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$N=1$ Super-Chern-Simons Coupled to Parity-Preserving Matter from Atiyah-Ward Space-Time

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Abstract

In this letter, we present the Parkes-Siegel formulation for the massive Abelian $N=1$ super-QED_{2+2} coupled to a self-dual supermultiplet, by introducing a chiral multiplier superfield. We show that after carrying out a suitable dimensional reduction from (2+2) to (1+2) dimensions, and performing some necessary truncations, the simple supersymmetric extension of the $\tau_3\mathrm{QED}_{1+2}$ coupled to a Chern-Simons term naturally comes out.

Key-words: Supersymmetry; Chern-Simons; Parity-Preserving Matter.

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The issue of self-duality has deserved a great deal of attention since a self-dual Yang-Mills theory in Atiyah-Ward space-time (2+2 dimensions) [1] has been pointed out as a source for various integrable models in lower dimensions, according to a conjecture by Atiyah and Ward [2].

Recently, the simple supersymmetric version of the self-dual Yang-Mills theory and self-dual supergravity model in Atiyah-Ward space-time has been formulated by Gates, Ketov and Nishino [3]. Also, by a suitable dimensional reduction proposed by Nishino, N=1 and N=2 super-Chern-Simons theories in D=1+2 were generated from N=1 and N=2 super-self-dual Yang-Mills theories in D=2+2 [4].

In the last years, 3-dimensional field theories [5] have been well-motivated in view of the possibilities of providing a gauge-theoretical foundation for the description of condensed matter phenomena, such as high-T_c superconductivity [6], where the QED$_3$ and τ_3QED$_{1+2}$ [6, 7] are some of the theoretical approaches used to understand more deeply about high-T_c materials. The finiteness on the Landau gauge of Chern-Simons theories [8] is also an interesting result that motivates the study of 3-dimensional gauge theories.

The relationship between massive Abelian N=1 super-QED$_{2+2}$ in Atiyah-Ward space-time and N=1 super-τ_3QED in D=1+2 has already been investigated by carrying out a dimensional reduction *à la* Scherk from (2+2) to (1+2) dimensions and by performing some suitable supersymmetry-preserving truncations [9, 10].

The purpose of this letter is to show that N=1 super-τ_3QED coupled to a super-Chern-Simons term in D=1+2 can be generated from the massive Abelian N=1 super-QED$_{2+2}$ [9, 10] coupled to a self-dual supermultiplet by using the Parkes-Siegel formulation in D=2+2 [11]. The dimensional reduction used here to show the relationship between the models previously mentioned was proposed by Nishino in Ref.[4]. Also, some suitable supersymmetry-preserving truncations are needed in order to suppress non-physical propagating modes as well as to keep a simple supersymmetry in D=1+2.

To introduce mass to the matter sector in D=2+2, without breaking gauge-symmetry, we have to deal with four scalar superfields: a pair of chiral and a pair of anti-chiral superfields; the members of each pair have opposite $U(1)$-charges [9, 10]. The Parkes-Siegel formulation for the massive Abelian N=1 super-QED$_{2+2}$ coupled to a self-dual supermultiplet, by introducing a chiral multiplier superfield, is described by the action : [1]

$$S^{\rm SD}_{\rm SQED} = -\int ds\; \Xi^{c} W + \int dv\; \left(\Psi^{\dagger}_{+} e^{4qV}\widetilde{X}_{+} + \Psi^{\dagger}_{-} e^{-4qV}\widetilde{X}_{-}\right) + \\ + i\,m\left(\int ds\; \Psi_{+}\Psi_{-} - \int d\widetilde{s}\; \widetilde{X}_{+}\widetilde{X}_{-}\right) + \text{h.c.} \quad , \tag{1}$$

where q is a real dimensionless coupling constant and m is a real parameter with dimension of mass. The $+$ and $-$ subscripts in the matter superfields refer to their respective $U(1)$-charges.

