# The Binary Pulsar Is Not the Ultimate Test for the Theory of Gravity

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## Abstract

General relativity can be formulated either as in its original geometrical version (Einstein, 1915) or as a field theory (Feynman, 1962). In the Feynman presentation of Einstein theory an hypothesis concerning the interaction of gravity to gravity, which was hidden in the original version, becomes explicit. This is nothing but the assumed extension of the validity of the equivalence principle not only for matter-gravity interaction, but also for gravity-gravity. Recently we have presented a field theory of gravity (from here on called the NDL theory) which does not contain such a hypothesis. We have shown that, for this theory, both the cosmological structure and the PPN approximation for the solar tests are satisfied.

The proposal of this paper is to go one step further and to show that NDL theory is able to solve the problem of radiation emission by a binary pulsar in the same degree of accuracy as it was done in the GR theory. In the post-Newtonian order of approximation we show that the quadrupole formula of this theory is equal to the corresponding one in general relativity. Thus, the unique actual observable distinction of these theories concerns the velocity of gravitational waves, which becomes then the true ultimate test for gravity theory.

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# 1   Introduction

## 1.1   Introductory Remarks

The general relativity (GR) description of the observations concerning the rate of the energy lost by the binary pulsar increased enormously the status of this theory. Thus, any theory that dares to be competitive with GR should at least be able to provide the same degree of accuracy in the explanation of this phenomenon (besides, of course, all remaining standard observational tests, which means PPN and cosmology).

Although GR is presented as an universal modification of the metrical properties of spacetime, an alternative way to describe GR as a field theory in the same lines as any other interaction was revived by some authors [1, 2]. The idea goes back to Feynman[1] investigation [5]. Indeed, in his 1962 *Lectures on Gravitation* it has been shown that a field theoretical approach of gravity should be possible and its basic ingredients should deal, besides the field $\varphi_{\mu\nu}$ itself, with two metric tensors: an auxiliary one $\gamma_{\mu\nu}$ — which is not observable — and an effective one $g_{\mu\nu}$ related by $g_{\mu\nu} = \gamma_{\mu\nu} + \varphi_{\mu\nu}$.

The basic hypothesis of GR concerns the extension of the equivalence principle beyond its original domain of experimental evidence, that concerns material substance of any form, the adoption of its validity not only by matter or non-gravitational energy of any sort but also by gravity energy itself. Such an universality of interaction is precisely the cornerstone that makes possible the identification of a unique overall geometry of spacetime $g_{\mu\nu}$. In Eintein GR the properties of gravity are associated to the Riemannian curvature, which becomes then the equivalent substitute of gravitational *forces*. We remark that, although such geometrization scheme is permissible, it is by no way mandatory. All observable characteristics and properties of Einstein theory can be well described in terms of a field $\varphi_{\mu\nu}$. Indeed, the lesson we learn from Feynman approach is this: **the geometrical description of GR is nothing but a choice of representation**. Let us emphasize that such an alternative description of GR in no way sets a restriction on it, but only enlarges its power of understanding.

From this approach it follows that contrary to a widespread belief, GR can be described in terms of a two metric structure[2]. Furthermore, Feynman has shown that the coherence of a spin-2 theory that starts with the linear Fierz-Pauli [6] equation written in terms of the symmetric field $\varphi_{\mu\nu}$ in a Minkowskian spacetime requires, in a very natural way, due to the self-interaction process described above, the use of an induced metric tensor, the quantity $g_{\mu\nu}$. Thus, as we announced above, the field-theoretical way of treating GR appeals to a two metric structure. This is the *standard* procedure. Nevertheless, and just by tradition, this is not the way that Einstein theory is presented in the text books[3]. This interpretation allows us to state that two-metric theories of gravity are less exotic than it is usually displayed [7]. Let us emphasize that the second metric is nothing but a convenient auxiliary tool of the theory. It is not observable (neither in general relativity

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[1]The original issues seems to be firstly worked out by Gupta [3] and Kraichnan [4].

[2]This is also the case in NDL theory.

[3]The absence in the literature of such alternative but equivalent way to present Einstein theory of gravity seems to be the main responsible for the young students of theoretical physics to understand GR as a completely separate and different theory from any other field.

nor in NDL theory) and as such can be eliminated from a description made only in terms of observable quantities. In textbook presentations of GR one makes the choice of a unique geometry. This, of course, does not preclude an alternative equivalent description [2].

Recently we have exploited some consequences of such field theoretical description of gravity adding a new ingredient: we do not require the extrapolation to gravitational energy of the hypothesis of universality of the equivalence principle (EP) although, as we shall see, it contains many of the ingredients of GR. The main lines of NDL theory can be synthesized in the following statements:

- Gravity is described by a symmetric second order tensor $\varphi_{\mu\nu}$ that satisfies a non-linear equation of motion;

- Matter couples to gravity in an universal way. In this interaction, the gravitational field appears only in the combination $g_{\mu\nu} = \gamma_{\mu\nu} + \varphi_{\mu\nu}$. Such tensor $g_{\mu\nu}$ acts as the true metric tensor of the spacetime as seen by matter or energy of any form except gravitational energy;

- The self interaction terms of the gravitational field break the universal modification of the spacetime geometry.

