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Localization and the interface between quantum mechanics, quantum field theory and quantum gravity

dedicated to the memory of Rob Clifton

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Abstract

We show that there are significant conceptual differences between QM and QFT which make it difficult to view QFT as just a relativistic extension of the principles of QM. The root of this is a fundamental distiction between Born-localization in QM (which in the relativistic context changes its name to Newton-Wigner localization) and *modular localization* which is the localization underlying QFT, after one liberates it from its standard presentation in terms of field coordinates. The first comes with a probability notion and projection operators, whereas the latter describes causal propagation in QFT and leads to thermal aspects.

Taking these significant differences serious has not only repercussions for the philosophy of science, but also leads to a new structural properties as a consequence of vacuum polarization: the area law for *localization entropy* near the the causal localization horizon and a more realistic cutoff independent setting for the cosmological vacuum energy density which is compatible with local covariance.

Contents

1 Introductory remarks

Ever since QM was discovered, the conceptual differences between classical theory and QM have been the subject of fundamental investigations with profound physical and philosophical consequences. But the conceptual relation between QFT and QM which is at least as challenging and rich of surprises has not received the same amount of attention and scrutiny. Apart from some admirable work on the significant changes which the theory of measurements must undergo in order to be consistent with the structure of QFT [1], I am not aware of in-depth attempts, although physicists occasionally investigated special problems as e.g. the issue of *Bell states* in *local quantum physics* (LQP[1]) [2] or the important relations between causal disjointness with the existence of uncorrelated states and the issue of statistical independence [4].

On the other side one should mention that some spectacular misunderstanding of conceptual properties in passing from QM to LQP led to incorrect results about alleged violations of the velocity of light remaining a limiting velocity in the quantum setting (the famous Fermi Gedankenexperiment) which, as a result of a publication in Phys. Lett. [5] and a simultaneous article in *Nature* on the prospects of time machines, created quite a stir at the time and led to a counter article [6]. Since the LQP presentation of the Fermi Gedankenexperiment is one of the strong motivations for non-experts to engage with its conceptual setting and therefore has a high pedagogical value in the present context, it is natural that it will obtain some space in this article.

The reader who expects an axiomatic setting for a new LQP-based approach to the measurement issue will be disappointed; before one does something ambitious like this one must take stock of the conceptual problems and this can only be achieved by going somewhat beyond the present "shut up and calculate" attitude. But let me emphasize again that there is no difference in content between QFT and LQP. I use LQP instead of QFT whenever I think that something may not be found in the standard textbooks, and this is certainly the case with some of the material in this paper. There is of course one recommendable exception, namely Rudolf Haag's book "Local Quantum physics"; but in a fast developing area of particle physics two decades (referring to the time it was written) is a long time.

The paper consists of two main parts, the first is entirely dedicated to the exposition of the differences between QM and LQP, whereas the second deals with thermal consequences of vacuum polarization caused by causal localization.

The first part starts with a subsection on *direct particle interactions* (DPI), a framework which incorporates all those properties of a relativistic theory which one is able to formulate solely in terms of particles (most of them already appearing in the S-matrix work of E.C.G. Stückelberg). However the enforcement of the cluster factorization property (the spatial aspect of macro-causality) in DPI requires more involved arguments; it is

[1] We use this terminology instead of QFT if we want to direct the reader's attention away from the textbook Lagrangian quantization towards the underlying principles [3]. QFT (the content of QFT textbooks) and LQP deal with the same physical principles but LQP is less comitted to a particular formalism (Lagrangian quantization, functional integrals) and rather procures always the most adaequate mathematical concepts for their implementation. It includes of course all the results of the standard perturbative Lagrangian quantization but presents them in a conceptually and mathematically more satisfactory way. Most of the subjects in this article are outside of textbook QFT.

not automatic as in nonrelativistic QM, and as a result DPI does not allow a second quantization presentation. Most particle physicists seem to be unaware of its existence and tend to believe that a relativistic particle theory, which is consistent with macro-causality and has a Poincaré-invariant S-matrix, must be equivalent to QFT[2].

Since the ideas which go into its construction are important for appreciating the conceptual differences of relativistic QM to QFT, we will at least sketch some of the arguments showing that DPI theories fulfill all the physical requirements which one is able to formulate about relativistic particles, as Poincaré covariance, unitary and macro-causality of the resulting S-matrix (which includes cluster factorization). In contradistinction to nonrelativistic mechanics for which clustering follows trivially from the additivity of pair-(or higher-) particle potentials, and also in contradistinction to QFT where the clustering is a rather straightforward consequence of locality and the energy positivity, the implementation in the relativistic DPI setting is much more subtle and this is related to the lack of a second quantization reformulation of multi-particle interactions. The important point in the present context is that there exists a quantum mechanical relativistic theory which implements interaction without using fields in which the S-matrix is Poincaré invariant and fulfills macro-causality.

In this way one learns to appreciate the fundamental difference between quantum theories which have no maximal velocity and those which have. DPI only leads to finite velocity propagation for asymptotically large time-like separations, so the causal propagation is valid only in the sense of asymptotically large timelike distances between asymptotically separated Born-localized events. Saying that DPI is macro- but not micro-causal implies that it cannot be used to study properties of local propagation over finite distances; the incorrect contradiction against the Fermi-Gedankenexperiment mentioned before resulted from ignoring this conceptually important point.

At the root of this difference is the existence of two very different concepts of localization namely the *Born localization* which is the only localization for QM, and the *modular localization* which is the one underlying the locality notion in QFT and which is relevant for causal propagation over finite distances. The justification and understanding of this terminology will be one of the main points of the present paper. Whereas QM only knows the Born localization, QFT requires both, Born-localization for (the wave functions of) particles before and after a scattering event, and modular localization in connection with fields and local observables[3]. Without Born localization and the associated projectors, there would be no scattering theory leading to cross sections and QFT would loose its most prominent observables and become just a mathematical playground. In contradistinction to QM and DPI, in QFT there is no way in which in the presence of interactions the notion of *particles at finite times* can be saved. The statement that an isolated relativistic particle cannot be localized below its Compton wave length refers to the (Newton-Wigner adaptation of the) Born localization. These structural limitations do not exclude that a QFT phenomenon may present itself like in QM in the sense of FAPP (for all practical

[2]The related folklore dictum one finds in the literature is: relativistic quantum theory of particles + cluster factorization property = QFT. Apparently this goes back to S. Weinberg.

[3]Particles are objects with a well-defined ontological status whereas (composite) fields form an infinite set of coordinatizations which generate the local algebras. Modular localization is the localization property which is independent of what field coordinatization has been used.

purposes) as this statement about the Compton wave length being a limit in a FAAP sense. The question when the QM setting can be applied in QFT problems in the FAAP sense is, similar to the problem of validity of quasi-classical approximations in QM, a highly artistic and complex issue.

The first part also contains a section which focusses on the radical difference between the Newton-Wigner (NW) localization (the name for the Born localization adapted to the relativistic particle setting) and the localization which is inherent in QFT, which in its intrinsic form, i.e. liberated from singular pointlike field coordinatization, will be referred to as *modular localization* [7][8][11]. The terminology has its origin in the fact that it is backed up by a mathematical theory within the setting of operator algebras which bears the name Tomita-Takesaki[4] *modular theory,* although, within the setting of thermal QFT, physicists independently discovered various aspects of it [3]. Its relevance for causal localization was only spotted a decade later [10] and the appreciation of its use in problems of thermal behavior at causal- and event- horizons and black hole physics had to wait another decade.

The last section of the first part shows the enormous conceptual distance between QM and LQP by presenting the world of LQP as the result of *relative positioning* of a finite (and rather small) number of *monads* within a Hilbert space. Here we are using the terminology introduced by Leibniz in a philosophical context. In fact it turns out that the LQP adaptation of this setting goes even somewhat beyond what Leibniz had in mind when he imagined physical reality arising from interrelations between monads with spacetime serving as ordering device for the monads (in the present particle physics context monads are copies of the unique hyperfinite type III_1 factor algebra) which is the abstract form of quantum matter substrate. In other words the full physical content of LQP i.e. the material substrate together with Minkowski spacetime symmetry is encoded into the relative positioning of a finite number of monads in a Hilbert space. The algebraic structure of QM, relativistic or not, has no such monad structure; the global algebra as well as all Born-localized subalgebras are always of type I which is either the algebra of all bounded operators $B(H)$ in a Hilbert space (in case of irreducible ground state representations) or multiples or tensor products thereof.

The second part addresses two important astrophysical consequences of vacuum polarization, the first section deals with *localization entropy* and recalls its area proportionality which is a more recent result [47][48]. We will explain why entanglement in QFT is very different from the better known entanglement in quantum mechanics. The second subsection also contains some new remarks about the cosmological constant problem. In both cases these results of LQP cast serious doubts on whether what was hitherto perceived to be the interface between LQP and QG has been placed correctly. In particular it is questionable that the thermal aspects of black holes, in particular the area proportionality of entropy, need the inference of QG instead of being fully understood in terms of QFT in CST.

