CBPF-NF-029/96

On the Dalitz Plot Approach in Non-leptonic Charm Meson Decays

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ABSTRACT

We show that the non-resonant contribution to non-leptonic charm meson decays cannot be considered constant in the phase space of the reaction as it usually is. We argue that this is relevant for any weak reaction. We discuss in detail the decay $D^+ \to K^-\pi^+\pi^+$.

Key-words: Weak decays; Charm meson; Dalitz plot.

Non-leptonic charm meson decays have been extensively studied both from the theoretical and the experimental side. They are essentially dominated by two body decays. In the majority of them, at least one of the decaying particle is a resonance, i.e., one detects three or more particles. As a consequence, when analysing a given three body final state, for example, one has to consider all the possible two body intermediate states yielding such a final state, together with the corresponding direct non-resonant contribution. The comprehension of the whole non-leptonic decay pattern of charm mesons and the knowledge of the precise decay partial widths is essential to the understanding of many open problems, as for example the old D mesons lifetime puzzle.

Dalitz plot analysis[1] is a widely used technique particularly well suited for the experimental study of these kind of decays. Dalitz plot brings information on both the kinematics and the dynamics of a given many body reaction. As it is weighted by the squared amplitude of the reaction, a flat Dalitz plot corresponds to a decay with a constant dynamics in the whole phase space[1]. Within this technique, intermediate resonances appear in a simple way, and possible interferences between them are taken into account. Amplitude and phase of each intermediate resonance, and then its corresponding partial width decays, can thus be found. This technique has indeed been used by experimental collaborations that have measured many body decays of charm mesons within the last fifteen years.

The aim of this letter is to discuss some of the hypothesis assumed in these analysis. We claim that the usual assumption of a flat non-resonant contribution to the decay is not adequate and we show the importance of a correct parametrization of this contribution.

In the past, Dalitz plots have proven to be very useful to describe reaction dominated by the strong interaction. This analysis technique is particularly suited for reactions where the dynamics is dictated by a dominant behavior like the spin of a decaying particle or the emergence of resonances — both in scattering and in decay channels. These reactions have clear signatures in the plot. This is why the interest has never been put on the remaining overall contribution to the dynamics which has always been simply parametrized as a uniform, flat amplitude. Indeed, experimental results had supported this assumption. In other words, both in many body decays and hadron-hadron interactions the dynamics are almost constant in the phase space, except for the contribution dictated by the spin of the decaying particle or the existence of resonances.

Analogously, when experimental data on non-leptonic decays of charm mesons became available, the natural approach to Dalitz plot analysis for the study of resonant substructures was to assume that the non-resonant contribution to the dynamics has no variations in the phase space. Thus, during the last fifteen years, experimental data on non-leptonic decays have been fitted using Breit-Wigner functions[3] for each possible resonance whereas the non-resonant part has been fitted as a *constant.*

This parametrization has been used by many experimental teams[4, 5, 6, 7, 8]. However, it has proven to be not totally adequate to describe non-leptonic charm meson decays. Indeed, a very poor overall fit quality is reported. These poor results do not improve when having a high statistics or when considering a larger amount of resonances[8]. Moreover, this problem appears in all the $D \to K\pi\pi$ decay channels already measured ($D^0 \to \bar{K}^0\pi^+\pi^-$, $D^+ \to \bar{K}^0\pi^+\pi^0$, $D^+ \to K^-\pi^+\pi^+$ and $D^0 \to K^-\pi^+\pi^0$) [9] and the worst fit is obtained for $D^+ \to K^-\pi^+\pi^+$, where the non-resonant contribution dominates[8].

(In this case, with 29 degrees of freedom, the χ^2 per degree of freedom is as bad as 3.01.)

When the non-resonant contribution to the decay amplitude is not negligible, its correct parametrization becomes crucial. An incorrect parametrization will certainly influence the fit of the resonances and consequently the values of their amplitudes and phases. Thus, the comprehension of the whole decay pattern strongly depends on an acceptable form of the non-resonant part. As an example, MarkIII reported[5] on an incompatibility on the measurement of the branching ratio (BR) of $D^+ \to \bar{K}^*\pi^+$: The experimental value for this BR when detected from the final state $K^0\pi^0\pi^+$ is $5.3 \pm 0.4 \pm 1.0$ whereas its value is $1.8 \pm 0.2 \pm 1.0$ when the final state is $K^-\pi^+\pi^+$. Note that while the non-resonant contribution to the first final state is of the order of 15% of the total partial decay width, in the second it is as large as 80%.