It has been conjectured [7] that the observation of the binary pulsar should be the ultimate test of gravity theory. This is due to the fact that so far the alternative models that have been proposed to explain gravitational processes did not succeeded in provide coherent explanation of observations, mainly concerning the pulsar PSR 1913 + 16.

However this statement seems to be not true. Indeed we shall prove in the present paper that NDL theory is able to provide a description of the gravitational radiation emitted by the pulsar.

## 1.2   Synopsis

In Section 2 we introduce the definitions and symbols we are using.

In Section 3 we summarize the standard Gupta-Feynman-Deser (GFD) approach for the field theory of gravitation that led to general relativity.

In Section 4 we make a short review of our previous paper [8] and compare with GFD approach. We show the main features of NDL theory with emphasis on the propagation of gravitational waves.

In Section 5 we present a short derivation of the gravitational quadrupole formula to radiation emission by a binary system. We compare then our results with the GR.

We conclude in Section 6 with some comments concerning both theories, mainly with respect to the classical test and binary pulsar. We end with some comments and perspectives for future investigations.

## 2   Definitions and Notations

In order to exhibit the complete covariance of the theory all quantities will be described in an arbitrary system of coordinates. In the auxiliary background geometry of Minkowski

spacetime of metric $\gamma_{\mu\nu}$ the covariant derivative, represented by a semi-comma, is

$$V_{\mu;\nu} = V_{\mu,\nu} - \Delta_{\mu\nu}^{\alpha} V_{\alpha} \tag{1}$$

in which the associated Christoffel symbol $\Delta_{\mu\nu}^{\alpha}$ is given by

$$\Delta_{\mu\nu}^{\alpha} = \frac{1}{2}\gamma^{\alpha\beta}\left(\gamma_{\beta\mu,\nu} + \gamma_{\beta\nu,\mu} - \gamma_{\mu\nu,\beta}\right). \tag{2}$$

The corresponding curvature tensor vanishes identically, that is

$$R_{\alpha\beta\mu\nu}(\gamma_{\epsilon\lambda}) = 0. \tag{3}$$

We define a three-index tensor $F_{\alpha\beta\mu}$, which we will call the **gravitational field**, in terms of the symmetric standard variable $\varphi_{\mu\nu}$ (which will be treated as the potential) to describe spin-two fields, by the expression

$$F_{\alpha\beta\mu} = \frac{1}{2}(\varphi_{\mu[\alpha;\beta]} + F_{[\alpha}\gamma_{\beta]\mu}) \tag{4}$$

where we are using the anti-symmetrization symbol $[\,]$ like

$$[x,y] \equiv xy - yx. \tag{5}$$

We use an analogous form for the symmetrization symbol $(\,)$

$$(x,y) \equiv xy + yx. \tag{6}$$

The quantity $F_{\alpha}$ is the trace

$$F_{\alpha} = F_{\alpha\mu\nu}\gamma^{\mu\nu} \tag{7}$$

that is,

$$F_{\alpha} = \varphi_{,\alpha} - \varphi_{\alpha}{}^{\epsilon}{}_{;\epsilon}. \tag{8}$$

From the above definition it follows that this quantity $F_{\alpha\beta\mu}$ is anti-symmetric in the first pair of indices and obeys the cyclic identity, that is

$$F_{\alpha\mu\nu} + F_{\mu\alpha\nu} = 0, \tag{9}$$

and

$$F_{\alpha\mu\nu} + F_{\mu\nu\alpha} + F_{\nu\alpha\mu} = 0. \tag{10}$$

From the field variables we can form the invariants[4]:

$$
\begin{aligned}
A &\equiv F_{\alpha\mu\nu}F^{\alpha\mu\nu}, \\
B &\equiv F_{\mu}F^{\mu}.
\end{aligned} \tag{11}
$$

Greek indices run into the set $\{0,1,2,3\}$, while Latin indices run into the set $\{1,2,3\}$. Finally, the quantity $\kappa$ is related to Newton's constant $G_{\rm N}$ and the velocity of light $c$ by the definition

$$\kappa = \frac{16\pi G_{\rm N}}{c^4}. \tag{12}$$

We set $G_{\rm N} = 1$ and $c = 1$.

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[4]Note that, besides this invariants, it is possible to define a quantity $C$, constructed with the dual, that is $C \equiv F^{*}_{\alpha\mu\nu}F^{\alpha\mu\nu}$. We will not deal with such quantity here.

