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Ohm's Law: a Path-Integral Study at Classical and Quantum Level

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We present a path integral study of the classical and quantum electric current Ohm's law in resistive mediums.

Introduction

One of the most important macroscopic law's of electromagnetism is the well known Ohm's Law relating the current flow density to the applied steady electric field by a linear relationship where the proportionality constant is connected to the resistive medium characteristics.

It is very important to remark that up to the author's knowledgement there is not yet a clear deduction of this macroscopic charge flow law from a microscopic theory. For example in classical charges conduction, one always present the Ohm's law from the phenomenological point of view by introducing by hand a damping force, acting against the charge flow velocity, proportional to velocity and taking into account the collisions with the "resistive" medium by means of a pure damping term. According to the above pointed out phenomenological analysis, one obtains the Ohm's law as the steady limit of the charge-current flow velocity. I clarify and generalize that classical-phenomenological framework in section 1 by modelling the resistive medium as a classical "(statistically chaotic) bath of one-dimensional harmonic (vibrations) medium field", as firstly proposed by myself in my previous work ([1]) and related to D-dimensional isotropic resistive mediums.

The situation on the quantum regime is, at least, heavily phenomenological from first principles. Let me point out that in the usual text books treatment ([8]), one considers the acceleration of an electron in a *periodic* (perfect) lattice by means of the time variation of the electron band vector wave. One, thus, taken into account the interaction of the electron Block states with impurities and lattice vibrations by a pure phenomenological procedure which, by its turn, consists by defining a characteristic time between the electron collisitions. Equivalently: a phenomenological damping term is considered in the classical band electron acceleration equation in order to simulate the medium resistance. Note that in the presence of a steady external applied electric field, the cristal electron Schrödinger equation does not satisfies the Block theorem and, thus, one only has the Band-structure energy level concepts in a perturbative or phenomenological approach and thus, implying by its turn a very weak external applied electric field situation. I present in section 2 a general Ohm's law deduction in the quantum regime by introducing explicitly the interaction of the quantum electron with a reservoir in a Langevin-Hydrodynamic framework similar to that exposed in section 1 and ref. [2]. Note that my proposed procedure has a general validity including the case of quantum electron motion in metallic cristals, in polar cristals, quantum ions motions in fluids, etc... Finally, I close section 2 by presenting a deduction of Ohm's law directly from the one-body Caldirola-Kanai quantum theory and the magneto-resistance case.

Section 1 – The Classical Ohm's Law in the Path-Integral Framework – the one dimensional case

Let us, thus, start our study by modelling the resistive medium by a one-dimensional scalar medium field (phonons, etc...) with the following dynamic wave equation in the range $t \in [0, \infty)$ and $x \in (-\infty, \infty)$ (see [1])

$$\frac{\partial^2 \phi(x,t)}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} \tag{1}$$

Let us impose the medium statistics chaos by means of the following random initial conditions

$$\phi(x,0) = 0 \tag{2}$$
$$\frac{\partial \phi}{\partial t}(x,0) = f(x) \tag{3}$$

where $f(x)$ represents the statistical chaotic initial velocities satisfying the Gaussian statistics and leading, thus, to the stochastic behaviour for the resistive medium as in the usual baths in Brownian physics ([4])

$$\langle F(x)F(x')\rangle = \gamma \delta(x - x') \tag{4}$$

Here the effective medium velocity is given by $\sigma/m = \frac{1}{c^2}$ where m denotes the medium "atoms" harmonic oscillators mass, σ is the lattice strength of the medium oscillators and γ the medium randomness intensity.

Let me introduce the usual plane wave expansion for this resistive medium field generated by the medium harmonic oscillator "atoms"

$$\phi(x,t) = \int_{1/\Lambda<|k|<\Lambda} dk\ \phi_k(t) e^{ikx} \tag{5}$$

with

$$\phi_{-k}(t) = \phi_k^*(t) \tag{6}$$

Here Λ is an intrinsic wave vector cut-off setting a scale for the particle damping constant as it will be shown later on (the Debye frequency).