These experimental results suggest that in non-leptonic charm meson decays one has to study the non-resonant contribution more carefully and eventually it has to be fitted using another parametrization.

Our claim is that the physics of non-leptonic charm meson decays is essentially different from that of the reactions dominated by the strong interaction. In weak interactions between quarks and leptons helicity plays an important role. Consequently, one expects a significant dependence of the weak amplitudes on the momenta of the interacting particles. Thus, the dynamics of these reactions should vary from one point of the phase space to another, the amount of this variation being dependent on the specific physical reaction.

This should be particularly important in weak decays of charm mesons. Indeed, on the one side, the large value of the charm quark mass allows for a quasi perturbative treatment of QCD. On the other side, charm quark decays into light quarks and this enhances the importance of helicity. One could make a first approach to the weak partonic mechanism responsible for the Cabibbo favored D meson decays, i.e. $c \to su\bar{d}$, by analysing the decay of τ leptons, $\tau \to \mu\bar{\nu}_\mu\nu_\tau$, which are essentially similar. Moreover, it is the simplest weak decay one can describe, i.e. a pure leptonic one, and will shed some light on the dependence of a weak reaction on its phase space.

The theoretical Dalitz plot corresponding to the decay $\tau \to \mu\bar{\nu}_\mu\nu_\tau$ can be obtained by taking the well known decay amplitude of pure leptonic decays[10]. This decay amplitude can be written as a function of two invariant variables defining a Dalitz plot, e.g., $m^2_{\mu\bar{\nu}_\mu} \equiv (p_\mu + p_{\bar{\nu}_\mu})^2$ and $m^2_{\mu\nu_\tau} \equiv (p_\mu + p_{\nu_\tau})^2$ to give

$$|\mathcal{M}_{\tau\to\mu\bar{\nu}_\mu\nu_\tau}|^2 \propto m^2_{\mu\nu_\tau}(m^2_\tau - m^2_{\mu\nu_\tau}) \tag{1}$$

where m_τ is the τ mass.

The dynamics of the reaction has thus a quadratic dependence on the variable $m^2_{\mu\nu_\tau}$. As the Dalitz plot is weighted by $|\mathcal{M}_{\tau\to\mu\bar{\nu}_\mu\nu_\tau}|^2$, equation (1) shows that a Dalitz plot of a pure weak decay has indeed significant variations along the phase space.

Obviously, due to the hadronization procedure of partons after their weak interaction, the result of the previous example cannot be simply translated to hadronic decays. In the latter case, one has to deal with non-perturbative QCD effects, related to the final hadronic state formation after the pure partonic interaction. However, one can consider an approximate method which has been successfully used to describe D meson decays. The method is based on both the factorization technique [11] and an effective Hamiltonian [12, 13] for the partonic interaction.

As we are interested in the non-resonant contributions, let us analyse the channel $D^+ \to K^-\pi^+\pi^+$, which seems to have a very large non-resonant part, as was quoted above. The effective Hamiltonian includes one gluon exchange corrections to the tree level weak vertex $c \to su\bar{d}$ [12, 13]

$$\mathcal{H}_{eff} = (\frac{G_F}{\sqrt{2}})\cos^2\theta_c[a_1 : (\bar{s}c)(\bar{u}d) : +a_2 : (\bar{s}d)(\bar{u}c) :] \tag{2}$$

where $(\bar{q}q')$ is a short-hand notation for $\bar{q}\gamma^\mu(1-\gamma_5)q'$. The coefficients a_1 and a_2 characterize the contribution of the effective charged and neutral currents respectively, which include short-distance QCD effects. Their values have been fitted in the case of charm meson two body decays (see for example reference [12]). The diagrams contributing to the decay $D^+ \to K^-\pi^+\pi^+$ are shown in Figure (). Using factorization one obtains the following decomposition for the hadronic amplitude

$$\begin{aligned}\mathcal{M}_{D^+\to K^-\pi^+\pi^+} &= (\frac{G_F}{\sqrt{2}})\cos^2\theta_c[a_1\langle K^-\pi_1^+|\bar{s}c|D^+\rangle\langle\pi_2^+|\bar{u}d|0\rangle \\ &\quad +a_2\langle K^-\pi_1^+|\bar{s}d|0\rangle\langle\pi_2^+|\bar{u}c|D^+\rangle \ + \ (\pi_1^+ \leftrightarrow \pi_2^+)] \ .\end{aligned} \tag{3}$$