# 3    The GR Theory of Gravity: A Short Resume

General relativity takes for granted that gravity is nothing but the fact that all exist-
ing form of energy/matter interacts through the modification of the universal geometry.
However, such a view is not exclusive and it is conceivable to try to use two metrics to
describe in an equivalent way all content of such theory. There is no simpler and more
direct way to prove this statement than the one set forth by Feynman. It is worth to
remark that a such duplication causes no further difficulties when one realizes that the
second auxiliary metric $\gamma_{\mu\nu}$ is unobservable.

Let us pause for a while and make, just for completeness, a summary of the principal
features of this equivalent scheme. The theory starts with the Fierz-Pauli linear equation

$$G^{(\mathrm{L})}_{\mu\nu} = -\,\kappa\,T_{\mu\nu} \tag{13}$$

in which $T_{\mu\nu}$ is the matter energy-momentum tensor and $G^{(\mathrm{L})}_{\mu\nu}$ is a linear operator defined
by:

$$G^{(\mathrm{L})}_{\mu\nu} \equiv \Box\phi_{\mu\nu} - \phi^{\alpha}{}_{\mu;\alpha\nu} - \phi^{\alpha}{}_{\nu;\alpha\mu} + \phi^{\alpha}{}_{\alpha;\mu\nu} - \gamma_{\mu\nu}\bigl(\Box\phi^{\alpha}{}_{\alpha} - \phi^{\alpha\beta}{}_{;\alpha\beta}\bigr). \tag{14}$$

The action for this linear theory is given in terms of the invariants of the field $F_{\alpha\mu\nu}$
— defined by Eq. (4) — that is:

$$S^{(\mathrm{L})} = \int \mathrm{d}^4x\,\sqrt{-\gamma}\,(A - B). \tag{15}$$

Since $G^{(\mathrm{L})}_{\mu\nu}$ is divergence-free it follows for coherence that the matter energy momentum
tensor should also be divergence-free. However this is in contradiction with the fact that
gravity may exchange energy with matter. To overcome such situation, one introduces
an object which we call *Gupta-Feynman gravitational energy tensor* $t^{(g)}_{\mu\nu}$ — a cumbersome
non linear expression in terms of $\varphi_{\alpha\beta}$ and its derivatives — that is to be added to the
right hand side of Eq. (13) in order to obtain a compatible set of equations:

$$G^{(\mathrm{L})}_{\mu\nu} = -\,\kappa\,\left[ t^{(g)}_{\mu\nu} + T_{\mu\nu} \right]. \tag{16}$$

Note that, instead of using the standard procedure (as it happens in others nonlinear
theories) — which in the case we examine here, asks for the introduction of a nonlinear
functional of the invariants $A$ and $B$, dealt with in the linear case — in order to obtain
the dynamics of GR, one must use other functionals of the basic field $\varphi_{\mu\nu}$ which are not
present in the linear case, that means, they are not displayed in terms of the invariants $A$
and $B$. We do not intend to repeat here the whole procedure[5], but only to call the reader's
attention to such an unusual treatment of dealing with a nonlinear process. The origin of
this approach goes back to the hypothesis of the validity of the equivalence principle for
gravitational energy. In the next section, we will show that NDL theory follows a more
traditional way of generalization to a nonlinear theory by the assumption of nonlinear
functional of the basic invariants dealt with in the linear case. Why does GR break this
symmetry? What is its motivation? The answer to this we can find by the assumption of
the general validity of the equivalence principle for all forms of energy, including gravity.

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[5]The reader may consult the references [1] and [2] for more details.

In general relativity since the identification of gravitational processes to the modified geometry is postulated *a priori* there is no room for the suspection of the assumption of such hypothesis. It is only in its Feynman version that it appears netly.

Now it is straightforward to show that, for a convenient choice of the expression of the *Gupta-Feynman gravitational energy*, the equation (16) is nothing but Einstein dynamics. The main steps can be synthesized as follows.

From the equivalence principle, the observable geometry is given by the quantity[6]

$$\sqrt{-g}g^{\mu\nu} = \sqrt{-\gamma}\left(\gamma^{\mu\nu} + \varphi^{\mu\nu}\right). \tag{17}$$

Define the tensor $K^{\alpha}{}_{\mu\nu}$ as[7]:

$$K^{\alpha}{}_{\mu\nu} = \frac{1}{2}g^{\alpha\epsilon}\left[g_{\mu\epsilon;\nu} + g_{\nu\epsilon;\mu} - g_{\mu\nu;\epsilon}\right], \tag{18}$$

and obtain the Ricci contracted curvature tensor as:

$$R_{\mu\nu} = -\frac{1}{2}K_{(\mu\,;\nu)} + K^{\alpha}{}_{\mu\nu\,;\alpha} - K^{\alpha}{}_{\mu\lambda}\,K^{\lambda}{}_{\nu\,\alpha} + K^{\lambda}{}_{\mu\nu}\,K_{\lambda}. \tag{19}$$

