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Electron-electron attractive interaction in Maxwell-Chern-Simons QED$_3$ at zero temperature

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One discusses the issue of low-energy electron-electron bound states in the Maxwell-Chern-Simons model coupled to QED$_3$ with spontaneous breaking of a local $U(1)$-symmetry. The scattering potential, in the non-relativistic limit, steaming from the electron-electron Møller scattering, mediated by the Maxwell-Chern-Simons-Proca gauge field and the Higgs scalar, might be attractive by fine-tuning properly the physical parameters of the model.

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I. INTRODUCTION

In the latest years, planar QED (QED_3) [1-11] has been object of intensive investigation, both by its formal aspects and by the possibilities of application to important phenomena in the realm of Condensed Matter Physics, namely high-T_c superconductivity and quantum Hall effect. The first phenomenon, as well-known, is related to the existence of electron-electron bound states, but the nature of these pairs belonging to high-T_c phase is not still set up. The absence of a ultimate theory for high-T_c superconductivity has attracted attention of a large number of condensed matter and field theorists.

The search for a mechanism inducing the formation of electron-electron bound states has also passed through QED_3, since high-T_c superconductivity is supposed to be a quasi-planar phenomenon. Moreover, it is known that the Coulombian interaction in three space-time dimensions leads to a confining potential rather than a condensating one, which indicates the necessity of a finite range interaction. One should stress here that, in spite of some claims found out in the literature, the electromagnetic potential cannot be of the $1/r$-type in three space-time dimensions, for it would demand a highly non-local action, leading to serious troubles as long as causal propagation of particles is concerned; on the other hand, it does not lead to bound states [12], contrary to what happens in four space-time dimensions. The idea of providing mass to the photon was then proposed as an attempt to try to by-pass this difficulty. In this sense, the Maxwell-Chern-Simons (MCS) model [1] was adopted as a mechanism for generating (topological) mass for the photon. A deconfining potential consequently emerges and the quest for electron-electron bound states turns out to be a sensible matter. All these aspects have been embrassed in [2], where the MCS model coupled to QED_3 is considered as a main tool for investigation of fermion-fermion scattering processes mediated by a topological massive gauge boson. The issue of electron-electron bound states, in the MCS QED_3, has been taken into account for the first time by numerical simulations in Ref. [3], however, their result is characterized by the fact that just one-photon exchange diagrams have been taken into account, leading to an incomplete Aharonov-Bohm potential term [4]. The authors of Ref. [5] comment on the results presented in [3] asserting that they hold on for small k (statistics parameter), nevertheless in this regime perturbation theory breaks down and higher order contributions to the electron-electron scattering amplitude become also important, so that the term in $1/k^2$, stemming from the two-photon exchange diagrams, could not be neglected. The solution to this controversy consists in considering the two-photon exchange diagrams [5,6], whose contribution to the order $1/k^2$ to the scattering potential restores the gauge invariance in the non-relativistic limit [13] of the theory and circumvents the erroneous conclusion of an attractive centrifugal barrier. Indeed, the work of Ref. [5] displays a number of interesting limits where e^-e^- and e^+e^+ bound state formation may be analyzed and the important outcome is that the Pauli dipole interaction among the electrons, due to their magnetic moment, may, in a suitable limit, dominate over the charge-charge repulsive interaction, leading to bound state formation, which has been also addressed in [7]. It is also concluded that, in the case of light gauge bosons, the MCS model minimally coupled to QED_3 does not provide electron-electron bound states. The MCS model non-minimally coupled to fermions and bosons carrying an anomalous magnetic moment and within the perturbative region $1/k \ll 1$ has been analyzed in [8]. The presence of this non-minimal coupling is pointed out to be a key factor for the appearance of an attractive potential between charges of same sign.

Until the present moment, one observes that all the quoted works concerning electron-electron bound states make use of the Chern-Simons term as the only mechanism yielding the photon mass. In our work, one employs a different theoretical approach to generate photon mass (beyond the topological one) and possibly the electron-electron bound states. Specifically, one adopts a Maxwell-Chern-Simons model minimally coupled to QED_3 with spontaneous breaking of a local $U(1)$-symmetry. Similarly, in Refs. [9,10], the Higgs mechanism has been used in the framework of a parity-preserving QED_3 in searching for electron-electron bound states. The symmetry breaking is accomplished by a sixth-power potential, where a Higgs scalar and a massive gauge boson (Maxwell-Chern-Simons-Proca) stem as a by-product from the breaking of a local $U(1)$-symmetry. As we shall present here, the low-energy Møller scattering mediated by these two quanta points to the real possibility of an attractive e^-– e^- scattering potential. Thus, it becomes manifest that the Higgs mechanism has the relevant role of allowing electron-electron pair condensation. In fact, our proposal is based upon the Higgs exchange to bind the electron pair rather than on a mass relationship that leads to a dominance of magnetic moment interaction over the charge repulsion.

