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The Program of the Eternal Universe

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Abstract

We present a short review of cosmological models that do not contain a global singularity. They represent scenarios of an *eternal* universe.

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1 Introduction

It has been said that the most crucial of the unsolved problems in the Einstein Cosmological Program can be stated as follows:

- Is the Universe *eternal* or did it have a *beginning*?

In other words, has space-time always been there or was there a time in which physical reality was not reducible to a succession of events represented in a four-dimensional continuum?

The need for physicists to go into such unusual question appeared more drastically in the last decade as a byproduct of the quantum analysis of the cosmological gravitational field. During the seventies the idea that the inevitability of the presence of a singular origin of the universe was a net consequence of the laws of Physics spread through the scientific community. The basis that supported such a view was provided by a series of theorems[1]. Although these theorems do not show that the gravitational field, say the curvature of spacetime, attained an infinite value (which one should naturally expect in order to characterize a given geometry as *singular*) they led to the belief that General Relativity plus some further conditions induce the presence of particular domains in spacetime in which properties related to the continuity of the geometry would no more be reliable. This program did not succeed. It was realized later that the possibility of a rational description of the universe should not be based on inaccesible initial conditions. However, it seems worth, just for the sake of completeness, to consider a typical example of the theorems that contributed a lot for the sustain of such an ideology.

2 The Singularity Theorems: Mathematical Basis for a Singular Nature of the Universe

There are many different approaches to the mathematical analysis of the singularity. A typical example was provided by S. Hawking who proved the theorem that follows:

The following requirements on a space-time $\mathcal{M}$ are mutually inconsistent:

- There exists a compact spacelike hypersurface (without boundary) $\mathcal{H}$;
- The divergence Θ of the unit normals to $\mathcal{H}$ is positive at every point of $\mathcal{H}$;
- $R_{\mu\nu}\, v^\mu\, v^\nu < 0$ *or* $= 0$ for each timelike vector v^μ;
- $\mathcal{M}$ is geodesically complete in past timelike directions.

Let us analyse briefly some of these requirements. Condition (i) is an hypothesis on the global behavior of the universe. It assumes that there is no closed timelike curves, for instance. Condition (ii) rests on the observational fact that our Universe is indeed expanding. This is the Hubble effect. Condition (iii) seems the weakest point. The possibility of its validity in our world rests on its identification to the positivity of the

[1] See for instance Penrose in ([1].

energy. It was formulated as an extension of the application of Einstein equation of GR under the presence of a perfect fluid. We shall see that different sources of gravity can violate such condition.

3 Historical Note

There is no doubt that the above mathematical scheme, if the conditions of their applicability were fulfilled in the real world, would imply the existence of peculiar regions of spacetime. One could even believe that some sort of drastic event could exist near these domains, like for instance the appearance of an enormous curvature. If this should be true then one could accept that these *singularity theorems* should indeed be an important achievement of classical General Relativity. Although its unquestionable beauty, simplicity and mathematical insight on the metrical properties of certain classes of Riemannian geometries, I think that summing up all analysis made during the last years, it is fair to state that these theorems are of very little help in a complete description of the actual Universe.

The main reason for this is due to the fact that it is far from being a definitive true that all conditions required from the theorems are fulfilled in the actual universe. It could appear strange, to the historian of this period of the scientific activity that it took almost twenty years for the relativist community to emphasize such doubt. The situation can be synthesized in the following way. The singularity theorems were so simple, their demonstration so well and elegantly presented by the authors, that soon they become identified with *the truth* concerning the actual properties of the universe. Even today there are not few theoreticians that still believe that a cosmological singular origin of our Universe is an inevitable consequence of the theory of gravity. Neverthless, for many different reasons[2] the general feeling today does not agree any longer with such an idea.

4 Non Singular Universe

We limit all our considerations here regarding the problem of the singularity in the realm of the above theorems. This means that we would concentrate our analysis to the exam of the following question:

- Are the conditions of applicability of the *singularity theorems* fulfilled in our Universe?

We will exhibit some examples that have been proposed and that answers negatively to this question. We describe only simple examples of a few proposals that are present in the literature concerning nonsingular cosmologies. In this choice we are guided by simplicity and restrained by the lack of space here. We will present a more thorough investigation of others distinct proposals elsewhere. The basis of such behavior can be associated among others, to one of the following schemes:

[2] A more complete history of such situation will be presented in a forthcoming book.

- Non-minimal coupling of gravity to other fields;
- Modification of the Riemannian structure of space-time, e.g., the Wist (Weyl Integrable SpaceTime);
- Modification of Einstein equation of gravity;
- Violation of the condition $R_{\mu\nu}\, v^{\mu}\, v^{\nu} < 0$.
- Quantum creation of the Universe.

Let us describe some examples of these alternatives models that seems worth to be analysed and that were presented at the Marcell Grossmann Meeting in Jerusalem, 1997.

5 Non Minimal Coupling

It has been argued that the Equivalence Principle should play the role of the true guide in the search of the manner in which matter fields couple to gravity. Such description dominated the scenario of the scientific community along decades. However, in the last years there has been a severe criticism on this and a lot of new arguments have been presented that goes beyond this approachs. For instance, it has been claimed that a scalar field should couple conformally to gravity, that is, its interaction Lagrangian should contain a non-minimal term involving the scalar of curvature R.

Once we accept that the Equivalence Principle should not be extrapolated to become a generator of physical laws, the actual coupling between matter fields and gravity should be founded elsewhere. The question we face is this:

- How do matter fields couple to gravity?

There is no unique answer to this question. We concentrate here on the examination of some possibilities that have been employed in cosmology. We shall see that this choice will have a crucial effect in the question of the cosmological singularity.

5.1 The Scalar Field

There are two principal models of coupling a scalar field to gravity that have been used. They are:

- Minimal Coupling.
- Conformal Coupling.

Since the minimal interaction does not produce a non singular cosmology, let us concentrate our analysis here on the conformal coupling. In order to apply the singularity theorems one has to analyse the sign of the quantity $R_{\mu\nu}\, v^{\mu}\, v^{\nu}$. The total Lagrangian (that is, gravity plus the scalar field) is given by

$$\mathcal{L} = \frac{1}{k}\, R + \partial_{\mu}\,\varphi\,\partial_{\nu}\,\varphi\, g^{\mu\nu} - \frac{1}{6}\, R\,\varphi^{2} + V(\varphi) \qquad (1)$$

The net effect of this coupling is to make two crucial modifications. The first one concerns the change of the gravitational coupling k which becomes the spacetime dependent effective coupling:

$$\frac{1}{k_{REN}} = \frac{1}{k} - \frac{1}{\varphi^2}.$$

Besides this, the righ-hand side is no more the energy-momentum tensor $t_{\mu\nu}$ but the extended conformal energy-momentum tensor $T_{\mu\nu}$:

$$T_{\mu\nu} = t_{\mu\nu} + \frac{1}{6}\,\Box\,\varphi^2 - \frac{1}{6}\partial_\mu\,\partial_\nu\,\varphi^2$$

in which $t_{\mu\nu}$ is given by

$$t_{\mu\nu} = \partial_\mu\,\varphi\partial_\nu\,\varphi - \frac{1}{2}g_{\mu\nu}\left(\partial_\lambda\,\varphi\partial^\lambda\,\varphi + V(\varphi)\right)$$

6 Eternal Universes Generated by Scalar Fields

A typical solution of the cosmological geometry free of singularity was proposed independently by many authors. Just to provide one single example of these models let us consider the following case described by Melnikov and Orlov in 1979.

Guided by the features of the mechanism of spontaneous symmetry breaking they tried to find a geometry such that the in the semi-classical regime[3] the scalar field lies on its fundamental state given by

$$< \, 0\,\varphi\,0 \, >= \nu\,\frac{f(\eta)}{A(\eta)} \tag{2}$$

in which η is the conformal time of an open Friedmann universe.

$$ds^2 = A^2\left\{\,d\eta^2 - d\chi^2 - sinh^2\chi(d\theta^2 + sin^2\theta\,d\Phi^2\right\} \tag{3}$$

The radius of the universe takes the simple non singular form in terms of the global time:

$$A(t) = \sqrt{t^2 + Q^2}$$

in which Q is a constant.

7 Eternal Universes Generated by Electromagnetic Field

There are seven possible cases of non-minimal coupling of a vector field with gravity. They can be divided into two classes:

[3]The scalar field was treated as a quantum field although the geometry was taken as a classical object.