Another fact which corroborates the necessity for change in thinking comes from the fact that the prohibitively large value for the vacuum energy, which has been erroneously

[4]Tomita was a Japanese mathematician who discovered some properties of the theory in the first half of the 60s, but it needed a lot of polishing in order to be accepted by the mathematical community, and this is where the name Takesaki entered.

attributed to QFT, is in reality based on the level occupation in relativistic QM. The standard estimate, which led to the cosmological constant problem, violates *local covariance* (local diffeomorphism equivalence) which is one of QFT in CST most cherished principles. We indicate how a calculation without cutoff and in agreement with local covariance may look like. With this critical comments I am in good company. In a paper by Hollands and Wald [12] such critical thinking even entered the title of their article: *Quantum Field Theory Is Not Merely Quantum Mechanics Applied to Low Energy Effective Degrees of Freedom.*

2 The interface between quantum mechanics and quantum field theory

Shortly after the discovery of field quantization in the second half of the 1920s, three viewpoints about its content and purpose emerged. There were those who pointed out that particles were already "quantum" ever since Heisenberg's discovery of quantum mechanics, and therefore there is no sufficient reason for quantizing their quantum description a second time. On the opposite side was Pascual Jordan, who, following de Broglie, not only almost single-handedly created QFT (against his critics from the first group), but also defended the (in those days) radical point of view that electromagnetism as well as matter must be described by a unified formalism of quantum fields. Dirac's position was somewhere between these extremes in that quantum theory should mean *quantizing a true classical reality*[5] reflected his conviction that quantization denotes quantizing the classical description which is wave-like for classical electromagnetism and corpuscular for classical particles; thus rejecting the field quantization for massive particles for which no classical wave reality exists (only in the 50s he fully embraced field quantization).

The unified quantized wave field point of view of Jordan finally won the argument, but there is some irony in the fact that the idea of antiparticles entered QFT through Dirac's hole theory and not through Jordan's field quantization. It was also the hole theory in which the first perturbative QED computations (which entered the textbooks of Heitler and Wenzel) were done before it was recognized that this setting was not really consistent. This insight only surfaced when it was found out that the hole formalism is unsuitable for problems involving renormalization in which *vacuum polarization* plays the essential role. The latter had been discovered already in the 30s by Heisenberg [13] in connection with states obtained by applying composites of free fields (as conserved currents) to the vacuum. Shortly afterwards Furry and Oppenheimer [14] studied perturbative interactions of Lagrangian fields and noticed to their amazement that the perturbatively interacting Lagrangian field applied to the vacuum created inevitably some "stuff" in addition to the expected one-particle state. Unlike the case of composite free fields, the number of the particles/antiparticles in that state increases with perturbative order, pointing towards the fact that one has to deal with infinite polarization clouds. It is these (at first sight innocent looking) interaction-induced polarization clouds which, as will be shown in the

[5] Jordan's extreme formal positivistic point of view allowed him to quantize everything which fitted into the classical Lagrangian field formalism independent of whether it had a classical reality or not.

subsequent sections, separate QM from LQP in a conceptually dramatic way. In fact these polarization clouds are more characteristic of LQP than the actual creation of particles in scattering processes which is usually cited as the main distinction between LQP and QM. This is because the DPI setting can be generalized to incorporate particle creation through scattering [19], whereas vacuum polarization is a rather direct consequence of quantum field theoretic localization which is incompatible with QM.

2.1 Direct particle interactions, relativistic QM

In the following we address the question why, inspite of the mentioned inconsistency of attempts at particle theories as the hole theory, there can be at all a consistent relativistic particle theory. By this we mean a quantum theory of interacting particles which fulfills all physical requirements which one can formulate solely in terms of particles; this includes in addition to unitary representation of the Poincaré group also macro-causality, but certainly not micro-causality which has no place in a particle-based theory would force us away from a pure particles setting.

A positive answer was given in terms of Coester's *direct particle interactions* (DPI) where direct means "not field-mediated". This idea was first formulated in the non-trivial context of 3-particle systems [15] and then generalized (in collaboration with Polyzou [16]) to arbitrary high particle number. As a pure relativistic particle theory without vacuum polarization, it turns out to have no natural second quantization setting. But on the other hand it fulfills all properties which are expressible in terms of particle concepts without the use of fields. In particular these theories fulfill the cluster separability properties of the associated Poincaré invariant unitary S-matrix which no particle-based S-matrix approach was able to implement before, neither that of Heisenberg nor that of Chew's bootstrap approach and also not the dual model/string approach.

It has been known since the early days of particle physics that an interacting relativistic 2-particle system of massive particles (for simplicity of equal mass) is simply described by going into the c.m. system and modifying the mass operator in the following way

$$M = 2\sqrt{p^2 + m^2} + v, \quad H = \sqrt{P^2 + M^2} \tag{1}$$

The interaction v may be taken as a *local* function in the relative coordinate which is conjugate to the relative momentum p in the c.m. system; but since the scheme does not lead to local differential equations, there is not much to be gained from such a choice. One may follow Bakamjian and Thomas [17] and choose the Poincaré generators in such a way the interaction does not affect them directly apart from the Hamiltonian.. Denoting the interaction-free generators by a subscript 0 one arrives at the following system of two-particle generators

$$\begin{aligned} \vec{K} &= \frac{1}{2}(\vec{X}_0 H + H\vec{X}_0) - \vec{J} \times \vec{P}_0 (M+H)^{-1} \\ \vec{J} &= \vec{J}_0 - \vec{X}_0 \times \vec{P}_0 \end{aligned} \tag{2}$$

where the Wigner canonical spin J commutes with $\vec{P} = \vec{P}_0$ and $\vec{X} = \vec{X}_{.0}$ and $W_\mu = \varepsilon_{\mu\nu\kappa\lambda} P^\nu M^{\kappa\lambda}$ is the Pauli-Lubanski vector which is useful for the covariant description of spin. We leave the check that the commutation relations of the Poincaré generators result from the above definitions together with the canonical commutation relations of the single particle canonical variable (which furnish a complete irreducible set of operators in terms of which any operator in the Hilbert space may be written) to the reader. As in the nonrelativistic setting, short ranged interactions v lead to Møller operators and S-matrices via a converging sequence of unitaries formed from the free and interacting Hamiltonian

$$\begin{aligned}\Omega_{\pm}(H,H_0) &= \lim_{t\to\pm\infty} e^{iHt}e^{-H_0 t} \\ \Omega_{\pm}(M,M_0) &= \Omega_{\pm}(H,H_0) \\ S &= \Omega_+^*\Omega_-\end{aligned} \tag{3}$$

The identity in the second line is the consequence of a theorem which say that the limit is not affected if instead of M we take a positive function of M as $H(M)$, as long as H_0 is the same function of M_0. This insures the frame-independence of the Møller operators and the L-invariance of the S-matrix. Apart from this identity the rest behaves just as in nonrelativistic scattering theory. As in standard QM, the 2-particle cluster property is the statement that $\Omega_{\pm}^{(2)} \to \mathbf{1}$, $S^{(2)} \to \mathbf{1}$ in the limit of infinite spatial separation of the centers of wave packets of the two particles. The cluster factorization follows from the same kind of short range assumption that already assured the validity of the asymptotic convergence.

The BT form for the generators can be achieved inductively for an arbitrary number of particles. As will be seen, the advantage of this form of the generators is that in passing from n-1 to n-particles the interactions simply add and one ends up with Poincaré group generators for an interacting n-particle system. But for $n > 2$ the aforementioned subtle problem with the cluster property arises: whereas this iterative construction in the nonrelativistic setting complies with cluster separability, this is not the case in the relativistic context. This problem shows up for the first time in the presence of 3 particles [15]. The BT iteration from 2 to 3 particles gives the 3-particle mass operator

$$\begin{aligned}M &= M_0 + V_{12} + V_{13} + V_{23} + V_{123} \\ V_{12} &= M(12,3) - M_0(12;3),\ M(12,3) = \sqrt{\vec{p}_{12,3}^{\,2} + M_{12}^2} + \sqrt{\vec{p}_{12,3}^{\,2} + m^2}\end{aligned} \tag{4}$$

and the $M(ij,k)$ result from cyclic permutations Here $M(12,3)$ denotes the 3-particle invariant mass in case the third particle is a “spectator” (which by definition does not interact with 1 and 2). The momentum in the last line is the relative momentum between the (12)-cluster and particle 3 in the joint c.m. and M_{12} is the associated two-particle mass (invariant energy in the (12) c.m system). As in the nonrelativistic case, one can always add a totally connected contribution. Setting this contribution to zero, the 3-particle mass operator only depends on the two-particle interaction v. But contrary to the nonrelativistic case, the BT generators constructed with M do not fulfill the cluster separability requirement as it stands. The latter demands that if the interaction between two clusters is removed, the unitary representation factorizes into that of the product of the two clusters. One expects that shifting the third particle to infinity will render it a spectator and result in a factorization $U_{12,3} \rightarrow U_{12} \otimes U_3$. Unfortunately what really happens is that the (12) interaction also gets switched off i.e. $U_{123} \rightarrow U_1 \otimes U_2 \otimes U_3$. The reason for this violation of the cluster separability property, as a simple calculation using the transformation formula from c.m. variables to the original p_i, i = 1, 2, 3 shows [16], is that the spatial translation in the original system (instead of the $12,3$ c.m. system) does remove the third particle to infinity as it should, but unfortunately it also drives the two-particle mass operator (with which it does not commute) towards its free value which violates clustering.