Our paper is organized as follows. In Section II, the Maxwell-Chern-Simons model coupled to QED_3 with spontaneous breaking of a $U(1)$-symmetry is introduced. The low-energy electron-electron scattering potential, in the Born approximation, is derived and discussed in Section III. The final conclusions are left to Section IV. In the Appendix, Section V, general physical properties of planar QED are alucidated.

II. THE MAXWELL-CHERN-SIMONS QED AND THE HIGGS MECHANISM

The action for the Maxwell-Chern-Simons model coupled to QED_3 with a local $U(1)$-symmetry is given by:

$$S_{\mathrm{QED}} = \int d^3x \Big\{ -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\overline{\psi}\gamma^\mu D_\mu\psi + \frac{1}{2}\theta\epsilon^{\mu\nu\alpha}A_\mu\partial_\nu A_\alpha - m_e\overline{\psi}\psi - y\overline{\psi}\psi\varphi^*\varphi + D^\mu\varphi^* D_\mu\varphi + - V(\varphi^*\varphi)\Big\} , \tag{1}$$

where the $V(\varphi^*\varphi)$ is a sixth-power potential, being the most general renormalizable $U(1)$-invariant potential in three dimensions [9,10]:

$$V(\varphi^*\varphi) = \mu^2\varphi^*\varphi + \frac{\zeta}{2}(\varphi^*\varphi)^2 + \frac{\lambda}{3}(\varphi^*\varphi)^3 . \tag{2}$$

The covariant derivatives are defined as follows:

$$D_\mu\psi = (\partial_\mu + ieA_\mu)\psi \quad \text{and} \quad D_\mu\varphi = (\partial_\mu + ieA_\mu)\varphi . \tag{3}$$

In the action $\mathrm{S_{QED}}$, Eq.(1), $F_{\mu\nu}$ is the usual field strength for A_μ, ψ is a spinor field describing a fermion with positive spin polarization (spin up) and an anti-fermion with negative spin polarization (spin down)

[9,10], whereas φ is a complex scalar field. In three space-time dimensions, the positive- and negative-energy solutions have their polarization fixed by the signal of mass in the Dirac mass term [9,10,14]. The conventions ** adopted here are stated in the Appendix, where the mass dimensions of all the fields and parameters are displayed in the Table II.

The sixth-power potential is the responsible for breaking the electromagnetic $U(1)$-symmetry. Analyzing the structure of the potential $V(\varphi^*\varphi)$, one must impose that it is bounded from below and it yields only stable vacua (metastability is ruled out). These requirements reflect on the following conditions on the parameters μ, ζ and λ [9,10]:

$$\lambda > 0 \ , \quad \zeta < 0 \quad \text{and} \quad \mu^2 \leq \frac{3\zeta^2}{16\lambda} \ . \tag{4}$$

Considering $\langle\varphi^*\varphi\rangle = v^2$, the vacuum expectation value for the scalar field product $\varphi^*\varphi$ is given by

$$\langle\varphi^*\varphi\rangle = v^2 = -\frac{\zeta}{2\lambda} + \sqrt{\left(\frac{\zeta}{2\lambda}\right)^2 - \frac{\mu^2}{\lambda}} \ , \tag{5}$$

while the minimum condition reads

$$\mu^2 + \zeta v^2 + \lambda v^4 = 0 \ . \tag{6}$$

In order to preserve the manifest renormalizability of the model, one adopts the 't Hooft gauge:

$$S_{\mathrm{R}_\xi} = \int d^3x \Big\{ -\frac{1}{2\xi}(\partial^\mu A_\mu - \sqrt{2}\xi M_A \chi)^2 \Big\} \ . \tag{7}$$

Then, by adding it up to the action (1), and assuming the following parametrization for the scalar field,

$$\varphi = v + H + i\chi \ , \tag{8}$$

where H represents the Higgs scalar and χ the would-be Goldstone boson, the Maxwell-Chern-Simons QED$_3$ action with the $U(1)$-symmetry spontaneously broken is as follows

$$\begin{aligned} S_{\mathrm{QED}}^{\mathrm{broken}} = \int d^3x \Big\{ & -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}M_A^2 A^\mu A_\mu - \frac{1}{2\xi}(\partial^\mu A_\mu)^2 + \overline{\psi}(i\gamma^\mu D_\mu - m)\psi + \frac{1}{2}\theta\epsilon^{\mu\nu\alpha}A_\mu\partial_\nu A_\alpha + \\ & + \partial^\mu H \partial_\mu H + \partial^\mu\chi\partial_\mu\chi - \xi M_A^2\chi^2 - y\overline{\psi}\psi(2vH + H^2 + \chi^2) + 2eA^\mu(H\partial_\mu\chi - \chi\partial_\mu H) + \\ & + e^2 A^\mu A_\mu (2vH + H^2 + \chi^2) - \mu^2((v+H)^2 + \chi^2) - \frac{\zeta}{2}((v+H)^2 + \chi^2)^2 - \frac{\lambda}{3}((v+H)^2 + \chi^2)^3 \Big\} \ , \end{aligned} \tag{9}$$

where the mass parameters M_A^2, m and M_H^2, read

$$M_A^2 = 2v^2e^2 \ , \quad m = m_e + yv^2 \quad \text{and} \quad M_H^2 = 2v^2(\zeta + 2\lambda v^2) \ . \tag{10}$$

**The metric adopted is $\eta^{\mu\nu} = \mathrm{diag}(+,-,-)$ and the γ-matrices are taken as $\gamma^\mu = (\sigma_z, i\sigma_x, -i\sigma_y)$.

