CBPF-NF-026/96

hep-th/9605158

UGVA-DPT-1996/05-925

Note on Constrained Cohomology[1*]

by

François Delduc[2], *Nicola Maggiore*[3,4], *Olivier Piguet*[5,6] and *Sylvain Wolf*[3]

Centro Brasileiro de Pesquisas Físicas - CBPF
Departamento de Campos e Partículas
Rua Dr. Xavier Sigaud, 150
22290-180 – Rio de Janeiro, RJ – Brazil

Abstract

The cohomology of the BRS operator corresponding to a group of rigid symmetries is studied in a space of local field functionals subjected to a condition of gauge invariance. We propose a procedure based on a filtration operator counting the degree in the infinitesimal parameters of the rigid symmetry transformations. An application to Witten's topological Yang-Mills theory is given.

Key-words: Cohomology; Algebraic renormalization; BRS.

PACS numbers: 11.15.-q (gauge field theories), 03.65.Fd (algebraic methods), 03.70. (theory of quantized fields)

---

[1]Supported in part by the Swiss National Science Foundation and by OFES contract 93.0083 and Human Capital and Mobility, EC contract ERBCHRXCT920069

[2]Laboratoire de Physique Théorique ENSLAPP, URA 14-36 du CNRS, associee à l'ENS de Lyon, à l'université de Savoie et au LAPP, groupe de Lyon, ENS Lyon, Allée d'Italie 46, F-69364 Lyon, France

[3]Département de Physique Théorique, Université de Genève, quai E. Ansermet 24, CH-1211 Genève 4, Switzerland

[4]On leave of absence from Università degli Studi di Genova, Dipartimento di Fisica, Italy

[5]Instituto da Física, Universidade Católica de Petrópolis, 25610-130 Petrópolis, RJ, Brazil and Centro Brasileiro de Pesquisas Físicas (CBPF), Rua Xavier Sigaud 150, 22290-180 Urca, RJ, Brazil

[6]On leave of absence from Département de Physique Théorique, Université de Genève, Switzerland. Supported in part by the Brazilian National Research Council (CNPq)

[*]To be published in Phys. Lett. B.

1 Introduction

All the symmetries of a given quantum field theory, including possibly the gauge invariance, may be grouped together in a single extended BRS operator D [1]. Here, whereas the ghosts associated to the gauge symmetry are represented by local fields, those associated with rigid symmetries are constant[7]. Both types of ghosts have a Grassmann parity opposite to that of the corresponding generator. The proof of renormalizability is then a matter of computing the cohomology of D in the space of local field polynomials with dimensions restricted by power counting [3].

A typical example is provided by the supersymmetric Yang-Mills theories quantized in the Wess-Zumino gauge. There, such a procedure appears to be necessary [4, 5] since the supersymmetry generators do not form a closed algebra because of the presence of gauge transformations [6].

Very often, it may be desirable to single out the individual symmetries from this unifying picture. This is needed, for instance, in the construction of gauge invariant operators belonging to some representation of a rigid symmetry. This is a problem of constrained cohomology, i.e. of the cohomology of D in a space constrained by the requirement of gauge invariance.

In the case of the super Yang-Mills theories, the rigid symmetry is the supersymmetry itself, and an important set of gauge invariant operators is given by the components of the supercurrent multiplet [7], i.e. the supermultiplet of currents which includes the R-axial current, the spinor current and the energy-momentum tensor. The constrained cohomology approach we want to describe here was introduced [5] in this context, in order to construct the supercurrent [8].

It turns out that the structure involved is of the type studied by Witten [9] in the framework of topological Yang-Mills theories.

The method and its relevance for quantum field theory are described in Section 2. In particular, we reproduce the algebraic structure, called "BRS extension", found by Henneaux [10] in a Hamiltonian approach.

In Section 3 we briefly discuss the example of Witten's topological Yang-Mills theory. The reader may look at [5] for the supersymmetric gauge theoretical example.

[7] See [2] for an earlier example of such a treatment for a rigid symmetry.