In other words the BT produces a Poincaré covariant 3-particle interaction which is additive in the respective c.m. interaction terms (4), but the Poincaré representation U of the resulting system will not be cluster-separable. However, as shown first in [15], at least the 3-particle S-matrix computed in the additive BT scheme turns out to have the cluster factorization property. But without implementing the correct cluster factorization not only for the S-matrix but also for the 3-particle Poincaré generators there is no chance to proceed to a clustering 4-particle S-matrix.

Fortunately there always exist unitaries which transform BT systems into cluster-separable systems *without affecting the S-matrix*. Such transformations are called *scattering equivalences.* they were first introduced into QM by Sokolov [18] and their intuitive content is related to a certain insensitivity of the scattering operator under quasilocal changes of the quantum mechanical description at finite times. This is vaguely reminiscent of the insensitivity of the S-matrix in QFT against local changes in the interpolating field-coordinatizations[6]. The notion of scattering equivalences is conveniently described in terms of a subalgebra of *asymptotically constant operators* C defined by

$$\begin{aligned} \lim_{t\rightarrow\pm\infty} C^{\#} e^{iH_0 t}\psi = 0 \\ \lim_{t\rightarrow\pm\infty} \left(V^{\#} - 1\right) e^{iH_0 t}\psi = 0 \end{aligned} \tag{5}$$

where $C^{\#}$ stands for both C and C^*. These operators, which vanish on dissipating free wave packets in configuration space form a *-subalgebra which extends naturally to a C^*-algebra $\mathcal{C}$. A scattering equivalence is a unitary member $V \in \mathcal{C}$ which is asymptotically equal to the identity (which is the content of the second line). Applying this asymptotic equivalence relation to the Møller operator one obtains

$$\Omega_\pm(VHV^*, VH_0V^*) = V\Omega_\pm(H, H_0) \tag{6}$$

so that the V cancels out in the S-matrix. Scattering equivalences do however change the interacting representations of the Poincaré group according to $U(\Lambda, a) \rightarrow VU(\Lambda, a)V^*$.

The upshot is that there exists a clustering Hamiltonian H_{clu} which is unitarily related to the BT Hamiltonian H_{BT} i.e. $H_{clu} = BH_{BT}B^*$ such that $B \in \mathcal{C}$. is uniquely determined in terms of the scattering data computed from H_{BT}. It is precisely this clustering of H_{clu} which is needed for obtaining a clustering 4-particle S-matrix which is cluster-associated the $S^{(3)}$. With the help of M_{clu} one defines a 4-particle interaction following the additive

[6] In field theoretic terminology this means changing the pointlike field by passing to another (composite) field in the same equivalence class (Borchers class) or in the setting of AQFT by picking another operator from a local operator algebra.

BT prescription; the subsequent scattering formalism leads again to a clustering 4-particle S-matrix and again one would not be able to go to n=5 without passing from the BT to the cluster-factorizing 4-particle Poincaré group representation. Coester and Polyzou showed [16] that this procedure can be iterated and hence one arrives at the following theorem

Theorem: *The freedom of choosing scattering equivalences can be used to convert the Bakamijan-Thomas presentation of multi-particle Poincaré generators into a cluster-factorizing representation. In this way a cluster-factorizing S-matrix $S^{(n)}$ associated to a BT representation H_{BT} (in which clustering mass operator $M_{clu}^{(n-1)}$ was used) leads via the construction of $M_{clu}^{(n)}$ to a S-matrix $S^{(n+1)}$ which clusters in terms of all the previously determined $S^{(k)}, k < n$. The use of scattering equivalences impedes the existence of a 2^{nd} quantized formalism.*

For a proof we refer to the original papers [16][19]. In passing we mention that the minimal extension (the one determined uniquely in terms of the two-particle interaction v) from n to n+1 for $n > 3$ contains *connected 3-and higher particle interactions* which are nonlinear expressions (involving nested roots) in terms of the original two-particle v.This is another unexpected phenomenon as compared to the nonrelativistic case, although less surprising from a QFT position.

This theorem shows that it is possible to construct a relativistic theory which only uses particle concepts, thus bringing to an end an old folklore which says relativity + clustering = QFT. Whether one should call this DPI theory "relativistic QM" is a matter of taste, it depends on what significance one attributes to those scattering equivalences. But in any case it is a *relativistic S-matrix setting which goes beyond the prior attempts by Heisenberg who missed out on the cluster factorization. In this context one should also mention that Chews S-bootstrap never tried to implement clustering and of course none of these important properties have been checked in the dual model/string theory.* Taking into considerations the sophistication one needs in order to implement macrocausality in a particled based theory outside the micro-causal setting of QFT, the possibility that a multiparticle S-matrix constructed according to the prescriptions of string-theory by some stroke of luck fulfills these requirements is smaller than finding a needle in a haystack; but maybe it is possible to modify string theory so that it complies with them.

Coester and Polyzou also showed that this relativistic setting can be extended to processes which maintain cluster factorization in the presence of a finite number of creation/annihilation channels, showing, as mentioned before, that the mere occurrence of particle creation is not characteristic for QFT. Different from the nonrelativistic Schroedinger QM, the superselection rule for masses of particles which results from Galilei invariance does not carry over to the relativistic setting, which therefore is less restrictive.

Certain properties which are automatic consequences of locality in QFT and can be expressed in terms of particles as TCP symmetry, the existence of anti-particles, the spin-statistics connection, can be added "by hand". Other properties which are on-shell relics of locality which QFT imprints on the S-matrix and which require the notion of analytic continuation in on-shell particle momenta as the crossing property, cannot be implemented in the QM setting of DPI.

2.2 First brush with the intricacies of the particles-field problems in QFT

QFT, apart from free fields QFT, in contrast to QM (Schrödinger- or relativistic DPI-QM), does not admit a particle interpretation at finite times. If it would not be for the asymptotic scattering interpretation in terms of incoming/outgoing particles associated with the free in/out fields, there would be hardly anything of a non-fleeting nature which can be measured. In QFT in CST and thermal QFT where this particle concept is missing, the set of conceivable measurements is very meagre and is essentially reduced to energy- and entropy- densities as in thermal systems and black hole radiation.

Since the notion of particle is often used in a more general sense than in this paper, it may be helpful to have an interlude on this topic. By particle I mean an asymptotically stable object which forms the tensor product basis for an asymptotically complete description; in other words the particle concept equips the QFT with a (LSZ, Haag-Ruelle) complete asymptotic particle interpretation[7] which imposes a Fock space tensor structure on the Hilbert space of the interacting system. The physics behind it is the idea that if we were cobbling the asymptotic spacetime region with counters and monitor coincidences of localization events, then an n-fold coincidence would remain stable if the far removed localization centers would move freely. QFT achieves this asymptotic completeness structure through asymptotic Born localization. The particle concept in QFT is precisely applicable where it is needed namely asymptotically and where the non-covariant aspect of an individual Born (=NW) localization becomes irrelevant (since the asymptotic geometric relations between counters is described by covariant mass-shell momenta).

Thus the invariant S-matrix has no memory about the reference-system-dependent Born localization of particle counters. Tying the particle concept to asymptotically stable counter-coincidences can be traced back to a seminal paper by Haag and Swieca [20]. The Fock representation of free fields is the only model which admits this interpretation for all times; by passing to inequivalent representations of the underlying CCR or CAR C^*-algebra the particle interpretation is lost even at asymptotic times.

It is this asymptotic particle structure which leads to the observational richness of QFT. Once we leave this setting by going to curved spacetime, to QFT in KMS thermal states representations, or if we restrict a Minkowski spacetime theory to a Rindler wedge (with the Hamiltonian being the boost operator with its two-sided spectrum), we are loosing this observational wealth. The restriction to the Rindler world inherits of course the particle structure of the say free field Minkowski QFT, but this is not intrinsic in the Rindler sense[8] since the Minkowski vacuum is now a thermal state and there is no particle scattering theory in the boost time in such a thermal situation. Of course there remains the possibility to measure thermal excitations in an *Unruh counter*, to use a counter for

[7] The asymptotic completeness property was for the first time established (together with a recent existence proof) in a family of factorizing two-dimensional models (see the section on modular localization) with nontrivial scattering.

[8] There is of course the mathematical possibility of choosing a groundstate representation for a Rindler world instead of restricting the Minkowski vacuum. In that case it is not clear whether in the presence of interactions the exitations above this ground state have the Haag-Swieca asymptotic localization stability i.e. whether scattering theory applies to such a situation. It would be interesting to (dis)prove the validity of Haag-Ruelle scattering theory in such a situation.

observing Hawking radiation or to determine the energy density in a cosmological reference state (see also last section). But this is done by placing a counter in a thermal medium associated with a state on a Rindler or Hawking black hole system which measures an ensemble of excitations which are present in such a (possibly homogeneous) reference state. Physicists who work on QFT in CST use this more general notion of particles [24] in the sense of Unruh thermal excitations. But given the impossibility of measuring quantum fields directly, the question whether there exist any other measurements besides the thermal radiation measurements remains open. On may use the word particle in any way one wants as long as one accounts for the observable consequences.