Let us first discuss the term driven by a_1, i.e, the one of Figure (.a). The most general form to decompose the first matrix element can be written in terms of four form factors[14]. Using the parametrization of reference [15] we can write:

$$\langle K^-\pi_1^+|\bar{s}c|D^+\rangle = A_1^\mu F_1 + A_2^\mu F_2 + iV_3^\mu F_3 + A_4^\mu F_4 \tag{4}$$

where

$$\begin{aligned}A_1^\mu &= p_K^\mu + p_D^\mu - Q^\mu\frac{Q\cdot(p_K+p_D)}{Q^2} \\ A_2^\mu &= p_{\pi_1}^\mu + p_D^\mu - Q^\mu\frac{Q\cdot(p_{\pi_1}+p_D)}{Q^2} \\ V_3^\mu &= \epsilon^{\mu\alpha\beta\gamma}p_K^\alpha p_{\pi_1}^\beta p_D^\gamma \\ A_4^\mu &= Q^\mu = p_K^\mu + p_{\pi_1}^\mu - p_D^\mu = -p_{\pi_2}^\mu \ .\end{aligned}$$

The terms proportional to F_1, F_2 and F_4 originate from the axial vector part of the matrix element whereas the one proportional to F_3 originates from the vector part; the terms proportional to F_1, F_2 and F_3 correspond to spin 1 and F_4 to spin 0. The four form factors depend on three variables $m_1^2 = (p_k + p_{\pi_1})^2$, $m_2^2 = (p_k + p_{\pi_2})^2$ and Q^2 which is a constant (m_π^2) in this case.

The second matrix element in equation (3) has the well known form

$$\langle\pi_2^+|\bar{u}d|0\rangle = if_\pi p_{\pi_2}^\mu \ . \tag{5}$$

The only contributing term in equation (4) after multiplying it by equation (5), is the axial spin 0 term, i.e.,

$$\langle K^-\pi_1^+|\bar{s}c|D^+\rangle\langle\pi_2^+|\bar{u}d|0\rangle = (p_{\pi_2\ \mu}\ F_4)\,(p_{\pi_2}{}^\mu\ f_\pi) = f_\pi m_\pi^2 F_4 \ . \tag{6}$$

Concerning the contribution of Figure (.b), one can use the well known expressions[16]

$$\begin{aligned}\langle\pi_2^+|\bar{u}c|D^+\rangle &= \left[(p_D+p_{\pi_2})^\mu - \frac{m_D^2-m_\pi^2}{q^2}(p_D-p_{\pi_2})^\mu\right]F_{D\pi}^{1^-}(q^2)\\ &\quad + \frac{m_D^2-m_\pi^2}{q^2}(p_D-p_{\pi_2})^\mu F_{D\pi}^{0^+}(q^2)\end{aligned}$$

and

$$\begin{aligned}\langle K^-(p_K)\pi_1^+|\bar{s}d|0\rangle &= \langle\pi_1^+|\bar{s}d|K^+(-p_K)\rangle = \\ &\left[(-p_K+p_{\pi_1})^\mu - \frac{m_K^2-m_\pi^2}{q^2}(-p_K-p_{\pi_1})^\mu\right]f_+(q^2)\\ &+\frac{m_K^2-m_\pi^2}{q^2}(-p_K-p_{\pi_1})^\mu f_0(q^2)\ .\end{aligned}$$

In both equations above, $q^2=(p_D-p_{\pi_2})^2=(-p_K-p_{\pi_1})^2$ while the functions $F_{D\pi}^{J^P}(q^2)$ — corresponding to a current of spin parity J^P — and $f_+(q^2)$ and $f_0(q^2)$ are form factors. We will be back to them bellow.

One then finds for the second contribution in equation (3),

$$\begin{aligned}&\langle\pi^+|\bar{u}c|D^+\rangle\langle K^-\pi^+|\bar{s}d|0\rangle = F_{D\pi}^{1^-}(m_1^2)f_+(m_1^2)\left(m_D^2+m_K^2+2m_\pi^2-2m_2^2-m_1^2\right)\\ &+\ \left[F_{D\pi}^{1^-}(m_1^2)f_+(m_1^2)-F_{D\pi}^{0^+}(m_1^2)f_0(m_1^2)\right]\frac{(m_D^2-m_\pi^2)(m_K^2-m_\pi^2)}{m_1^2}\\ &+\ (m_1^2\leftrightarrow m_2^2)\end{aligned}\tag{7}$$

where we have explicitly introduced the Dalitz plot variables m_1^2 and m_2^2 defined above.

