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GAP ROAD: MAIN PROPERTIES*

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ABSTRACT

We present the phase diagram, Liapunov exponent and multifractality associated with a asymmetric map, which generate a new road to chaos.

Key-words: Chaos; Asymmetric Maps; Multifractality; Liapunov Exponent.

The three standard universal roads to chaos, namely period-doubling, intermittency and quasiperiodicity, associated with continuous differentiable maps were studied extensively in the last decade. Nevertheless, the variety of routes to chaos can be much wider than those in continuous differentiable maps. New universal roads to chaos associated with maps with a singularity at the extremum are now being object of increasing interest [1,4]. For example, maps with a discontinuity at the extremum can be generated by appropriate Poincaré sections in flows where typical trajectories on or near the attractor pass close to a saddle point[2]. In this situation the evolution of the dynamical variable depends on the sign of its preimage. The standard example of such systems is the Lorenz model, where the origin is a hyperbolic point. A typical map generated on this model is the following

$$x_{t+1} = \begin{cases} 1-\varepsilon_1-a_1|x_t|^{z_1} & \text{if } x>0 \\ 1-\varepsilon_2-a_2|x_t|^{z_2} & \text{if } x\leq 0 \end{cases} \tag{1}$$

We showed that a new universal road to chaos is associated with such maps. In the present communication we exhibit the main features of this road, namely the phase-diagram, the Liapunov exponent and multifractality.

To study the a-evolution (with $a_1=a_2\equiv a$) of the attractor, we chose a typical example, namely $(\varepsilon_1,\varepsilon_2)=(0,0.1)$. For increasing a, after a period-doubling bifurcation, we see the appearance of sequence of *inverse cascades*. The first cascade is $\ldots 12\rightarrow 10\rightarrow 8\rightarrow 6\rightarrow 4$; it accumulates on $a=(1-\varepsilon_1)^{1-z_1}=1$. Immediately above this cascade we observe a couple of standard pitchfork bifurcations and, further on, another inverse cascade $\ldots 25\rightarrow 21\rightarrow 17\rightarrow 13\rightarrow 9$, and again a pitchfork bifurcation (the last one before entrance into chaos) into period 18. Then a great amount of inverse cascades are observed. The adding constant of any cascade is the period of the attractor that exists immediately below its accumulation point. Indeed a very fine structure is present. Between any two consecutives elements of a cascade there is always another cascade, whose periods grow with the rule mentioned above. Now, if we fix a and study the ε-evolution (with $\varepsilon_2=0$ and $\varepsilon_1\equiv\varepsilon$) of the attractor we see, as shown in Fig. 1, a similar structure.

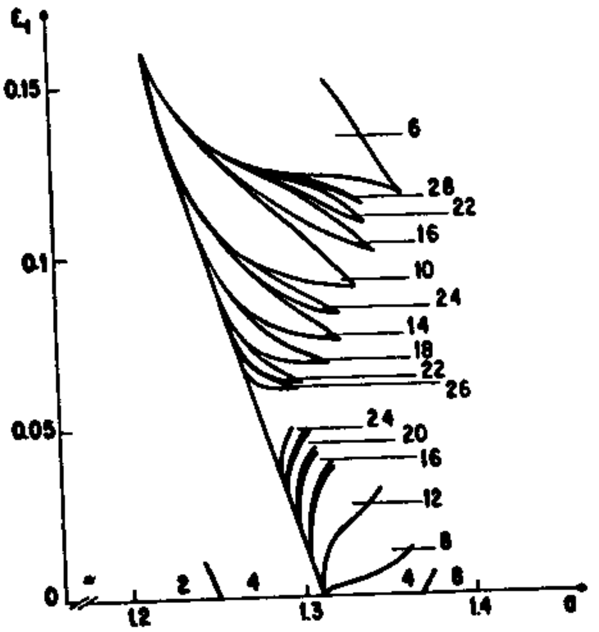

Fig. 1. Phase-diagram for $z_1=z_2=2$ and $\varepsilon_2=0$. The numbers indicate the period of the attractor. For $\varepsilon_1=0$ we recover the well known period-doubling sequence. We used $x_0=0.5$

For fixed $(\varepsilon_1,\varepsilon_2)$ there are a minimal value a_k^m and a maximal value a_k^M where a k-cycle of a cascade loses its stability. The following laws are observed

$$|a_k^m - a_{k+1}^m| \sim |a_{k-1}^m - a_k^m|^{z_1} \qquad (2.a)$$

as well as

$$|a_k^m - a_\infty^m| \sim |a_{k-1}^m - a_\infty^m|^{z_1} \qquad (2.b)$$

for k large enough. The same laws hold for $\{a_k^M\}$, for all cascades, for all values of $(\varepsilon_1,\varepsilon_2)$, such that $\varepsilon_1\neq\varepsilon_2$, in the presence or absence of high order terms in Eq. (1), and also if we fix a and vary $(\varepsilon_1,\varepsilon_2)$.