Naive intuition suggests of course that in the "for all practical purposes sense" one should be able to use the idealized setting of scattering theory also for non-asymptotic intermediate settings as long as the curvature does not require a quantum treatment. But there is presently no concept which makes this vague idea precise (scattering theory in the flat tangent space?).

Recent developments have led to view QFT in a functorial setting as a functor from globally hyperbolic Lorentz manifolds to C*- or operator algebras [25]. In other words the same abstract matter substrate may be ordered using different spacetime ordering devices. In this descriptive functorial sense a QFT in CST has an associated Minkowski spacetime QFT with a particle interpretation in the above sense. But the particle structure requires the presence of asymptotic regions and precisely that is not covered by the functorial local relation between the observables belonging to locally isometric regions; i.e. just where the functorial relation would be needed for transplanting the notion of particles it breaks down.

One could argue that the observational indigence on the side of particles may be counterbalance by measuring quantum fields directly, but which fields and how? Quantum fields have, in contradistinction to their measurable classical counterparts, no "individuality"; they are just coordinatizing local algebras; there are infinitely many of them and there is no intrinsic hierarchy between elementary and composite besides that of their superselected charges. One is as good as the other as long as it creates from the vacuum the correct charge which the particle carries. *The main purpose of fields is to interpolate asymptotic particles and implement the local covariance principle*. Even the setting of *nets of local field algebras* (which is the field-coordinate-free implementation of the locality requirement) is an "as if" world of objects. No quantum field theorist will loose time thinking about ontological aspects of individual operators in a local algebra inasmuch as no experimentalist insists to know (apart from the sensitivity of his counter) his counter's detailed inner workings. The essential aspect for both, the theoretician as well as the experimentalist, is the localization of events and not the assignment of an event to a particular field or to a particular brand of counter. In some sense the world of the *infinitely many* (composite) pointlike fields is the prize to pay for being able to implement the local covariance principle in LQP.

Compare the confusing plurality of fields with the simplicity of particles which are uniquely determined by their Poincaré representation properties[9]. But without the enforcement of the local covariance via a net of local algebras or via generating pointlike

[9] It will be shown in a later section that modular localization which is the localization concept underlying local covariance, is unique despite the plethora of pointlike fields.

covariant fields it is impossible to understand those subtle properties of the S-matrix which have been verified in experiments. Each attempt to implement those properties which the S-matrix inherits from the locality principle directly by hand (i.e. avoiding local nets and local fields) has ended in failure: Heisenberg's S-matrix approach, the Chew bootstrap and in my opinion, despite its popularity, also the dual model/string theory setting. In fact DPI it is besides QFT the only theory known to date in which the validity of the minimal requirements on particles and their interaction can be fulfilled.

Our rather detailed presentation[10] of the setting of DPI in the previous section serves exclusively to highlight this significant conceptual difference between (relativistic) QM and QFT; there was no intention here to proselyte for DPI, although the protagonists of this setting, who are mathematical nuclear physicists [16], advocate an extended form of DPI (with creation channels) for medium energy particle phenomenology in energy ranges where only a few mesons are created.

The conceptual differences between a DPI relativistic QM and QFT are enormous, but in order to appreciate this, one has to become acquainted with structural properties of QFT which are somewhat removed from the standard properties and unfortunately have not entered textbooks; it is the main purpose of the following sections to highlight these contrasts by going more deeply into QFT.

There are certain folkloric statements about the relation QM–QFT whose dismissal does not require any conceptual sophistication. For example in trying to make QFT more susceptive to newcomers it is sometimes said that a free field is nothing more than a collection of infinitely many coupled oscillators. Although not outright wrong, this characterization misses the most characteristic property of how spacetime enters as an ordering principle into QFT. It would not help any newcomer who knows a quantum oscillator, but has not met a free field before, to construct a free field from those words This is somewhat reminiscent of equating QM via Schrödinger's formulation with classical wave theory. What may be gained for a newcomer by appealing to his computational abilities acquired in classical electrodynamics is more than lost in the conceptual problems which he confronts later when facing the subtleties of quantum physics.

2.3 Quantum mechanical Born localization versus covariant localization in LQP

Let us know come to the main point namely the difference between QM and LQP in terms of their localization concept. We will use the word *Born localization* for the localization probability density of the Schroedinger wave function $p(x) = |\psi(x)|^2$ (or its Newton-Wigner counterpart).

As a historical curiosity we mention that Born's original publication [21] does not deal with localization properties in QM, rather it introduces the probability concept for a scattering amplitude in the Born approximation, i.e. it preempted the notion of cross section. Wave function localization in conjunction with probability only entered two years later starting with a paper by Pauli (without reference to Born's work); hence Born

[10] The reason for not just referring to the original papers is that this setting does not seem to be known outside a small circle of mathematical inclined nuclear physicists.

localization is Born's probability rule as extended from scattering theory to the x-space Schrödinger wave function. Being a bona fide probability, one may characterize the Born localization in a spatial region $R \in \mathbb{R}^3$ at a given time in terms of a localization projector $P(R)$. The standard version of QM and the various settings of measurement theory rely heavily on these projectors. Without Born localization and the ensuing projectors it would be impossible to formulate the conceptual basis for the time-dependent scattering theory.

The adaptation of Born localization to the setting of relativistic particles and their direct interactions is known under the name *Newton-Wigner Localization* because these authors [22] introduced a frame-dependent selfadjoint localization operator and its family of projections. Its lack of covariance in finite time propagation leads to frame-dependence and superluminal contributions, which is why the terminology "relativistic QM" has to be taken with a grain of salt. However in the asymptotic limit of large timelike separation as required in scattering theory, the covariance, frame-independence and causal relations are recovered. With other words one obtains a Poincaré-invariant unitary S-matrix whose DPI construction guaranties also the validity of all the macro-causality requirements (spacelike clustering, absence of timelike precursors) which can be formulated in a particle setting without taking recourse to interpolating local fields. Even though the individual particle localizations are frame-dependent, the asymptotic relation between two NW events is given in terms of the geometrically associated covariant on-shell momenta or 4-velocities. In fact all observations on particles always involve Born-localization measurements. We will often use the name NW and Born interchangeably for the localization in the relativistic particle context.

It is not accidental that the increasing popularity of nonlocal and noncommutative models occurs at a time in which the understanding for the physical relevance on causality issues has been on a down turn. Instead considerable attention is focussed on mathematical technicalities about what is the best way to implement noncommutativity by using star-products and similar modifications. Practically no consideration is given to the physically crucial questions of cluster factorization and absence of timelike precursors for the resulting S-matrix. It seems that these matters which enjoyed a prominence at the time of Stückelberg and during the heydays of dispersion relations have vanished from the collective knowledge.

In comparing QM with QFT it is often convenient in discussions about conceptual issues to rephrase the content of QM in terms of operator algebras and states (expectation value functionals on operator algebras); in this way one also achieves more similarity with the formalism of QFT where this abstraction becomes important. In QFT the identification of pure states with state-vectors of a Hilbert space has no intrinsic meaning and often cannot be maintained in concrete situations. For the same reasons of achieving a unified description we use the multi-particle (Fock space) setting instead of the Schroedinger formulation. This multiplicative Fock space setting is not available for DPI, in which case a comparison of concepts becomes less elegant.

The global algebra which contains all observables independent of their localization is the algebra $B(H)$ of all bounded operators in Hilbert space. Physically important unbounded operators are not members but rather have the mathematical status of being affiliated with $B(H)$ and its subalgebras; this bookkeeping makes it possible to apply powerful theorems from the theory of operator algebras (whereas unbounded operators are treated on a case to case basis). $B(H)$ is the correct global description whenever the physical system under discussion arises as the weak closure of a ground state representation of an irreducible system of operators[11]. According to the classification of operator algebras, $B(H)$ and all its multiples are of Murray von Neumann type I_∞ whose characteristic property is the existence of minimal projectors (in the irreducible case these are the one-dimensional projectors belonging to measurements which cannot be refined).

The differences between QM and LQP emerge as soon as one uses localization in order to provide a physical substructure to $B(H)$. It is well kown that a dissection of space into nonoverlapping spatial regions i.e. $\mathbb{R}^3 = \cup_i R_i$ implies via Born localization a tensor factorization of $B(H)$ and H

$$B(H) = \bigotimes_i B(H(R_i)) \tag{7}$$
$$H = \bigotimes_i H(R_i),\ \ P(R_i)H = H(R_i)$$

Hence there is orthogonality between subspaces belonging to localizations in nonoverlapping regions and one may talk about states which are pure in R_i. A pure state in the global algebra $B(H)$ may not be of the tensor product form but may rather be a superposition of factorizing states. In that case the reduced density matrix obtained by averaging outside a region R_i leads to the phenomenon of *entanglement*. Although one may relate this quantum mechanical entanglement with the notion of entropy, it is an entropy in the sense of information theory and not in the thermal sense of thermodynamics, i.e. one cannot assign a temperature as a quantitative measure of the degree of quantum mechanical entanglement which results from restricting pure global states. The trivial net structure of $B(H)$ in terms of the $B(H(R_i))$ is of a kinematical kind. The quantum mechanical dynamics through a Hamiltonian shows that the tensor factorization from Born localization at one time is almost instantaneously lost in the time-development, as expected of a theory of without a maximal propagation speed.