2 Covariance of Gauge Invariant Operators Under a Group of Rigid Symmetries

The quantization of gauge theories possessing a rigid symmetry is conveniently performed by introducing an extended nilpotent BRS operator D which, besides the BRS operator s associated with gauge invariance, also includes the infinitesimal transformations of the rigid symmetry [4, 5]. Let us write the infinitesimal transformations of the fields φ^i, in the classical theory, as[8]

$$\delta_{\rm rigid}\,\varphi^i := \varepsilon^A \delta_A \varphi^i \ , \tag{2.1}$$

with, in the most general case[9]

$$[\delta_A, \delta_B] = -c_{AB}{}^C \delta_C \quad + \ \text{eqs. of motion} + \text{gauge transf.} \ , \tag{2.2}$$

$$[\delta_A, s] = 0 \ . \tag{2.3}$$

$\delta_{\rm rigid}$ may act nonlinearly on the fields, and the algebra may happen to close only modulo equations of motion and field-dependent gauge transformations [10]. The extended BRS operator is then defined by

$$D := s + \varepsilon^A \delta_A + \frac{1}{2} c_{AB}{}^C \varepsilon^A \varepsilon^B \frac{\partial}{\partial \varepsilon^C} + O(\varepsilon^2) \ , \tag{2.4}$$

where the infinitesimal parameter ε^A is a constant ghost: it is an anticommuting (resp. commuting) number if the generator δ_A is a bosonic (resp. fermionic) operator. The term involving the structure constants $c_{AB}{}^C$ ensures the nilpotency of D:

$$D^2 = 0 \quad \text{(modulo eqs. of motion)} \ , \tag{2.5}$$

which expresses in a compact way the whole algebra of infinitesimal generators. Terms quadratic in ε are present in (2.4) if the rigid algebra (2.2) closes modulo field dependent gauge transformations, as it is the case, for instance, in the supersymmetric gauge theories in the Wess-Zumino gauge [4, 5].

---

[8] Summation over repeated indices is always understood.

[9] In the case of a superalgebra, the bracket $[\cdot,\cdot]$ is a graded commutator, i.e. an anticommutator if both its arguments are fermionic, and a commutator otherwise. We shall keep this notation throughout the paper.

[10] In the special case where the action of δ_A is linear:

$$\delta_A \varphi^i = T_A{}^i{}_j \varphi^j \ ,$$

and the algebra closes off-shell, this corresponds to the matrix algebra

$$[T_A, T_B] = c_{AB}{}^C T_C \ .$$

The invariance of the quantum theory under the extended BRS transformations (2.4), i.e. under the gauge BRS symmetry s and the rigid symmetry δ_A, is expressed by a Slavnov-Taylor identity – we assume that there is no anomaly –

$$\mathcal{S}(\Gamma) := \int dx \sum_i \frac{\delta\Gamma}{\delta\varphi_i^*(x)} \frac{\delta\Gamma}{\delta\varphi^i(x)} + \frac{1}{2} c_{AB}{}^C \varepsilon^A \varepsilon^B \frac{\partial\Gamma}{\partial\varepsilon^C} = 0 \ . \tag{2.6}$$

Here $\Gamma(\varphi, \varphi^*)$ is the vertex functional, i.e. the generating functional of the 1-particle-irreducible, amputated Green functions, and the φ_i^* are external fields coupled to the extended BRS transformations of the fields φ^i, introduced at the classical level by adding to the invariant action the terms

$$S_{\rm ext} = \int dx \sum_i \varphi_i^* D\varphi^i + O((\varphi^*)^2) \ . \tag{2.7}$$

The terms quadratic in the external sources are necessary if the whole algebra, as expressed by the nilpotency of D, holds only on-shell [11].

The Slavnov-Taylor identity also defines the quantum form of the field transformations. The extended BRS transformations of a local quantum composite field operator $\mathcal{O}$, whose vertex functions are generated by the insertion functional $\mathcal{O} \cdot \Gamma(\varphi, \varphi^*)$, is defined by

$$\mathcal{B}_\Gamma(\mathcal{O} \cdot \Gamma) \ , \tag{2.8}$$

where the Γ-dependent linear functional operator $\mathcal{B}_\Gamma$ is the linearized Slavnov-Taylor operator

$$\mathcal{B}_\Gamma := \int dx \sum_i \left(\frac{\delta\Gamma}{\delta\varphi_i^*(x)} \frac{\delta}{\delta\varphi^i(x)} + \frac{\delta\Gamma}{\delta\varphi(x)} \frac{\delta}{\delta\varphi_i^*(x)} \right) + \frac{1}{2} c_{AB}{}^C \varepsilon^A \varepsilon^B \frac{\partial}{\partial\varepsilon^C} \ . \tag{2.9}$$

This operator is automatically nilpotent:

$$(\mathcal{B}_\Gamma)^2 = 0 \ , \tag{2.10}$$

if Γ obeys the Slavnov identity (2.6). The action of $\mathcal{B}_\Gamma$ on the fields φ and φ^* defines their extended BRS transformations at the quantum level.

