example. On the other hand, the $y-$parameter measures the coupling between the fermions and the Higgs scalar, working in fact as an effective constant that embodies contributions of all possible mechanisms of the electronic interaction via Higgs-type (scalar) excitations, as the spinless bosonic interaction mechanisms: phonons, plasmons, and other collective excitations. This theoretical similarity suggests an identification of the field theory parameter with an effective electron-scalar coupling (instead of an electron-phonon one): $y \rightarrow \lambda_{es}$.

The numerical analysis is developed by means of the implementation of the variational method on the Schrödinger equation, supplemented by the degenerated potential. The procedure is initiated by the use of the an s-wave trial function: $g\,(r) = r^{1/2}e^{-\beta r}$, given by Eq. (35). Tables I and II exhibit the values of the e^-e^- bound state and the average length of the e^-e^- state (ξ_{ab}) for V_{eff}, in accordance with the input parameters $(\nu^2, Z, \alpha, y, \zeta)$, for $l = 0$. The degenerated potential obviously assures the following equality: $\mathrm{E}_{ee\uparrow\uparrow} = \mathrm{E}_{ee\downarrow\downarrow} = \mathrm{E}_{ee\uparrow\downarrow}$, $\xi_{ab\uparrow\uparrow} = \xi_{ab\downarrow\downarrow} = \xi_{ab\uparrow\downarrow}$. Table III contains numerical data generated by the variational method, for $l = 1$, starting from the following trial function: $\varphi\,(r) = r^{3/2}e^{-\beta r}$, given by Eq. (36).

TABLE I. Input parameters: $\nu^2, l_\perp, \alpha, \zeta, M_H^2 = \nu^2|\zeta|$ and $l = 0$; output numerical data: $E_{e^-e^-}$ and ξ_{ab}. Trial Function: $\varphi\,(r) = r^{1/2}e^{-\beta r}$

v^2(meV)	$l_\perp$(Å)	y	α	ζ (eV)	$M_H = \sqrt{\nu^2\lvert\zeta\rvert}$	β	$E_{e^-e^-}$(meV)	ξ_{ab}(Å)
47.0	10.0	4.0	1.0	4.0	433.0	51.1	−59.2	19.3
47.0	10.0	4.0	0.5	4.0	433.0	51.8	−23.7	19.0
48.0	10.0	4.0	0.5	4.0	438.0	29.8	−50.2	16.6
48.0	10.0	3.9	1.0	4.0	438.0	29.8	−24.8	33.1
60.0	8.0	4.0	1.0	8.0	693.0	71.1	−33.3	13.9
60.0	8.0	4.0	0.5	6.0	600.0	69.2	−32.8	14.3
60.0	8.0	3.9	1.0	5.0	548.0	27.1	−30.4	36.4
70.0	7.0	4.0	0.4	7.0	700.0	89.2	−62.7	11.6
70.0	7.0	4.0	0.6	8.0	748.0	87.5	−54.0	11.3
70.0	7.0	3.9	1.0	7.0	700.0	51.2	−32.3	19.3
70.0	7.0	3.9	0.5	5.0	590.0	50.8	−38.5	19.4

TABLE II. Input parameters: $\nu^2, l_\perp, \alpha, \zeta, M_H^2 = \nu^2|\zeta|$ and $l = 0$; output numerical data: $E_{e^-e^-}$ and ξ_{ab}. Trial Function: $\varphi(r) = r^{1/2}e^{-\beta r}$

v^2 (meV)	$l_\perp$(Å)	y	α	ζ (eV)	$M_H = \sqrt{2\nu^2\|\zeta\|}$	β	$E_{e^-e^-}$(meV)	ξ_{ab}(Å)
40.0	12.0	4.0	1.0	2.0	400.0	56.1	-54.1	17.6
40.0	12.0	4.0	0.5	2.0	400.0	59.2	-24.5	16.7
40.0	12.0	4.0	0.3	2.0	400.0	58.1	-17.2	17.0
40.0	12.0	4.0	1.0	2.5	447.2	57.9	-31.4	17.0
50.0	10.0	4.0	1.5	6.3	793.7	79.1	-41.1	12.5
50.0	10	4.0	1.5	5.3	728.0	79.1	-63.1	12.5
60.0	8.0	4.0	0.5	3.0	600.0	69.2	-32.8	14.3
60.0	8.0	3.9	0.1	2.0	489.9	51.2	-38.6	19.3
60.0	8.0	3.9	1.0	2.0	489.9	27.2	-62.8	36.3
80.0	6.0	4.0	0.5	4.0	800.0	79.1	-40.2	12.5
80.0	6.0	4.0	0.1	3.0	692.8	78.1	-76.7	12.6
80.0	6.0	3.9	0.5	2.5	632.5	27.1	-36.0	36.4
80.0	6.0	3.9	0.6	2.5	632.5	29.8	-45.7	33.1