The LQP counterpart of the Born-localized subalgebras at a fixed time are the observable algebras $\mathcal{A}(\mathcal{O})$ for causally completed ($\mathcal{O} = \mathcal{O}''$, the causal complement taken twice) spacetime regions $\mathcal{O}$; they form what is called in the terminology of LQP a *local net* $\{\mathcal{A}(\mathcal{O})\}_{\mathcal{O}\subset M}$ of operator algebras indexed by regions in Minkowski spacetime $\cup\mathcal{O} = \mathcal{M}$ which is subject to the natural and obvious requirements of isotony ($\mathcal{A}(\mathcal{O}_1) \subset \mathcal{A}(\mathcal{O}_2)$ if $O_1 \subset O_2$) and causal locality (the algebras commute for spacelike separated regions).

The connection with the standard formulation of QFT in terms of pointlike fields is that smeared fields $\Phi(f) = \int \Phi(x)f(x)d^4x$ with $supp f \subset \mathcal{O}$ under reasonable general conditions generate local algebras. Pointlike fields, which themselves are too singular to be operators (even if admitting unboundedness), have a well-defined mathematical meaning as operator-valued distributions. But as mentioned before, there are myriads of fields which generate the same net of local operator algebras, hence they play a similar role in LQP as coordinates in modern differential geometry i.e. they coordinatize the net of

[11]The closure in a thermal equilibrium state associated with a continuous spectrum Hamiltonian leads to a unitarily inequivalent (type III) operator algebra without minimal projectors.

spacetime indexed operator algebras and only the latter has an intrinsic meaning. But as the use of particular spatial coordinates often facilitates calculations, the use of particular fields with e.g. the lowest short-distance dimension within the infinite charge equivalence class of fields can greatly simplify calculations in QFT. Therefore it is a problem of practical importance to construct a covariant basis of locally covariant pointlike fields of an equivalence class. For massive free fields and massless fields of finite helicity such a basis is especially simple since the Wick-basis of composite fields still follows in part the logic of classical composites. This remains so in the presence of interactions in which case the Wick-ordering gets replaced by the technically more demanding "normal ordering". For free fields in CST and the definition of their composites it is important to require the *local covariant transformation behavior* under local isometries [23]. The conceptual framework for the general case with interactions has also been understood [25]. The different field coordinates (the analog of the free field and its Wick-composites when interactions are present) with a cyclic action on a vacuum-like reference state carry the same localization information as the algebraic net.

After these remarks about the relation of fields in QFT with the local net of LQP, we now return to the main question namely what changes if we pass from the Born localization of QM to the causal localization of LQP? The crucial property is that a localized algebra $\mathcal{A}(\mathcal{O}) \subset B(H)$ together with its commutant $\mathcal{A}(\mathcal{O})'$ (which under very general conditions[12] is equal to algebra of the causal disjoint of $\mathcal{O}$ i.e. $\mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}'))$ are two von Neumann factor algebras i.e.

$$B(H) = \mathcal{A}(\mathcal{O}) \vee \mathcal{A}(\mathcal{O})',\ \mathcal{A}(\mathcal{O}) \cap \mathcal{A}(\mathcal{O})' = \mathbb{C}\mathbf{1} \tag{8}$$

But in contrast to the QM algebras the local factor algebras are not of type I and $B(H)$ does *not tensor-factorize* in terms of them, in fact they cannot even be embedded into a $B(H_1) \otimes B(H_2)$ tensor product. The prize to pay for ignoring this important fact and imposing wrong structures is the appearance of spurious ultraviolet divergences. On the positive side, as will be seen later, without this significant change in the nature of algebras there would be no holography onto causal horizons, no thermal behavior caused by localization and a fortiori no area-proportional localization entropy.

In QM a pure state vector, which with respect to a distinguished tensor product basis in $H(R) \otimes H(R \backslash \mathbb{R}^3)$ is a nontrivial superposition of tensor-basis states, will be generally an impure state if restricted to $B(H(R))$; in the standard formalism (where only pure states are represented by vectors) it is described by a density matrix. This phenomenon of *entanglement* is best described by the *information theoretic notion of entropy*. On the other hand each pure state on $B(H(R))$ or $B(H(R\backslash\mathbb{R}^3))$ originates from a pure state on $B(H)$. The situation in LQP is radically different since the local algebras as $\mathcal{A}(\mathcal{O})$ have *no pure states at all*; so the dichotomy between pure and mixed states breaks down and the kind of entanglement caused by field theoretic localization is much more violent then that coming from Born-localization (see below).

The situation does not change if one takes for $\mathcal{O}$ a region R at a fixed time; in fact in a theory with finite propagation one has$\mathcal{A}(R) = \mathcal{A}(D(R))$ where $D(R)$ is the

[12] In fact this duality relation can always be achieved by a process of maximalization (Haag dualization) which increases the degrees of freedom inside $\mathcal{O}$. A pedagogical illustration based on the "generalized free field" can be found in [26]..

diamond shaped double cone subtended by R (the causal shadow of R). Even if there are no pointlike generators and if the theory only admits a macroscopically localized net of algebras (e.g. a net of non-trivial wedge-localized factor algebras $\mathcal{A}(W)$ but trivial double cone algebras $\mathcal{A}(\mathcal{O})$), the algebras would not tensor factorize i.e. $B(H) \neq \mathcal{A}(W) \otimes \mathcal{A}(W')$, so these properties are not directly related to the singular nature of generating fields. It turns out that there is a hidden singular aspect in the sharpness of the $\mathcal{O}$-localization which generates infinitely large vacuum polarization clouds on the causal horizon of the localization.

Many divergencies in QFT are the result of conceptual errors in the formulation resulting from tacitly identifying QFT with some sort of relativistic QM[13], especially in computations with pointlike localized fields. Conceptual mistakes are facilitated by the fact that even nonlocal but covariant objects are singular; this is evident from the Kallen-Lehmann representation of a covariant scalar object

$$\langle A(x)A(y)\rangle = \int \Delta_+(x-y,\kappa^2)\rho(\kappa^2)d\kappa^2 \tag{9}$$

which was proposed precisely to show that even without demanding locality, but retaining only covariance and the Hilbert space structure (positivity), a certain singular behavior is unavoidable. In the DPI scheme this was avoided because there are simply no pointlike covariant objects in such a setting; the emphasis there is on generators of the Poicaré group and *invariant global* operators as the Møller operators and the S-matrix. In the algebraic formulation the covariance requirement refers to the geometry of the localization region $\mathcal{A}(\mathcal{O})$ i.e.

$$U(a,\Lambda)\mathcal{A}(\mathcal{O})U(a,\Lambda)^* = \mathcal{A}(\mathcal{O}_{a,\Lambda}) \tag{10}$$

whereas no requirement about the transformation behavior under finite (tensor, spinor) Lorentz representations (which would bring back the unboundedness and thus prevent the use of powerful theorems in operator algebras) is imposed for the individual operators. The singular nature of pointlike generators (if they exist) is then a purely mathematical consequence.

We have seen that although QM and QFT can be described under a common mathematical roof (C^*-algebras with a state functional), as soon as one introduces the physically important localization structure, significant conceptual differences appear. These differences are due to the presence of vacuum polarization in QFT as a result of causal localization, and they have dramatic consequences; the most prominent ones will be presented in this and the subsequent sections, as well as in the second part.

The net structure of the observables allows a local comparison of states: two states are locally equal in a region $\mathcal{O}$ if and only if the expectation values of all operators in $\mathcal{A}(\mathcal{O})$ are the same in both states. Local deviations from any state, in particular from the vacuum state, can be measured in this manner, and states that are indistinguishable from the vacuum in the causal complement of some region ('strictly localized states' [27]) can be defined. Due to the unavoidable correlations in the vacuum state in relativistic

[13] The correct treatment of perturbation theory which takes into account the singular nature of pointlike quantum fields may yield more free parameters than in the classical setting, but one is never required to confront infinities or cut-offs.

quantum theory (the Reeh-Schlieder property [3]), the space $H(\mathcal{O})$ obtained by applying the operators in $\mathcal{A}(\mathcal{O})$ to the vacuum is, for any open region $\mathcal{O}$, dense in the Hilbert space and thus far from being orthogonal to $H(\mathcal{O}')$. This somewhat counter-intuitive fact is inseparably linked with a structural difference between the local algebras and the algebras encountered in non-relativistic quantum mechanics (or the global algebra of a quantum field associated with the entire Minkowski space-time) as mentioned in connection with the breakdown of tensor-factorization (8).

The result is a particular benevolent form of Murphy's law: *everything which is not forbidden (by superselection rules) to couple is coupled.* On the level of particles this is called *nuclear democracy*: Any particle whose superselected charge is contained in the spectrum of fused charges of a cluster of particles can be viewed as a bound state of that cluster. This renders QFT conceptually much more attractive and fundamental than QM, but it also contributes to its computational complexity if one tries to access it using operator or functional methods from QM. Any violation of this law also violates the setting of QFT, the only known approach to particle physics which is not subject to this law and at least maintains macro-causality is the before presented quantum mechanical DPI setting. Whereas the latter has minimal projections (corresponding to optimal observations), this is not so for the local algebras which turn out to be of type III (the terminology of Murray and von Neumann); in these algebras every projection is isometrically equivalent to the largest projector which is the identity operator. Some physical consequences of this difference have been reviewed in [28].