At this point, I consider the following set of equations for a charged particle of mass M in the presence of an external one-dimensional applied electric field added with the resistive medium interaction which is taken to be of the simplest linear form with a medium-particle interaction couplign constant $g > 0$ ([1])

$$M\frac{d^2Q}{dt^2}(t) = -eE - gk\phi_k(t) \tag{7}$$

$$\frac{d^2\phi_k}{dt^2} = -\frac{k^2}{c^2}\phi_k(t) - gkQ(t) \tag{8}$$

By considering the above differentials equations of motion in the frequency domain by means of a Laplace transform, I get the following algebraic equations in the frequency domain in place of eqs. (7)-(8)

$$M\tilde{s}\tilde{Q}(s) = -\frac{eE}{s} - gk\tilde{\phi}_k(s) \tag{9}$$

$$s^2\tilde{\phi}_k(s) - \tilde{f}_k(s) = -\frac{k^2}{c^2}\tilde{\phi}_k(s) - gk\tilde{Q}(s) \tag{10}$$

By making use of the identity ([2])

$$\frac{1}{s^2+\frac{k^2}{c^2}} = \left(1 - \frac{s^2}{s^2+\frac{k^2}{c^2}}\right)\frac{1}{\frac{k^2}{c^2}} \tag{11}$$

I can re-write eq. (9) in the following suitable form where Λ the cut-off of the resistive medium wave vectors harmonic oscillators $1/\Lambda < |k| < \Lambda$

$$Ms^2\tilde{Q}(s) = -\frac{eE}{s} - \frac{g}{\left(\int_{1/\Lambda}^{\Lambda} dk\right)}\int_{1/\Lambda<|k|<\Lambda} dkK\left[\frac{\tilde{f}_k(s) - gk\tilde{Q}(s)}{s^2+\frac{k^2}{c^2}}\right] \tag{12}$$

I note that the $\tilde{Q}(s)$ term, coming from the last equation in the right-hand side of eq. (12) is given by

$$(c^2g^2)\tilde{Q}(s) - \frac{(2\pi g^2 c)}{\left(\int_{1/\Lambda<|k|<\Lambda} dk\right)}(s\tilde{Q}(s)) \tag{13}$$

The time domain equation associated with eq. (13) is, thus, given by

$$M\ddot{Q}(t) = (c^2g^2)Q(t) - eE - \nu^{(\Lambda)}\frac{dQ}{dt}(t) + F(t) \tag{14}$$

where I have firstly, a restauring force counter-term piece which I neglect in what follows since the motion frequency $w_0^2 = \frac{c^2g^2}{M} << 1$; and besides its presence does not influences my results, secondly a induced (not put by hand!) damping term with a friction coefficient $\nu^{(\Lambda)} = \frac{2\pi g^2 c}{\Lambda}$ and finally, and external random force $F(t)$ coming from the statistical chaotic resistive medium oscillators initial velocities and satisfying the Gaussian Statistics

$$\begin{aligned}\langle F(t)F(t')\rangle &= \frac{g^2}{\Lambda^2}\gamma c^2 \int_{1/\Lambda<|k|<\Lambda} dk\ sen\left(\frac{kt}{c}\right)\ sen\left(\frac{kt'}{c}\right) \\ &\sim \frac{g^2\gamma c^2\cdot c}{\Lambda^2}\ \delta(t-t') \stackrel{def}{=} \bar{\gamma}\delta(t-t')\end{aligned} \tag{15}$$

Note that a new strenght disorder $\bar{\gamma}$ is related to the old strenght disorder γ harmonic oscillators medium initial velocities by $\bar{\gamma} \stackrel{def}{\equiv} \frac{c^3\gamma g^2}{\Lambda^2}$.

Let me, thus, deduce the Ohm's law from the Langevin equations ((14)-(15)).

In this classical framework, the current density is $I(t) = \rho\ \frac{dQ}{dt}(t)$ with ρ is the free charge density of the charge flowing in the resistive medium which will be taken to be unity for simplicity in what follows.