III. THE LOW-ENERGY ELECTRON-ELECTRON SCATTERING POTENTIAL

The issue of electron-electron bound states in the Maxwell-Chern-Simons model coupled to planar QED has been addressed to in the literature since the end of the eighties [2–5], motivated by possible applications to the parity-breaking high-T_c superconductivity phenomenon.

In this Section, we shall present the evaluation of the electron-electron scattering potential in the low-energy approximation. The Møller electron-electron scattering process is mediated by the Higgs scalar and the Maxwell-Chern-Simons-Proca gauge field. In order to compute the scattering potential through the Møller electron-electron amplitude, we show the propagators associated to the Higgs (H), the fermion (ψ) and the massive gauge boson (A_μ), which stem straightforwardly from the action (9), as presented below

$$\langle\overline{\psi}(k)\psi(k)\rangle = i\frac{k+m}{k^2-m^2}\ ,\quad \langle H(k)H(-k)\rangle = \frac{i}{2}\frac{1}{k^2-M_H^2}\quad \text{and}$$
$$\langle A_\mu(k)A_\nu(-k)\rangle = -i\left\{\frac{k^2-M_A^2}{(k^2-M_A^2)^2-k^2\theta^2}\left(\eta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)+\frac{\xi}{(k^2-\xi M_A^2)}\frac{k_\mu k_\nu}{k^2}+\frac{\theta}{(k^2-M_A^2)^2-k^2\theta^2}i\epsilon^{\mu\alpha\nu}k_\alpha\right\}\ . \quad (11)$$

The propagator of the Maxwell-Chern-Simons-Proca field given above can be rewritten in the following way

$$\begin{aligned}\langle A_\mu(k)A_\nu(-k)\rangle = & -i\left\{\left[\frac{C_+}{k^2-M_+^2}+\frac{C_-}{k^2-M_-^2}\right]\left(\eta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)+\frac{\xi}{(k^2-\xi M_A^2)}\frac{k_\mu k_\nu}{k^2}+\right.\\ & \left.+\left[\frac{C}{k^2-M_+^2}-\frac{C}{k^2-M_-^2}\right]i\epsilon_{\mu\alpha\nu}k^\alpha\right\}\ ,\end{aligned} \quad (12)$$

where the positive definite constants C_+, C_-, C, and the squared masses M_+^2 and M_-^2, are given by:

$$C_\pm = \frac{1}{2}\left[1\pm\frac{\theta}{\sqrt{4M_A^2+\theta^2}}\right]\ ,\quad C=\frac{1}{\sqrt{4M_A^2+\theta^2}}\ , \quad (13)$$

$$M_\pm^2 = \frac{1}{2}\left[2M_A^2+\theta^2\pm|\theta|\sqrt{4M_A^2+\theta^2}\right]\ , \quad (14)$$

with the massive poles, M_+^2 and M_-^2, corresponding to the two massive propagating quanta. It can be readily checked that both of them are physical states in that the residues at the poles are positive-definite. From the action $S_{\rm QED}^{\rm broken}$, given by Eq.(9), it can be derived the Feynman rules associated to the electromagnetic and Yukawa interactions, $\mathcal{V}_{\psi H\psi} = 2ivy$ and $\mathcal{V}_{\psi A\psi} = ie\gamma^\mu$, respectively.

Let us now start the derivation of the electron-electron scattering potential through the total Møller scattering amplitude ($\mathcal{M}_{\rm total}$) in the low-energy approximation, *i.e.*, the non-relativistic limit ($\mathcal{M}_{\rm total}^{\rm nr}$). The scattering potential is nothing but the two-dimensional Fourier transform of the lowest-order $\mathcal{M}_{\rm total}$-matrix element:

$$V(r) = \int\frac{d^2\vec{k}}{(2\pi)^2}\ \mathcal{M}_{\rm total}^{\rm nr}\ e^{i\vec{k}\cdot\vec{r}}\ . \quad (15)$$

In the case we are analyzing here (the electron-electron scattering, e^-– e^-, mediated by the Higgs, H, and the massive gauge boson, A_μ), the matrix $\mathcal{M}_{\rm total}^{\rm nr}$ that appears in Eq.(15) is precisely the part of the covariant matrix element which corresponds to direct scattering, s-channel. This can be understood in view of the fact that antisymmetric wave functions in non-relativistic quantum mechanics automatically take care of the contributions resulting from the exchange scattering. The s-channel amplitudes for the e^-– e^- scattering mediated by the Higgs and the gauge field, with the corresponding Feynman diagrams displayed in Fig.1, are listed below:

1. Scattering amplitude with the Higgs exchange:

$$-i\mathcal{M}_{e^-He^-} = \overline{u}(p_1)(2ivy)u(p_1')\langle H(k)H(-k)\rangle\overline{u}(p_2)(2ivy)u(p_2') \ , \tag{16}$$

2. Scattering amplitude with the massive gauge boson exchange:

$$-i\mathcal{M}_{e^-Ae^-} = \overline{u}(p_1)(ie\gamma^\mu)u(p_1')\langle A_\mu(k)A_\nu(-k)\rangle\overline{u}(p_2)(ie\gamma^\nu)u(p_2') \ , \tag{17}$$

where $k^2 = (p_1' - p_1)^2$ is the invariant squared momentum transfer. In the partial scattering amplitudes $\mathcal{M}_{e^-He^-}$ and $\mathcal{M}_{e^-Ae^-}$, given by Eqs.(16) and (17), respectively, the spinor $u(p)$ is the positive-energy solution of the Dirac equation for ψ, satisfying the following normalization condition stated in the Appendix:

$$\overline{u}(p)u(p) = 1 \ . \tag{18}$$

The momenta configuration in the center-of-mass frame (c.m.) of the two interacting electrons, as well as the momentum transfer, are chosen as

$$\begin{aligned} p_1 = (E, p, 0) \ , \quad p_1' = (E, p\cos\phi, p\sin\phi) \ , \\ p_2 = (E, -p, 0) \ , \quad p_2' = (E, -p\cos\phi, -p\sin\phi) \quad \text{and} \\ k = p_1' - p_1 = (0, p(\cos\phi - 1), p\sin\phi) \ , \end{aligned} \tag{19}$$

where ϕ is the c.m. scattering angle, which is defined as the angle between the directions in the center-of-mass frame of the two incoming (initial state) and outgoing (final state) electrons.

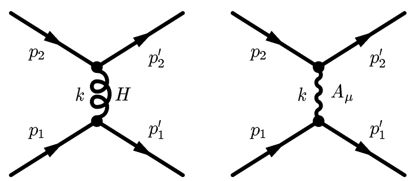

FIG. 1. Feynman diagrams associated to the e^-– e^- scattering mediated by the Higgs and the massive gauge boson.

Assuming the momenta configuration above (19), the total scattering amplitude in the low-energy approximation, $\mathcal{M}^{\rm nr}_{\rm total}$, can now be derived from the partial ones, $\mathcal{M}^{\rm nr}_{e^- He^-}$ and $\mathcal{M}^{\rm nr}_{e^- Ae^-}$:

$$\mathcal{M}^{\rm nr}_{\rm total} = \mathcal{M}^{\rm nr}_{e^- He^-} + \mathcal{M}^{\rm nr}_{e^- Ae^-} \ , \tag{20}$$

where

$$\mathcal{M}^{\rm nr}_{e^- He^-} = -2v^2y^2 \frac{1}{\vec{k}^2 + M_H^2} \ , \tag{21}$$

and

$$\begin{aligned} \mathcal{M}^{\rm nr}_{e^- Ae^-} &= \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_{\rm AB} \\ &= e^2 \left[\frac{C_+}{\vec{k}^2 + M_+^2} + \frac{C_-}{\vec{k}^2 + M_-^2} \right] + e^2 4 \frac{p^2}{2m}(1 - \cos\phi) \left[\frac{C}{\vec{k}^2 + M_+^2} - \frac{C}{\vec{k}^2 + M_-^2} \right] + \\ &\quad + ie^2 4 \frac{p^2}{2m} \sin\phi \left[\frac{C}{\vec{k}^2 + M_+^2} - \frac{C}{\vec{k}^2 + M_-^2} \right] \ . \end{aligned} \tag{22}$$

Notice that the first two terms of the massive gauge field amplitude, $\mathcal{M}^{\rm nr}_{e^- Ae^-}$, given in Eq.(22), $\mathcal{M}_1$ and $\mathcal{M}_2$, are the real part of the Møller scattering amplitude, whereas the last one, $\mathcal{M}_{\rm AB}$, which is imaginary, is the Aharonov-Bohm amplitude for the fermions [2,5,8,11]. The total Møller scattering amplitude in the non-relativistic limit reads as below:

$$\begin{aligned} \mathcal{M}^{\rm nr}_{\rm total} = & -2v^2y^2 \frac{1}{\vec{k}^2 + M_H^2} + e^2 \left[\frac{C_+}{\vec{k}^2 + M_+^2} + \frac{C_-}{\vec{k}^2 + M_-^2} \right] + e^2 4 \frac{p^2}{2m}(1 - \cos\phi) \left[\frac{C}{\vec{k}^2 + M_+^2} - \frac{C}{\vec{k}^2 + M_-^2} \right] + \\ & + ie^2 4 \frac{p^2}{2m} \sin\phi \left[\frac{C}{\vec{k}^2 + M_+^2} - \frac{C}{\vec{k}^2 + M_-^2} \right] \ . \end{aligned} \tag{23}$$