In the action (1), the chiral ($\Psi_{\pm}$), the anti-chiral ($\widetilde{X}_{\pm}$) and the chiral multiplier (Ξ) superfields are defined as follows:

$$\Psi_{\pm}(x,\theta,\widetilde{\theta}) = e^{i\tilde{\theta}\tilde{\partial}\theta}\left[A_{\pm}(x) + i\theta\psi_{\pm}(x) + i\theta^{2}F_{\pm}(x)\right] \quad , \quad \widetilde{D}_{\dot{\alpha}}\Psi_{\pm} = 0 \quad , \tag{2}$$

$$\widetilde{X}_{\pm}(x,\theta,\widetilde{\theta}) = e^{i\theta\partial\!\!\!/\tilde{\theta}}\left[B_{\pm}(x) + i\widetilde{\theta}\widetilde{\chi}_{\pm}(x) + i\widetilde{\theta}^{2}G_{\pm}(x)\right] \quad , \quad D_{\alpha}\widetilde{X}_{\pm} = 0 \quad , \tag{3}$$

$$\Xi_{\alpha}(x,\theta,\widetilde{\theta}) = e^{i\tilde{\theta}\tilde{\partial}\theta}\left[A_{\alpha}(x) + \theta^{\beta}\left(\epsilon_{\alpha\beta}E(x) - \sigma^{\mu\nu}_{\alpha\beta}H_{\mu\nu}(x)\right) + i\theta^{2}F_{\alpha}(x)\right] \quad , \quad \widetilde{D}_{\dot{\beta}}\Xi_{\alpha} = 0 \quad , \tag{4}$$

where, $A_{\pm}$ and $B_{\pm}$ are complex scalars, $\psi_{\pm}$ and $\widetilde{\chi}_{\pm}$ are Weyl spinors, $F_{\pm}$ and $G_{\pm}$ are complex auxiliary scalars, A_{α} is a Weyl spinor, E is a complex scalar, $H_{\mu\nu}$ is a complex antisymmetric rank-2 tensor and F_{α} is a Weyl auxiliary spinor.

In the Wess-Zumino gauge [12], a complex *vector* superfield, V, is written as

$$V(x,\theta,\widetilde{\theta}) = \frac{1}{2}i\theta\sigma^{\mu}\widetilde{\theta}B_{\mu}(x) - \frac{1}{2}\widetilde{\theta}^{2}\theta\lambda(x) - \frac{1}{2}\theta^{2}\widetilde{\theta}\widetilde{\rho}(x) - \frac{1}{4}\theta^{2}\widetilde{\theta}^{2}D(x) \quad , \tag{5}$$

[1] We are adopting in this letter, $\eta_{\mu\nu}$=$(+,-,-,+)$, for the A-W space-time metric, $ds{\equiv}d^4xd^2\theta$, $d\widetilde{s}{\equiv}d^4xd^2\widetilde{\theta}$ and $dv{\equiv}d^4xd^2\theta d^2\widetilde{\theta}$, where θ and $\widetilde{\theta}$ are Majorana-Weyl spinors. Also, the supersymmetry covariant derivatives are defined by : D_{α}=$\partial_{\alpha} - i\partial\!\!\!/_{\alpha\dot{\alpha}}\widetilde{\theta}^{\dot{\alpha}}$ and $\widetilde{D}_{\dot{\alpha}}$=$\widetilde{\partial}_{\dot{\alpha}} - i\widetilde{\partial\!\!\!/}_{\dot{\alpha}\alpha}\theta^{\alpha}$. For more details about notational conventions in D=2+2 and D=1+2, see ref.[9, 10].

where D is a complex auxiliary scalar, λ and $\widetilde{\rho}$ are Weyl spinors and B_μ is a *complex* vector field.

The field-strength superfields, W_α and $\widetilde{W}_{\dot\alpha}$, defined by

$$W_\alpha = \frac{1}{2}\widetilde{D}^2 D_\alpha V \quad \text{and} \quad \widetilde{W}_{\dot\alpha} = \frac{1}{2}D^2\widetilde{D}_{\dot\alpha}V \quad , \tag{6}$$

respectively, satisfy the chiral and anti-chiral conditions, $\widetilde{D}_{\dot\beta}W_\alpha{=}0$ and $D_\beta\widetilde{W}_{\dot\alpha}{=}0$.