Now, a simple functional integral shift as exposed in my earlier papers ([5]), leads to the following path-integral representation for the Gaussian stochastic process defined by our Langevin equation ((14) (see ref. [6]).

$$\begin{aligned} Z[J(t)] &= \frac{1}{Z(0)}\int D^F[I(t)]exp\left\{-\frac{1}{2\bar{\gamma}}\int_0^\infty dt\left[\frac{d}{dt}I(t) - \frac{e}{M}E - \frac{\nu^{(\Lambda)}}{M}I(t)\right]^2\right\} \\ & exp\left(i\int_0^\infty I(t)J(t)dt\right)\end{aligned} \tag{16}$$

Now it is straightforward calculation to evaluate the averaged current and, thus, obtain the Ohm's law in this classical situation

$$\bar{I} = \langle I(t')\rangle = \frac{\delta Z[J(t)]}{\delta J(t')}\Big|_{J(t)=0} = \bar{\gamma}\ \frac{\nu^{(\Lambda)}}{M}\left(\frac{e}{M}\right)E\int_0^\infty ds\left[\frac{e^{-\frac{\nu^{(\Lambda)}}{M}(s-t')}\theta(s-t')}{\nu^{(\Lambda)}/M}\right]$$

$$= \bar{\gamma} \frac{M}{\nu^{(\Lambda)}} \frac{e}{M} E = \frac{e\bar{\gamma}}{\nu^{(\Lambda)}} E \tag{17}$$

By introducing the potential $V = E.d$, where d is now the resistive medium lenght, one obtains the following formulae for the medium resistance parameter in terms of our microscopic constants (see eq. (17) and eq. (15)).

$$\begin{aligned} R_{medium} &= \frac{\nu^{(\Lambda)} d}{e\bar{\gamma}} = \frac{2\pi g^2 c d}{e\bar{\Lambda}[c^3 \gamma g^2/\Lambda^2]} \\ &= \frac{2\pi \Lambda d}{e c^2 \gamma} \end{aligned} \tag{18}$$

Let me close this section with the following physical remarks on eq. (19). Firstly if one increases the interaction g of the charged particle with the medium "atoms" the R_{medium} will be insensitive since it does not depends on g. However, in this strong coupling situation it certainly will be necessary to take into account non-linear terms of the medium wave field eq. (1)-eq.(2) and consequently these new non-linear terms will leads to deviations of the Linear Ohm's law eq. (17). For instance in the usual classical framework one should expects a medium resistance proportional to square of the velocity instead of the usual linear damping term. It is a straightforward calculation to see that the steady solution of the charged particle velocity equation satisfies now $\frac{\alpha}{\alpha^m} v^2 = eE = \frac{eV}{d}$ which naturally leads to a *square root Ohm's law* $V = \frac{d}{e}\frac{\nu}{m}\left(\frac{I}{Nes}\right)^2$ with a new "resistivity" parameter σ which is exactly given by $\sigma = \frac{d}{e}\frac{\alpha}{m}\frac{1}{N^2 e^2 s^2}$. Here s the medium area and N the number of carrier charges on the medium.

Finally, I point out that the analysis in the full 3D-medium (not factorizable in one-dimensional mediums) leads to the super-Ohmic case (see ref. [1] for details).

Section 2 – Ohm's law in Brownian Quantum Mechanics – the 3D Case

Let me start this section by reviewing my proposed D-dimensional Brownian quantum mechanics ([3]). Brownian quantum mechanics is an effective closed quantum system description of the interaction of a quantum single particle with a thermal reservoir by a Langevin-Euler equation for the quantum particle probability current

$$\vec{j}(\vec{x},t) = \psi(\vec{x},t) i\hbar \vec{\nabla}\psi^*(\vec{x},t) - \psi^*(\vec{x},t) i\hbar \vec{\nabla}\psi^*(\vec{x},t) \tag{19}$$

The main point in my proposed Brownian quantum mechanics is to introduce dissipation by considering a W.K.B pure phase approximation for the one-dimensional particle Schrödinger wave equation, namelly $\psi(x,t) = A\ exp\left(\frac{i}{\hbar}S(x,t)\right)$ with A constant, and, thus, I replace the Schrödinger wave equation by the Langevin-Euler equation for the W.K.B quantum current $\vec{j}(x,t) = \vec{\nabla}\cdot S(\vec{x},t)$, i.e.:

$$\frac{\partial}{\partial t}\vec{j}(x,t) + \frac{1}{2M}\vec{j}(x,t)\cdot\vec{\nabla}j(\vec{x},t) = -\nu\vec{j}(\vec{x},t) + \vec{F}(\vec{x},t) \tag{20}$$

Here $F(x,t)$ is the usual Langevin-bath random force satisfying the white noise statistics $\langle F_i(x,t)F_j(x',t')\rangle = D\delta(x-x')\delta(t-t')\delta_{ij}$. ([3])

Let me deduce the Ohm's law from the complete stochastic Langevin equation (20) solved exactly by a path integral procedure borrowed from similar procedure of that implemented in section 1, eq. (16) and at the one loop level approximation in the theory's coupling constant D (the noise bath strenght).