- Class One

$$L_1 = R\, W_\mu\, W^\mu$$

$$L_2 = R_{\mu\nu}\, W^\mu\, W^\nu$$

- Class Two

$$L_3 = R\, F_{\mu\nu}\, F^{\mu\nu}$$

$$L_4 = R\, F^*_{\mu\nu}\, F^{\mu\nu}$$

$$L_5 = R_{\mu\nu}\, F^\mu_\alpha\, F^{\alpha\nu}$$

$$L_6 = W_{\mu\nu\alpha\beta}\, F^{\mu\nu}\, F^{\alpha\beta}$$

$$L_6 = W^*_{\mu\nu\alpha\beta}\, F^{\mu\nu}\, F^{\alpha\beta}$$

The first class breaks the gauge invariance but has the right dimension. The second class needs the introduction of a constant of dimension of lenght. The consequences of these possible candidates have been examined in the literature[4]. For our purposes here, it is enough to limit ourselves to just one case.

From the fact that it is not possible to compatibilize the spatial homogeneity and isotropy of the standard FRW model with the presence of a vector field involving a privileged direction, we must set that the Electric and the Magnetic parts of the field must vanishes. This imposes that the coupling of the field with gravity must be of the first class[5]. We set the Lagrangian to be

$$L = \frac{1}{\kappa}\, R - \frac{1}{4}\, F_{\mu\nu}\, F^{\mu\nu} + \beta\, R\, W_\mu\, W^\mu \tag{4}$$

with $F_{\mu\nu} = \partial_\nu\, W_\mu - \partial_\mu\, W_\nu$.

The equations of motion that follow are given by:

$$(\frac{1}{\kappa} + \beta\, W^2) G_{\mu\nu} = \beta\, \Box\, W^2 g_{\mu\nu} - \beta\ \nabla_\mu\ \nabla_\nu W^2 - \beta\, R\, W_\mu\, W_\nu - E_{\mu\nu} \tag{5}$$

and

$$\nabla_\nu\ F^{\mu\nu} = -\beta\, R\, W^\mu \tag{6}$$

[4]See ([?]) for more informations and references.

[5]This is due to the fact that the *potential vector* W_μ must then be an observable quantity: the electromagnetic vector field at these cosmical conditions must violate gauge invariance.

in which $E_{\mu\nu}$ is nothing but Maxwell stress tensor

$$E_{\mu\nu} = F^{\alpha}_{\mu}\, F_{\alpha\nu} + \frac{1}{4}\, g_{\mu\nu} F_{\mu\alpha}\, F^{\mu\alpha}.$$

In a FRW geometry

$$ds^2 = dt^2 - A^2(t)\, d\sigma^2$$

we set the ansatz

$$W_\mu = W(t)\, \delta^0_\mu.$$

Then, it follows that $F_{\mu\nu}$ vanishes. A solution was found such that the coupled set of gravity plus electrodynamics non-minimally coupled is given by

$$W^2(t) = \frac{1}{\kappa}(1 - \frac{t}{A})$$

and for the radius of the universe:

$$A(t) = \sqrt{t^2 + Q^2}.$$

in which Q is a constant that measures the minimum radius of the universe. For $Q = 0$ the system reduces to the empty Minkowski spacetime in Milne coordinates.

7.1 Weyl restricted geometry

In this section I present an example of a cosmological modification of the Riemannian nature of spacetime. it can be alternatively interpreted as the geometrization of a scalar field, the dilaton.

The surmise of a Weyl geometrical background configuration leads to a non-standard cosmological scenario which, in the case of a homogeneous and isotropic line element, admits a non-singular, eternal Friedman-like solution exhibiting the following main features:

1. the evolution of the Universe begins at the infinitely remote past due to the unstability of a spatially infinite, empty Minkowski space-time;

2. this matter-free open Universe, driven by the energy associated to a geometrized homogeneous Weyl field $\omega(t)$, collapses adiabatically until a minimum radius a_0 is approached;

3. in the course of this everlasting collapse, the Universe is always accelerated (or "inflationary") and any occasional matter fluctuation is exponentially suppressed;

4. near to the phase of maximum contraction, the cosmic evolution enters in a non-adiabatical regime in which the collapse is reverted to an expansion;

5. this bouncing may be associated to the propagation of a Weyl instanton ("Wiston") in an Euclideanized, classically-forbidden region;

6. as the expanding phase initiates, matter (e.g., photons) and entropy fluctuations are exponentially amplified at the expenses of the energy of the Weyl field;

7. an eventual baryon excess taking place at the start of the expanding era may be amplified as well;

8. this mechanism of matter-entropy production saturates soon, and the cosmic evolution attains a standard, radiation-dominated Friedman configuration.

Given the properties outlined above, this Friedman-like cosmological model describes an eternal, bouncing Universe, created from a Minkowskian "Nothing", in which the singularity, horizon and flatness problems of standard Cosmology do not occur; the model also provides a geometry-driven mechanism able to control the production of large amounts of matter and

entropy, due to the amplification of vacuum fluctuations. Once the environment temperature is always bounded, this creation process stands for a "Big—but finite—Bang" event. The observed presence of a baryon excess also fits naturally within the proposed scheme.

To achieve the demonstration of the above statements, the contents of the present paper are arranged as follows. In Section II we discuss the physical and cosmological motivations and present the basic assumptions of the proposed scenario. Section III provides a brief survey on the necessary mathematical machinery of Weyl space theory. In Section IV we derive a Friedman-like non-singular solution and comment upon its properties. In Section V, the construction of a mechanism of matter-entropy production is detailed and other issues concerning thermodynamical processes are discussed. Section VI, finally, contains a short account of the results obtained previously and some concluding remarks.

8 Motivations

8.1 Primordial Cosmology

In standard HBB models, with conventional matter as source, the Universe has a singular origin. This means that the scale factor $a\,(t)$ of a spatially homogeneous and isotropic FRW line element

$$ds^2 = dt^2 - a^2(t)d\sigma^2 \tag{7}$$

vanishes at a finite time t_0 in the past[10]. Due to this distinctive feature, in addition to its well-known observational successes—the incorporation, in a natural way, of the evidence concerning the Hubble expansion, the presence of a cosmic microwave background radiation and the primordial relative abundances of the chemical elements—the HBB program also leads to a bunch of difficult questions. In the literature, the lists of "standard troubles" usually comprise the following items:[10,11]

- the occurrence of causal limitations to the cosmic homogeneity ("horizon" problem);

- the apparently Euclidean nature of space ("flatness" problem);

- the explanation of the prevalence of matter against anti-matter and of the observed ratio of entropy per baryon ("baryon asymmetry" problem);

- the elaboration of an accurate perturbative scheme to allow for galaxy formation.

These issues are seen to be related to the fact that very specific initial conditions are required in order to guarantee a proper cosmic evolution to the later stage we observe today; and, particularly remarkable,

the "singularity" problem, which concerns the absolutely unscrutable provenance of the physical world from the HBB initial singularity: no causal description of the behavior of the Universe could be expected to include the singular origin in view of the divergent (infinite) values assumed by physical quantities at the creation instant t_0. The application of the notion of thermodynamical equilibrium to the cosmic "fluid" under such extreme conditions also seems doubtful.[12].

In recent years the horizon and "flatness" problems have been attacked by means of various sorts of conjectures based on a rapidly expanding primordial phase ("inflation") of the cosmic evolution, associated to a De Sitter solution[7]. Most inflationary scenarios have in common the fact that the De Sitter phase starts from a non-vanishing value of the cosmic radius. The introduction of such finite radius, however, does not necessarily contradict the occurrence of an initial singularity; in many approaches the inflationary phase is spread between two standard radiation-dominated eras, eventually preceded by a standard singularity. Nevertheless, different authors were motivated to consider non-singular, inflationary models in order to explore the appealing possibility of generating a classical structure—such as the De Sitter space-time—from a typically quantum process such as quantum vacuum tunneling[4,5]. Let us sketch briefly their argumentation.

The De Sitter solution is provided by Einstein's equations for the "vacuum", generically represented by a (positive) cosmological constant $\Lambda = 3\zeta^2$. Accordingly, the scale factor $a(t)$ satisfies the Friedman equation

$$\dot{a}^2 - \zeta^2 a^2 = -\varepsilon \tag{8}$$

where $\varepsilon = \pm 1$. Some authors privilege closed worlds ($\varepsilon = 1$) since basic material properties such as total mass and charge can be made null[3,18]. In the closed case, a typical De Sitter solution is obtained as

$$a(t) = 1/\zeta \cosh(\zeta t) \tag{9}$$

Now this solution exhibits an apparent deficiency: the occurrence of a primeval collapsing phase of *infinite* duration, thus implying an everlasting cosmic history. This is an already traditional difficulty of bouncing eternal universe models in general, irrespective of whether one or many bounces are allowed[19]. The problem is to conceive the behavior of matter in the course of, say, a collapse-expansion sequence of unlimited duration. If gravity can somehow produce particles, for instance, an infinite amount of matter—and entropy—must have been produced during the past collapsing phase. Thus such eternal longevity, whereas it could be helpful in resolving some "standard troubles" (the horizon problem, for example)[11,20], is hardly conciliated with finite values of entropy and matter production, unless some type of saturation mechanism has been in action throughout the infinite past evolution.