The Reeh-Schlieder property also implies that the expectation value of a projection operator localized in a bounded region cannot be interpreted as the probability of detecting a particle-like object in that region, since it is necessarily nonzero if acting on the vacuum state. In the context of Born-localization one would refer to the uncertainty relation, but our later study reveals that the restriction of the vacuum (or any other global finite energy state) to $\mathcal{A}(\mathcal{O})$ is entangled in a much more radical sense namely it has transmuted into a KMS thermal state at a appropriately normalized (Hawking) temperature[14]. The intrinsically defined *modular "Hamiltonian"* associated via modular operator theory to standard pair $(\mathcal{A}(\mathcal{O}), \Omega_{vac})$ allows a physical interpretation only in those rare cases when it coincides with one of the global spacetime generators (e.g. the Lorentz boost for the wedge region in Minkowski spacetime, a conformal transformation for the double cone region in a conformal theory). This phenomenon has the same origin as the later discussed universal area proportionality of localization entropy (which is the entropic side of the same thermal coin associated with modular localization).

There exists in fact a whole family of modular Hamiltonians since the operators in $\mathcal{A}(\mathcal{O})$ naturally fulfill the KMS condition of any standard pair $(\mathcal{A}(\check{\mathcal{O}}), \Omega_{vac})$ for $\check{\mathcal{O}} \supset \mathcal{O}$: how the different modular thermal states physically "out themselves" depends on which larger system one wants the operators in $\mathcal{A}(\mathcal{O})$ to be associated with, i.e. it depends on who declares himself to be the observer. The system itself has no preference, it fulfills all those different KMS properties with respect to all those infinitely many different modular Hamiltonians simultaneously. In certain cases there is a preferred region and this situation

[14]The effects we are concerned with are ridiculously small and probably never mearurable, but here we are interested in principle aspects of the most successfull and fundamental theory and not in FAPP issues.

of extreme virtuality caused by vacuum polarization passes to real physics. The interesting and most prominent case comes about when spacetime curvature is creating a black hole[15]. In such a situation the fleeting "as if" aspect of a causal localization horizon changes to give room for a more real *event horizon*. For computations of thermal properties however, including thermal entropy, it does not matter whether the horizon is a fleeting causal localization horizon or a "real" curvature generated black hole event horizon. This leads to a picture about the LQP-QG interface which is somewhat different from that in most of the literature; we will return to these issues in connection with the presentation of the *split property* in the section on algebraic modular aspects.

A direct comparison with NW-localization can be made in the case of free fields which are well defined as operator valued distributions in the space variables at a fixed time. The one-particle states that are NW Born-localized in a given space region at a fixed time are not the same as the states obtained by applying field operators smeared with test functions supported in this region to the vacuum. The difference lies in the non-local energy factor $\sqrt{p^2+m^2}$ linking the non-covariant NW states with the states defined in terms of the covariant field operators. Causality in relativistic quantum field theory is mathematically expressed through local commutativity, i.e., mutual commutativity of the algebras $\mathcal{A}(\mathcal{O})$ and $\mathcal{A}(\mathcal{O}')$.

There is an intimate connection of this property with the possibility of preparing states that exhibit no mutual correlations for a given pair of causally disjoint regions. In fact, in a recent paper Buchholz and Summers [4] show that local commutativity is a necessary condition for the existence of such uncorrelated states. Conversely, in combination with some further properties (split property [30], existence of scaling limits), that are physically plausible and have been verified in models, local commutativity leads to a very satisfactory picture of statistical independence and local preparabilty of states in relativistic quantum field theory. We refer to [31][32] for thorough discussions of these matters and [28][11] for a brief review of some physical consequences. The last two papers explain how the above mentioned concepts avoids the defects of the NW localization and resolve spurious problems rooted in assumptions that are in conflict with basic principles of relativistic quantum physics. In particular it can be shown how an alleged difficulty [5][6] with Fermi's famous Gedankenexperiment [29] which Fermi proposed in order to show that the velocity of light is also the limiting propagation velocity in quantum electrodynamics can be resolved by taking [28] into account the progress on the conceptual issues of causal localization and the gain in mathematical rigor since the times of Fermi.

After having discussed some significant conceptual differences between QM and LQP, one naturally asks for an argument why and in which way QM appears as a nonrelativistic limit of LQP. The standard kinematical reasoning of the textbooks is acceptable for fermionic/bosonic systems in the sense of FAPP, but has not much strength on the conceptual level. To see its weakness, imagine for a moment that we would live in a 3-dim. world of anyons (abelian plektons, where plektons are Wigner particles with braid group statistics). Such relativistic objects are by their very statistics so tightly interwoven that there simply are no compactly localized free fields which only create a localized anyon without a vacuum polarization cloud admixture. In such a world no nonrelativistic limit

[15] Even in that case there is no difference whether one associates the localization property with the outside, inside, or with the horizon of the black hole.

which maintains the spin-statistic connection could lead to QM, the limiting theory would rather *remain a nonrelativistic QFT*, there is simply no Schrödinger equation for plektonic particle-like objects which carry the spin/statistics properties of anyons. In 4-dimensional spacetime there is no such obstacle against QM, simply because there exist relativistic free fields whose application to the vacuum generates a vacuum-polarization-free one-particle state and the spin-statistics structure does not require the permanence of polarization clouds in the nonrelativistic limit.

2.4 Modular localization

In the previous sections we mentioned on several occasions that the localization underlying QFT can be separated from the locality associated with a particular field, in other words it can be liberated from properties of special field coordinatization. This is achieved by a marvelous and really impressive theory within the setting of operator algebras which was independently discovered by mathematicians and physicists in the middle of the 60ies. It becomes especially accessible (at least for physicists) if one introduces it first in its more limited spatial- instead of its full algebraic- context. Since it merits more attention than it hitherto received from the particle physics community, I will present some of its methods and achievements.

Modular localization of single particle states is a concept that is intrinsically defined within the representation theory of the Poincaré group. It is determined by the PCT operator multiplied with a rotation (it is a reflection along the edge of a wedge) and the generator of the Lorentz boosts associated with the wedge. It has been realized in recent years by Brunetti, Guido and Longo [7] and with somewhat different motivations by myself [8], that by appealing to this interpretation of the Tomita involution for wedges and using the spatial counterpart of Tomita-Takesaki theory [33], it is possible to partially invert the above procedure of passing from local algebras $\mathcal{A}(\mathcal{O})$ to localized state vectors $H(\mathcal{O})$; namely there is a natural localization structure on the representation space for any positive energy representation of the proper Poincaré group. Upon second quantization gives rise to a local net of operator algebras on the Fock space over the representation Hilbert space.

In the context of Wigner's description of elementary relativistic systems, the starting point is an irreducible representation $U_1$of the Poincaré´group on a Hilbert space H_1 that after second quantization becomes the single-particle subspace of the Hilbert space (Fock-space) H of the field[16]. The construction then proceeds according to the following steps [7][34][11]. To maintain simplicity we limit our presentation to the bosonic situation.

One first fixes a reference wedge region, e.g. $W_0 = \{x \in \mathbb{R}^d, x^{d-1} > |x^0|\}$ and considers the one-parametric L-boost group (the hyperbolic rotation by χ in the $x^{d-1} - x^0$ plane) which leaves W_0 invariant as well as the reflection j_{W_0} across the edge of the wedge (i.e. along the coordinates $x^{d-1} - x^0$). The Wigner representation $U(a, \Lambda)$ is then used to define

$$\Delta^{it} = U(0, \Lambda_{W_0}(\chi = \text{-}2\pi t)),\ J_{W_0} = U(0, j_{W_0}) \tag{11}$$

where attention should be paid to the fact that in a positive energy representation any

[16]The construction works for arbitrary positive energy representations, not only irreducible ones.

operator which involves a time inversion is necessarily antilinear. A one- parametric subgroups of geometric origin as Δ^{it} permits an analytic continuation in t in the form of unbounded densely defined positive operators Δ^{τ}. With the help of such an unbounded operator we define the unbounded antilinear operator which has still a dense domain

$$S_{W_0} = J_{W_0}\Delta_{W_0}^{\frac{1}{2}} \tag{12}$$

Using the group theoretical geometric properties one finds that this operator has the remarkable property of being a closed operator with $S_{W_0}^2 \subset \mathbf{1}$. Such operators which are unbounded and yet involutive on their domain occur only in modular theory and are called Tomita involutions; Tomita discovered them in a vastly more general algebraic context which will be mentioned later. The idempotency means that $S_{W_0}^2$ has ± 1 eigenspaces; since S_{W_0} is antilinear the $+$-space multiplied with i changes the sign and hence it suffices to introduce a notation for just one eigenspace

$$\begin{aligned} K(W_0) &= \{domain\ of\ \Delta_{W_0}^{\frac{1}{2}}, S_{W_0}\psi = \psi\} \\ J_{W_0}K(W_0) &= K(W_0') = K(W_0)',\ duality \\ \overline{K(W_0) + iK(W_0)} &= H_1,\ K(W_0) \cap iK(W_0) = 0 \end{aligned} \tag{13}$$

It is important to be aware that, unlike QM, we are here dealing with real subspaces of the complex one-particle Wigner representation space H_1. An alternative which avoids the use of real subspaces is to directly deal with complex dense subspaces. Introducing the graph norm of the dense space S_{W_0} the dense complex subspace $K(W_0)+iK(W_0)$ becomes a Hilbert space in its own right. The second and third line require some explanation. The upper dash on regions denotes the causal disjoint (which is the opposite wedge) whereas the dash on real subspaces means the symplectic complement with respect to the symplectic form $Im(\cdot,\cdot)$ on H_1. The two properties in the third line are the defining property of what is called the standardness property of a real subspace; any standard K permits to define an abstract S

$$\begin{aligned} S(\psi + i\varphi) &= \psi - i\varphi \\ S &= J\Delta^{\frac{1}{2}} \end{aligned} \tag{14}$$

whose polar decomposition (written in the second line) yields two modular objects, a unitary modular group Δ^{it} and a antiunitary reflection which generally have however no geometric significance. The domain of S is the same as the domain of $\Delta^{\frac{1}{2}}$ namely $K + iK$ which in the context of the Wigner theory is determined by group representation theory only.