By proceeding as in ref. [5]-[6], we can write the characteristic funcional for eq. (20) as the following Euclidean field path integral

$$\begin{aligned} &Z[\vec{k}(x,t)] = \frac{1}{Z(0)}\int D^F[\vec{j}(x,t)] \\ &exp\left\{-\frac{1}{D}\int_0^\infty dt\int_{-\infty}^{+\infty}dx\left(\frac{\partial\vec{j}}{\partial t} + (\vec{j}\cdot\vec{\nabla})\vec{j} + \nu\vec{j} - \frac{e}{M}\vec{E}\right)^2(\vec{x},t)\right\} \\ &exp\left\{i\int_0^\infty dt\vec{j}(\vec{x},t)\vec{k}(\vec{x},t)\right\} \end{aligned} \tag{21}$$

Let me evaluate eq. (21) at one-loop level around the static Ohm's law classical configuration $\vec{j}_{classical} = \frac{e}{M\nu}\vec{E}$

$$\vec{j}(\vec{x},t) = \frac{e}{M\nu}\vec{E} + \sqrt{\frac{D}{2}}\overrightarrow{\hat{j}}(x,t) \tag{22}$$

Note that the classical Ohm's law configuration is given by the steady (space-time independent) solution of the classical equation associated to the action weight in the path integral eq. (21).

We arrive, thus, at the following Gaussian functional integral as my proposed one-loop approximation

$$\begin{aligned} Z[\vec{K}(\vec{x},t)] &= \frac{1}{Z[0]}\int D^F[\vec{j}(x,t)] \\ exp &\left\{ -\frac{1}{2}\int_0^\infty dt \int_{-\infty}^{+\infty} dx \left(\frac{\partial \overrightarrow{\hat{j}}}{\partial t} + \frac{e}{M\nu}(\vec{E}\cdot\vec{\nabla})\overrightarrow{\hat{j}} + \nu\,\overrightarrow{\hat{j}} \right)^2 (\vec{x},t) \right\} \\ exp &\left(i\int_0^\infty dt \int_{-\infty}^{\infty} dx \vec{K}(\vec{x},t) \left(\frac{e\vec{E}}{M\nu} + \sqrt{\frac{D}{2}}\overrightarrow{\hat{j}}(\vec{x},t) \right) \right) \end{aligned} \tag{23}$$

It is a simple evaluation to write the causal Green function of the kinetic operator associated to eq. (23) in order to have the exactly expression for the current-current two point correlation

$$\langle \hat{j}_i(x,t)\hat{j}_p(x',t')\rangle = \frac{D\delta_{ip}}{4}\left[\delta\left((\vec{x}-\vec{x}\,') - \frac{e\vec{E}}{M\nu}(t-t')\right)\frac{e^{-\nu(t-t')}}{\nu}\theta(t-t')\right] + \frac{e^2E_iE_p}{M^2\nu^2} \tag{24}$$

I conclude, thus, that the current charge transient flow is propagating by a damped wave process at a speed $\frac{e\vec{E}}{\nu M}$ towards to its steady Ohm's law value $\langle\vec{j}(\infty)\rangle = \frac{e}{m\nu}\vec{E}$.

At this point, I consider for mathematical completeness of my path integral study in this section 2, the usual hydrodynamical shear stress damping term $\alpha\Delta\vec{j}(\vec{r},t)$ in eq. (20) instead of the usual Brownian term proportional to velocity $-\nu\vec{j}(\vec{r},t)$ previously analyzed on that master equation.