The problem we want now to solve is: how to extract the gauge-BRS and the rigid transformation laws of the quantum fields, together with their algebra, from the nilpotent extended BRS transformations containing all the symmetries of the theory ?

The answer begins by introducing a filtration operator

$$\mathcal{N} := \varepsilon^A \frac{\partial}{\partial\varepsilon^A} \ , \tag{2.11}$$

whose eigenvalues are the nonnegative integers, its application being restricted to the polynomials in φ, φ^* and ε. The vertex functional and the quantum BRS operator

(2.9) may be expanded according to these eigenvalues:

$$\Gamma = \sum_{n\geq 0}\Gamma_n \ , \quad \mathcal{B}_\Gamma = \sum_{n\geq 0}\mathcal{B}_n \ , \tag{2.12}$$

where

$$\mathcal{N}\Gamma_n = n\Gamma_n \ , \quad [\mathcal{N},\mathcal{B}_n] = n\mathcal{B}_n \ . \tag{2.13}$$

The nilpotency of $\mathcal{B}_\Gamma$ implies, at order 0, 1 and 2, the equations

$$\begin{aligned} &(\mathcal{B}_0)^2 = 0 \ , \\ &[\mathcal{B}_0,\mathcal{B}_1] = 0 \ , \\ &(\mathcal{B}_1)^2 + [\mathcal{B}_0,\mathcal{B}_2] = 0 \ . \end{aligned} \tag{2.14}$$

$\mathcal{B}_0$ is interpreted as the gauge-BRS operator. It is natural to interpret $\mathcal{B}_1$ as the operator of rigid transformations. More precisely, we can write

$$\mathcal{B}_1 =: \varepsilon^A X_A + \frac{1}{2}c_{AB}{}^C\varepsilon^A\varepsilon^B\frac{\partial}{\partial\varepsilon^C}, \quad \mathcal{B}_2 =: \frac{1}{2}\varepsilon^A\varepsilon^B X_{AB} \ . \tag{2.15}$$

The first equation defines the generators of the rigid transformations we were looking for. The second equation defines "second order" transformations. The algebra of the gauge-BRS and rigid operators is easily deduced from (2.14):

$$\begin{aligned} &(\mathcal{B}_0)^2 = 0, \\ &[\mathcal{B}_0, X_A] = 0 \ , \\ &[X_A, X_B] + c_{AB}{}^C X_C + [\mathcal{B}_0, X_{AB}] = 0 \ . \end{aligned} \tag{2.16}$$

One sees that the rigid generators X_A do not fulfill in general a closed (super)Lie algebra[11]. But this will however be the case if we restrict their action to the space of gauge invariant operators. A gauge invariant local field operator $[\mathcal{O}]$ is defined as a cohomology class of invariant composite insertion $\mathcal{O}\cdot\Gamma$ (c.f. (2.8)) :

$$\mathcal{B}_0(\mathcal{O}\cdot\Gamma) = 0, \tag{2.17}$$

with

$$[\mathcal{O}] = 0 \quad \text{if} \quad \mathcal{O}\cdot\Gamma = \mathcal{B}_\Gamma(\hat{\mathcal{O}}\cdot\Gamma) \ \ \forall\, \mathcal{O}\in[\mathcal{O}] \ . \tag{2.18}$$

There is a natural definition of the action of the rigid generators X_A on these cohomology classes, since these generators commute with the gauge-BRS operator (see (2.16)). It is then clear that

$$(\mathcal{B}_1)^2[\mathcal{O}] = [0] \ , \tag{2.19}$$

or, equivalently,

$$\left([X_A, X_B] + c_{AB}{}^C X_C\right)[\mathcal{O}] = [0] \ , \tag{2.20}$$

[11]The third of eqs. (2.16) precisely represents the algebraic structure one encounters in supersymmetry [5, 6], as mentioned in the introduction.

on any gauge invariant operator $[\mathcal{O}]$.