This difficulty can be easily surmounted through the assumption that the infinitely old collapsing era simply did not exist, due to quantum effects dominating the cosmic behavior near the region of maximum contraction. The overall picture is that the Universe began its spatio-temporal evolution directly in a classical metric configuration of the De Sitter type, endowed with a minimum radius a_0. Once this typical dimension a_0 is taken to be of the order of the Planck length L_P, quantum processes shall be invoked to lay the physical foundations for the emergence of the classical De Sitter stage. An ingenious way to supply a framework in which the quantum generation of a classical structure can be depicted is to resort to a quantum version of the theory embodied in Eq. (2). A semi-classical approximation of this theory[21] is provided by an Euclideanization procedure in which Eq. (2) is interpreted as a dynamical process consisting of a particle with position $q = a$, submitted to a potential $V(q) = -\zeta^2 q^2$. One then obtains that the minimum value q_0 classically allowed is given by $a_0 = 1/\zeta$, and the corresponding classically forbidden region is described by the Euclideanized equation

$$\dot{a}^2 = 1 - \zeta^2 a^2. \tag{10}$$

In this way, a quantum tunneling process—represented by an instanton solution of Eq. (4) — may provide a connection between the quantum and the classical regimes: a De Sitter space-time appears as the "terminal point" of a De Sitter instanton propagating in the classically forbidden region[4]. The interpretation of the physical status of the Euclideanized region is controversial: some authors argue in favor of its physical reality—therefore admitting, implicitly, a non-Lorentzian phase of the cosmic evolution[22]—whereas others support the opinion that such region is purely virtual once it does not define an actual space-time structure, at least in a classical sense.[23] According to this view, the virtual Euclidean stage is to be associated to a structureless quantum vacuum state denominated "Nothing"[3,4]. Thus, in this view, the Universe was created, through a quantum tunneling process, from a "Nothing" state identified to a instanton solution of the Euclideanized equation Eq. (4).

8.2 Unstabilities of Minkowski space-time

In the quantum creation models outlined above the Universe seemingly manifests a very specific preference to tunnelate into a De Sitter configuration—and not, for instance, into Minkowski space-time, which in the general-relativistic context is understood as an empty (i.e., completely matter-free) Universe, and so, in this sense, a truly "fundamental" state of Einstein's dynamics[24]. Both configurations, moreover, display the maximum number of symmetries admitted by Einstein's theory. A possible explanation is that the De Sitter solution (due to the presence of a cosmological constant term) allows for the occurrence of vacuum fluctuations—which is a indispensable condition for the ulterior appearance of matter—while Minkowski space is classically as well as quantum-mechanically stable against statistical perturbations.[25]

However, the statements about the stability of the Minkowski vacuum quoted above are ultimately model-dependent, since they rely on perturbative schemes related, in different ways, to specific descriptions of matter properties. On the other hand, it could be argued that *unstable* Minkowski spaces constitute very appealing candidates to perform the role of a cosmic proto-structure: since Minkowski spaces bear no distinctive trace, they possess no causal "memory"—whatever the conjectural process leading to a Minkowski vacuum, its effects would be utterly erased of any causal chain established subsequently. Therefore, an unstable Minkowski configuration is indeed "original"; the choice of the precise type of perturbations yielding such unstability requires, of course, further discussion.

The problem may be posed as follows: if the assumption that the actual, observed Universe developed from fluctuations of an unstable empty space-time is accepted, could we devise a sufficiently generic (i.e., independent of matter properties) perturbative scheme so as to provide for a smooth evolution to the present Friedmanian era? This question probably does not have an unique, definitive answer; nonetheless, we may attempt to shed some light upon it through the examination of a particular model.

According to the Special Theory of Relativity, Minkowski space-time constitutes the fundamental descriptive arena in which inertial observers shall compare their measurements of distances and durations in order to supply an absolute meaning to the laws of Physics[26]. Once Minkowski spaces are devoid of any matter-energy content, their characterization depends exclusively on the spatiotemporal determination of physical events by a class of ideal observers through the *gedanken* exchange of light signals. Given the assumption that, in the sake of generality, fluctuations of the matter-energy content shall be discarded, the only remaining physical system available to be perturbed consists of the basic framework of the measurement procedure itself, that is, the idealized apparatuses of clocks and rods employed to quantify separations and intervals. *Our working hypothesis therefore addresses the induction of unstabilities of Minkowski space through perturbations of the system of measure units.* More specifically, we will consider "structural" fluctuations of Minkowski geometry in the general form

$$\delta \left(g_{\mu\nu;\lambda} \right) = \left(\delta \omega_\lambda \right) g_{\mu\nu} \tag{11}$$

in which $\omega_\lambda = \partial_\lambda \omega, \omega(x)$ being a scalar field defined on the background manifold, and the semi-colon stands for covariant differentiation. In Section III we will show how fluctuations of this type may be ascribed to variations of measuring scales.

8.3 The "Structural Problem"

In the context of the standard formulation of space-time dynamics put forth by Einstein's General Theory of Relativity the hypothesis underlying Eq. (5) is rendered inconsistent from the outset, once no room is left for variations of measuring apparatuses of the kind outlined above. This may be seen as a consequence of the stringent requirements imposed upon the characteristics of space-time by the rules of General Relativity, according to which the behavior of clocks and measuring rods must be determined exclusively by the metric properties (i.e., the metric tensor) of the underlying manifold. This implies, in turn, that the structure of physical space-time must correspond unequivocally to that of a Riemannian manifold, in which covariant derivatives of the metric tensor vanish (i.e., $g_{\mu\nu;\lambda} = 0$). Indeed, if this condition is fulfilled, the manifold affine connections $\Gamma^{\alpha}_{\mu\nu}$ become identical to the Christoffel symbols $\left\{ {\alpha \atop \mu\nu} \right\}$ of Riemann geometry[26]. The same result may be obtained *a posteriori* by means of the Palatini variational method (see Section III).

Therefore, the adoption of perturbations of Minkowski space-time in the form of Eq. (5) requires (or, conversely, induces) modifications of the affine nature of space-time—which, according to the rules of General Relativity, should be (either on *a priori* or *a posteriori* grounds) strictly Riemannian. In the course of the last decades, however, these requirements have been questioned both from the axiomatical and the observational standpoints. Indeed, both types of approaches lead to the conclusion that space-time structure is *not* completely described by a simple Riemannian manifold. Following the attempts of Ehlers, Pirani and Schild[27] to supply, through the consideration of ideal operations of elementary clocks and rods, an axiomatical foundation for the geometrical nature of space-time, one is led to the assertion that the "...Weylian structure of space-time is axiomatically well-founded, whereas its Lorentzian structure is not"[28]. On the other hand, the *observational* determination of the behavior of measuring instruments, besides the metric tensor $g_{\mu\nu}(x)$, must also involve a scalar function $\omega(x)$ in order to guarantee the conformal invariance of null light cones (not to be confused with the principle of conformal invariance of *all* physical laws—see Section III)—which constitute the most important observational aspects of the background geometry; in consequence, a *conformally-Riemannian* structure is involved, rather than a Riemannian one[29].

These considerations compel us to conclude that a new, much deeper problem is embodied in an eventual change of the affine nature of space-time, as suggested in Eq. (5): accepting that currently the structure of space-time is indeed Riemannian, are there sound reasons to believe that it has always been so? Or, in a more rigorous, formal sense: given the fact that space-time structure is such that on a certain hypersurface Σ_0 the covariant derivative of the metric tensor vanishes, $g_{\mu\nu;\lambda}(\Sigma_0) = 0$—which corresponds to a Riemannian configuration—what can be said about the value of $g_{\mu\nu;\lambda}(\Sigma_1)$ on another hypersurface Σ_1? Thus, in addition to the "standard troubles" quoted before, a new component must be brought to our cosmological investigations: the determination of the evolutionary pattern of the geometrical background affine character. This is a restricted form of what we may call a *"structural problem"*, which in its broadest scope concerns any kind of possible variations of the basic nature of the structure of space-time.

There is, in principle, an unlimited number of physical scenarios in which some kind of structural change might take place—so, put in a completely unrestricted form, "structural problems" seem hopelessly vague. In order to address them in a proper way, a definite conceptual context—*i.* e., a cosmological model—for the description of such structural transitions must be provided. In the literature, different sorts of modifications have been proposed, within either classical or quantum approaches, thereby resulting effects such as, for instance, variations of topological properties[30] or changes in the signature of the metric[31]. In the present paper, likewise, the hypothesis conveyed in Eq. (5) with respect to scale fluctuations of the Minkowski vacuum corresponds to a specific assumption about the evolution of the value of $g_{\mu\nu;\lambda}(\Sigma)$ and, accordingly, of the non-Riemannian character of the geometrical background. In view of the axiomatical and observational arguments mentioned previously, and in accordance with Eq. (5), we will assume thereof that the basic structure of space-time is of a *conformally-Riemannian* type. Conformally-Riemannian geometries are more commonly acknowledged as *Weyl-integrable space-times* (WISTs). In Section III we will supply a brief account of the essentials of the theory of Weyl spaces[32].