The contribution of diagram (.a), given by equation (6) is proportional to $f_\pi m_\pi^2$. Thus, unless the form factor F_4 were unacceptably large ($F_4\sim 10^3$), we can safely neglect this contribution in favor of that of diagram (.b); the latter contains m_D^2 — see equation (7) — and turns out to be much larger than the one of equation (6). In fact, one can hint that the non-resonant part of the decay $D^+\to K^-\pi^+\pi^+$ is large precisely because the contribution of diagram (.b) is *not* small. This conclusion can be safely extrapolated to the other D decay channels. The various consequences of this remark are discussed elsewhere[19].

The non-resonant contribution to the amplitude of the decay $D^+\to K^-\pi^+\pi^+$ can thus be simply written replacing equation (7) in (3), neglecting the contribution of Figure (.a). The final expression thus depends on the effective coefficient a_2 and the four form factors. The two $D\pi$ form factors $F_{D\pi}^{J^P}(q^2)$, have well stablished expressions[13] :

$$F_{D\pi}^{J^P}(q^2)=\left(1-\frac{q^2}{M_{D\pi,J^P}^2}\right)^{-1}\tag{8}$$

where $M_{D\pi,1^-}=2.01$ GeV and $M_{D\pi,0^+}=2.2$ GeV. They have been successfully used in the kinematical range we are considering here. The poles lie outside our kinematical region. Concerning $K\pi$ form factors $f_+(q^2)$ and $f_0(q^2)$ they can be extracted from the semi-leptonic decays $K \to \pi l \nu$, with $l = e, \mu$. Nevertheless, it is not clear that the usual parametrization[10]

$$f_+(q^2) = f_+(0)\left(1 + \lambda_+ \frac{q^2}{m_\pi^2}\right) \ , \ f_0(q^2) = f_0(0)\left(1 + \lambda_0 \frac{q^2}{m_\pi^2}\right) \qquad (9)$$

could be valid in the whole kinematic region of our reaction. As we are considering a non-resonant decay, these expressions obviously do not contain the poles. In equation (9) $f_+(0) = f_0(0) = 1$; the other coefficients have been measured to be[17] $\lambda_+ \approx 0.03$ independently of the measured channel, whereas the extracted value of λ_0 depends on the decay: $\lambda_0 \approx 0$ (0.025) when extracted from $K^- \to \pi^0 \mu^- \nu$ ($K^0 \to \pi^+ \mu^- \nu$).

In order to check the validity of this calculation scheme, we have evaluated the non-resonant partial decay width $\Gamma(D^+ \to K^- \pi^+ \pi^+)_{NR}$ using the expressions above. With $\lambda_0 = 0$ and the value of a_2 extracted from two body decay[12], we found a branching ratio (BR) of 9% which is close to the reported experimental value[17] $7.3 \pm 1.4\%$ — despite this value has been obtained fitting the non-resonant contribution to a constant. We studied the stability of this result under the change of the parameters λ_+ and λ_0 : If we take the various values extracted from different channels we found that the BR varies less than 30%. Allowing a larger variation of these parameters — including $\lambda_+ = \lambda_0 = 0$, i.e. constant form factors — the BR remains however of the same order of magnitude.

Figure () shows the Dalitz plot for the non-resonant contribution to the decay $D^+ \to K^- \pi^+ \pi^+$ as a function of the variables m_1^2 and m_2^2. It has been generated via Monte Carlo with a weight proportional to the square of the amplitude in equation (3), using equation (7). We have considered the same central value of the parameters as above. As one can see from equation (7) and Figure (), according to this calculation the matrix element describing the dynamics of the non-resonant contribution to the decay $D^+ \to K^- \pi^+ \pi^+$ significantly varies along the phase space of the reaction. Its shape remains almost the same for other values of the parameters of the $K\pi$ form factor. This is still valid even if we take the four form factors as constants. Thus, a fit parametrizing the non-resonant contribution as a constant cannot be correct in this case.