Remark: The nilpotency condition of $\mathcal{B}_1$ on the gauge invariant operators may be called a constrained nilpotency condition. The computation of the cohomology of $\mathcal{B}_1$ in the space of the gauge invariant operators is known as a problem of invariant cohomology [12], or "constrained cohomology". We shall apply the above formalism in the next section to a simple derivation of Witten's observables [9] in four-dimensional topological Yang-Mills theory.

3 Application: Topological Yang-Mills Theory

Witten's topological Yang-Mills field theory [9] in $D = 4$ Euclidean space is described by the following set of fields: a gauge field A_μ with its associated ghost c, and a fermionic vector field ψ_μ with its associated ghost φ.

These fields take their value in the adjoint representation of the gauge group, chosen as an arbitrary compact Lie group with structure constants[12] $f_{ab}{}^c$.

The invariances of this theory, namely the gauge symmetry and a supersymmetric-like shift symmetry, are grouped together in an extended BRS operator D, whose action on the fields is given by

$$\begin{aligned}
DA_\mu &= -\nabla_\mu c + \varepsilon\psi_\mu \ , \\
D\psi_\mu &= [c, \psi_\mu] - \varepsilon\nabla_\mu\varphi \ , \\
D\varphi &= [c, \varphi] \ , \\
Dc &= c^2 - \varepsilon^2\varphi \ , \\
D\varepsilon &= 0 \ ,
\end{aligned} \tag{3.1}$$

with

$$D^2 = 0 \ . \tag{3.2}$$

∇_μ is the covariant derivative: $\nabla_\mu \cdot = \partial_\mu \cdot + [A_\mu, \cdot]$. The supersymmetry infinitesimal parameter ε is taken to be commuting and plays the role of a constant ghost. Usually, ε is absorbed in a redefinition of the fields ψ_μ and φ, which accordingly acquire ghost numbers 1 and 2, respectively. We shall nevertheless keep ε explicit to better illustrate our procedure. We remark that the presence of the term quadratic in ε in the transformation law of the ghost field c is necessary for the nilpotency of D. In this parametrization all the fields introduced up to now have ghost number 0 except c and ε, of ghost number 1.

[12] We use a matrix notation, e.g. $A_\mu = A^a_\mu \tau_a$, with $[\tau_a, \tau_b] = f_{ab}{}^c\tau_c$, and with the trace normalized in such a way that $\mathrm{Tr}\, \tau_a\tau_b = \delta_{ab}$.

The classical action reads [9]

$$S = \frac{1}{4g^2}\mathrm{Tr}\int d^4x\ \left(F^{+\mu\nu}F^{+}_{\mu\nu} + \cdots\right)\ , \tag{3.3}$$

where

$$F^{+}_{\mu\nu} = \frac{1}{2}\left(F_{\mu\nu} - \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}\right)\ , \tag{3.4}$$

is the anti-selfdual part of the Yang-Mills field strength. The dots represent the terms needed for the gauge fixing [14, 15].

This theory is renormalizable, and even ultraviolet finite. The proof [15] is based on the triviality of the cohomology of the extended BRS operator D. However, in order to maintain the exposition simple, we shall keep ourselves at the level of the classical theory described by the action (3.3), the generalization to the quantum theory being straightforward.

The filtration operator (2.11) and the expansion (2.12) take here the form

$$\mathcal{N} = \varepsilon\frac{\partial}{\partial\varepsilon}\ , \tag{3.5}$$

$$D = D_0 + \varepsilon D_1 + \varepsilon^2 D_2\ , \tag{3.6}$$

where we factorized the powers of ε. The transformation laws of the operators D_n and their algebra – up to order 2 – read

$$\begin{array}{lll}
D_0A_\mu = -\nabla_\mu c & D_1A_\mu = \psi_\mu\ , & D_2A_\mu = 0\ , \\
D_0\psi_\mu = [c,\psi_\mu]\ , & D_1\psi_\mu = -\nabla_\mu\varphi\ , & D_2\psi_\mu = 0\ , \\
D_0\varphi = [c,\varphi] & D_1\varphi = 0\ , & D_2\varphi = 0\ , \\
D_0c = c^2\ , & D_1c = 0\ , & D_2c = -\varphi\ ,
\end{array} \tag{3.7}$$

$$(D_0)^2 = 0\ ,\quad [D_0,D_1] = 0\ ,\quad (D_1)^2 + [D_0,D_2] = 0\ . \tag{3.8}$$