The Liapunov exponent λ as a function of a for $(\varepsilon_1,\varepsilon_2)=(0,0.1)$ is depicted in Fig. 2. We observe a roughly self-similar structure. For a given cascade the curves become narrower when the periods grow and shift towards negative values of λ, thus exhibiting, at the accu-

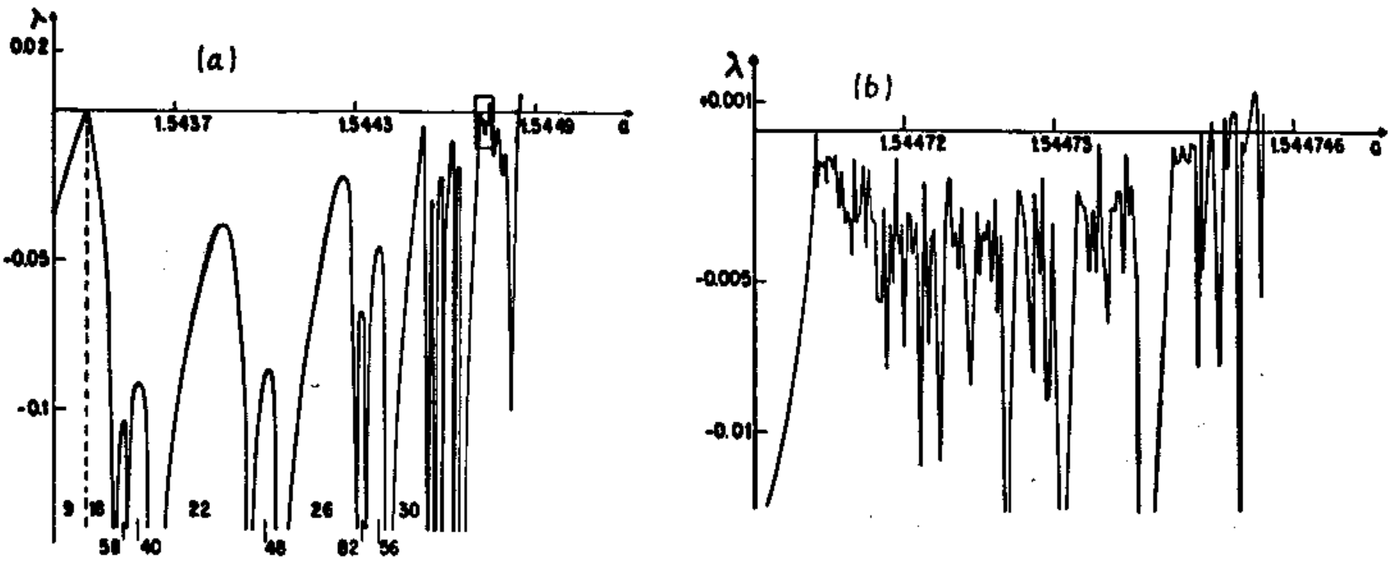

Fig. 2. Evolution of the Liapunov exponent as a function of a for $(\varepsilon_1,\varepsilon_2)=(0,0.1)$, $z_1=z_2=2$ and $x_0=0.5$. The numbers inside the curves in (a) indicate the period of the attractor; (b) is the expansion of the small rectangle in (a)

mulation point, presumably *infinitely large periods with no chaos.* Chaos first appears at the accumulation point of the accumulation points. The maxima of the λ vs. a curves with lowest large periods approach $\lambda=0$ and drive the system into chaos.

The attractor at the entrance into chaos is a multifractal. In Fig. 3 we show the corresponding function $f(\alpha)$[5] for $(\varepsilon_1,\varepsilon_2)=(0,0.1)$. We found $D_0\simeq0.95$, $D_{-\infty}\simeq5.7$ and $D_{\infty}\simeq0.45$.

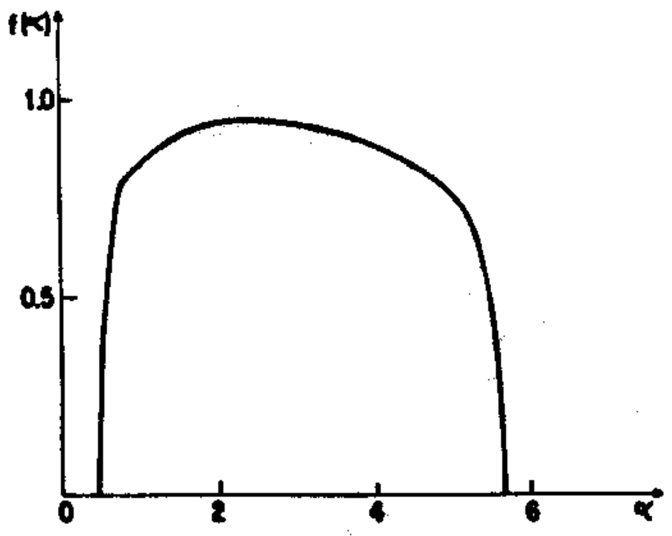

Fig. 3. Multifractal function $f(\alpha)$ for $(\varepsilon_1,\varepsilon_2)=(0,0.1)$, $z_1=z_2=2$ and $x_0=0.5$ (chaos appears at $a\simeq1.5447414$)

Experimental realizations of the features herein mentioned would be extremely welcome.

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