Now, bearing in mind that the non-relativistic scattering potential in the Born approximation is obtained from the scattering amplitude (23) through the Fourier transform given by Eq.(15), one gets:

$$V(r) = V_{\mathrm{Higgs}}(r) + V_{\mathrm{gauge}}(r) \ , \tag{24}$$

$$V_{\mathrm{Higgs}}(r) = -\frac{1}{2\pi} 2v^2 y^2 K_0(M_H r) \ , \tag{25}$$

$$\begin{aligned} V_{\mathrm{gauge}}(r) &= V_1(r) + V_2(r) + V_{\mathrm{AB}}(r) \\ &= \frac{e^2}{2\pi}\left[C_+ K_0(M_+ r) + C_- K_0(M_- r)\right] - \frac{e^2}{2\pi}\frac{C}{m}\left[M_+^2 K_0(M_+ r) - M_-^2 K_0(M_- r)\right] + \\ &+ 2\frac{e^2}{2\pi}\frac{l}{mr} C\left[M_+ K_1(M_+ r) - M_- K_1(M_- r)\right] \ . \end{aligned} \tag{26}$$

Therefore, the e^-– e^- low-energy scattering potential, $V(r)$, is given by

$$\begin{aligned} V(r) = & -\frac{1}{2\pi} 2v^2 y^2 K_0(M_H r) + \frac{e^2}{2\pi}\left\{\left[C_+ - \frac{C}{m} M_+^2\right] K_0(M_+ r) + \left[C_- + \frac{C}{m} M_-^2\right] K_0(M_- r) + \right. \\ & \left. + 2\frac{l}{mr} C\left[M_+ K_1(M_+ r) - M_- K_1(M_- r)\right]\right\} , \end{aligned} \tag{27}$$

where, K_0 and K_1 are the zeroth- and first-order modified Bessel functions of the second kind, respectively, and l is the angular momentum.

It should be stressed here that the low-energy electron-electron scattering potential we are deriving is valid only in the perturbative regime, where loop corrections are negligible if compared to the semi-classical approximation. Perturbation theory is realized whenever dimensionless parameters are kept much smaller than one. At the broken-symmetry phase, the Maxwell-Chern-Simons model coupled to planar QED has four dimensionless parameters, e^2/m, e^2/M_H, e^2/M_+ and e^2/M_-. Nevertheless, the masses M_H and M_- vanish in the unbroken-symmetry phase (when $v^2 \to 0$), in this way e^2/m and e^2/M_+ remain the natural dimensionless parameters respect to which perturbation theory shall be performed. For our purposes here (where the low-energy electron-electron potential is derived through the Born approximation of the Møller scattering amplitude in the non-relativistic limit), since the electron is the heaviest particle (electron effective mass (10), $m \approx 0,5 MeV$) with the Higgs (in condensed matter phenomena, $M_H \approx meV$) and the massive gauge boson ($M_\pm \approx meV$) being the intermediate quanta, we can ensure confidence on the perturbative regime by assuming e^2/m and $y \ll 1$ provided that $e^2/M_+ \ll 1$.

Non-trivial aspects of the Galilean (non-relativistic) limit of a gauge theory are discussed in the work of Hagen [13]. In the non-relativistic limit, even though the perturbative regime is considered, besides the one-photon exchange diagrams, one has to take into account two-photon exchange contributions so as to preserve gauge invariance (the non-relativistic Hamiltonian is quadratic in momentum), as presented by the authors of Refs. [5,6] in the framework of a Maxwell-Chern-Simons model minimally coupled either to fermions or to fermions and scalars. The non-relativistic scattering potential for the MCS QED_3 model has been derived in Ref. [5], there the perturbative regime is established by the statistics parameter k (in our case it is given by $4\pi\theta/e^2$) whenever $1/k \ll 1$. In order to guarantee gauge invariance in the low-energy approximation, despite of $1/k \ll 1$, two-photon exchange diagrams have to be taken into account as well, which leads to the correct low-energy electron-electron scattering potential for the MCS model coupled to planar QED [5] as follows:

$$V_{\mathrm{MCS}}(r) = \frac{e^2}{2\pi}\left[1 - \frac{\theta}{m}\right] K_0(\theta r) + \frac{1}{mr^2}\left\{l - \frac{e^2}{2\pi\theta}[1 - \theta r K_1(\theta r)]\right\}^2 \ . \tag{28}$$

For feasible applications to Condensed Matter Physics, which should require $\theta \ll m$, the non-relativistic MCS QED$_3$ scattering potential, given above by Eq. (28), results to be repulsive, where its first term corresponds to the electromagnetic potential whereas the last one includes the Aharonov-Bohm, the centrifugal barrier and the two-photon exchange contributions.