In summary, we have shown that in non-leptonic charm meson decays the hypothesis of a uniform non-resonant contribution is far to be acceptable. Experimental teams that have measured these decays had already called the attention on this fact. In this letter we show that the variations can be very important. It is not possible to predict a general form for these contributions as they depend on the particular reaction. Nevertheless, in the case here studied the amplitude can be fairly approximated by a simplified linear function of the Dalitz plot variables m_1^2 and m_2^2, i.e., giving rise to a quadratic shape dependence in the Dalitz plot. In other words, we propose to fit data using, at least, a linear function $(a + bm_1^2 + cm_2^2)$ for the non-resonant part of the amplitude, where a, b and c are real numbers.

An adequate parametrization of this contribution is essential. On the one side, an incorrect parametrization of the non-resonant part will certainly influence the fit of the resonances; thus the whole decay pattern extracted from the fit could be wrong[19]. On the other side, this non-resonant contribution contains many information on the physics of the decay. Particularly, form factors can be measured along the phase space of the reaction. The measurement of the precise form of the non-resonant contribution is thus of great importance.

Concluding, we have shown that weak interaction leads to a large dependence of the dynamics on the phase space of the reaction. Particularly, for non-leptonic heavy flavor decays — where factorization procedure is currently used — even the dynamics of the non-resonant contribution should have an important variation in the Dalitz plot. Non-perturbative QCD effects should wash out this structure through the multiple exchange of soft gluons. Thus, the appearance of these structures in the non-resonant contribution to the Dalitz plot can be seen as an indication of the validity of the factorization technique[19], which explicitely neglect soft gluons corrections.

We acknowledge Alberto Reis for a usefull discussion. One of us (RMG) wants to thank the warm hospitality at LAFEX-CBPF.

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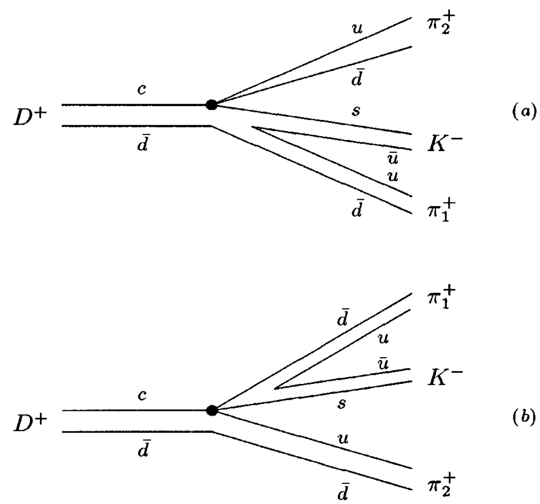

Figure 1: The two diagrams contributing to the decay $D^+ \to K^-\pi^+\pi^+$ according to the effective Hamiltonian of equation (2).

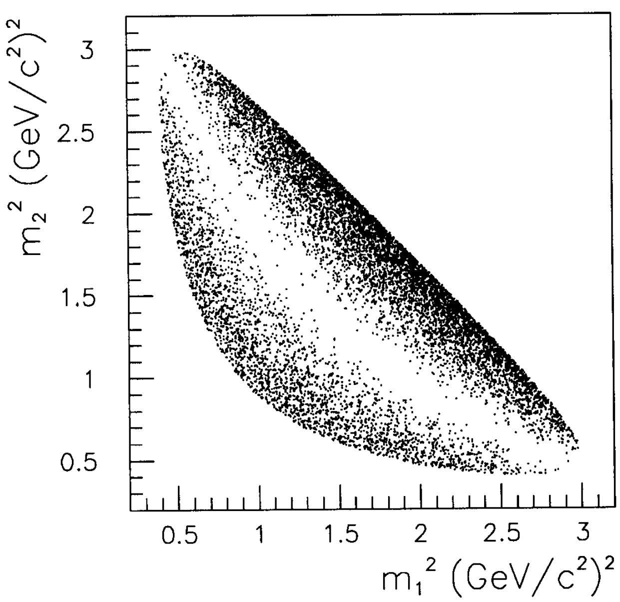

Figure 2: The Dalitz plot of the decay $D^+ \to K^-\pi^+\pi^+$, weighted by $|\mathcal{M}_{D^+\to K^-\pi^+\pi^+}|^2$ as in equations (3) and (7), generated via Monte Carlo. The Dalitz plot variables are $m_1^2 \equiv (p_K + p_{\pi_1})^2$ and $m_2^2 \equiv (p_K + p_{\pi_2})^2$.