By adopting the Wess-Zumino gauge and considering the superfields defined above, the following component-field action stems from the superspace action of eq.(1) :

$$\begin{aligned} S^{\rm SD}_{\rm SQED} = & \int d^4x \left\{ -\frac{1}{2}H^*_{\mu\nu}\left(G^{\mu\nu} - \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G_{\rho\sigma}\right) - i\left(A^{\rm c}\partial\!\!\!/\widetilde{\rho} + F^{\rm c}\lambda\right) - E^*D + \right. \\ & -F^*_+G_+ - A^*_+\Box B_+ - \frac{1}{2}i\psi^{\rm c}_+\partial\!\!\!/\widetilde{\chi}_+ - qB_\mu\left(\frac{1}{2}i\psi^{\rm c}_+\sigma^\mu\widetilde{\chi}_+ + A^*_+\partial^\mu B_+ - B_+\partial^\mu A^*_+\right) + \\ & +iq\left(A^*_+\widetilde{\chi}_+\widetilde{\rho} + B_+\psi^{\rm c}_+\lambda\right) - \left(qD + q^2B_\mu B^\mu\right)A^*_+B_+ + \\ & -F^*_-G_- - A^*_-\Box B_- - \frac{1}{2}i\psi^{\rm c}_-\partial\!\!\!/\widetilde{\chi}_- + qB_\mu\left(\frac{1}{2}i\psi^{\rm c}_-\sigma^\mu\widetilde{\chi}_- + A^*_-\partial^\mu B_- - B_-\partial^\mu A^*_-\right) + \\ & -iq\left(A^*_-\widetilde{\chi}_-\widetilde{\rho} + B_-\psi^{\rm c}_-\lambda\right) + \left(qD - q^2B_\mu B^\mu\right)A^*_-B_- + \\ & \left. +m\left(\frac{1}{2}i\psi_+\psi_- - \frac{1}{2}i\widetilde{\chi}_+\widetilde{\chi}_- - A_+F_- - A_-F_+ + B_+G_- + B_-G_+\right)\right\} + \text{h.c.} \quad , \end{aligned} \tag{7}$$

where $G_{\mu\nu}$ is the usual field-strength associated to B_μ.

Therefore, it can be easily seen, from (7), that the field equation for $H^*_{\mu\nu}$ gives the self-duality of the field-strength $G^{\mu\nu}$:

$$\frac{\delta S^{\rm SD}_{\rm SQED}}{\delta H^*_{\mu\nu}} = 0 \quad \Longrightarrow \quad G^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G_{\rho\sigma} \quad . \tag{8}$$

Since we are adopting the Wess-Zumino gauge, we can read directly from the matter sector of (1), the following set of local $U(1)_\alpha{\times}U(1)_\gamma$ transformations [9, 10] :

$$\delta_g A^*{}_\pm = \pm iq\beta(x)A^*{}_\pm \quad , \quad \delta_g \overset{\rm c}{\psi}_\pm = \pm iq\beta(x)\overset{\rm c}{\psi}_\pm \quad \text{and} \quad \delta_g F^*{}_\pm = \pm iq\beta(x)F^*{}_\pm \quad ; \tag{9}$$

$$\delta_g B_\pm = \mp iq\beta(x)B_\pm \quad , \quad \delta_g\widetilde{\chi}_\pm = \mp iq\beta(x)\widetilde{\chi}_\pm \quad \text{and} \quad \delta_g G_\pm = \mp iq\beta(x)G_\pm \quad , \tag{10}$$

where $\beta{\equiv}\alpha{-}i\gamma$ is an arbitrary infinitesimal complex function. The transformations for the gauge superfield components surviving the Wess-Zumino gauge are as follows :

$$\delta_g\lambda = \delta_g\widetilde{\rho} = 0 \quad , \quad \delta_g D = 0 \quad \text{and} \quad \delta_g B_\mu = i\,\partial_\mu\beta \quad . \tag{11}$$

Also, for the component fields of the multiplier superfield (4), since $\delta_g\Xi_\alpha{=}0$, we have :

$$\delta_g A_\alpha = \delta_g F_\alpha = 0 \quad , \quad \delta_g E = 0 \quad \text{and} \quad \delta_g H_{\mu\nu} = 0 \quad . \tag{12}$$

Therefore, in the Wess-Zumino gauge, the $U(1)_\gamma$-symmetry is gauged by the real part of B_μ with real gauge function γ, whereas the $U(1)_\alpha$-symmetry is gauged by its imaginary part with real gauge function α. By analysing the transformations (9) and (10), a local Weyl-like symmetry $U(1)_\gamma$ naturally comes out as one of the actual symmetries of the action (7). However, the gauge field (the real part of B_μ) that gauges this symmetry will be supressed in the process of dimensional reduction, then, such a symmetry, will not persist in $D{=}1{+}2$ [9].