Following the lines drawn in the previous section for the general case, we consider the space $\mathcal{H}_{(0)}$ of gauge invariant local operators, i.e. the cohomology of D_0. Within this space, the analogous of (2.19) reads

$$D_1^2\Delta = 0\ ,\quad \forall\,\Delta\in\mathcal{H}_{(0)} \tag{3.9}$$

which must be intended according to cohomology classes.

As it is shown in the Appendix, it follows from the transformation laws (3.7) that the the constrained cohomology of D_1, i.e. the cohomology of D_1 in $\mathcal{H}_{(0)}$, is given by the D_0-cohomology classes represented by the functions

$$F\left(P_{\mathrm{inv}}(c),\, Q_{\mathrm{inv}}(\varphi)\right)\ , \tag{3.10}$$

whose arguments are group invariant polynomials which depend only on the fields c and φ without their derivatives.

The result (3.10) shows that, to the contrary of the cohomology of D, the constrained cohomology is not empty. The filtration we introduced in the global ghosts ε allowed to see this easily.

According to the definition of [9], the observables $\mathcal{O}$ of the model should be independent of c, thus excluding for instance terms like $\mathrm{Tr}\, c^{2n+1}$. The consequence of the c-independence is that the elements of the constrained space in which calculating the cohomology of D_1 are invariant under D_2. This property, together with the third of eqs. (3.8), implies the *exact* nilpotency of D_1 (and not in the sense of the cohomology classes):

$$D_0\mathcal{O} = D_2\mathcal{O} = 0 \rightarrow (D_1)^2\mathcal{O} = 0 \ . \tag{3.11}$$

This means that the zero ghost cohomology of D in $\mathcal{H}_{(0)}$ is identical to that of D_1. ¿From (3.10), requiring c-independence, we recover the Witten's observables of the topological Yang-Mills theory:

$$\{\mathcal{O}_{\mathrm{Witten}}\} = \{Q_{\mathrm{inv}}(\varphi)\} \ , \tag{3.12}$$

which are invariant polynomials of the field φ only, with the exclusion of its derivatives. As a comment, we remark that the above result (3.12) has been obtained after a simple cohomology calculation in a restricted functional space (see the Appendix), while usually the Witten's observables are characterized as what is called the "equivariant", or "basic", cohomology [16], which requires a specific, and more complicated, analysis [17].

4 Conclusions

We have been able to extract the individual symmetries, namely the gauge invariance and the rigid symmetries, from the general BRS operator of the theory. We have shown in particular that, although the generators constructed in this way do not form a closed algebra, they do so if their action is restricted to the space of gauge invariant quantities defined by the cohomology of the gauge BRS operator. Only in this restricted space it makes sense to study the cohomology corresponding to the rigid invariance, and we stress that this holds for any gauge theory characterized by an additional rigid invariance satisfying the general algebra (2.2). As an example, we recovered the Witten's observables of the topological Yang-Mills theory, which are usually found in the more complicated context of the equivariant cohomology. Moreover, the formalism proposed in this paper is well adapted to the quantum theory, since the constraints such as gauge invariance, as well as the symmetry generators, are expressed in a functional way, i.e. as constraints and operations on the generating functionals of Green functions.

Acknowledgments: One of the authors (O.P.) is very indebted to the Conselho Nacional de Pesquisa e Desevolvimento (Brazil) for its financial support, to the Institute of Physics of the Catholic University of Petrópolis (Brazil) and its head Prof. R. Doria, as well as to the CBPF (Rio de Janeiro), where part of this work has been done, for their hospitality. We are grateful for interesting discussions with M.A. De Andrade, C. Becchi, O.M. Del Cima, N. Dragon and S.P. Sorella on this work, and we are glad to thank R. Stora and M. Henneaux for a critical reading of the manuscript and for drawing our attention to refs. [16] and [10].