At this point one has to make a definite choice for $t_{\mu\nu}^{(g)}$ in terms of the quantities $\varphi_{\mu\nu}$ and $K^{\alpha}{}_{\mu\nu}$. A rather long but tedious calculation shows that in order to arrive at Eisntein's equations of motion one must choose (see for instance GPP [2])

$$\kappa\,t_{\mu\nu}^{(g)} = -(K\,K)_{\mu\nu} + \frac{1}{2}\gamma_{\mu\nu}(K\,K)^{\alpha}{}_{\alpha} + Q^{\lambda}{}_{\mu\nu;\lambda} \tag{20}$$

in which

$$(K\,K)_{\mu\nu} \equiv K^{\alpha}{}_{\mu\nu}K_{\alpha} - K^{\alpha}{}_{\mu\beta}K^{\beta}{}_{\nu\alpha} \tag{21}$$

and

$$\begin{aligned}
Q^{\lambda}{}_{\mu\nu} \equiv{} & \frac{1}{2}\left\{-\gamma_{\mu\nu}\varphi^{\alpha\beta}K^{\lambda}{}_{\alpha\beta} + \varphi_{\mu\nu}K^{\lambda} - \varphi_{(\mu}{}^{\lambda}K_{\nu)} + \varphi^{\beta\lambda}K^{\alpha}{}_{\beta(\mu}\gamma_{\nu)\alpha}\right. \\
& \left. + \varphi_{(\mu}{}^{\beta}\left[K^{\lambda}{}_{\nu)\beta} - \gamma_{\nu)\alpha}\gamma^{\sigma\lambda}K^{\alpha}{}_{\beta\sigma}\right]\right\}.
\end{aligned} \tag{22}$$

Using the expression (20) into the formula (16) one obtains finally

$$R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu} = -\frac{\kappa}{2}T_{\mu\nu}. \tag{23}$$

Note that this expression assures the validity of equivalence principle not only for all matter and energy, but also by gravitational energy.

We can synthesize, this procedure by the statement:

- The interaction of matter and the gravitational energy is nothing but a universal modification of spacetime geometry.

Could it be possible to follow another path to deal with a nonlinear extension of Fierz original model? The answer is yes and led us to the NDL model. Let us see how this can be made in a straightforward way.

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[6]In this section we follow the convention as in [2]. We could use instead $g^{\mu\nu} = \gamma^{\mu\nu} + \varphi^{\mu\nu}$ or $g_{\mu\nu} = \gamma_{\mu\nu} + \varphi_{\mu\nu}$, which provide non-equivalent theories.

[7]We remind the reader of our convention that the semi-comma is the covariant derivative with respect to Minkowski metric in an arbitrary coordinate system.

# 4    The NDL Theory of Gravity: A Short Resume

The NDL theory starts at the same point as GR, that is, Fierz linear theory for spin two field. However, instead of breaking the symmetry displayed in the linear regime, presented in the combination of the invariants under the form $A - B$, as it was done in the Einstein case, NDL assumes that this symmetry is maintained even after the introduction of nonlinearities.

In a previous paper [8] the Feynman approach to nonlinear field theory of gravity that led to general relativity through an infinite series of self-interaction processes has been re-examined.

We extended the standard Feynman-Deser approach of field theoretical derivation of Einstein's gravitational theory. It was then possible to show how to obtain a theory that incorporates a great part of general relativity and can be interpreted in the standard geometrical way like GR, as far as the interaction of matter to gravity is concerned. The most important particularity of the new theory concerns the gravity to gravity interaction. This theory satisfies all standard tests of gravity and lead to new predictions about the propagation of gravitational waves. Since there is a large expectation that the detection of gravitational waves will occur in the near future, the question of which theory describes nature better will probably be settled soon.

The Lagrangian for the gravitational field in the NDL theory is given by:

$$L = \frac{b^2}{\kappa} \left\{ \sqrt{1 - \frac{U}{b^2}} - 1 \right\},$$ (24)

where $U$ is defined by

$$U \equiv A - B.$$ (25)

The gravitational action is expressed as:

$$S = \int \mathrm{d}^4 x \sqrt{-\gamma}\, L,$$ (26)

where $\gamma$ is the determinant of the Minkowskian spacetime metric $\gamma_{\mu\nu}$ in an arbitrary coordinate system. Taking the variation of the gravitational action (26) with respect to the potential $\varphi_{\mu\nu}$, result in the following equations of motion:

$$\left[ L_U F^{\lambda(\mu\nu)} \right]_{;\lambda} = -\frac{1}{2} T^{\mu\nu}$$ (27)

where $L_U$ represents the derivative of the Lagrangian with respect to the invariant $U$, and $T^{\mu\nu}$ is the energy-momentum tensor density of the matter contents.