In summary, we will deal here with a specific structural problem in which the evolution of the Universe is provoked by unstabilities of an "original" empty Minkowski space, due to measuring scales perturbations associated to a WIST background manifold. On the grounds of the good experimental status currently enjoyed by Einstein's General Theory of Relativity, objections could be raised against the surmise of the abandonment of the Riemannian configuration—particularly if modifications of the well-tested *local* characteristics of the gravitational field are implied. However, the enlargement of the structure of space-time to a WIST configuration proposed here does not lead to a new theory of gravitational phenomena, once *global* effects only (in the sake, say, of the inclusion of a cosmological constant term into Einstein's equations) are induced—as we will see in what follows.

9 A Brief Review of the Theory of Weyl Spaces

9.1 Introduction to Weyl spaces

In view of the reasons put forth in the previous section, we are interested in exploring the suggestion that space-time structure exhibits a Weylian character. A Weyl geometry is an affine manifold specified by a metric tensor $g_{\mu\nu}(x)$ and a "gauge" vector $\omega_\mu(x)$ which participate in the definition of the manifold affine connection $\Gamma^\alpha_{\mu\nu}(x)$. Besides the MMG group of Riemannian structures, Weyl geometries admit internal ("gauge") transformations which are intimately connected to point-dependent variations of measuring scales. Due to this property, such geometries have been considered, for example, in abelian gauge theories—as in Weyl's original attempt to unify, on a geometrical basis, electromagnetism and gravitation[32]; and in theories addressing the conformal invariance of physical processes—such as in the scale-invariant theories of Dirac[33] and Canuto[34]. It is important to remark that both attempts have failed, mainly due to the fact that physical laws are *not* conformally invariant.[35] Moreover, the most generic cases of Weyl geometries provoke the so-called "second clock effect", leading to observational inconsistencies. Before demonstrating how such difficulties can be circumvented, let us provide the reader with some necessary definitions and notations.

In Weyl geometries the rule of parallel transport of a given vector requires a non-vanishing covariant derivative of the metric tensor $g_{\mu\nu}$:

$$g_{\mu\nu;\lambda} = g_{\mu\nu}\omega_\lambda \tag{1}$$

in which $\omega_\lambda(x)$ is the gauge vector and the semi-colon denotes covariant differentiation in a general affine sense. This implies that vector lengths may vary along transport or, equivalently, that the units of measure may change locally. Remark, in contrast, that one of the attractive results of Einstein's theory of gravitation is that it contains *a posteriori* the Riemannian characterization of space-time structure. The argument is simple and is commonly related to the Palatini variational procedure[36] as follows:

Consider the theory given by Einstein's Lagrangian

$$L_E = \sqrt{-g}R \tag{2}$$

varying in the Palatini fashion, that is, taking both the metric tensor $g_{\mu\nu}$ *and* the (as yet unspecified) affine connection $\Gamma^\alpha_{\mu\nu}$ as independent geometric variables, one obtains

$$[\delta g_{\mu\nu}] : \qquad R_{\mu\nu} = 0 \tag{3}$$

$[\delta\Gamma^{\alpha}_{\mu\nu}]$: $$g_{\mu\nu\|\lambda} = 0 \tag{4}$$

where $R_{\mu\nu}$ is the Ricci tensor and the double bar denotes covariant differentiation in a Riemannian sense, i.e., making use of the Christoffel symbols

$$\left\{ {\alpha \atop \mu\nu} \right\} \equiv 1/2 g^{\alpha\lambda} \left[g_{\mu\lambda},_{\nu} + g_{\nu\lambda},_{\mu} - g_{\mu\nu},_{\lambda} \right] \tag{5}$$

of Riemann geometry (commas indicate simple differentiation). Therefore, a Riemann configuration—characterized by Eq. (4), which implies that vector lengths do not change under parallel transport—is obtained as a direct consequence of the variational procedure.

9.2 WIST

However, this is a model-dependent result. Other Lagrangians will yield different geometrical configurations. Consider, for instance, the theory of a scalar field $\phi(x)$ in the form

$$L = \sqrt{-g} f(\phi) R + \mathcal{L}(\phi) \tag{6}$$

Variation *a la* Palatini (with ϕ, $g_{\mu\nu}$ and $\Gamma^{\alpha}_{\mu\nu}$ as independent variables) now gives Eq. (1) in place of Eq. (4), with

$$\omega_{\lambda} = -\left[\ln f(\phi)\right],_{\lambda} \tag{7}$$

Thus the variational principle leads to a special kind of Weyl geometry and *not* to a Riemann space[36]. This particular type of Weyl geometries—in which the gauge vector is the gradient of a scalar function—is called a *conformally-Riemannian* or *Weyl-integrable space-time* (WIST), and in fact constitutes the basis of the cosmic scenario examined here. Its fundamental importance for the present developments stems from the following reason: according to the definition of a Weyl space, variations of the units of measure are controlled by the gauge vector $\omega_{\mu}(x)$. Weyl suggested that in the course of an infinitesimal parallel transport dx^{α} the length $L = g_{\mu\nu}\ell^{\mu}\ell^{\nu}$ of a given vector $\ell^{\mu}(x)$ is changed according to the first-order expression

$$dL = L\omega_{\alpha}dx^{\alpha} \tag{8}$$

This result implies, in general, observational difficulties. Suppose, for instance, that at a given space-time point A two identical clocks are synchronized. According to General Relativity, if these two clocks travel to another point B through distinct paths, gravitational effects may cause them to lose their synchronization. This is the “first clock effect”. In Weyl spaces, due to the distinct variation of the units of measure along the two different paths, the discrepancy between time measurement units at B might add a supplementary contribution to the loss of synchronization—called the “second clock effect”. This effect was the root of Einstein’s criticism against Weyl’s original proposal of unification, once in the case of *closed* circuits such additional synchronization loss would disagree with well-known observations.[28] To overcome this objection, one has to impose the coincidence of the units of measure of both observers at A, regardless of the particular closed path chosen; this implies that

$$\oint dL = 0. \tag{9}$$

But according to Stoke's theorem this condition leads precisely to the result that $\omega_{\mu},_{\nu} - \omega_{\nu},_{\mu} = 0$, that is,

$$\omega_{\mu} = \omega,_{\mu} . \qquad (10)$$

Thus the corresponding Weyl structure is characterized by a gauge vector which is the gradient of a scalar function—a Weyl geometry in which length variations are integrable along closed paths or, in short, a WIST. It is interesting to remark that the variational procedure sketched above (Eq. (6)) does not lead to a general Weyl space, but specifically to a WIST configuration, in which the "second clock effect" results eliminated.

9.3 Conformal invariance

From Eq. (1) it is simple to derive the expression of the Weyl affine connection $\Gamma^{\alpha}_{\mu\nu}$:

$$\Gamma^{\alpha}_{\mu\nu}(x) = \left\{ {}^{\alpha}_{\mu\nu} \right\} - 1/2 \left[\omega_{\mu}\delta^{\alpha}_{\nu} + \omega_{\nu}\delta^{\alpha}_{\mu} - g_{\mu\nu}\omega^{\alpha} \right] \qquad (11)$$

Consider now a conformal mapping of the metric tensor $g_{\mu\nu}$ such as

$$\widetilde{g}_{\mu\nu} = \Omega^2(x)\, g_{\mu\nu} \qquad (12)$$

in a given *Riemann* geometry. It then follows that the corresponding transformed connection is given by

$$\widetilde{\Gamma}^{\alpha}_{\mu\nu} = \left\{ {}^{\alpha}_{\mu\nu} \right\} + (1/\Omega) \left[\Omega,_{\mu} \delta^{\alpha}_{\nu} + \Omega,_{\nu} \delta^{\alpha}_{\mu} - g_{\mu\nu} g^{\alpha\varepsilon} \Omega,_{\varepsilon} \right] \qquad (13)$$

Setting $\omega(x) = -\ln \Omega^2(x)$, one obtains that connections Eqs. (11, 13) are equivalent when Eq. (10) holds. Thus Weyl-integrable space-times are also called conformally-Riemannian, since a conformal transformation maps a Riemann geometry into a WIST one. If the laws of physics were invariant with respect to conformal transformations, the WIST scalar function $\omega(x)$ would be unobservable and both structures could not be distinguished by any physical effect. The hypothesis of the conformal invariance of all physical processes, in fact, provided the basis for the approaches of Dirac ("Large Number Hypothesis") and Canuto and co-workers ("Scale-invariant theory"), who advocated the introduction of a new general symmetry (besides MMG) in Physics: the gauge invariance of measuring units[34]. Despite the elegancy of these proposals, eventually astrophysical observations brought in decisive evidence against the assumption of a general conformal symmetry of physical laws[35]. Therefore the WIST field $\omega(x)$ cannot, in principle, be discarded by a convenient gauge choice; it suffices to dynamically break the global conformal invariance of a given WIST theory in order to distinguish it, under conformal transformations, from its Riemannian counterpart. In consequence, $\omega(x)$ constitutes a true (i.e., observable) field and Riemann and WIST configurations are physically distinguishable.