I get, thus, the following quantum mechanical current equation in the range $0 \le t \le \infty$ in place of eq. (21)

$$\frac{\partial\vec{j}(\vec{r},t)}{\partial t} + \frac{1}{2M}(\vec{j}\cdot\vec{\nabla})\vec{j}(\vec{r},t) - \alpha\Delta\vec{j}(\vec{r},t) = -e\vec{E} + \vec{F}(\vec{r},t) \tag{25}$$

By proceeding as in the text, I obtain the stochastic process characteristic functional associated to the random stirred Burger equation (25) in the form of a quantum field path integral similar in its structure to eq. (21), namelly:

$$Z[\vec{K}(\vec{r},t)] = \frac{1}{Z(0)} \int D^F[\vec{j}(\vec{r},t)]$$
$$exp\left\{ -\frac{1}{2D} \int_0^\infty dt \int_{-\infty}^{+\infty} dx \left(\frac{\partial}{\partial t}\vec{j}(\vec{r},t) + \frac{1}{2M}(\vec{j}\vec{\nabla})\vec{j}(\vec{r},t) - \alpha\Delta\vec{j}(\vec{r},t) + e\vec{E} \right)^2 \right\}$$
$$exp\left\{ i \int_0^\infty dt\ \vec{j}(\vec{r},t)\vec{K}(\vec{r},t) \right\} \tag{26}$$

In the one-loop approximation to eq. (26) around the classical Ohm's law configuration

$$\vec{j}(\vec{r},t) = \sigma\vec{E} + \frac{1}{\sqrt{D}} \overrightarrow{\hat{j}}_{fl}(\vec{r},t) \tag{27}$$

the "transient" two-point correlation function is easily obtained

$$\langle \overrightarrow{\hat{j}}_i(\vec{r},t) \overrightarrow{\hat{j}}_\ell(\vec{r}\,',t') \rangle = \left\{ \frac{\delta_{i\ell}\theta(t-t')}{\alpha} \int_{-\infty}^{+\infty} d^3\vec{p}\ exp\left\{ \vec{p}\cdot[(\vec{r}-r') - \sigma\vec{E}(t-t')] \right\} \frac{1}{|\vec{p}|} \right.$$
$$\left. exp\left\{ -\alpha(\vec{p})^2(t-t') \right\} \right\} = \frac{\theta(t-t')}{\alpha^2(t-t')} \left[exp - \left(\frac{[(\vec{r}-\vec{r}\,') - \sigma\vec{E}(t-t')]^2}{4\alpha(t-t')} \right) \right]$$
$$\left\{ {}_1F_1\left(\frac{1}{2}; \frac{3}{2}; \frac{[(\vec{r}-\vec{r}\,') - \sigma\vec{E}(t-t')]^2}{4\alpha(t-t')} \right) \right\} \delta_{i\ell} \tag{28}$$

By analyzing eq. (28), one is lead to conclude that the only difference between the usual and the hydrodynamical damping terms are related to the different transient behaviours eq. (24) and eq. (28) to arrive at Ohm's law. However, next loop corrections will be entirely different since the associated theory's "propagators" (eq. (24) and eq. (28) are different in its mathematical form).

Finally, let me put in the usual path integral formalism the usual wisdom of quantum damping by means of a second-quantized Schrödinger field theory interacting with phonos (the quantized medium vibrations) ([1]). In this framework, the first basic object to be studied is the generating functional of the phonons Green's functions at a temperature T. ([1]).

$$Z_{phonos}[j_i(\vec{x},t)] = \frac{1}{Z(0)} \int D^F[r^i(\vec{x},t)]\ exp\left\{ -\frac{1}{kT} \int_0^{\frac{1}{kT}} dt \right.$$
$$\left. \int_{-\infty}^{+\infty} d^3\vec{x}\ c_{ijk\ell} U_{ij}(\vec{x}) \cdot U_{k\ell}(x) \right\}\ exp\left\{ i \int_0^{\frac{1}{kT}} dt \int_{-\infty}^{+\infty} d^3x\ j_i(\vec{x},t) r_i(x,t) \right\} \tag{29}$$

Here $\{r_i(\vec{x},t)\}$ denotes the medium displacement second quantized phonon field, the symmetric matrix $U_{ij}(\vec{x}) = \frac{1}{2}\left(\frac{\partial}{\partial x_i} r_j + \frac{\partial}{\partial x_j} r_i\right)(\vec{x},t)$ is the medium strain tensor, $c_{ijk\ell}$ denotes the elastic medium tensor and the path integral weight in eq. (29) is the second order Hook's law.