Let us pause for a while in order to make contact with GR. For this, we express Eq. (27) under the form

$$G^{(\mathrm{L})}_{\mu\nu} = \chi_{\mu\nu} + \frac{1}{2L_U} T_{\mu\nu}$$ (28)

where the quantity $\chi_{\mu\nu}$ is provided by

$$\chi_{\mu\nu} \equiv \frac{L_{UU}}{L_U} U_{,\alpha} F^{\alpha}{}_{(\mu\nu)}.$$ (29)

One should compare this expression with the corresponding one (Eq. (16)) in GR. It seems worth to remark that in the corresponding expression for GR in place of $\chi_{\mu\nu}$ it appears precisely the *Gupta-Feynman gravitational energy.*

Let us make a short analysis of the wave propagation description in this theory just for completeness. In what follows the symbol $[J]_\Sigma$ represents the discontinuity of the function $J$ through the surface $\Sigma$.

We set the following Hadamard's [9] discontinuity conditions :

$$[F_{\mu\nu\alpha}]_\Sigma = 0 \tag{30}$$

and

$$[F_{\mu\nu\alpha;\lambda}]_\Sigma = f_{\mu\nu\alpha}k_\lambda, \tag{31}$$

where $k_\alpha$ represents the wave vector normal to the surface of discontinuity $\Sigma$. The quantity $f_{\alpha\beta\gamma}$ has the same symmetries of $F_{\alpha\beta\gamma}$. Taking the discontinuity of the equation of motion (27) we obtain[8]:

$$f_{\mu(\alpha\beta)}k^\mu + 2\frac{L_{UU}}{L_U}(\eta - \zeta)F_{\mu(\alpha\beta)}k^\mu = 0 \tag{32}$$

in which the quantities $\eta$ and $\zeta$ are defined by

$$\begin{aligned}
\eta &\equiv F_{\alpha\beta\mu}f^{\alpha\beta\mu}, \\
\zeta &\equiv F_\mu f^\mu.
\end{aligned} \tag{33}$$

Considering the discontinuity relation and using the identities (9) and (10), after some algebraic manipulations it results:

$$k^\mu k^\nu [\gamma_{\mu\nu} + \Lambda_{\mu\nu}] = 0, \tag{34}$$

in which the quantity $\Lambda_{\mu\nu}$ is written in terms of the gravitational field as:

$$\Lambda_{\mu\nu} \equiv 2\frac{L_{UU}}{L_U}[F_\mu{}^{\alpha\beta}F_{\nu(\alpha\beta)} - F_\mu F_\nu]. \tag{35}$$

Note that the gravitational disturbances propagate in a modified geometry, changing the background geometry $\gamma_{\mu\nu}$, into an effective one $g_{\mu\nu}$, which depends on the energy distribution of the field $F_{\alpha\beta\mu}$. This fact shows that such a property stems from the structural form of the Lagrangian[9].

Differently from general relativity, in the NDL theory the characteristic surfaces of the gravitational waves propagate on the null cone of an effective geometry distinct of that observed by all other forms of energy and matter. This result gives a possibility to choose between these two theories just by observations of the gravitational waves. This is a challenge that is expected to be solved in the near future.

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[8]Note that this equation has a misprint in a original formula as it appeared in Ref. [8].

[9]Indeed, we have shown recently that the same occurs for spin-1 field. See Ref. [10].

# 5   Gravitational Quadrupole Emission

It has been observed that the orbital period $P_b$ of a binary system has a secular decrease. A plethora of effects may cause this, but the most important one is the emission of gravitational radiation [11]. The measurement of the change of the orbital period of this system due to radiation damping is in good agreement with the prediction of gravitational quadrupole emission of general relativity. On the other hand, as it was pointed out by Eardley [12] and Will [13], the corresponding analysis of binary systems undertaken in the realm of most alternative theories of gravity, predict gravitational dipole radiation. This is a heavy drawback of these theories since the dipole contribution exceeds the corresponding general relativity quadrupole emission, making this test a fundamental one.

In this section we will show that, in the NDL theory, the gravitational radiation has a quadrupole origin and can be evaluated in a very analogous way as in the GR theory. We decided here to present this evaluation step by step in order to compare with the standard evaluation formula from GR.

## 5.1   Gravity Energy Momentum Tensor

Since NDL is a field theory of gravity we can define its corresponding gravitational energy momentum tensor through the standard definition:

$$t_{\mu\nu} = \frac{2}{\sqrt{-\gamma}} \frac{\delta L \sqrt{-\gamma}}{\delta \gamma^{\mu\nu}}. \tag{36}$$

From Lagrangian (24) we obtain:

$$t_{\mu\nu} = -L\gamma_{\mu\nu} + 2L_U \left\{ 2F_{\mu\alpha\beta}F_\nu{}^{\alpha\beta} + F_{\alpha\beta\mu}F^{\alpha\beta}{}_\nu - F^\alpha F_{\alpha(\mu\nu)} - F_\mu F_\nu \right\}. \tag{37}$$