TABLE III. Input parameters: $\nu^2, l_\perp, \alpha, \zeta, M_H^2 = 2\nu^2|\zeta|$ and $l = 1$; output data: $E_{e^-e^-}$ and ξ_{ab}. Trial function: $\varphi(r) = r^{3/2}e^{-\beta r}$

v^2(meV)	$l_\perp$(Å)	y	α	ζ (eV)	$M_H = \sqrt{2\nu^2\|\zeta\|}$	β	$E_{e^-e^-}$ (meV)	ξ_{ab}(Å)
30.0	16.0	4.0	2.0	-2.0	489.9	55.1	-71.5	53.7
30.0	15.5	4.0	2.0	-3.0	489.9	40.7	-23.2	72.7
30.0	15.5	4.0	3.0	-4.0	489.9	42.2	-56.2	70.1
32.0	15.0	4.0	2.0	-3.0	438.2	70.7	-49.5	41.9
32.0	15.0	4.0	1.0	-2.0	357.8	51.1	-18.0	58.9
50.0	10.0	4.0	1.5	-5.3	728.0	80.9	-43.9	36.6
50.0	10.0	4.0	1.5	-4.0	632.4	79.1	-77.3	37.4
50.0	10.0	4.0	0.8	-3.0	547.7	72.4	-49.5	40.9
50.0	10.0	4.0	0.5	-3.0	547.7	42.9	-25.0	45.0
80.0	6.5	3.8	1.0	-4.0	800.0	61.3	-21.6	48.3
80.0	6.5	3.8	0.5	-3.0	692.8	50.7	-18.8	58.4
80.0	6.5	3.8	0.5	-2.5	632.5	51.8	-52.3	57.1

From the data of the Tables I, II, III, it is possible to get an understanding of the influence of the parameters on the values of the e^-e^- energy and ξ_{ab}. When $|\zeta|$ and ν^2 increase, the Higgs mass grows up, reducing the range of the attractive interaction, which is noticed through reduction of the bound state energy. In the same way, the rising of the $\alpha-$parameter implies a larger Chern-Simons mass and a reduction of the repulsive interaction range, determining an increment of the bound state energy. The parameter $l_\perp$ acts directly in the coupling constant e_3: the bigger is $l_\perp$, the smaller is gauge coupling, and the smaller the repulsive interaction, favoring again the increase of bound state energy. The parameters ν^2 and y act on the Higgs interaction coupling, in such a way to promote a sensitive raising of the biding energy. In the particular case of Table III, it is evident a sensitive enhancement in the value of ξ_{ab}, a consequence of the isotropic trial function that behaves as $r^{3/2}$ at the origin . This isotropic character results in a non-realistic approximation, since the angular momentum state $l = 1$ must exhibit some anisotropy. This observation attributes to the data of Table III a more qualitative aspect without invalidating the fundamental result of this section: by means of a suitable fitting of the parameters, it is possible to obtain values of the energy and the correlation length for the pairs e^-e^- inside a scale usual for some solid state systems.

V. GENERAL CONCLUSIONS

The electron-electron interaction potentials, derived from a MCS Electrodynamics with spontaneous symmetry breaking, puts in evidence the physical possibility of electronic pairing and the formation of bound states. This theoretical prediction occurs when the parameters of the model are so chosen that the contribution stemming from the scalar (Higgs) sector overcomes the contribution induced by the gauge boson exchange (always repulsive in the energy scale relevant for the solid state excitations, $\theta \ll m_e$). The numerical results, displayed in Tables I, II and III, reveal the achievement of binding energies in the $meV-$scale, and correlation lengths in the scale $10-30$Å, which is a possible argument in favour of the MCS QED$_3$ adopted here to address the electronic pairing process in the realm of some Condensed Matter planar systems, with manifestation of parity-breaking, such as the Hall systems (there are also some references that discuss the nonconservation of parity symmetry in the context of the high-T$_c$ superconductors [11]). Finally, we must observe that the present MCS model bypasses the difficulties found by several other models [4], [6], [8], [9] that attempted to obtain e^-e^- bound states considering only the exchange of vector bosons. The v^2-values disposed in Tables I, II, III reconfirm the energy scale $(10-100meV)$ for the breaking of U(1)-local symmetry obtained in the framework of planar superconductors [15], and in the case of a parity-preserving electronic pairing [16].

