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# EFFECTIVE POTENTIAL USING UNCONSTRAINED CHIRAL SUPERFIELD PROPAGATORS

by

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Abstract

S. Weinberg's tadpole method is implemented to calculate the effective potential in supersymmetric theories using superfield formulation. The simplified unconstrained chiral superfield propagators found recently[4] when the mass parameters are constant chiral superfields render the computation very simple in the case of Wess-Zumino model considered here as an illustration even in higher loops.

Soon after the supersymmetry discussed by Wess and Zumino[1] the superfields realizing the supersymmetry algebra were introduced by Salam and Strathdee[2] who also formulated superfield Feynman rules[2]. Grisaru, Roček and Siegel[3] managed to simplify the original rather cumbersome propagators for chiral superfields with constant mass parameters so that higher loop calculations became manageable.

In the calculation of effective potential using superfield formulation we require propagators of chiral superfields with mass parameters which are constant chiral superfields†. In a recent paper the author[4] has given a very simple expressions for the propagators in this case compared to those available in the literature[5]. It was pointed out that the simplification arises naturally if we introduce unconstrained superfield potentials like in electrodynamics. The vertex rules and the use of unconstrained superfield propagators can be read off from the interaction term for chiral superfields[4]. It becames now possible to calculate rather easily effective potential to higher loops using superfield formulation. We will illustrate the general procedure for the case of Wess-Zumino model[6]. We remark that higher loop corrections to effective potential are now required in connection with some SUSY-GUT model.

† We consider here only non-gauge theory.

The effective potential may be computed by adapting for the superfields the methods developed for the conventional field theory by Coleman and Weinberg(7), S. Weinberg(8) or the functional method of Jackiw(9). We will use for simplicity S. Weinberg's tadpole method(7) which has been used in several conventional field theories(10). In superfield formulation we need to calculate very few tadpole supergraphs and the calculation is further made easy if we use simplified propagators(4).

The renormalizable action for chiral superfields is given by§(11)

$$I[\Phi,\Phi^\dagger] = \int d^8z\, \Phi^\dagger\Phi + \left[\int d^6s\left(\frac{\kappa}{2}\Phi^2 + \frac{g}{2}\Phi^3\right) + h.c\right] \qquad (1)$$

where $\kappa$ is constant mass parameter. We make the shifts $\Phi \to \Phi + C(\theta)$, $\Phi^\dagger \to \Phi^\dagger + C^\dagger(\bar\theta)$ where $C(\theta) = (a + f\theta^2)$, $C^\dagger(\bar\theta) = a^* + f^*\bar\theta^2$ are constant chiral (external) superfields with vanishing spinor components. The free action of the shifted theory is given by

$$I_0' = \int d^8z\, \Phi^\dagger\Phi + \left[\int d^6s\; m(\theta)\, \Phi^2 + h.c\right] \qquad (2)$$

where $m(\theta) = \tilde a + \tilde f\theta^2 = (\kappa + 2ag) + 2gf\theta^2$, while

$$I_{int}' = \int d^6s\left[\frac{g}{3}\Phi^3 + \kappa\, C(\theta)\,\Phi + g\, C^2(\theta)\,\Phi\right] + \int d^8z\, C^\dagger(\theta)\Phi + h.c \qquad (3)$$

§We follow the notation of ref.(11).

The propagators for unconstrained superfields S, $S^\dagger$ defined by $\Phi=-\frac{1}{4}\bar{D}^2 S$, $\Phi^\dagger=-\frac{1}{4}D^2 S^\dagger$ may be derived from $I_0'$ and were shown to take the simple form[(4)§]

$$\Delta^{SS^\dagger} = i\left[\square - m^\dagger P_1 m\right]^{-1} \delta^8(z-z')$$
$$\Delta^{S^\dagger S^\dagger} = \frac{m}{4\square}\bar{D}^2 \Delta^{SS^\dagger} \tag{4}$$

with analogous expressions for $\Delta^{SS}$ and $\Delta^{S^\dagger S}$. The Feynman rules may be easily found from $I'_{int}$ as in conventional field theory.

In the zero loop approximation the contribution to the effective action from the shifted Lagrangian is simply

$$\Gamma_0^{(1)} = \int d^6 s\left[\kappa\, C(\theta) + g\, C^2(\theta)\right]\Phi + \int d^8 z\, C^\dagger \Phi + \text{h.c.}$$
$$= \int d^2\theta\left[\kappa\, C(\theta) + g\, C^2(\theta)\right]\tilde{\Phi}(0,\theta,\bar{\theta}) + \int d^4\theta\, C^\dagger(\bar{\theta})\, \tilde{\Phi}(0,\theta,\bar{\theta}) + \text{h.c.} \tag{5}$$

where $\tilde{\Phi}(p,\theta,\bar{\theta})$ is the Fourier transform of $\Phi$. We find

$$\Gamma_0^{(1)} = \tilde{a} f\, \tilde{A}(0) + (\kappa a + g a^2 + f^*)\, \tilde{F}(0) + \text{h.c.} \tag{6}$$