Since $\tau_3\text{QED}_{1+2}$ coupled to a topological model in $D{=}1{+}2$ has been used in some theoretical approaches in Condensed Matter Physics [6, 7] (and we are interested to obtain its $N{=}1$ supersymmetric version), it will be interesting to perform the dimensional reduction proposed by Nishino [4] on the action given by eq.(7). Bearing in mind that this process should yield extended supersymmetry [13, 14], some truncations will be needed in order to remain with an $N{=}1$ supersymmetry in $D{=}1{+}2$, as well as to suppress unphysical modes that will certainly appear after the dimensional reduction are performed [9]. These modes correspond to negative-norm 1-particle states (ghosts) and they will be unavoidable in 3 dimensions, for the kinetic terms of the action (7) are totally off-diagonal.

We perform the dimensional reduction[2] *à la* Nishino [4] from D=2+2 to D=1+2 on the action (7). As a result, it can be found the following supersymmetric action in D=1+2 :

$$\begin{aligned} S^{D=3} \; = \; \int d^3\hat{x} \Big\{ & \frac{\mu}{2}\epsilon^{klm} B_k^* G_{lm} + i\frac{\mu}{2}\overline{A}\gamma^m\partial_m\rho - \frac{\mu}{2}\overline{F}\lambda + \frac{\mu}{2}E^* D + \\ & - F_+^* G_+ - A_+^*\Box B_+ - \frac{1}{2}i\overline{\psi}_+\gamma^m\partial_m\chi_+ - qB_m\left(\frac{1}{2}i\overline{\psi}_+\gamma^m\chi_+ + A_+^*\partial^m B_+ - B_+\partial^m A_+^*\right) + \\ & + q\left(A_+^*\overline{\chi}_+^c\rho - B_+\overline{\psi}_+\lambda\right) - \left(qD + q^2 B_m B^m\right) A_+^* B_+ + \\ & - F_-^* G_- - A_-^*\Box B_- - \frac{1}{2}i\overline{\psi}_-\gamma^m\partial_m\chi_- + qB_m\left(\frac{1}{2}i\overline{\psi}_-\gamma^m\chi_- + A_-^*\partial^m B_- - B_-\partial^m A_-^*\right) + \\ & - q\left(A_-^*\overline{\chi}_-^c\rho - B_-\overline{\psi}_-\lambda\right) + \left(qD - q^2 B_m B^m\right) A_-^* B_- + \\ & - m\left(\frac{1}{2}\overline{\psi}_+^c\psi_- + \frac{1}{2}\overline{\chi}_+^c\chi_- + A_+F_- + A_-F_+ - B_+G_- - B_-G_+\right)\Big\} + \text{h.c.} \quad , \end{aligned} \tag{13}$$

where the real parameter, μ, has dimension of mass. Notice that after the dimensional reduction, the coupling constant q has acquired dimension of $(\text{mass})^{\frac{1}{2}}$.

Since the spectrum of the action given by eq.(13) will be spoiled by the presence of negative-norm states, truncations will be needed in order to suppress these unphysical modes. However, to identify the ghost fields to be truncated, we must to diagonalize the whole free sector of the action (13).

To perform the diagonalization of the free action (13), we need to find some linear combinations of the fields. Therefore, by the same procedure used for the case presented in Ref.[9], we have found the following transformations :

1. gauge sector :

$$A = \frac{1}{\sqrt{2}}(\xi + \eta) \quad \text{and} \quad \rho = \frac{1}{\sqrt{2}}(\xi - \eta) \quad ; \tag{14}$$

$$F = \sqrt{2}(\varphi + \phi) \quad \text{and} \quad \lambda = \sqrt{2}(\varphi - \phi) \quad ; \tag{15}$$

$$E = \frac{1}{\sqrt{2}}\left(\widehat{E} + \widehat{D}\right) \quad \text{and} \quad D = \frac{1}{\sqrt{2}}\left(\widehat{E} - \widehat{D}\right) \quad ; \tag{16}$$