VI. APPENDIX

In this Appendix one presents the spinor algebra $so(1,2)$ that generates the Dirac spinors, solutions of the Dirac equation in $D = 1+2$ dimensions. The adopted metric is $\eta^{\mu\nu} = (+,-,-)$, and the Dirac equation is written as:

$$(\not{p} - m)\, u_+(p) = 0, \tag{42}$$
$$(\not{p} + m)\, u_-(p) = 0, \tag{43}$$

where $u_+(p)$, $u_-(p)$ stands for the positive energy spinors with polarization "up" and "down" respectively. The solution of the equations (42,43) are given by:

$$u_+(p) = \frac{\not{p} + m}{\sqrt{2m(E+m)}} u_+(m, \overrightarrow{0}), \tag{44}$$
$$u_-(p) = \frac{\not{p} - m}{\sqrt{2m(E+m)}} u_+(m, \overrightarrow{0}), \tag{45}$$

where $u_+(m, \overrightarrow{0})$ and $u_-(m, \overrightarrow{0})$ represent an up-electron and down-electron (respectively) in the rest frame:

$$u_+(m, \overrightarrow{0}) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; \qquad u_-(m, \overrightarrow{0}) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{46}$$

In $D = 1+2$, the generators of the group SO(1,2) are given by:

$$\Sigma^{jl} = \frac{1}{4}[\gamma^j, \gamma^l], \tag{47}$$

where the γ matrices must satisfy the $so(1,2)$ algebra

$$[\gamma_\mu, \gamma_\nu] = 2i\epsilon_{\mu\nu\alpha}\gamma^\alpha, \tag{48}$$

and are taken by: $\gamma^\mu = (\sigma_z, -i\sigma_x, i\sigma_y)$.

Using this convention, the spinors $u_+(p)$, $u_-(p)$ are written at the form:

$$u_+(p) = \frac{1}{\sqrt{2m(E+m)}}\begin{bmatrix} E+m \\ -ip_x - p_y \end{bmatrix}; \overline{u}_+(p) = \frac{1}{\sqrt{2m(E+m)}}\begin{bmatrix} E+m & -ip_x + p_y \end{bmatrix}, \tag{49}$$

$$u_-(p) = \frac{1}{\sqrt{2m(E+m)}}\begin{bmatrix} ip_x - p_y \\ E+m \end{bmatrix}; \overline{u}_-(p) = \frac{1}{\sqrt{2m(E+m)}}\begin{bmatrix} -ip_x - p_y & E+m \end{bmatrix}, \tag{50}$$

They obviously satisfy the normalization condition: : $\overline{u}_+(p)u_+(p) = 1$ and $\overline{u}_-(p)u_-(p) = -1$.

In the center of mass frame, the 4-momenta of the scattered electrons (elastic scattering hypothesis) can be written as:

$$\begin{aligned} p_1 &= (E, p, 0), \quad p_1' = (E, p\cos\phi, p\sin\phi), \\ p_2 &= (E, -p, 0), \quad p_2' = (E, -p\cos\phi, -p\sin\phi), \\ k &= p_1' - p_1 = (0, p(\cos\phi - 1), p\sin\phi), \end{aligned}$$

where ϕ is the angle defined (in relation to the initial direction) by the particles after the scattering.

Adopting this convention, the current terms are evaluated:

$$\left[\overline{u}_+(p_1')\gamma_0 u_+(p_1)\right] = \frac{(E+m)^2 + p^2 e^{i\theta}}{2m(E+m)} = \left[\overline{u}_+(p_2')\gamma_0 u_+(p_2)\right]; \tag{51}$$

$$\left[\overline{u}_+(p_1')\gamma_1 u_+(p_1)\right] = -\frac{p}{2m}(1 + e^{i\theta}) = -\left[\overline{u}_+(p_2')\gamma_1 u_+(p_2)\right]; \tag{52}$$

$$\left[\overline{u}_+(p_1')\gamma_2 u_+(p_1)\right] = \frac{-ip}{2m}(1 - e^{i\theta}) = -\left[\overline{u}_+(p_2')\gamma_2 u_+(p_2)\right]; \tag{53}$$

$$\left[\overline{u}_-(p_1')\gamma_0 u_-(p_1)\right] = \frac{(E+m)^2 + p^2 e^{-i\theta}}{2m(E+m)} = \left[\overline{u}_-(p_2')\gamma_0 u_-(p_2)\right]; \tag{54}$$

$$\left[\overline{u}_-(p_1')\gamma_1 u_-(p_1)\right] = -\frac{p}{2m}(1 + e^{-i\theta}) = -\left[\overline{u}_-(p_2')\gamma_1 u_-(p_2)\right]; \tag{55}$$

$$\left[\overline{u}_-(p_1')\gamma_2 u_-(p_1)\right] = \frac{ip}{2m}(1 - e^{-i\theta}) = -\left[\overline{u}_-(p_2')\gamma_2 u_-(p_2)\right] \tag{56}$$

These current terms were used in the evaluation of the scattering amplitudes in the nonrelativistic approximation: $p^2 \ll m^2$. Finally, given the correlation between mass and spin [17], valid in QED_3, it is reasonable to enquire if the spinor $u_-(p)$ does not represent an antiparticle rather than the spin-down particle. This issue is solved in the Appendix of Ref. [12], where one shows that the charge of the spinor $u_-(p)$ is equal to the one of the spinor $u_+(p)$.

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