9.4 Some useful quantities

Given the Weyl connection Eq. (11), it is straightforward to write Weylian expressions for geometrical objects with the use of the corresponding Riemannian formulae; the covariant differentiation of a vector field V^α reads, for instance,

$$V^\alpha_{;\mu} = V^\alpha{}_{,\mu} + \left\{ {}^{\alpha}_{\mu\nu} \right\} V^\upsilon - 1/2 \left[\omega_\mu \delta^\alpha_\nu + \omega_\nu \delta^\alpha_\mu - g_{\mu\nu}\omega^\alpha \right] V^\nu = \\ = V^\alpha_{||\nu} - 1/2 \left[\omega_\mu V^\alpha + \omega_\nu V^\nu \delta^\alpha_\mu - g_{\mu\nu} V^\nu \omega^\alpha \right] \tag{14}$$

In particular, the contracted (Ricci) curvature tensor $R_{\mu\nu} \equiv R^\alpha_{\mu\alpha\nu}$ can be written in terms of its Riemannian counterpart $\widehat{R}_{\mu\nu}$ and the gauge vector ω_μ as follows:

$$R_{\mu\nu} = \widehat{R}_{\mu\nu} - \frac{3}{2}\omega_{\mu||\nu} + \frac{1}{2}\omega_{\mu||\nu} - \frac{1}{2}\omega_\mu\omega_\nu - \frac{1}{2}g_{\mu\nu}\left[\omega^\lambda_{||\lambda} - \omega_\lambda\omega^\lambda\right] \tag{15}$$

which, in the case of a WIST, reduces to

$$R_{\mu\nu} = \widehat{R}_{\mu\nu} - \omega_{\mu||\nu} - \frac{1}{2}\omega_\mu\omega_\nu - \frac{1}{2}g_{\mu\nu}\left[\widehat{\Box}\,\omega - \omega_\lambda\omega^\lambda\right] \tag{16}$$

where $\omega_\mu = \partial_\mu\omega, p$ is the D Alembertian operator and the symbol $\hat{}$ denotes objects constructed in the associated Riemannian structure (i.e., making use of Christoffel symbols only). Contracting Eq. (16) one obtains the WIST scalar curvature R :

$$R = \widehat{R} - 3\omega^\mu_{||\mu} + \frac{3}{2}\omega_\mu\omega^\mu = \widehat{R} - 3\,\widehat{\Box}\,\omega + \frac{3}{2}\omega_\mu\omega^\mu \tag{17}$$

10 A Friedman-like Cosmological Model

10.1 Dynamical equations

Let us then proceed to the implementation of the investigative program discussed in Section II. We consider the veritable primordial phase of the evolution of the Universe to correspond to a "Nothing" state described by an empty Minkowski space-time. In order to provoke the unstability of this basic configuration we resort to perturbations of the system of measuring units as in the form given in Eq. (2.5),

$$\delta\left(g_{\mu\nu;\lambda}\right) = \left(\delta\omega_\lambda\right)g_{\mu\nu} \tag{1}$$

Since this is a particular case of Eq. (3.1), we are explicitly assuming that the background geometry is endowed with a Weylian structure.

The subsequent evolution of the Cosmos depends, of course, on the behavior of the perturbations $\delta\omega_\lambda$. Thus a dynamical framework is required in which the pair $(g_{\mu\nu}, \omega_\lambda)$ constitutes the set of fundamental geometrical variables. A simple action involving this pair is given by

$$S_c = \int \sqrt{-g}\left[R + \xi\omega^\lambda_{;\lambda}\right] \tag{2}$$

in which ξ is a dimensionless parameter. Two points deserve comment here: firstly, in view of the arguments put forth in Section III about the "second clock" effect, we will restrict our considerations to a WIST configuration, *i. e.*, we assume that $\omega_\lambda(x) = \partial_\lambda\omega(x)$ in *Eq.* (2). Then the set of independent variables is actually reduced to $(g_{\mu\nu}, \omega)$. Secondly, the attentive reader will be aware of the presence of a total divergence term in the action S_c; in the usual Riemannian context of General Relativity, the contribution of total divergence terms to the dynamical equations vanishes. Remark, however, that according to Eq. (3.14) one has for the divergence of the gauge vector $\omega_\lambda(x)$ the expression

$$\omega^{\lambda}_{;\lambda} = \omega^{\lambda}_{\lambda} - 1/2\omega^{\lambda}\omega_\lambda = 1/\sqrt{-g}\left(\sqrt{-g}\omega^{\lambda}\right),_{\lambda} - 1/2\omega^{\lambda}\omega_\lambda \tag{3}$$

in the WIST case, and so a non-vanishing contribution to the dynamics is obtained. Decomposition Eq. (3) also shows that if a term proportional to $\omega^\lambda\omega_\lambda$ is included in the Lagrangian in Eq. (2) the net result is just a renormalization of parameter ξ.

Variation of action S_c with respect to the pair $(g_{\mu\nu}, \omega)$ of independent WIST variables yields the equations of motion

$$R_{\mu\nu} - 1/4Rg_{\mu\nu} + \omega,_{\mu\|\nu} = 0 \tag{4}$$

and, consequently,

$$\widehat{\Box}\,\omega = 0, \tag{5}$$

in which the double bar denotes Riemannian covariant differentiation and $\widehat{p}$ is the D' Alembertian operator written in the associate Riemann space-time, *i.* e., Eq. (5) reads

$$\widehat{\Box}\,\omega = 1/\sqrt{-g}\left(\sqrt{-g}\omega,_{\alpha} g^{\alpha\beta}\right),_{\beta} = 0 \tag{6}$$

Let us at this point remind the reader that we do not aim to associate action S_c to a new theory of gravity. We treat S_c, instead, as an effective canonical action which results of a combination of geometrical components (metric $g_{\mu\nu}$ and gauge vector ω_λ) of distinct nature. Nevertheless, in order to simplify our understanding of the cosmological consequences of the present model it is useful to recast the set Eqs. (4,5) of WIST dynamical equations in terms of a Riemannian configuration plus an external source term represented by the scalar field $\omega(x)$. This re-interpretation is legitimate due to the decomposition Eq. (3.16) of the WIST contracted curvature tensor $R_{\mu\nu}$ in terms of the associated Ricci tensor $\widehat{R}_{\mu\nu}$ and functions of the scalar field $\omega(x)$. In this vein, Eqs. (4, 5) can be rewritten as follows:

$$\widehat{R}_{\mu\nu} - 1/2\widehat{R}g_{\mu\nu} - \lambda^2\omega_\mu\omega_\nu + \lambda^2/2\omega_\alpha\omega^\alpha g_{\mu\nu} = 0 \tag{7}$$

$$\widehat{\Box}\,\omega = 0 \tag{8}$$

in which $\lambda^2 = \left[\frac{4\xi-3}{2}\right]$.[37] Eq. (7) is thus equivalent to an Einstein equation in which the WIST field ω provides the source of the Riemannian curvature.

Once in the present paper we will be concerned exclusively with spatially homogeneous FRW cosmologies, described by the line element Eq. (2.1), it is natural to make the WIST field ω a function of the cosmic time t only: $\omega = \omega(t)$. The gauge vector ω_λ then becomes

$$\omega_\lambda = \partial_\lambda\omega(t) = \dot{\omega}\,\delta^0_\lambda \tag{9}$$

where the dot denotes simple differentiation with respect to the time variable. In this case, the ω-dependent "source" term in the Einstein equation Eq. (7) may be seen to represent a "stiff matter" state of a perfect fluid, endowed with a *negative* energy, once if Eq. (7) is rewritten in the form

$$\widehat{R}_{\mu\nu} - 1/2\widehat{R}g_{\mu\nu} = -T_{\mu\nu}\,(\omega) = -\left[(\rho_\omega + p_\omega)\,V_\mu V_\nu - p_\omega g_{\mu\nu}\right] \tag{10}$$

in which $\{V_\mu\}$ is a set of unit time-like vectors, one obtains for the energy density ρ_ω and the isotropic pressure p_ω the values

$$\rho_\omega = p_\omega = -\lambda^2/2\dot{\omega}^2 \tag{11}$$

and so the equation of state of the "ω-fluid" indeed corresponds to a "stiff matter" state[38].