It is easy to obtain a net of $K(W)'s$ by $U(a,\Lambda)$-transforming $K(W_0)$. A bit more tricky is the construction of sharper localized subspaces via intersections

$$K(\mathcal{O}) = \bigcap_{W\supset\mathcal{O}} K(W) \tag{15}$$

This intersection may not be standard, in fact they may be zero. There are three classes of irreducible positive energy representation, the family of massive representations $(m > 0, s)$

with half-integer spin s and the family of massless representation which consists really of two subfamilies with quite different properties namely the $(0,h)$, h half-integer class (often called the neutrino, photon class), and the rather large class of $(0,\kappa>0)$ infinite helicity representations parametrized by the continuous-valued Casimir invariant κ. For the first two classes the $K(\mathcal{O})$ is standard for arbitrarily small $\mathcal{O}$ but this is definitely not the case for the infinite helicity family for which the compact localization spaces turn out to be trivial[17]. Their tightest nontrivial localization is a spacelike cone with an arbitrary small opening angle and after second quantizations (see next subsection) they lead to semi-infinite spacelike strings.

A different kind of spacelike string-localization arises in d=1+2 Wigner representations with anomalous spin [35]. The amazing power of this modular localization approach is that it preempts the spin-statistics connection in the one-particle setting, namely if s is the spin of the particle (which in d=1+2 may take on any real value) then one finds for the connection of the symplectic complement with the causal complement the generalized duality relation $K(\mathcal{O}')=ZK(\mathcal{O})'$ where the square of the twist operator $Z=e^{2\pi is}$ is easily seen (by the connection of Wigner representation theory with the two-point function) to lead to the statistics phase: $Z^2=$ statistics phase [35]. The fact that one never has to go beyond string localization (and fact, apart from those mentioned cases, never beyond point localization) in order to obtain the generating fields for a QFT is remarkable in view of the many attempts to introduce extended objects into QFT.

It should be clear that modular localization which goes with real subspaces (or dense complex subspaces) cannot be connected with probabilities and projectors. It is rather related to causal localization aspects and the standardness of $K(\mathcal{O})$ is nothing else then the one-particle version of the Reeh-Schlieder property. As will be seen in the next section it is an important tool in the non-perturbative construction of models.

2.5 Algebraic aspects of modular theory

A net of real subspaces $K(\mathcal{O})\subset H_1$ for an finite spin (helicity) Wigner representation can be "second quantized"[18] via the CCR (Weyl) respectively CAR quantization functor; in this way one obtains a covariant $\mathcal{O}$-indexed net of von Neumann algebras $\mathcal{A}(\mathcal{O})$ acting on the Fock space $H=Fock(H_1)$ built over the one-particle Wigner space H_1. For integer spin/helicity values the modular localization in Wigner space implies the identification of the symplectic complement with the geometric complement in the sense of relativistic causality, i.e. $K(\mathcal{O})'=K(\mathcal{O}')$ (spatial Haag duality). The Weyl functor takes the spatial version of Haag duality into its algebraic counterpart. One proceeds as follows: For each Wigner wave function $\varphi\in H_1$ the associated (unitary) Weyl operator is defined as

$$\begin{aligned} Weyl(\varphi) &:= expi\{a^*(\varphi)+a(\varphi)\}, Weyl(\varphi)\in B(H) \\ \mathcal{A}(\mathcal{O}) &:= \{Weyl(\varphi)|\varphi\in K(\mathcal{O})\}'', \;\; \mathcal{A}(\mathcal{O})'=\mathcal{A}(\mathcal{O}') \end{aligned} \tag{16}$$

[17] It is quite easy to prove the standardness for spacelike cone localization just from the positive energy property which is shared by all three families.

[18] 2^{nd} quantization is a misdemeanor since it is a functor and has little in common with the artful parallellism to classical theory called "quantization", or in Edward Nelson's words: (first) quantization is a mystery, but second quantization is a functor.

where $a^*(\varphi)$ and $a(\varphi)$ are the usual Fock space creation and annihilation operators of a Wigner particle in the wave function φ. We then define the von Neumann algebra corresponding to the localization region $\mathcal{O}$ in terms of the operator algebra generated by the functorial image of the modular constructed localized subspace $K(\mathcal{O})$ as in the second line. By the von Neumann double commutant theorem, our generated operator algebra is weakly closed by definition.

The functorial relation between real subspaces and von Neumann algebras via the Weyl functor preserves the causal localization structure and hence the spatial duality passes to its algebraic counterpart. The functor also commutes with the improvement of localization through intersections $\cap$ according to $K(\mathcal{O}) = \cap_{W\supset O} K(W)$, $\mathcal{A}(\mathcal{O}) = \cap_{W\supset O}\mathcal{A}(W)$ as expressed in the commuting diagram

$$\begin{array}{ccc} \{K(W)\}_W & \longrightarrow & \{\mathcal{A}(W)\}_W \\ \downarrow\cap & & \downarrow\cap \\ K(\mathcal{O}) & \longrightarrow & \mathcal{A}(\mathcal{O}) \end{array} \tag{17}$$

Here the vertical arrows denote the tightening of localization by intersection whereas the horizontal ones denote the action of the Weyl functor.

The case of half-integer spin representations is analogous [34], apart from the fact that there is a mismatch between the causal and symplectic complements that is taken care of by a *twist operator* $\mathcal{Z}$ and as a result one has to use the CAR instead of the Weyl functor. In case of the large family of irreducible zero mass infinite spin representations in which the lightlike little group is faithfully represented, the finitely localized spaces $K(\mathcal{O})$ are trivial and the *most tightly localized nontrivial spaces are* $K(\mathcal{C})$ for $\mathcal{C}$ a *spacelike cone*. As the core of arbitrarily small double cones shrinks to a point, that of arbitrarily thin spacelike cones is a *covariant spacelike semiinfinite string*. The above functorial construction works the same and the generators of these algebras are singular spacelike semiinfinite string fields. Point- (or string-) like covariant fields are singular generators of these algebras and stringlike generators, which are also available in the pointlike case, turn out to have an improved short distance behavior [11]. They can be constructed from the unique Wigner representation by so called intertwiners between the canonical and the many possible covariant (dotted-undotted spinor finite representations of the L-group) representations. The Euler-Lagrange aspect plays no role in these construction since the causal aspect of hyperbolic differential propagation are fully taken care of by modular localization.

A basis of local covariant field coordinatizations is then defined by Wick composites of the free fields. The string-like fields do not follow the classical behavior, already before introducing Wick composites one has a continuous family of intertwiners between the unique Wigner representation and the continuously many covariant string interwiners. Their non-classical aspects are the reason why they have been discovered only recently and not at the time of Jordan's field quantization.

In order to avoid confusion with different usage of the same mathematical symbol, let us temporarily change our notation and write the one-particle operators with small letters as δ, j, s, serving the capital letters for the second quantized objects Δ, J, S. Using the standard notation Γ for the second quantization functor one obtains the Tomita Takesaki theory for the local algebras of the interaction-free local algebra $(\mathcal{A}(\mathcal{O}), \Omega)$ in standard position[19]

$$H_{Fock} = \Gamma(H_1) = e^{H_1},\ \left(e^h, e^k\right) = e^{(h,k)} \quad (18)$$
$$\Delta = \Gamma(\delta),\ J = \Gamma(j),\ S = \Gamma(s)$$
$$SA\Omega = A^*\Omega,\ A \in \mathcal{A}(O),\ S = J\Delta^{\frac{1}{2}}$$

With this we are getting to the core statement of the Tomita-Takesaki theorem which is a statement about the two modular objects Δ^{it} and J on the algebra

$$\sigma_t(\mathcal{A}(\mathcal{O})) \equiv \Delta^{it}\mathcal{A}(\mathcal{O})\Delta^{-it} = \mathcal{A}(\mathcal{O}) \quad (19)$$
$$J\mathcal{A}(\mathcal{O})J = \mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}')$$

in words: the reflection J maps an algebra (in standard position) into its von Neumann commutant and the unitary group Δ^{it} defines an one-parametric automorphism-group σ_t of the algebra. In this form (but without the last geometric statement involving the geometrical causal complement $\mathcal{O}'$) the theorem hold in complete mathematical generality for standard pairs $(\mathcal{A}, \Omega)$. The free fields and their Wick composites are ""coordinatizing" singular generators of this $\mathcal{O}$-indexed net of algebras in the sense that the smeared fields $A(f)$ with $suppf \subset \mathcal{O}$ are (unbounded operators) affiliated with $\mathcal{A}(\mathcal{O})$.