The usual interaction of the (scalar) Schrödinger field $\{\psi(\vec{r},t);\psi^*(\vec{r},t)\}$ with the medium phonons is given, thus, by the usual medium-electron potential interaction where $V(\vec{x})$ denotes the potential felt by the electron by the medium "atoms". In the path integral scheme I have the following path-integral for the interacting electronic field

$$
\begin{aligned}
&Z[K(\vec{x},t),K^*(\vec{x},t)] = \frac{1}{Z(0,0)} \int D^F[\psi^*(\vec{x},t)] \\
&exp\left\{-\frac{1}{kT}\int_0^{\frac{1}{kT}} dt \int_{-\infty}^{+\infty} d^3x \psi^*(\vec{x},t)\left[i\hbar\frac{\partial}{\partial t} - \frac{\hbar^2}{2M}\Delta_{\vec{x}}\right]\psi(\vec{x},t)\right\} \\
&exp\left\{-\frac{1}{KT}\int_0^{\frac{1}{KT}} dt \int_{-\infty}^{+\infty} d^3x(\psi i\hbar\nabla\psi^* - \psi^* i\hbar\nabla\psi)_i(\vec{x},t)V(\vec{x}-\vec{y})r_i(\vec{y},t)\right\} \\
&exp\left\{i\int_0^{\frac{1}{KT}} dt \int_{-\infty}^{+\infty} d^3x[\psi(\vec{x},t)K^*(\vec{x},t) + \psi^*(\vec{x},t)K(\vec{x},t)]\right\} \qquad (30)
\end{aligned}
$$

The complete microscopic system electron-phonon generating functional is thus, given by the combination of eq. (29) and eq. (30). Feynman diagrams may be easily implemented by following my previous studies ([1]).

In order to deduce the Ohm's law from eq. (27) - eq. (28), one should introduce an external homogeneous electric field $\{E_j\}$ in the system and try to evaluate the quantum current $\vec{j}(\vec{r},t)$ given by eq. (19) by means of Feynmans diagrams. Unfortunatelly, one will never get the usual macroscopic Ohm's law $\int_0^\infty dt \int d^3r\ j_i(\vec{r},t) = \sigma_{ij}E_j$ without putting by hand in the calculations the phenomenological "collision time" ([7]).

A possible microscopic mechanism leading to the Langevin eq. (20) is to consider a interaction of the Quantum Mechanical current $\vec{j}(\vec{y},t)$ given by eq. (19) directly with (the quantized phonon) field $\vec{r}(\vec{y},t)$, which by its turn, has a quartic self-interacting potential possessing a non-zero (positive) vacuum expectation value, namelly, at the leading order the above cited microscopic interaction will takes the macroscopic damping form $\vec{j}(\vec{y},t)\cdot\vec{r}(\vec{y},t) \approx \vec{j}(\vec{y},t)\cdot\langle\vec{r}(\vec{y},t)\rangle \simeq \nu\vec{j}(\vec{y},t)$ plus fluctuations. At this point we remark that in this friction mechanism framework the effective damping term comes from the phonon self-interaction broken-phase instead of the usual process of the direct scattering of the electron probability field density $(\psi\psi^*)(\vec{y},t)$ by the phonon field $\vec{r}(\vec{y},t)$ ([8]).

An alternative one-body phenomenological study to deduce the Ohm's law from quantum mechanics is to start from the begining directly of the (formal) Caldirola-Kanai lagrangean associated to a damped charged particle in the presence of an electric field ([4]), i.e.

$$\mathcal{L}[x(\sigma)] = \int_0^T d\sigma \left(e^{\nu\sigma} \left[\frac{1}{2}\, m_e \left(\frac{dx}{d\sigma}\right)^2 - eEx(\sigma) \right] \right) \tag{31}$$

According to my previous studies ([3]), an "effective" wave function can be associated to the dissipative system with a mass time-dependent term involving the damping reaction of a reservoir on the quantum electron

$$i\hbar \, \frac{\partial \psi^{ins}(x,t)}{\partial t} = \left(-\frac{1}{2e^{\nu t} m_e} \, \frac{d^2}{dx^2} - ee^{\nu}\; Ex \right) \psi^{ins}(x,t), \tag{32}$$

Here the initial condition

$$\psi^{ins}(x,0) = \delta(x) \tag{33}$$

is related to the fact that the electron is in a rest-origin situation at $t=0$.