2. fermionic and bosonic matter sector :

$$\psi_\pm = \frac{1}{\sqrt{2}}\left(\widehat{\psi}_\pm \mp \widehat{\psi}_\mp^c + \widehat{\chi}_\pm \pm \widehat{\chi}_\mp^c\right) \quad \text{and} \quad \chi_\pm = \frac{1}{\sqrt{2}}\left(\widehat{\chi}_\pm \pm \widehat{\chi}_\mp^c - \widehat{\psi}_\pm \pm \widehat{\psi}_\mp^c\right) \quad ; \tag{17}$$

$$A_\pm = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\left(\breve{A}_\pm \mp \breve{A}_\mp^*\right) - \widehat{B}_\pm\right] \quad \text{and} \quad B_\pm = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\left(\breve{A}_\pm \mp \breve{A}_\mp^*\right) + \widehat{B}_\pm\right] \quad ; \tag{18}$$

$$F_\pm = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\left(\breve{F}_\pm \mp \breve{F}_\mp^*\right) + \widehat{G}_\pm\right] \quad \text{and} \quad G_\pm = -\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\left(\breve{F}_\pm \mp \breve{F}_\mp^*\right) - \widehat{G}_\pm\right] \quad . \tag{19}$$

By replacing these field redefinitions into the action (13), one ends up with a diagonalized action, where the fields, η, $\widehat{\chi}_+$, $\widehat{\chi}_-$, $\widehat{B}_+$ and $\widehat{B}_-$ appear like ghosts in the framework of an N=2-supersymmetric model. Therefore, in order to suppress these unphysical modes, truncations must be performed. Bearing in mind that we are looking for an N=1 supersymmetric 3-dimensional model (in the Wess-Zumino gauge), truncations have to be imposed on the ghost fields, η, $\widehat{\chi}_+$, $\widehat{\chi}_-$, $\widehat{B}_+$ and $\widehat{B}_-$. To keep N=1

[2]After the dimensional reduction is performed, the 3-dimensional metric becomes η_{mn}=$(+,-,-)$. Note that, λ, ρ, A, F, ψ and χ are now Dirac spinors in D=1+2.

supersymmetry in the Wess-Zumino gauge, we must simultaneously truncate the component fields, $\widehat{G}_+$, $\widehat{G}_-$, $\widehat{D}$, $\widehat{E}$, ξ, ϕ, a_m and τ [3] . Now, the choice of truncating a_m, instead of A_m, is based on the analysis of the couplings to the matter sector: A_m couples to both scalar and fermionic matter and we interpret it as the photon field in 3 dimensions.

After performing these truncations, and omitting the ($\hat{}$) and ($\check{}$) symbols, we find the following action in D=1+2 :

$$\begin{aligned} S^{\rm SCS}_{\tau_3{\rm QED}} = \int d^3\hat{x}\, \{ & \mu\epsilon^{klm} A_k F_{lm} - 2\mu\overline{\lambda}\lambda + \\ & - A^*_+ \Box A_+ - A^*_- \Box A_- + i\overline{\psi}_+\gamma^m\partial_m\psi_+ + i\overline{\psi}_-\gamma^m\partial_m\psi_- + F^*_+F_+ + F^*_-F_- + \\ & - qA_m\left(\overline{\psi}_+\gamma^m\psi_+ - \overline{\psi}_-\gamma^m\psi_- + iA^*_+\partial^m A_+ - iA^*_-\partial^m A_- - iA_+\partial^m A^*_+ + iA_-\partial^m A^*_-\right) + \\ & - iq\left(A_+\overline{\psi}_+\lambda - A_-\overline{\psi}_-\lambda - A^*_+\overline{\lambda}\psi_+ + A^*_-\overline{\lambda}\psi_-\right) + q^2 A_m A^m \left(A^*_+A_+ + A^*_-A_-\right) + \\ & - m\left(\overline{\psi}_+\psi_+ - \overline{\psi}_-\psi_- + A^*_+F_+ - A^*_-F_- + A_+F^*_+ - A_-F^*_-\right)\Big\} \quad ; \end{aligned} \tag{20}$$

hence, we conclude that this is a supersymmetric extension of a parity-preserving action minimally coupled to a Chern-Simons field [5, 6, 7]. However, to render our claim more explicit, we are going next to rewrite (20) in terms of the superfields of N=1 supersymmetry in 3 dimensions.