Appendix

As said in Sect. 3, the operator D_1 is nilpotent within the space $\mathcal{H}_{(0)}$ of the local cohomology classes of the gauge BRS operator D_0 (see (3.9)). This space is isomorphic [18] to the space $\tilde{\mathcal{H}}_{(0)}$ of the gauge invariant polynomials generated by the invariant polynomials $P_{\rm inv}(c)$ and $Q_{\rm inv}(F,\psi,\varphi)$, the former depending on the field c but not on its derivatives, the latter depending on the fields $F_{\mu\nu}$, ψ_μ, φ and all their covariant derivatives. One notes that

$$(D_1)^2\tilde{\Delta} = 0 \ , \quad \forall\,\tilde{\Delta} \in \tilde{\mathcal{H}}_{(0)} \ , \tag{A.1}$$

and

$$D_1\tilde{\mathcal{H}}_{(0)} \subset \tilde{\mathcal{H}}_{(0)} \ . \tag{A.2}$$

Thus D_1 is a coboundary operator in $\tilde{\mathcal{H}}_{(0)}$.

The isomorphism of $\mathcal{H}_{(0)}$ and $\tilde{\mathcal{H}}_{(0)}$ implies that the image of D_1 in $\mathcal{H}_{(0)}$ is isomorphic to its image in $\tilde{\mathcal{H}}_{(0)}$:

$$\operatorname{Im} D_1|_{\mathcal{H}_{(0)}} \approx \operatorname{Im} D_1|_{\tilde{\mathcal{H}}_{(0)}} \ . \tag{A.3}$$

In order to see this, let us consider an arbitrary representative $\tilde{\Delta} + D_0(\cdots)$ of an element Δ of $\mathcal{H}_{(0)}$ represented by $\tilde{\Delta} \in \tilde{\mathcal{H}}_{(0)}$. Applying D_1 to it we get

$$D_1\left(\tilde{\Delta} + D_0(\cdots)\right) = D_1\tilde{\Delta} - D_0D_1(\cdots) \ ,$$

due to the anticommutativity of D_0 and D_1, i.e. its image is represented by $D_1\tilde{\Delta}$, which belongs to $\tilde{\mathcal{H}}_{(0)}$ due to the property (A.2).

Moreover, it also follows from the latter property that the kernel of D_1 in $\mathcal{H}_{(0)}$ is isomorphic to its kernel in $\tilde{\mathcal{H}}_{(0)}$:

$$\mathrm{Ker}\, D_1|_{\mathcal{H}_{(0)}} \approx \mathrm{Ker}\, D_1|_{\tilde{\mathcal{H}}_{(0)}} \ . \tag{A.4}$$

Indeed, the D_1-invariance of an element of $\mathcal{H}_{(0)}$ represented by $\tilde{\Delta} \in \tilde{\mathcal{H}}_{(0)}$ is expressed on any of its representatives by the condition

$$D_1 \left(\tilde{\Delta} + D_0(\cdots) \right) = D_0(\cdots) \ ,$$

which means that $D_1 \tilde{\Delta} = D_0(\cdots)$, i.e. $D_1 \tilde{\Delta} = 0$ since $\tilde{\mathcal{H}}_{(0)}$ does not contain D_0-exact elements.

The results (A.3) and (A.4) show that the cohomologies of D_1 in $\mathcal{H}_{(0)}$ and in $\tilde{\mathcal{H}}_{(0)}$ are isomorphic. We are thus left to compute the latter cohomology.