Let us quote here that the corresponding Noether energy momentum tensor

$$N^\alpha{}_\beta = \varphi_{\mu\nu,\beta} \frac{\partial L}{\partial \varphi_{\mu\nu,\alpha}} - \delta^\alpha{}_\beta L, \tag{38}$$

reduces in our case to

$$N_{\mu\nu} = -L\gamma_{\mu\nu} - 2L_U \varphi_{\alpha\beta,\nu} F_\mu{}^{\alpha\beta}. \tag{39}$$

The balance of energy between the gravitational field and its sources takes the simple expression:

$$N^\mu{}_{\nu;\mu} = \frac{1}{2} T^{\alpha\beta} \varphi_{\alpha\beta;\nu} . \tag{40}$$

## 5.2   Energy Radiation

In the evaluation of the quadrupolar radiation in NDL theory, we will follow a similar procedure as the corresponding calculation in GR on this subject. We decided to do so in order to exhibit the similitude and the different points concerning both theories.

The left hand side of Eq. (16) and (28) — the linear part of both theories — is gauge independent. We take, as usual, the gauge condition[10]:

$$\left( \varphi_\alpha{}^\beta - \frac{1}{2}\varphi\gamma_\alpha{}^\beta \right)_{;\beta} = 0. \tag{41}$$

Correspondingly we are thus led to define a new quantity $h_{\alpha\beta}$ by setting :

$$h_{\alpha\beta} = \varphi_{\alpha\beta} - \frac{1}{2}\varphi\gamma_{\alpha\beta}. \tag{42}$$

Using this into the equation of motion (Eq. (28)), we obtain:

$$\Box h_{\mu\nu} = \chi_{\mu\nu} + \frac{1}{2L_U}T_{\mu\nu}, \tag{43}$$

in which the D'Alambertian operator is taken in the Minkowski background and $\chi_{\mu\nu}$ is given by Eq. (29).

Using the associate Green's function the solution of $h_{\mu\nu}$ is

$$h_{\mu\nu}(\vec{x}, t) = \int \mathrm{d}^3x' Z_{\mu\nu}(\vec{x}', t^{ret} = t - |\vec{x} - \vec{x}'|), \tag{44}$$

where we defined

$$Z_{\mu\nu}(\vec{x}', t^{ret} = t - |\vec{x} - \vec{x}'|) = \frac{\left\{ \chi_{\mu\nu} + \frac{1}{2L_U}T_{\mu\nu} \right\}_{ret}}{\left| \vec{x} - \vec{x}' \right|}. \tag{45}$$

We then expand this expression in the far region to obtain the series:

$$h_{\mu\nu}(\vec{x}, t) = \frac{1}{R}\int \mathrm{d}^3x' Z_{\mu\nu}^{ret} + \frac{1}{R}\int \mathrm{d}^3x' Z_{\mu\nu}^{ret}\, \vec{x}.\vec{x}' + ... \tag{46}$$

where $R$ is the distance from the observer to the center of mass of the system.

### 5.2.1 Quadrupolar Radiation

Proceeding in analogy with GR (see Refs. [7, 14] for more details) we obtain the expression of the first order for the quantity $h_{kl}$:

$$h_{kl} = \frac{2\mu}{R}\frac{\partial^2}{\partial t^2}(x_k x_l) + O\left(\frac{1}{R}\right)^2, \tag{47}$$

where $\mu$ is the reduced mass of the system and $m$ is the total mass $m = m_1 + m_2$.

Note that, using the gauge condition, we can obtain the temporal derivatives of $h_{\mu 0}$ in terms of the corresponding derivatives of the spatial components, that is:

$$h^{k0}{}_{,0} = \hat{n}_j h^{jk}{}_{,0} \tag{48}$$

$$h^{00}{}_{,0} = \hat{n}_j \hat{n}_k h^{jk}{}_{,0} \tag{49}$$

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[10]We remind the reader that all formulas are taken in the Minkowski background in a complete covariant way, that is, in an arbitrary coordinate system.

in which $\hat{n}_k$ is the unitary vector directed from the source to the observer,

$$\vec{n} = \frac{\vec{x}}{R}. \tag{50}$$

In the case we are interested, that consists in the energy emitted by the system far from the source, only the gravitational contribution must be taken into account. This allows us to write:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -R^2 \oint_{\Omega} \mathrm{d}\Omega \, t^{0j} \, \hat{n}_j. \tag{51}$$