Let us now remind that our main task is to derive the gauge-invariant scattering potential in the non-relativistic limit for the model proposed here. In this way, this amounts in adding to Eq.(27) the centrifugal barrier and the one-loop corrections resulting from the two-photon exchange diagrams, by following the steps pointed out in Refs. [5,6] together with the general arguments on non-relativistic gauge theories analyzed in [13]. Therefore, as a final result, the non-relativistic effective scattering potential of the MCS QED$_3$ model with spontaneous symmetry breaking, $V_{\rm eff}(r)$, reads as below:

$$\begin{aligned} V_{\rm eff}(r) = & -\frac{1}{2\pi} 2v^2 y^2 K_0(M_H r) + \frac{e^2}{2\pi}\left\{\left[C_+ - \frac{C}{m}M_+^2\right] K_0(M_+ r) + \left[C_- + \frac{C}{m}M_-^2\right] K_0(M_- r)\right\} + \\ & + \frac{1}{mr^2}\left\{l + \frac{e^2}{2\pi} C r [M_+ K_1(M_+ r) - M_- K_1(M_- r)]\right\}^2 , \qquad (29) \end{aligned}$$

where $\frac{l^2}{mr^2}$ is the centrifugal barrier and the term in C^2 arises from the one-loop two-photon exchange diagrams [5,6]. It can be concluded from the effective electron-electron scattering potential $V_{\rm eff}(r)$, that the only attractive contribution to it comes from the Higgs interaction given by the first term in Eq.(29). However, the second term, which is proportional to $e^2/2\pi$, shows to be repulsive in the range of parameters we are restricting our model, whereas the last one has always the same behavior, namely, repulsive. In view of the attractive nature of the Yukawa interaction, by an appropriate fine-tuning of the parameters (coupling constants and masses) of the model, so as to compensate the repulsion caused by the electromagnetic interaction and the "effective" centrifugal barrier, the Møller scattering potential $V_{\rm eff}(r)$ turns out to be attractive. As a consequence, this might favor electron-electron bound states provided $V_{\rm eff}(r)$ is "weak" in the sense of Kato and satisfies the Setô bound as discussed by Chadan *et al.* [12] in the framework of low-energy scattering in three space-time dimensions, this issue is now under investigation [15].

IV. GENERAL CONCLUSIONS

The low-energy electron-electron scattering potential we have derived for the MCS QED$_3$ model with spontaneous symmetry breaking sets up the physical framework for the mechanism of an electron-electron pairing and the consequent formation of bound states. The Higgs contribution to the effective scattering potential reveals to be always attractive while the gauge boson contribution is repulsive in the range of parameters dictated by the condensed matter phenomena, namely, $\theta \ll m$. Therefore, one concludes that the e^-– e^- scattering potential, $V_{\rm eff}(r)$, given by Eq.(29), is always attractive whenever, by a properly fine-tuning of the parameters, the attraction caused by the Higgs mediation becomes stronger than the repulsion yields by the gauge field mediation and the "effective" centrifugal barrier. Thus, as a conclusion, the Higgs mechanism [9,10] provides a possible mechanism for an electron-electron attractive potential, and therefore sets up an effective possibility for pair condensation at the low-energy limit of a parity-breaking QED$_3$. Finally, one points out that this model bypasses the difficulties found by several authors [3] who tried to obtain electron-electron bound states in MCS QED$_3$ by only considering the exchange of gauge bosons.

It is important to observe that the gauge-mediated contribution, $V_{\rm eff}^{\rm gauge}(r)$ (the last three terms of Eq.(29)), to the scattering potential, $V_{\rm eff}(r)$,

$$V_{\rm eff}^{\rm gauge}(r) = \frac{e^2}{2\pi}\left\{\left[C_+ - \frac{C}{m}M_+^2\right]K_0(M_+r) + \left[C_- + \frac{C}{m}M_-^2\right]K_0(M_-r)\right\} + \\ + \frac{1}{mr^2}\left\{l + \frac{e^2}{2\pi}Cr[M_+K_1(M_+r) - M_-K_1(M_-r)]\right\}^2 , \tag{30}$$

reproduces the usual form for a vanishing Proca photon mass. In this limit

$$M_+ \longrightarrow \theta\ ,\quad M_- \longrightarrow 0\ ,\quad C_+ \longrightarrow 1\ ,\quad C_- \longrightarrow 0\ ,\quad K_1(M_-r) \longrightarrow \frac{1}{M_-r}\ ,\quad C \longrightarrow \frac{1}{\theta}\ , \tag{31}$$

such that one has

$$\lim_{M_A \longrightarrow 0} V_{\rm eff}^{\rm gauge}(r) = \frac{e^2}{2\pi}\left[1 - \frac{\theta}{m}\right]K_0(\theta r) + \frac{1}{mr^2}\left\{l - \frac{e^2}{2\pi\theta}[1 - \theta r K_1(\theta r)]\right\}^2 , \tag{32}$$

which is exactly the same as the one obtained in the works of Refs. [2,5].