¿From the observation that

$$\begin{aligned}
& D_1 \left(\nabla_{\mu_1} \cdots \nabla_{\mu_n} F_{\mu\nu} \right) = \\
& \quad \nabla_{\mu_1} \cdots \nabla_{\mu_n} \left(\nabla_\mu \psi_\nu - \nabla_\nu \psi_\mu \right) + \sum_{i=1}^{n} \nabla_{\mu_1} \cdots \nabla_{\mu_{i-1}} \left[\psi_{\mu_i} \, , \, \nabla_{\mu_{i+1}} \cdots \nabla_{\mu_n} F_{\mu\nu} \right] \ , \\
& D_1 \left(\nabla_{\mu_1} \cdots \nabla_{\mu_n} \left(\nabla_\mu \psi_\nu + \nabla_\nu \psi_\mu \right) \right) = \\
& \quad \nabla_{\mu_1} \cdots \nabla_{\mu_n} \left(- \nabla_\mu \nabla_\nu \varphi - \nabla_\nu \nabla_\mu \varphi + 2 \left[\psi_\mu \, , \, \psi_\nu \right] \right) \\
& \quad + \sum_{i=1}^{n} \nabla_{\mu_1} \cdots \nabla_{\mu_{i-1}} \left[\psi_{\mu_i} \, , \, \nabla_{\mu_{i+1}} \cdots \nabla_{\mu_n} \left(\nabla_\mu \psi_\nu + \nabla_\nu \psi_\mu \right) \right] \ , \\
& \left[\nabla_\mu , \nabla_\nu \right] \phi = \left[F_{\mu\nu} \, , \, \phi \right] \ , \quad \forall \, \phi \ ,
\end{aligned}$$

we see that we can choose, as independent variables, the fields

$$\{ X, \ D_1 X, \ \varphi, \ c \} \ , \tag{A.5}$$

with

$X :=$

$F_{\mu\nu}$ and its symmetric covariant derivatives ,
$\nabla_\mu \psi_\nu + \nabla_\nu \psi_\mu$ and its symmetric covariant derivatives ,
ψ_μ .

In this basis the BRS operators read

$$\begin{aligned}
D_0 &= \sum_X \left([c, X] \frac{\partial}{\partial X} + [c, D_1 X] \frac{\partial}{\partial (D_1 X)} + [c, \varphi] \frac{\partial}{\partial \varphi} + c^2 \frac{\partial}{\partial c} \right) \ , \\
D_1 &= \sum_X \left(D_1 X \frac{\partial}{\partial X} + [\varphi, X] \frac{\partial}{\partial (D_1 X)} \right) \ , \\
D_2 &= - \varphi \frac{\partial}{\partial c} \ ,
\end{aligned}$$

and obey the algebra (3.8). In particular D_1 is not nilpotent, when applied to the variables (A.5), although it does when applied to the elements of $\tilde{\mathcal{H}}_{(0)}$. Extending a well known argument [3] to the present case of an operator which is not nilpotent, we define the operators

$$D_1' := X\frac{\partial}{\partial(D_1X)}\ , \quad F := X\frac{\partial}{\partial X} + D_1X\frac{\partial}{\partial(D_1X)}\ ,$$

we check that

$$[D_0, F] = 0\ , \quad [D_1, F] = 0\ , \quad [D_1, D_1'] = F\ ,$$

and we note that

$$D_1'\tilde{\mathcal{H}}_{(0)} \subset \tilde{\mathcal{H}}_{(0)}\ .$$

For solving the cohomology equation

$$D_1\tilde{\Delta} = 0\ , \quad \tilde{\Delta} \in \tilde{\mathcal{H}}_{(0)}\ ,$$

we expand $\tilde{\Delta}$ according to the eigenvalues of the operator F:

$$\tilde{\Delta} = \sum_{n\geq 0}\tilde{\Delta}^{(n)}\ , \quad \text{with} \quad F\tilde{\Delta}^{(n)} = n\tilde{\Delta}^{(n)}\ .$$

We have then

$$D_0\tilde{\Delta}^{(n)} = 0\ , \quad D_1\tilde{\Delta}^{(n)} = 0\ , \quad \forall\, n \geq 0\ ,$$

and, defining

$$\tilde{\Delta}' := \sum_{n\geq 1}\frac{1}{n}D_1'\tilde{\Delta}^{(n)}\ ,$$

which belongs to $\tilde{\mathcal{H}}_{(0)}$, we get

$$\tilde{\Delta} = \tilde{\Delta}^{(0)} + D_1\tilde{\Delta}'\ .$$

This shows that the cohomology of D_1 in $\tilde{\mathcal{H}}_{(0)}$ is given by the X- and D_1X-independent $\tilde{\Delta}^{(0)}$, i.e. that it consists of the elements of $\tilde{\mathcal{H}}_{(0)}$ of the form (3.10). This is the result announced in Sect. 3, due to the isomorphism of the cohomologies of D_1 in $\tilde{\mathcal{H}}_{(0)}$ and in $\mathcal{H}_{(0)}$.