In order to analyze the electrical condutance in this *single particle closed quantum system approach*, we should evaluate the electron velocity quantum operator, the electronic current, in our proposed quantum damped state Eq. (32) i.e.,

$$j(t) = \int_{-\infty}^{+\infty} dx \psi^{ins}\; i\hbar\, \frac{\partial}{\partial x} (\psi^{ins}(x,t))^* \,. \tag{34}$$

Let us now solve exactly the Eq. (32)-eq. (33), by considering the new "time"variable change on those equations

$$\begin{aligned} \zeta &= \frac{1}{m_e \nu}(1 - e^{-\nu t}) \\ \tilde{\psi^{ins}}(x,\zeta) &= \psi^{ins}(x,t) \end{aligned} \tag{35}$$

The new Schrödinger equation takes, thus, the form of exactly soluble problem in the new coordinate system (x,ζ) (now with $m_e = 1$)

$$i\hbar \frac{\partial \tilde{\psi^{ins}}(x,\zeta)}{\partial \zeta} = \left[-\frac{1}{2}\, \frac{d^2}{dx^2} + eE \cdot x \left(\frac{1}{(1-\nu\zeta)^2} \right) \right] \tilde{\psi^{ins}}(x,\zeta) \tag{36}$$

with

$$\tilde{\psi}(x,\zeta)_{\zeta\to 0^+} = \delta(x). \tag{37}$$

The solution of Eq. (36) is well-known ([6]) for the initial condition Eq. (37), i.e.,

$$\tilde{\psi^{ins}}(x,\zeta) = \sqrt{\frac{1}{2\pi i\hbar\zeta}}$$
$$exp\ \frac{i}{2\hbar\zeta}\left[x^2 - 2x\int_0^\zeta\ j(s)sds - 2\int_0^\zeta j(s)\int_0^s ds'j(s')(\zeta-s)s'\right] \tag{38}$$

where we have introduced the simplified notation:

$$j(s) = eE\ \frac{1}{(1-\nu s)^2}\ . \tag{39}$$

It is a straightforward calculation to obtain the form of the electronic current per volume in our theoretical model for quantum dissipative system *after disregarding the current associated to the free case of my analysis.* It yields the following result:

$$j(t) = 2eE\nu\ \frac{t}{(1-e^{-\nu t})^2} - 4eE\ \frac{(arctan(1-e^{-\nu t}))}{(1-e^{-nut})^2}\ . \tag{40}$$

The Ohm's law will, thus, be given by the following integral ([8])

$$\bar{j} = \int_0^\infty dt\ j(t)e^{-\nu t} = \frac{2eE}{\nu}\left[\int_0^\infty ds\ \frac{e^{-s}s}{(1-e^{-s})^2}\right]$$
$$-\frac{4eE}{\nu}\left[\int_0^\infty ds\ \frac{[arctan(1-e^{-s})e^{-s}]}{(1-e^{-s})^2}\right] = \frac{E}{\nu}\bar{c} \tag{41}$$

I get, thus, the Ohm's law if I identify the macroscopic (medium) resistence R as proportional to the damping constant ν. Note that in the calculation above we have introduced the dissipative anomaly factor $e^{-\nu t}$ (see [3] and [4]).

It is worth remark the one can say that the macroscopic medium resistence *increases* with the temperature if the electronic flux has a macroscopic behaviour like a classic gas and *decreases* with the temperature if it has a behavior like a liquid (see ref. [7] - chapter 4).

The case of the magneto resistance will be our next study.