Use of Eq. (9) into the field equation Eq. (8) yields a first integral for the function $\omega\,(t)$:

$$\dot{\omega} = \gamma a^{-3} \tag{12}$$

where γ = constant. In turn, Einstein's equations Eq. (7) for the Friedman scale factor $a\,(t)$ consists of the system

$$\dot{a}^{\,2} + \varepsilon + \lambda^2/6\,(\dot{\omega}\;a)^2 = 0 \tag{13}$$

$$2a\ddot{a} + \dot{a}^2 + \varepsilon - \lambda^2/2\,(\dot{\omega}\;a)^2 = 0, \tag{14}$$

where $\varepsilon = (0,\,+1,\,-1)$ is the 3-curvature parameter of the FRW geometry. From Eqs. (13, 14) we see that if $(3-4\xi) = -\lambda^2 < 0$ an *open* Universe is obtained (i.e., $\varepsilon = -1$).

A combination of Eqs. (12) and (13) supplies the fundamental dynamical equation

$$\dot{a}^2 = 1 - [a_0/a]^4 \tag{15}$$

with a_0 = constant = $[\gamma^2\lambda^2/6]^{1/4}$. It is straightforward to show that *Eq.* (14) results of Eqs. (11, 12, 13).

Prior to the elaboration of the solution of the system of structural and dynamical equations Eqs. (12, 15) let us comment on the consequent cosmological model and list some interesting results.

10.2 Aspects of the model

The age of the Universe: it is an immediate consequence of Eq. (15) that the scale factor a(t) cannot attain values lesser than the minimum limit a_0. The singularity problem, one of the most fundamental difficulties of standard cosmologies, is solved in the present theory.

Let us consider a time reversal operation and run backwards into the past of the cosmic evolution. As the cosmic radius $a(t)$ decreases, the temperature of the material medium grows. In HBB models such increment is unlimited; in the present theory, on the other hand, there is an epoch of greatest condensation in the vicinity of the minimum radius a_0. Close to this period, there occurs a *continuous* "phase transition" in the geometrical background: a Weylian structure is activated, according to Eq. (12), and in consequence an unbounded growth of the temperature is inhibited. The Universe attains the minimum radius a_0 at $(t = 0)$, beckoning to a previous collapsing era. Once the Universe had this infinite collapsing era to become homogeneous, in the present scenario the "horizon" problem of Standard Cosmology does not happen also.

The Riemannian structure of space time: for very large times the scale factor behaves as $a \sim t$. Thus, asymptotically, the geometrical configuration assumes a Riemannian character (once $\omega^{\cdot} \rightarrow 0$) in the form of a flat Minkowski space (in Milne's coordinate system). In consequence, in the present model the evolution of the Cosmos may be assigned to a primordial unstability of Minkowski space, at the remote past, against Weylian perturbations of the Riemann structure in the guise of Eq. (2.5). In order to prescribe the behavior of these perturbations, knowledge is required on the time dependence of the gauge vector ω_λ. As we will see, once the WIST function $\omega^{\cdot}$ has a maximum at $(t = 0)$, the largest deviation of the Riemannian configuration corresponds to the epoch of greatest contraction near to the value a_0; we shall postpone this development, though, to a subsequent part of this section.

-*The flatness concern:* in the standard theory one faces the following problem. Defining the critical density ρ_c of the Friedman model as $\rho_c = 3H^2 = 3\left[a^{\cdot}/a\right]^2$, current observations show that the value of the ratio $\rho - \rho_c/\rho_c$ is rather large—where ρ is the density of the matter-energy sources. However, this quantity could assume a very small value at the beginning of the present expanding era. In fact, it is a consequence of Einstein's equations for radiation (according to the equation of state $p = 1/3\rho$) that $\rho - \rho_c/\rho_c \sim 1/a^{\cdot 2} \sim 0$ when $(t \rightarrow 0)$; this in turn implies a "flat" or Euclidean $(\varepsilon = 0)$ configuration. How could such an enormous difference have occurred? In other words, why should the Universe display such a fine-tuning (i.e., $\rho \sim \rho_c$) of its initial conditions?

In the present scenario this becomes a false problem. Indeed, from Eqs. (11, 13) it follows that close to the era of maximum condensation the matter-energy distribution is dominated by the energy of the WIST field $\omega(t)$—which, according to Eqs. (11, 12), is described by a "stiff matter" state such that $\rho_\omega = p_\omega \sim a^{-6}$. In this case, near to the minimum value a_0 one has, in view of Eq. (15),

$$\rho - \rho_c/\rho_c \sim 1/a^{\cdot 2} \sim \left(1 - \left[a_0/a\right]^4\right)^{-1} \tag{16}$$

which is a rather large quantity. To guarantee the compatibility of Einstein's equations, it suffices that $\varepsilon = -1$ (open solution). Hence, no resource to a specific set of initial conditions is required.

The accelerated Universe: in the present model the Universe starts to evolve due to Weylian perturbations of an empty Minkowski space-time; thus, the most remote image of the cosmic history is that of a collapsing primordial Universe of infinite radius. Throughout this collapsing era the cosmic evolution is driven by the energy of the WIST field $\omega(t)$; in consequence, in the course of the entire collapse the Universe is accelerated—or "inflationary"—once $a^{\cdot\cdot} = 2/a\left[a_0/a\right]^4 > 0$. In fact, were the Universe always dominated by the ω -energy only, it would accelerate forever. However, as we will see in the next section, in the neighborhood of the maximally condensed epoch a significant amount of matter may come to appear, therefore implying important modifications of space-time curvature in the ensuing expanding era. Nevertheless, it is remarkable that during the whole collapsing era the Universe manifested such inflationary behavior.

A quest for stability: among the difficult questions concerning "eternal", bouncing Universes one may count the problem of their survival with respect to eventual *metric* perturbations. With the use of Eqs. (12, 15), it is straightforward to show that way of the stage of greatest condensation the Universe is *stable*[39]. As we will show in the next section, this result is consistent with the behavior of matter fluctuations prior to the phase of greatest contraction.

11 Matter and Entropy Production

11.1 Matter and entropy production in the standard context

Until now we have been studying structural and gravitational aspects implied by the basic assumption of the present model: that space-time geometry is dynamically determined by the interaction of a pair of fundamental geometrical components, namely, the metric $g_{\mu\nu}(x)$ and the WIST field $\omega(x)$—which we assimilated to the geometrical background in the sense that its behavior effectively controls the affine character of space-time. According to the previous developments, the Universe evolved from scale fluctuations of a primordial Minkowski vacuum; by the same token, its matter content should also originate from dynamical processes involving the fundamental pair $(g_{\mu\nu}, \omega)$. Let us then turn our attention to the material substance of the Universe.

Let us first remind the reader that matter (and entropy) creation in the standard context relies ultimately on the occurrence of an initial singularity. Consider, for instance, the standard HBB model in which the Friedman scale factor is given by $a(t) \sim t^n$ with $n < 1$. Thus the Hubble expansion is represented by a monotonic function of a very regular behavior, corresponding to a completely adiabatic configuration. The initial singular state, then, is the only occasion in which there exists a non-monotonic behavior able to engender matter and entropy. Strictly speaking, when the cosmic temperature is within a few orders of magnitude of the Planck temperature $T_P \sim 10^{32} K$ the Friedman expansion is fast enough in order to allow for the creation of particle—anti-particle pairs[42]. However, such mechanism cannot explain the observed asymmetry between matter and anti-matter (one of the aspects of the "baryon asymmetry problem" of standard cosmology[10]) without making appeal to unaccessible initial conditions issued at the singularity.

In inflationary scenarios, the energy density associated with the "inflaton" scalar field dominates the evolution of the Universe at primordial epochs of great condensation. In homogeneous models, the behavior of the inflaton field $\phi(t)$ is described by the evolution equation

$$\dot{\rho_\phi} + 3H(\rho_\phi + p_\phi) = 0, \tag{1}$$

in which (ρ_ϕ, p_ϕ) are the energy density and the isotropic pressure associated to the inflaton and $H = [\dot{a}/a]$ is the Hubble parameter. In this case, couplings of the inflaton field $\phi(t)$ to other fields may give rise, via vacuum excitations, to particle generation, once as the inflaton oscillates its energy can be converted to produce other particles. This effect can be taken into account through the addition of a term such as $(-\Gamma_\phi \dot{\phi}^2)$ to the energy conservation equation, where Γ_ϕ is the total decay width of the inflaton field[43]; in the case of, for example, the production of relativistic particles (e.g., photons), the evolution equations for the energy densities ρ_ϕ of the inflaton and ρ_γ of the photons become, respectively,

$$\dot{\rho_\phi} = -(3H + \Gamma_\phi)\,\mu\rho_\phi, \tag{2}$$

$$\dot{\rho_\gamma} = -4H\rho_\gamma + \mu\Gamma_\phi\rho_\phi, \tag{3}$$

where use was made of the equations of state $p_\phi = (\mu - 1)\,\rho_\phi$ and $p_\gamma = 1/3\rho_\gamma$. Integrating these equations one obtains that if the value of the decay width Γ_ϕ is sufficiently large then the inflaton energy will be rapidly converted into photon energy, and its contribution to the total energy which drives the metric evolution will become negligible exponentially.