To conclude, we would like to stress that we shall next check whether or not low-energy electron-electron bound states stem from the MCS QED$_3$ model with spontaneous symmetry breaking. This shall be done by explicitly solving the Schrödinger equation with the help of numerical methods. Our results shall be reported elsewhere in a forthcoming paper [15].

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V. APPENDIX

Here we present some aspects of a massive Dirac spinor living in three space-time dimensions, like the positive and negative energy solutions to the Dirac equation satisfied by ψ. We present the Hamiltonian for ψ, and also compute explicitly the charges of the positive and negative energy wave functions associated to ψ.

A. Positive and negative energy solutions for ψ

Let us consider u and v, respectively, as the positive and negative solutions to the Dirac equations for ψ. Therefore, they satisfy the following equations in momentum space:

$$(p - m)u(p) = 0 \quad \text{and} \quad (-p - m)v(p) = 0 \ . \tag{33}$$

Their solutions are given by

$$u(p) = \frac{p + m}{\sqrt{2m(m+E)}}\, u(m, \vec{0}) \quad \text{and} \quad v(p) = \frac{-p + m}{\sqrt{2m(m+E)}}\, v(m, \vec{0}) \ , \tag{34}$$

where $E \equiv k^0 = \sqrt{\vec{k}^2 + m^2} > 0$. The wave functions $u(m, \vec{0})$ and $v(m, \vec{0})$ are the solutions of Eqs.(33) in the rest frame

$$u(m, \vec{0}) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad v(m, \vec{0}) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \ . \tag{35}$$

The positive and negative energy solutions given by Eqs.(34) are normalized to :

$$\overline{u}(p)u(p) = 1 \quad \text{and} \quad \overline{v}(p)v(p) = -1 \ . \tag{36}$$

B. The spin of u and v

Now, by considering the results of last subsection, one is able to determine the spins of the solutions u and v. We compute the spins in the particle rest frame, since we have in mind to explicitly exhibit the fact that the sign of the mass term fixes the polarization of the fermion.

In three space-time dimensions, the generators of the $\overline{SO(1,2)}$ group in the spinor representation read:

$$\Sigma^{kl} = \frac{1}{4}\, [\gamma^k, \gamma^l] \ , \tag{37}$$

where the γ-matrices are taken as $\gamma^\mu = (\sigma_z, i\sigma_x, -i\sigma_y)$.

The spin operator S^{12} is obtained from (37), and it reads

$$S^{12} = \frac{1}{2}\, \sigma_z \ . \tag{38}$$

Its action upon the rest frame wave functions given by Eqs.(35) is collected below:

$$S^{12}u(m, \vec{0}) = s^u u(m, \vec{0}) \quad \text{and} \quad S^{12}v(m, \vec{0}) = s^v v(m, \vec{0}) \ . \tag{39}$$

With the help of (35) and (38), we find the following values for the spin eigenvalues s^u and s^v:

$$s^u = \frac{1}{2} \quad \text{and} \quad s^v = -\frac{1}{2} \ . \tag{40}$$

An interesting point to stress here concerns the polarizations of a particle (u) and the corresponding anti-particle (v) belonging to the same Dirac spinor (ψ). As a typical feature of 3 space-time dimensions, if a particle has spin s, its anti-particle has spin $-s$.

C. The Hamiltonian for ψ

Now, considering the Dirac equation for ψ:

$$(i\partial\!\!\!/ - m)\psi = 0 \ , \tag{41}$$

it follows that

$$i\frac{\partial}{\partial t}\psi = \left(i\gamma^0\vec{\gamma}.\vec{\partial} + \beta m\right)\psi \equiv H_0\psi \ . \tag{42}$$

Therefore, for the general massive Dirac spinor, ψ, the free Hamiltonian operator in momentum space, H_0, is given by:

$$H_0\psi\equiv(\vec{\alpha}.\vec{p} + \beta m)\psi \ , \tag{43}$$

where

$$\vec{\alpha} = \gamma^0\vec{\gamma} \quad \text{and} \quad \beta = \gamma^0 \ . \tag{44}$$

D. The spin of u and v

Let us consider the spin operator given by Eq.(38):

$$S^{12} = \frac{1}{2}\ \sigma_z \ , \tag{45}$$

and the free Hamiltonian operator in momentum space for the spinor ψ (Eq.(43)):

$$H_0\psi\equiv(\vec{\alpha}.\vec{p} + \beta m)\psi \ , \tag{46}$$

where $\vec{\alpha}$ and β are given by Eqs.(44). It can be easily shown that the following commutator vanishes

$$\left[H_0, S^{12}\right] = 0 \ . \tag{47}$$

This result ensures that the eigenvalues (s^u and s^v) of the spin operator, S^{12}, corresponding respectively to the wave functions u and v are indeed good quantum numbers to label physical states.