References

[1] J.A. Dixon, *Commun. Math. Phys.* 140 (1991) 169;

[2] C. Becchi, A. Blasi, G. Bonneau, R. Collina and F. Delduc, *Commun. Math. Phys.* 120 (1988) 121;

[3] O. Piguet and S.P. Sorella, "Algebraic Renormalization", *Lecture Notes in Physics, Vol.* m28 (*Springer Verlag, Berlin, Heidelberg*, 1995);

[4] P.L. White, *Class. Quantum Grav.* 9 (1992) 413;
Class. Quantum Grav. 9 (1992) 1663;
N. Maggiore, *Int. J. Mod. Phys.* A10 (1995) 3781;
Int. J. Mod. Phys. A10 (1995) 3937;
N. Maggiore, O. Piguet and M. Ribordy, *Helv. Phys. Acta* 68 (1995) 264;

[5] N. Maggiore, O. Piguet and S. Wolf, *Nucl. Phys.* B458 (1996) 403;

[6] P. Breitenlohner and D. Maison, "Renormalization of supersymmetric Yang–Mills theories", *in Cambridge 1985, Proceedings: "Supersymmetry and its applications"*, p.309;
and "N=2 Supersymmetric Yang–Mills theories in the Wess–Zumino gauge", *in "Renormalization of quantum field theories with nonlinear field transformations", ed.: P. Breitenlohner, D. Maison and K. Sibold, Lecture Notes in Physics, Vol.* 303, p.64 (*Springer Verlag, Berlin, Heidelberg*, 1988);

[7] S. Ferrara and B. Zumino, *Nucl. Phys.* B87 (1975) 207;

[8] work in progress;

[9] E. Witten, *Commun. Math. Phys.* 117 (1988) 353;
Int. J. Mod. Phys. A6 (1991) 2775;

[10] M. Henneaux, *Nucl. Phys.* B308 (1988) 619;

[11] I.A. Batalin and G.A. Vilkovisky, *Phys. Lett.* 69B (1983) 309;
Phys. Rev. D28 (1981) 2567; *Phys. Lett.* 102B (1981) 27;

[12] R.Stora, private communication;

[13] D. Birmingham, M. Blau, M. Rakowski and G. Thompson,
Phys. Rep. 209 (1991) 129;

[14] S. Ouvry, R. Stora and P. van Baal, *Phys. Lett.* B220 (1989) 159;

[15] M.W. de Oliveira, *Phys. Lett.* B307 (1993) 347;
A. Brandhuber, O. Moritsch, M.W. de Oliveira, O. Piguet and M. Schweda, *Nucl. Phys.* B431 (1994) 173;

[16] J. Kalkman, *Commun. Math. Phys.* 153 (1993) 447;
R.Stora, "Equivariant cohomology and topological theories", Lecture given at the international symposium on BRS symmetry, September 1995, Kyoto, Japan, Enslapp-A-571/95;
R. Stora, F. Thuillier and J.-C. Wallet, "Algebraic structure of cohomological field theory models and equivariant cohomology", Lectures at the 1st Caribbean Spring School of Mathematics and Theoretical Physics, Saint François, Guadeloupe, 1993;

[17] S. Ouvry, R. Stora and P. van Baal, *Phys. Lett.* B220 (1989) 159;
A. Blasi and R. Collina, *Phys. Lett.* B222(1989)419

[18] G. Bandelloni, *J. Math. Phys.* 27 (1986) 2551;
J. Dixon, "Cohomology and renormalization of gauge theories", I, II, III, *unpublished preprints, 1976-1977*;
J. Dixon, *Commun. Math. Phys.* 139 (1991) 495;
M. Dubois-Violette, M. Henneaux, M. Talon and C.-M. Viallet, *Phys. Lett.* B289 (1992) 361;
F. Brandt, N. Dragon and M. Kreuzer, *Phys. Lett.* B231 (1989) 263; *Nucl. Phys.* B332 (1990) 224, 250;
F. Brandt, PhD Thesis (in German), *University of Hannover (1991), unpublished.*