11.2 Thermodynamical comments

In the present scenario, on the other hand, one should expect the creation of matter and entropy to occur in the course of the non-adiabatical regime correlate to the reversion of the collapsing to the expanding phase (Section IV). Indeed, according to Quantum Field Theory in curved space-times the number of particles of a given species is an adiabatical invariant. This means that if the effects of the background geometry can be characterized as an infinitesimally slow thermodynamical process, then no particles are created by the gravitational field. This result was shown[42] in the case of field theories minimally coupled to gravity within the standard Riemannian context, and it may be generalized immediately to arbitrary affine configurations as far as the main properties of the equations of motion of test-fields are retained. Nevertheless, in the present approach these standard adiabaticity arguments must be reconsidered in view of the non-adiabatical phase taking place when the Universe bounces at the minimum radius a_0, once in this case the energy of the WIST field $\omega\,(t)$ could be converted into matter.

What could we say, on an intuitive basis, about particle production in the eternal Universe dominated by the WIST function $\omega\,(t)$? The theory of chemical reactions, for instance, offers a model of a mechanism in which the variation of the number N of particles of a given chemical species is controlled by the environment temperature T and by the number N_0 of such particles already existing, i.e., $\Delta N \sim N_0 T$. From the standpoint of macroscopic Thermodynamics, in turn, taking into account the "stiff matter" behavior associate to the WIST field $\omega\,(t)$ a very naïve application of Gibb's law yields a temperature proportional to the inverse of the volume $V = a^3$, that is, $T \sim 1/a^3$. Combining these ideas, a rough estimate of the rate of particle creation in the course of the "structural transition" which characterizes the present scenario is given by

$$dN/dt = const.N/a^3. \tag{4}$$

Supposing that this is indeed the case, a straightforward use of solution Eq. (4.23) would give the total amount of particles produced in the present model. On the other hand, these considerations suggest the association of the WIST "energy" field $\dot{\omega}(t)$ to a thermal bath of temperature $T \sim a^{-3} \sim \dot{\omega}$ (note that once $\dot{\omega}(t)$ has a maximum at $(t = 0)$, temperature T is never divergent). As in conventional scalar field theory, this assumption leads to the induction of a Landau-type phase transition[44].

11.3 Particle creation in a WIST background

In order to provide more reliable arguments to support formula Eq. (3), however, we must consider once more the observed status of the known laws of physics with respect to point-dependent scale (conformal) transformations. According to the reasonings of Section III, in fact, convincing observational evidence indicates that physical quantities describing relevant properties of matter are *not* preserved under conformal mappings. This result is truly of importance once it guides us in the establishment of a gauge-independent methodology to generalize the Riemannian expressions of the laws of physics to the present WIST context. Indeed, following the standard prescription based on the Minimum Coupling Principle—which rules the extension of special-relativistic formulae, written in flat Minkowski space, to general-relativistic covariant expressions in curved space-time[20]—we will assume that in a WIST scenario the energy-momentum tensor $T^{\mu\nu}$, the entropy flux S^{μ} and the particle number current N^{μ} describing a given fluid satisfy the evolution relations

$$\begin{cases} T^{\mu\nu}{}_{;\nu} = 0 \\ S^{\mu}{}_{;\mu} = 0 \\ N^{\mu}{}_{;\mu} = 0 \end{cases} \tag{5}$$

where, as stipulated in Section III, the semi-colon denotes covariant differentiation in a Weyl manifold. In the particular case of a Riemann configuration, *i.* e., when the WIST field $\omega\,(t)$ vanishes, these expressions reduce to the usual conservation laws of General Relativity; for example, the particle current N^{μ} obeys in this case $N^{\mu}_{\|\mu} = 0$, and thus the conservation of the particle number density $n = N/V$, in a Friedman background, follows as usual:

$$\dot{n} + 3nH = 0 \tag{6}$$

In the WIST case, on the other hand, Eq. (4) yields precisely the intuitive formula Eq. (3) for particle production, due to the dissipative effects induced by the presence of the WIST field $\omega\,(t)$. Indeed, according to to the formula Eq. (3.14) of WIST covariant differentiation, in the case of a relativistic fluid (photons) in a Friedman background the evolution relations Eq. (4) may be written as

$$\begin{cases} \dot{\rho} + 4H\rho - 3\omega\dot{\rho} = 0 \\ \dot{n} + 3Hn - 2\omega\dot{n} = 0 \\ \dot{s} + 3Hs - 2\omega\dot{s} = 0 \end{cases} \tag{7}$$

in which $s = S/V$ is the entropy density. We see that the WIST "energy" function $\omega^{\cdot}\,(t)$ plays the role of a (time-dependent) total decay width Γ_{ω} of the bosonic field $\omega\,(t)$ into photons, in the likeness of the inflationary case described by Eq. (2). Integrating Eq. (6) one obtains

$$\begin{cases} \rho = \rho_0 \; a^{-4} \; exp\,[3\omega] \\ n = n_0 \; a^{-3} \; exp\,[2\omega] \\ s = s_0 \; a^{-3} \; exp\,[2\omega] \end{cases} \tag{8}$$

where the symbol (0) denotes the values of small fluctuations of these magnitudes that supposedly occurred at some occasion in the past. It is interesting to observe that in spite of the fact that the present theory has two free parameters, namely, the dimensionless parameter ξ in the Lagrangian Eq. (4.2) and the minimum radius a_0 of Eq. (4.15), according to the Wiston solution Eq. (4.26) the efficiency of the mechanism of matter-entropy production represented in Eq. (7) is sensitive only to the value of ξ—besides, of course, the seminal input supplied by the original fluctuations.

11.4 The baryon asymmetry problem

A remarkable consequence of the introduction of dissipative effects induced by the WIST character of the space-time background is the exponential dependence of matter properties on the behavior of WIST field $\omega(t)$. Indeed, according to the solution Eq. (4.24) it follows that any fluctuation $(\Delta\psi)_0$ experienced by a given matter field ψ at the remote past is strongly damped in the course of the collapsing phase $(t < 0)$; then there occurs a sudden transition from suppression to stimulation around $(t = 0)$, and a equally strong amplification begins as the expanding phase takes place $(t > 0)$. This production mechanism, however, saturates very rapidly, and for later times $(t >> 0)$ it becomes insignificant (note that these conclusions concern a Universe driven by a Wiston; in the case of an anti-Wiston, of course, this account shall be inverted). Thus, in distinction of other eternal, bouncing cosmologies, the infinite span of the contracting phase in the present model does not imply a boundless matter-energy production.

Due to the exponential damping of any primeval irregularity, only fluctuations taking place near $(t = 0)$ do care for the subsequent evolution; but these fluctuations are exponentially amplified for a short period, so as to allow for arbitrarily large amounts of matter — e.g., particles—and entropy to be created. This period of intense creation is tantamount to a non-equilibrium process; notwithstanding this fact, after the amplification mechanism has been shut down one might expect the WIST field declining contribution to the source of Einstein's equations to be rapidly outmatched by the newly produced matter content. In this way, the primordial "stiff matter" state associated to the the "Big—but not infinite—Bang" described here could be straightforwardly continued to a standard sequence of radiation-dominated and matter-dominated phases; the addition of a standard inflationary phase, if required, is also not excluded.

The operation of this amplification mechanism also provides a fresh perspective with which the standard baryon asymmetry problem may be envisaged. It is well known that the prevalence of matter (e.g., baryons) against anti-matter in the observed Universe—as well as the observed ratio of entropy per baryon—is not explained in standard cosmology except with the use of fine-tuned initial conditions[45]. In the present scenario, on the other hand, an eventual baryon excess fluctuation $\Delta N_0 = \left(N_B - N_{\overline{B}}\right)$ taking place shortly after the stage of maximum contraction at $(t = 0)$ may be exponentially increased up to a convenient amount, since in this case we have

$$\Delta N_B = \Delta N_0 \exp\left[2\omega\right]. \tag{9}$$

While the production rate depends on the free parameter ξ only, the initial spectrum of fluctuations which become the subject of the amplification mechanism must bear a relationship to the relative size of the Universe—and thus to the minimum radius a_0. Elementary particle theory, for instance, requires the set of specific baryonic species contained in ΔN_0 to be regulated by the environment temperature T_E—which in the present non-singular scenario is always bounded (that is, $T_E \leq T_M \sim a_0^{-3}$). According to the value chosen for the minimum radius a_0, different species may be selected for amplification; conversely, a particle physics analysis could in principle yield an evaluation of this parameter. We shall leave these matters, though, to a subsequent investigation.