Expressing the exact energy momentum tensor (37) in the new variables, results:

$$t_{\mu\nu} = -L\eta_{\mu\nu} + L_U \, Y_{\mu\nu} \tag{52}$$

where $Y_{\mu\nu}$ is given by:

$$
\begin{aligned}
Y_{\mu\nu} = {} & \left( 2h_\mu{}^{\beta,\alpha} h_{\nu\beta,\alpha} - h_\mu{}^{\alpha,\beta} h_{\alpha\beta,\nu} - h^{\beta\alpha}{}_{,\mu} h_{\nu\beta,\alpha} + h^{\alpha\beta}{}_{,\mu} h_{\alpha\beta,\nu} \right. \\
& \left. - h_{\mu\alpha,\beta} h_\nu{}^{\beta,\alpha} + \frac{1}{2} h^{,\alpha} h_{\alpha\mu,\nu} + \frac{1}{2} h^{,\alpha} h_{\alpha\nu,\mu} - h^{,\alpha} h_{\mu\nu,\alpha} - \frac{1}{2} h_{,\mu} h_{,\nu} \right).
\end{aligned} \tag{53}
$$

Note that since we are interested only on the post-Newtonian approximation, we can use the gauge condition (41) in order to simplify it. It is worth to remark that, in the second order of $h_{\alpha\beta}$, the NDL gravitational energy momentum tensor contains not only the associated gravitational energy momentum tensor of the general relativity, e.g., the Landau tensor, but extra terms.

Using the gauge condition (up to a total divergence) results (see appendix B for details):

$$
\begin{aligned}
t_{0i} = {} & -\frac{1}{32\pi} \left( -\frac{1}{2} \hat{n}_a \hat{n}_b \hat{n}_i \hat{n}_k \hat{n}_l h^{ab}{}_{,0} h^{kl}{}_{,0} + 2\hat{n}_a \hat{n}_b \hat{n}_i h^{ka}{}_{,0} h^b{}_{k,0} \right. \\
& \left. - \hat{n}_a \hat{n}_b \hat{n}_i h^{ab}{}_{,0} h^k{}_{k,0} - \hat{n}_i h^{kl}{}_{,0} h_{kl,0} + \frac{1}{2} \hat{n}_i h^k{}_{k,0} h^l{}_{l,0} \right).
\end{aligned} \tag{54}
$$

Performing the angular integrations and averaging over several oscillations of the sources yields[11]

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{R^2}{8} \left\langle \frac{2}{5} h^{kl}{}_{,0} h_{kl,0} - \frac{2}{15} h^k{}_{k,0} h^l{}_{l,0} \right\rangle. \tag{56}$$

At this point it is convenient to introduce the standard traceless momentum of inertia tensor:

$$I_{kl} \equiv \mu \left( x_k x_l - \frac{1}{3} \delta_{kl} x^2 \right). \tag{57}$$

---

[11]To perform the angular integration we use the following relations of the averages over the sphere:

$$\frac{1}{4\pi} \oint \mathrm{d}\Omega \hat{n}_k \hat{n}_l = \frac{1}{3} \delta_{kl}, \quad \frac{1}{4\pi} \oint \mathrm{d}\Omega \hat{n}_a \hat{n}_b \hat{n}_k \hat{n}_l = \frac{1}{15} \left( \delta_{ab} \delta_{kl} + \delta_{ak} \delta_{bl} + \delta_{al} \delta_{bk} \right). \tag{55}$$

After some algebraic manipulations we obtain the expression for the rate of gravitational energy lost,

$$\frac{dE}{dt} = -\frac{1}{5}\left\langle \dddot{I}^{kl}\, \dddot{I}_{kl}\right\rangle \tag{58}$$

a formula which is precisely the same as one obtained in GR in this order of approximation. Using the Newtonian result

$$\frac{dv^k}{dt} = -\frac{mx^k}{r^3} \tag{59}$$

we arrive at the Peter-Mathews (PM) [15] expression:

$$\frac{dE}{dt} = -\frac{8}{15}\left\langle \frac{\mu^2 m^2}{r^4}\left(12v^2 - 11\dot{r}^2\right)\right\rangle, \tag{60}$$

in which $v$ is the relative velocity and $\dot{r}$ is the temporal derivative of the orbital separation $r$.

By comparison of (60) and PM formula (see appendix C) we obtain the values $\kappa_1 = 1$, $\kappa_2 = 1$. Note that as in GR there is no dipole term for NDL theory.

# 6    Conclusion

In this paper we have continued the analysis of the NDL theory in what concerns the observational tests of gravity. Here we treated the emission of gravitational radiation by a binary system. We showed that, in the post-Newtonian approximation, the results of this theory are perfectly adjusted with the observational data, as well as in the general relativity.

In [16] Taylor claims that "the clock-comparison experiment for PSR 1913 + 16 thus provides direct experimental proof that changes in gravity propagate at the speed of light, thereby creating a dissipative mechanism in an orbiting system. It necessarily follows that gravitational radiation exists and has a quadrupolar nature". The second assertion (quadrupolar radiation) may be true independent from the first one (gravitational waves propagate at the velocity of light) — which is the case in our theory.