In order to formulate the N=1 super-Chern-Simons coupled to the τ_3QED (20) in terms of superfields, we refer to the work by Salam and Strathdee [15]. Extending their ideas to our case in D=1+2, the elements of superspace are labeled by (x^m, θ), where x^m are the space-time coordinates and the fermionic coordinates, θ, are Majorana spinors, θ^c=θ. [4]

Now, we define the N=1 complex scalar superfields with opposite $U(1)$-charges, $\Phi_\pm$, as

$$\Phi_\pm = A_\pm + \overline{\theta}\psi_\pm - \frac{1}{2}\overline{\theta}\theta F_\pm \quad \text{and} \quad \Phi^\dagger_\pm = A^*_\pm + \overline{\psi}_\pm\theta - \frac{1}{2}\overline{\theta}\theta F^*_\pm \quad , \tag{21}$$

where A_+ and A_- are complex scalars, ψ_+ and ψ_- are Dirac spinors and F_+ and F_- are complex auxiliary scalars. Their gauge-covariant derivatives read :

$$\nabla_a\Phi_\pm = (D_a \mp iq\Gamma_a)\,\Phi_\pm \quad \text{and} \quad \overline{\nabla}_a\Phi^\dagger_\pm = (\overline{D}_a \pm iq\overline{\Gamma}_a)\,\Phi^\dagger_\pm \quad , \tag{22}$$

where $D_a \equiv \overline{\partial}_a - i(\gamma^m\theta)_a\partial_m$ and $\overline{D}_a \equiv -\partial_a + i(\overline{\theta}\gamma^m)_a\partial_m$. The gauge superconnection, Γ_a, is written in the Wess-Zumino gauge as

$$\Gamma_a = i(\gamma^m\theta)_a A_m + \overline{\theta}\theta\lambda_a \quad \text{and} \quad \overline{\Gamma}_a = -i(\overline{\theta}\gamma^m)_a A_m + \overline{\theta}\theta\overline{\lambda}_a \quad , \tag{23}$$

with field-strength superfield, W_a, given by

$$W_a = -\frac{1}{2}\overline{D}_b D_a \Gamma_b \quad . \tag{24}$$

By using the previous definitions of the superfields, (21), (23) and (24), and the gauge-covariant derivatives, (22), we found how to build up the N=1 super-τ_3QED action coupled to a super-Chern-Simons term, in superspace; it reads :

$$S^{\rm SCS}_{\tau_3{\rm QED}} = \int d\hat{v}\left\{2\mu(\overline{\Gamma}W) + (\overline{\nabla}\Phi^\dagger_+)(\nabla\Phi_+) + (\overline{\nabla}\Phi^\dagger_-)(\nabla\Phi_-) + 2m(\Phi^\dagger_+\Phi_+ - \Phi^\dagger_-\Phi_-)\right\} , \tag{25}$$

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[3]The a_m field is the real part of B_m, since we are assuming B_m=a_m+iA_m. Also, as $\widehat{\varphi}$ is a Dirac spinor, it can be written in terms of two Majorana spinors in the following manner: φ=τ−$i\widehat{\lambda}$.

[4]The adjoint and charge-conjugated spinors are defined by $\overline{\psi}$=$\psi^\dagger\gamma^0$ and ψ^c=$-C\overline{\psi}^T$, repectively, where C=σ_y. The γ-matrices we are using arised from the dimensional reduction to D=1+2 are: γ^m=$(\sigma_x, i\sigma_y, -i\sigma_z)$. Note that for any spinorial objects, ψ and χ, the product $\overline{\psi}\chi$ denotes $\overline{\psi}_a\chi_a$.

where the superspace measure we are adopted is $d\hat{v}{\equiv}d^3\hat{x}d^2\theta$ and the Berezin integral is taken as $\int d^2\theta{=}-\frac{1}{4}\overline{\partial}\partial$.

Our final conclusion is that the massive Abelian $N{=}1$ super-QED$_{2+2}$ coupled to a self-dual supermultiplet as proposed in ref.[10], shows interesting features when an appropriate dimensional reduction is performed. The dimensional reduction *à la* Nishino we have applied to our problem becomes very attractive, since, after doing some truncations to avoid non-physical modes, $N{=}1$ super-Chern-Simons coupled to a parity-preserving matter sector (super-τ_3QED) is obtained as a final result.

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