The effective potential has the following expansion[(7)] $(\psi=\bar{\psi}=0)$

$$-V\left[A_c, F_c, A_c^\dagger, F_c^\dagger\right]$$
$$= \left[\tilde{\Gamma}_A^{(1)}(0;a,f,a^*,f^*)(A_c-a) + \tilde{\Gamma}_F^{(1)}(0,a,f,a^*,f^*)(F_c-f) + \text{h.c.}\right]$$
$$+ 2^{nd}\text{ order terms.} \tag{7}$$

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(§) For $m(\theta)=f\theta^2$ we find $\Delta^{S^\dagger S^\dagger} = \frac{-if}{(\square^2-|f|^2)}\theta^2\theta'^2\delta^4(x-x')$ and for constant m it effectively coincides with the expressions in Ref. 3. See also Ref. 11.

where $A_c$, $F_c$ etc. are classical fields and $\tilde{\Gamma}^{(n)}(0,0,\ldots 0;a,f,a^*,f^*)$ are the n-point functions in momentum space evaluated at zero external momenta. It follows that

$$
\begin{aligned}
-\left.\frac{\partial V}{\partial A_c}\right|_{\substack{A_c=a\\F_c=f}} &= \tilde{\Gamma}^{(1)}_{A}(0;a,f,a^*,f^*) \equiv \left.\frac{\delta\Gamma}{\delta A_c}\right|_{\substack{A_c=a\\F_c=f}} \\
-\left.\frac{\partial V}{\partial F_c}\right|_{\substack{A_c=a\\F_c=f}} &= \tilde{\Gamma}^{(1)}_{F}(0;a,f,a^*,f^*) \equiv \left.\frac{\delta\Gamma}{\delta F_c}\right|_{\substack{A_c=a\\F_c=f}}
\end{aligned}
\qquad (8)
$$

where $\Gamma$ is the effective action with spinor components set to zero. From Eqs. (6) and (8) we obtain

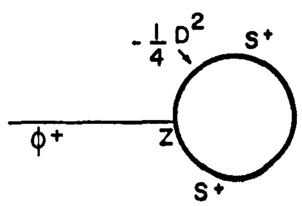
$$\frac{\partial V_0}{\partial a} = -\tilde{a}\, f \quad , \quad \frac{\partial V_0}{\partial f} = -(\kappa a+ga^2+f^*) \qquad (9)$$

together with their complex conjugate relations. On integration we obtain for the zero loop effective potential

$$V_0 = \left[|f|^2 + (ga^2+\kappa a)f + (ga^{*2}+\kappa a^*)f^*\right] \qquad (10)$$

where $a\to A_c$, $f\to F_c$ is to be understood. It reduces to $V_0=|F_c|^2$ if we use auxiliary field equations of motion and vanishes for $F_c=0$ corresponding to unbroken supersymmetry.

The computation of 1-loop effective potential is done in similar fashion. The $\Phi^\dagger$ tadpole contribution is



$$\Gamma_1^{(1)} = \frac{ig}{3} .3 \int d^8z\ \Phi^\dagger(z) \left[-\frac{1}{4} D^2 \Delta^{S^\dagger S^\dagger}(z,z')\right]_{z=z'}$$

$$= ig \int d^4\theta\ \tilde{\Phi}^\dagger(0,\theta,\bar{\theta}) \left[-\frac{1}{4} D^2 \Delta^{S^\dagger S^\dagger}(z,z')\right]_{z=z'} \tag{11}$$

A simple calculation using $\Delta^{S^\dagger S^\dagger}$ in Eq. (4) gives

$$\Gamma_1^{(1)} = \int \frac{d^4k}{(2\pi)^4} \left[g\tilde{f}\ \tilde{F}^\dagger(0) - \frac{2\tilde{a}g|\tilde{f}|^2}{(k^2+|\tilde{a}|^2)} \tilde{A}^\dagger(0)\right] \frac{1}{\left[(k^2+|\tilde{a}|^2)^2 - |\tilde{f}|^2\right]} \tag{12}$$

The partial derivatives of the effective potential are

$$\frac{\partial V_1}{\partial f^*} = -\int \frac{d^4k}{(2\pi)^4} \frac{g\tilde{f}}{\left[(k^2+|\tilde{a}|^2)^2 - |\tilde{f}|^2\right]}$$

$$\frac{\partial V_1}{\partial a^*} = \int \frac{d^4k}{(2\pi)^4} \frac{2g\tilde{a}|\tilde{f}|^2}{\left[(k^2+|\tilde{a}|^2)^2 - |\tilde{f}|^2\right]\left[k^2+|\tilde{a}|^2\right]} \tag{13}$$

plus their complex conjugates. Integrating we obtain the familiar result[12]

$$V_1 = \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \ell n \left[1 - \frac{|\tilde{f}|^2}{(k^2+|\tilde{a}|^2)^2}\right] \tag{14}$$

where $f \to F_c$, $a \to A_c$ is to be understood and it vanishes when $F_c = 0$. It is worth remarking at the ease of calculation in tadpole method even in higher loops. It is also possible to translate the conventional formulation of effective action in terms of superfields $\Phi_c$, $\Phi_c^\dagger$ but for practical purposes the procedure adopted above is simpler. The computation of effective potential to two loops and in gauge theories will be reported elsewhere.

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