From this remarks one can conclude that any theory which admits gravitational waves and gravitational radiation of quadrupolar nature is a good model. This does not prove that GR is correct, although it does prove that GR is a serious candidate to be the true theory of gravitational phenomena. From what we have shown in this paper, NDL theory is a good candidate too.

# 7    Acknowledgements

# Appendix A

From the structural form of Lagrangians of the Born-Infeld [17] type – as the one we have chosen to represent gravity processes – it follows that only even powers of the field variables $\varphi_{\mu\nu}$ appear in any polynomial-like expansion. This could be a drawback[12] of the theory if, in the future, observation asks for the presence of the odd terms. There is an easy way to solve this problem leaving the structural form of the theory intact. One has just to deal with a modification of the basic quantities by a re-definition of the field $F$ only through a change of $\varphi_{\mu\nu}$ by $\Psi_{\mu\nu}$, the nonlinear combination

$$\Psi_{\mu\nu} = \varphi_{\mu\nu} - \varphi_{\mu\alpha}\,\varphi^{\alpha}{}_{\nu}. \tag{61}$$

We leave the complete exam of this modification to a forthcoming paper. Let us only inform here that, as far as the standard classical tests (PPN, cosmology and the binary pulsar) are concerned, both theories, in the order of approximation dealt with, are undistinguishable.

# Appendix B

In this appendix we show explicitly the expansions of the terms appearing in the expression (52). Using the relations between the components of the field $h_{\mu\nu}$, given by (48) and (49), results:

$$
\begin{aligned}
h_0{}^{\beta,\alpha} h_{j\beta,\alpha} &= h_{00,0}h_{j0,0} + h_{k0,l}h_{jk,l} - h_{k0,0}h_{jk,0} - h_{00,l}h_{j0,l}\\
&= -\hat{n}_a\hat{n}_b\hat{n}_k h_{ab,0}h_{jk,0} - \hat{n}_l h_{kl,0}h_{jk,0} + \hat{n}_l h_{kl,0}h_{jk,0} + \hat{n}_a\hat{n}_b\hat{n}_k h_{ab,0}h_{jk,0}\\
&= 0;\\
h_0{}^{\alpha,\beta} h_{\alpha\beta,j} &= h_{00,0}h_{00,j} + h_{kl,j}h_{0k,l} - h_{k0,j}h_{0k,0} - h_{0l,j}h_{00,l}\\
&= -\hat{n}_a\hat{n}_b\hat{n}_k\hat{n}_l\hat{n}_j h_{ab,0}h_{kl,0} - \hat{n}_b\hat{n}_l\hat{n}_j h_{kl,0}h_{bk,0}\\
&\quad +\hat{n}_l\hat{n}_b\hat{n}_j h_{kl,0}h_{bk,0} + \hat{n}_a\hat{n}_b\hat{n}_k\hat{n}_l\hat{n}_j h_{kl,0}h_{ab,0}\\
&= 0;\\
h^{\beta\alpha}{}_{,0} h_{j\beta,\alpha} &= h_{00,0}h_{0j,0} + h_{kl,0}h_{jk,l} - h_{k0,0}h_{jk,0} - h_{0l,0}h_{j0,l}\\
&\quad -\hat{n}_a\hat{n}_b\hat{n}_k h_{ab,0}h_{jk,0} - \hat{n}_l h_{kl,0}h_{jk,0} + \hat{n}_l h_{kl,0}h_{jk,0} + \hat{n}_a\hat{n}_b\hat{n}_k h_{ab,0}h_{jk,0}\\
&= 0.
\end{aligned}
$$

In the same way we compute the other non vanishing terms:

$$
\begin{aligned}
h^{\alpha\beta}{}_{,0} h_{\alpha\beta,j} &= -\hat{n}_a\hat{n}_b\hat{n}_k\hat{n}_l\hat{n}_j h_{ab,0}h_{kl,0} + 2\hat{n}_a\hat{n}_b\hat{n}_j h_{ka,0}h_{kb,0} - \hat{n}_j h_{kl,0}h_{kl,0},\\
h_{,0}h_{,j} &= -\hat{n}_a\hat{n}_b\hat{n}_k\hat{n}_l\hat{n}_j h_{ab,0}h_{kl,0} + 2\hat{n}_a\hat{n}_b\hat{n}_j h_{ab,0}h_{kk,0} - \hat{n}_j h_{kk,0}h_{ll,0}.
\end{aligned}
$$

---

[12]In its quantum version this means that three-leg vertices are excluded.

# Appendix C

Just for completeness let us reproduce here the Peters-Mathews formula for radiation emitted quoted in the text:

$$\dot{E} = -\left\langle \frac{\mu^2 m^2}{r^4} \left[ \frac{8}{15} \left( \kappa_1 v^2 - \kappa_2 \dot{r}^2 \right) + \frac{1}{3} \kappa_D \sigma^2 \right] \right\rangle . \tag{62}$$

where the constants $\kappa_i$ depend on the each particular theory for gravity phenomena.

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