E. The charges of u and v

In order to determine the charges of the particles associated to the wave functions, u and v, it is necessary to compute the eigenvalues of the charge operator, Q, respected to the field operator, ψ. Its expansion in terms of the creation and annihilation operators reads as below:

$$\psi(x) = \int \frac{d^2\vec{k}}{(2\pi)^2}\frac{m}{k^0}\left[a(k)u(k)e^{-ik.x} + b^\dagger(k)v(k)e^{ik.x}\right] \ , \tag{48}$$

$$\overline{\psi}(x) = \int \frac{d^2\vec{k}}{(2\pi)^2}\frac{m}{k^0}\left[a^\dagger(k)\overline{u}(k)e^{ik.x} + b(k)\overline{v}(k)e^{-ik.x}\right] \ , \tag{49}$$

where the operators, $a^\dagger$ and $b^\dagger$, are the creation operators, and, a and b, are the annihilation operators.

With the help of the Dirac equation (33), the normalization conditions (36) and the relation

$$\left\{ p, \gamma^0 \right\} = 2p^0 \ , \tag{50}$$

the following equations are satisfied by the wave functions u and v:

$$u^\dagger(p)u(p) = \frac{p^0}{m} \quad \text{and} \quad v^\dagger(p)v(p) = \frac{p^0}{m} \ . \tag{51}$$

The microcausality fixes the following anticommutation relation:

$$\left\{ \psi(x), \psi^\dagger(y) \right\}_{x^0=y^0} = \delta^2(\vec{x} - \vec{y}) \ . \tag{52}$$

Now, by assuming the field operator expansions (48-49), and the normalization condition given by Eq.(51), the anticommutation relations between the creation and annihilation operators read:

$$\left\{ a(k), a^\dagger(p) \right\} = (2\pi)^2 \ \frac{k^0}{m} \ \delta^2(\vec{k} - \vec{p}) \quad \text{and} \quad \left\{ b(k), b^\dagger(p) \right\} = (2\pi)^2 \ \frac{k^0}{m} \ \delta^2(\vec{k} - \vec{p}) \ . \tag{53}$$

The charge operator, Q, associated to the field operator, ψ, is defined by the following normal ordering product:

$$Q = \int d^2\vec{x} : j^0(x) := -e \int d^2\vec{x} : \psi^\dagger(x)\psi(x) : \ , \tag{54}$$

which in terms of the creation and annihilation operators are given by

$$Q = -e \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{m}{k^0} \left[a^\dagger(k)a(k) - b^\dagger(k)b(k) \right] \ . \tag{55}$$

From the anticommutation relations (53) and the Eq.(55), for the charge operator Q, it can be easily shown that

$$\left[Q, a^\dagger(p) \right] = -e \ a^\dagger(p) \quad \text{and} \quad \left[Q, b^\dagger(p) \right] = +e \ b^\dagger(p) \ , \tag{56}$$

Let us denote the vacuum ground state by the “ket”, $|0\rangle$, such that

$$a(k)|0\rangle = 0 \quad \text{and} \quad b(k)|0\rangle = 0 \ , \tag{57}$$

where $\langle 0|0\rangle = 1$. Now, bearing in mind the commutation relations given by Eqs.(56), and applying them to the vacuum state, it follows that

$$Q|e^-\rangle = -e \ |e^-\rangle \quad \text{where} \quad |e^-\rangle = a^\dagger|0\rangle \ ; \tag{58}$$

$$Q|e^+\rangle = +e \ |e^+\rangle \quad \text{where} \quad |e^+\rangle = b^\dagger|0\rangle \ . \tag{59}$$

Due to these results, one concludes that:

1. $a^\dagger$ creates an electron (u) with spin $s^u = \frac{1}{2}$ and charge $-e$.

2. $b^\dagger$ creates a positron (v) with spin $s^v = -\frac{1}{2}$ and charge $+e$.

As a final conclusion, u is a wave function of an electron (e^-) with spin $s^u = \frac{1}{2}$, whereas v is a wave function of a positron (e^+) with spin $s^v = -\frac{1}{2}$. Some of the physical relevant results obtained in this Appendix are summarized in Table I.

Creation operator	Charge operator	Charge	Particle	Symbol	Wave function	Spin
$a^\dagger$	Q	$-e$	electron	e^-	u	$s^u = +\frac{1}{2}$
$b^\dagger$	Q	$+e$	positron	e^+	v	$s^v = -\frac{1}{2}$

TABLE I. Charge and spin of the particles associated to the field operator ψ.

	A_μ	ψ	φ	m_e	θ	e	y	μ	ζ	λ
d	1/2	1	1/2	1	1	1/2	0	1	1	0

TABLE II. Mass dimensions of the fields and parameters.

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