Let us consider eq. (20) in the presence of a constant electric field $E = (E_x, E_y)$ in the plane (x, y) and a constant magnetic field H in the z-direction as in the usual plasma-magnetohydrodynamics framework ([9])

$$\frac{\partial\vec{j}(\vec{r},t)}{\partial t} + \frac{1}{2M}(\vec{j}\cdot\vec{\nabla})\vec{j}(\vec{r},t) = -\nu\vec{j}(\vec{r},t) + \vec{F}(\vec{r},t) + e\vec{E} + \frac{1}{c}(\vec{j}(\vec{r},t)\times\vec{H}) \tag{42}$$

In a one-loop approximation around the magneto-resistance Ohm's law, we have the field decomposition

$$\vec{j}(\vec{r},t) = \vec{\phi} + \sqrt{D}\vec{j}(\overrightarrow{\hat{r}},t) \tag{43}$$

where the background steady current configuration satisfy the magneto-resistance Ohm's law

$$\nu\vec{\varphi} - e\vec{E} - \frac{1}{c}\vec{\phi}\times\vec{H} = 0 \tag{44}$$

After simple algebraic calculations as showed in the text, we arrive at the one-loop Gaussian action functional (see re. [9] for details)

$$S_{one-loop}[\overrightarrow{\hat{j}}(\vec{r},t)] = (\overrightarrow{\hat{j}}_1(\vec{r},t);(\overrightarrow{\hat{j}}_2(\vec{r},t);(\overrightarrow{\hat{j}}_3(\vec{r},t))$$

$$\begin{bmatrix} \left(-\left(\frac{\partial}{\partial t}+\phi_i\frac{\partial}{\partial x_i}\right)^2+\frac{e^2H^2}{c^2}+\nu^2\right) & \frac{2H}{c}\left(\frac{\partial}{\partial t}+\phi'\frac{\partial}{\partial x_i}-\nu\right) & 0 \\ -\frac{2H}{c}\left(\frac{\partial}{\partial t}+\phi^i\frac{\partial}{\partial x_i}-\nu\right) & \left(-\left(\frac{\partial}{\partial t}+\phi_i\frac{\partial}{\partial x_i}\right)^2+\nu^2\frac{e^2H^2}{c^2}\right) & 0 \\ 0 & 0 & \left(-\left(\frac{\partial}{\partial t}+\phi_i\frac{\partial}{\partial x_i}\right)^2+\nu^2\right) \end{bmatrix}$$

$$\begin{pmatrix} \hat{j}_1(\vec{r},t) \\ \hat{j}_2(\vec{r},t) \\ \hat{j}_3(\vec{r},t) \end{pmatrix} \tag{45}$$

A simple reading of eq. (45) give us the non-vanishing two-point correlation functions (the transient law)

$$\begin{aligned}\langle j_1(\vec{r},t);j_1(\vec{r}\,',t')\rangle &= \langle j_2(\vec{r},t);j_2(\vec{r}\,',t')\rangle \\ &= \delta^{(3)}((\vec{r}-\vec{r}\,')-\vec{\phi}(t-t'))\times F[(t-t'),\nu,H]\end{aligned} \tag{46}$$

Where the time-dependent form factor is given exactly by the following integral evaluated with the causality prescription of ref. [9]

$$\begin{aligned}F(t-t',\nu,H) &= \int_{-\infty}^{+\infty}\frac{e^{i\bar{w}(t-t')}\,d\bar{w}}{\bar{w}^2+\left(\nu^2+\frac{H^2}{c^2}-\frac{2i\nu H}{c}\right)-\frac{2H}{c}\bar{w}} \\ &-\frac{e^{-\nu}(t-t')}{\nu}\theta(t-t')e^{-i\frac{H}{c}t}\end{aligned} \tag{47}$$

Note that the magneto-conductivity Ohm's is given by the usual result ([8])

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \begin{bmatrix} \frac{e}{\nu}\left(\frac{1}{\left(1+\frac{eB}{\nu c}\right)^2}\right) & \frac{e}{\nu}\left(\frac{1}{\left(1+\frac{eB}{\nu c}\right)^2}\right)\left(\frac{eB}{\nu c}\right) & 0 \\ -\frac{e}{\nu}\left(\frac{1}{\left(1+\frac{eB}{\nu c}\right)^2}\right)\left(\frac{eB}{\nu c}\right) & \frac{e}{\nu}\left(\frac{1}{\left(1+\frac{eB}{\nu c}\right)^2}\right) & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} E_x \\ E_y \\ 0 \end{pmatrix} \tag{48}$$

Random mediums as proposed in ref. [9] and [10] will be the subject of paper to appear elsewhere.

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