12 Conclusion and Further Discussions

12.1 The cosmic evolution

On the basis of the above results, the complete history of the cosmic evolution in the case of an homogeneous and isotropic metric configuration may be outlined as follows: due to Weylian scale fluctuations ruled by Eq. (2.5), a primordial empty Minkowski space-time begins to collapse at a remote past. This collapsing phase of indefinite duration is driven by the WIST field $\omega(t)$, whose effects are thermodynamically equivalent to a "stiff matter" state of a perfect fluid with energy density given by $\rho_\omega \sim a^{-6}$ (Eq. (4.11)). Throughout the collapse, the Universe is accelerated—or "inflationary". In agreement with the stability arguments discussed in Section IV, any eventual matter-energy fluctuation is exponentially suppressed in the course of the entire collapsing phase. This resembles the "memory loss" of inflationary scenarios, in which the problem of the singular origin may be circumvented due to the presence of an effective horizon limiting the present observational scope.[7] The collapse proceeds adiabatically in a very slow pace until a stage of greatest condensation—corresponding to the minimum a_0 of the cosmic radius—is approached. In fact, in the neighborhood of this maximally condensed stage the contraction is accelerated to an acme and then decreases suddenly, reverting to an expansion when the minimum radius a_0 is attained.

In the likeness of quantum creation models, the infinite collapsing phase of the present scenario may be associated to the propagation of a Weyl instanton—or "Wiston"—in an Euclideanized, classically forbidden region (Section IV); according to this interpretation, the Universe—as a classical entity—emerged from "Nothing", endowed with a minimum radius a_0, in a "stiff matter" state characterized by the absence of a matter content (e.g. , baryons and leptons), except for small fluctuations. However, as the Universe begins to expand, a non-adiabatical amplification mechanism starts to operate, driven by the energy of the WIST field $\omega(t)$, in such a way that matter-energy fluctuations may come to be converted, in an exponential rate, into large amounts of particles and radiation. An eventual baryon excess may be amplified in the same fashion. This "Big—but not infinite -Bang" stage lasts for a very short period, once the energy of the produced material soon dominates the energy of the WIST field; in this way, the Universe enters in the Friedmanian radiation-dominated and matter-dominated regimes which characterize the standard evolution.

From an empirical point of view one could argue that the occurrence of a non-adiabatical cosmic stage is required by the observed existence of huge quantities of matter. However, according to each particular cosmological scenario, the details of matter production mechanisms can in principle be rather different. In effect, in order to excite a process in which enough entropy could be produced the standard HBB program makes appeal either to non-controlled initial conditions issued at the explosive beginning or, alternatively, to an intermediary instance provided by the inclusion of an inflationary era into the standard frame of the cosmic evolution.[7,10]

In the scenario proposed here, matter-entropy production appears as a natural consequence of a non-adiabatical regime driven by the brisk change from a collapsing phase to an expanding one, correlate to the maximal deviation from the Riemannian structure[46].

As we saw in Section V, a straightforward application of the Minimum Coupling Principle supplies a simple description of dissipative effects, induced by the WIST background, which appear in the evolution equations of matter fields. Similarly to inflationary approaches, these dissipative effects are expressed in terms of a (time-dependent) total decay width associated to the WIST field $\omega(t)$. Throughout the creation period, the environment temperature is never divergent; in this way, elementary particle problems—such as, for instance, quark confinement—can be addressed from a new angle.

With respect to the inclusion of a non-standard primordial "rigid matter" phase of the cosmic evolution, current trends in High Energy Physics suggest that the standard radiation-dominated era should be preceded by a primeval cosmic domain in which all fundamental interactions were unified. So far, the exact constituents of this unified state remain undetermined; nevertheless, given the circumstance that such unique mode of energy exchange should have a *global* character, in analogy with well-known issues of Field Theory[26] one is led to conceive that a "stiff" or "rigid matter" state, described by the ultra-relativistic equation of state $p = \rho$, might provide a suitable representation of this one-interaction configuration. In the present article we have shown that the hypothesis of a WIST background manifold leads to a dynamical scheme in which *geometry* itself accounts for the existence of a primordial "stiff" state[47].

In effect, in the present approach geometry generates *everything*. The conceptual cost to be paid for the election of one such unique physical matrix is, evidently, the widening of the traditional Riemannian space-time structure of General Relativity, once the geometrization of the basic inflaton-like field $\omega(t)$ implies the modification of the affine connections of the underlying manifold due to the contribution of ω-dependent terms; in turn, the behavior of the WIST "structural" function $\omega(t)$ is regulated, in a non-linear fashion, by the metric evolution (Section IV). The explicitly non-linear character of the system of fundamental dynamical equations describing the cosmic development, on the other hand, points to the most intriguing aspect put forth by the present WIST scenario: the equivalence of structural disturbances due to the curved WIST background (namely, the propagation of Wistons) to a semi-classical description of a *quantum* process.

Actually, Weyl spaces and quantum processes are not entirely foreign matters according to the literature. Since the early days of London[48], a curious connection of Weyl's length transport theory to certain aspects of quantum mechanics has been indicated, suggesting that quantum rules could be obtained from a classical formalism based on Weyl spaces. In other words, geometries in which length variation under transport may occur—such as Weyl's—seemingly constitute a well suited classical foundation upon which a successful interpretation of typical microscopic processes could be built. It has been shown, for instance, that the non-relativistic Schrödinger equation can be derived from a stochastic formulation in which quantum "forces" are due to curvature effects associated to the gauge vector ω_μ in a Weyl geometry. In this vein, quantum mechanical behavior would arise from a feedback relationship of the geometrical structure with dynamics.[49]

The analogies drawn in the text between Euclideanized solutions of quantum creation models and the present WIST theory, however, require further consideration. The dynamical activity of the WIST background is particularly pronounced close to the phase of greatest contraction, when the scale factor $a(t)$ approaches the minimum value a_0. The enlargement of the geometry to a WIST configuration could then be ascribed, if this value is sufficiently small, to a first approximation of a quantum description of gravitational processes. The excitation of a Weyl structure would therefore represent the initial response of the background manifold to the structural transition from classical to quantum regimes.

In the manner of Utiyama[50], one could also consider the engaging—but rather speculative—possibility that WIST effects could become relevant on *microscopic* dimensions. In the likeness of the "vacuum bubbles" examined in some current approaches of quantum gravity theory[51], such "Weyl bubbles" would constitute microscopic domains endowed with a non-Riemannian internal structure, which could interact with an embedding Riemann manifold either by analytical continuation or by discontinuous jumps at the frontier (for instance, while current observations conclude that conformal invariance is broken at large scales, no similar statement has been achieved with respect to the microworld; thus, localized microscopic domains of Weylian character, embedded in a Riemann background, could in principle exhibit an invariant behavior[52]). In a certain sense, one such WIST domain (of an infinite extension, though), enclosed between two asymptotic Riemann configurations, is described in the present scenario. Albeit unclear the relation of quantum processes to WIST structures may be as of now, the equivalence—at least in a particular case—of Wiston propagation to Euclideanized semi-classical solutions noticed in Section IV certainly deserve further investigation.

In conclusion, the assumption of a WIST manifold leads to a cosmological solution describing an eternal Friedman-like open Universe which does not exhibit the usual difficulties of standard models, e.g., the singularity, horizon and flatness problems. In fact, the presence of the WIST background accounts for both the unstability of a primordial Minkowskian "Nothing" and the operation of a matter-entropy creation mechanism, so as to dispense with the cumbersome standard initial singularity and/or fine-tuned initial conditions. This geometrization procedure also supplies a *dynamical* explanation of the origin of the observed baryon excess over anti-baryons in the Universe today; it may be conjectured, furthermore, that the occurrence of a WIST-driven non-adiabatical phase could provide a suitable basis for the derivation of an appropriate primordial spectrum of density fluctuations in order to allow for galaxy formation. This subject, as well as other complementary aspects of the scenario discussed here, shall be addressed in a forthcoming study.

13 Final Comments

The fact that there exist various alternative classical scenarios that do not contain a global singularity point out in the direction that the generic behavior of the gravitational field is not limited to the features presented in the classical singularity theorems. This means that the origin of the actual expansion phase of the universe should not be identified with a *primordial explosion*.

However, much of the effort of cosmologists during the last decades has been spent to *save* the Big Bang model, in the analysis of its difficulties, trying to answer the questions posed by this model. I think that the time has come to change this situation and to go deeply into a new road: to start a systematic search of solutions of the corresponding difficulties found in the Program of the Eternal Universe.

References

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