In the above second quantization context the origin of the T-T theorem and its proof is clear: the symplectic disjoint passes via the functorial operation to the operator algebra commutant and the spatial one-particle automorphism goes into its algebraic counterpart. The definition of the Tomita involution S through its action on the dense set of states (guarantied by the standardness of $\mathcal{A}$) as $SA\Omega = A^*\Omega$ and the action of the two modular objects Δ, J (18) is part of the general setting of the modular T-T theory; standardness is the mathematical terminology for the Reeh-Schlieder property i.e. the existence[20] of a vector $\Omega \in H$ with respect to which the algebra acts cyclic and has no "annihilators" of Ω. Naturally the proof of the abstract T-T theorem in the general setting of operator algebras is more involved.

The important property which renders this useful beyond free fields as a new constructive tool in the presence of interactions, is that for $(\mathcal{A}(W), \Omega)$ the antiunitary involution J depends on the interaction, whereas Δ^{it} continues to be uniquely fixed by the representation of the Poincaré group i.e. by the particle content. In fact it has been known for some [8] time that J is related via scattering theory to the S-matrix with its free counterpart J_0

$$J = J_0 S_{scat} \quad (20)$$

The physically relevant facts emerging from modular theory can be compressed into the following statements[21]

---

[19]The functor Γ preserves the standardness which thus passes from one-particle to the Fock space.

[20]In QFT any finite energy vector (which of course includes the vacuum) has this property as well as any nondegenerated KMS state. In the mathematical setting it is shown that standard vectors are "$\delta-$dense" in H.

[21]Alain Connes would like to see a third spatial decomposition in that list namely the decomposition of K into a certain positive cone and its opposite. With such a requirement one could obtain the entire *algebra strucure from that of states.* This construction has been highly useful in Connes classification of von Neumann algebras, but it has not been possible to relate this with physical concepts.

- *The domain of the unbounded operators $S(\mathcal{O})$ is fixed in terms of intersections of the wedge domains associated to $S(W)$; in other words it is determined by the particle content alone and therefore of what one usually calls of a kinematical nature. These dense domains change with O i.e. the dense set of localized states has a bundle structure.*

- *The complex domains $DomS(\mathcal{O}) = K(\mathcal{O}) + iK(\mathcal{O})$ decompose into real subspaces $K(\mathcal{O}) = \mathcal{A}(\mathcal{O})^{sa}\Omega$. This decomposition contains dynamical information which in case $\mathcal{O} = W$ reduces to the S-matrix (20). Assuming the validity of the crossing properties for formfactors, the S-matix fixes $\mathcal{A}(W)$ uniquely [9].*

The remainder of this subsection contains some comments about a remarkable constructive success of these modular methods. For this we need some additional terminology. Let us enlarge the algebraic setting by admitting unbounded operators with Wightman domains which are affiliated to $\mathcal{A}(\mathcal{O})$ and just take about "$\mathcal{O}$-localized operators" when we do not want to distinguish between bounded and affiliated unbounded operators. We call an $\mathcal{O}$-localized operators a vacuum **p**olarization **f**ree **g**enerator (PFG) if applied to the vacuum it generated a one particle state without vacuum-polarization cloud admixture. The the following two theorems have turned out to be useful in a constructive approach based on modular theory.

Theorem: *The existence of an $\mathcal{O}$-localized PFG for a subwedge causally complete $\mathcal{O} \subset W$ implies the freeness of the theory.*

Theorem: *Modular theory for wedge algebras insures the existence of PFGs even in the presence of interactions (at least if one relaxes the standard domain requirements for FPGs). Hence the wedge region permits the best compromise between interacting fields and one-particle states.*

Theorem: *Wedge localized PFGs with Wightman-like domain properties ("tempered" PFGs) lead to the absence of particle creation (pure elasic S_{scat}) which is only possible in d=1+1 and leads to the factorizing models (which hitherto were studied in the setting of the bootstrap-formfactor program). The compact localized subalgebra $\mathcal{A}(\mathcal{O})$ have no PFGs and possess the full interaction-induced vacuum polarization clouds.*

Some comments will be helpful. The first theorem gives an intrinsic (not dependent on any Lagrangian or other extraneous properties) local definition of the presence of interaction although it is not capable to differentiate between different kind of interactions (which would be reflected in the shapes of interaction-induced polarization clouds). The other two theorems suggest that the knowledge of the wedge algebra $\mathcal{A}(W) \subset B(H)$ may serve as a useful starting point for classifying and constructing models of LQP in a completely intrinsic fashion[22].

Such a program is well underway in the context of context of the factorizing models in the third theorem. Tempered PFGs which generate wedge algebra for factorizing have a rather simple algebraic structure. Their Fourier transforms (rewritten in terms of momentum space rapidities) $\tilde{Z}(\theta), \tilde{Z}^*(\theta)$ obey the Zamolodchikov-Faddeev commutation relations. Vice versa the formal Z-F computational device for the first time received a profound spacetime interpretation. Conceptualwise they are somewhere between Heisenberg-

[22] In particular the above commuting square remains valid in the presence of interactions if one changes $\mathcal{O} \to W$.

and incoming- fields. The simplicity of the wedge generators enable the computation of a spacetime double cone $\mathcal{D}$ affiliated space of operators. In contrast to the standard formalism of QFT this sharpening of localization from W to $\mathcal{D}$ is done by intersecting two W operator spaces. The resulting operator space is of the form

$$A(x) = \sum \frac{1}{n!} \int d\theta_1 ... \int d\theta_n e^{-ix\sum p(\theta_i)} a(\theta_1, ...\theta_n) \tilde{Z}(\theta_1)...\tilde{Z}(\theta_1) \tag{21}$$

where for the purpose of a compact notation we view the creation part $\tilde{Z}^*(\theta)$ is written as the $\tilde{Z}(\theta + i\pi)$ i.e. as the Z on the upper boundary of a strip[23]. This is similar to the old Glaser-Lehmann-Zimmermann representation for the interacting Heisenberg field [49] in terms of incoming free field (in which case the spacetime dependent coefficient functions turn out to be on-shell restrictions of Fourier transforms of retarded functions), except that instead of the on-shell incoming fields one takes the on-shell Z operators and the coefficient functions are the (connected part of the) multiparticle formfactors. As was the case with the GLZ series, the convergence of the formfactor series has turned out to be an intractable problem and like many other series in QFT (e.g. the perturbation series) and one would be well-advised to be prepared for the worst i.e. the divergence of the series.

The main property one has to establish if one's aim is to secure the existence of a QFT with local observables, is the standardness of the double cone intersection $\mathcal{A}(\mathcal{D}) = \cap_{W\supset\mathcal{D}}\mathcal{A}(W)$. Based on nuclearity properties of degrees of freedom in phasespace discovered by Buchholz and Wichmann [38], Lechner has found a method within the modular operator setting of factorizing models which achieves precisely this [39]. For the first time in the history of QFT one now has a construction method which goes beyond the Hamiltonian- and measure theoretical approach of the 60s [36]. The old approach could only deal with superrenormalizable models i.e. models whose basic fields did not have a short distance dimension beyond that of a free field.

At this point it is instructive to recall that QFT not only has been the most successful of all physical theories, but in comparison to all other theories in the pantheon of theoretical physics also the most shaky concerning its conceptual and mathematical foundations. Looking at the present sociological situation it seems that the past success in form of the standard model has generated an amnesia about this problem. But this issue is not going away, it is particularly visible in the fact that the perturbative series are divergent; even in those cases where one was able to establish Borel summability one knows nothing about the status of the theory without a priori knowledge about its existence. The great achievement for factorizing models is that one does not only know that these models exist as QFT, but one also has the explicit form of their S-matrix and formfactors; knowing that their on-shell observables are analytic around zero coupling it would therefore be very interesting to study the convergence/resummability status of the off-shell correlation functions.

The very existence of these theories, whose fields have anomalous trans-canonical short distance dimensions with interaction-dependent strengths, shows that there is nothing intrinsic about the ultraviolet problems posing an impediment; they are simply the

[23]The notation is suggested by the the strip analyticity coming from wedge localization. Of course only certain matrix elements and expectation values, but not field operators or their Fourier transforms, can be analytic; therefore the notation is symbolic.

unavoidable price to pay if one enters QFT via the classical quantization parallelism i.e. the standard approach which worked so well for passing from mechanics to quantum mechanics, but needs a lot of repair[24] (infinite "renormalization") if one ignores the very singular nature of quantum fields. That the problem-creating singular behavior of fields may be a description-dependent aspect had already been suspected by the protagonist of QFT Pascual Jordan who, as far back as 1929, pleaded for a formulation "without classic crutches" [40]. The fact that in the above construction of factorizing models one finds that for most of them there is not even a Lagrangian name illustrated the seriousness of Jordan's plea.

Since modular theory continues to play an important role in the physical results of the two remaining sections, I should be very careful in avoiding potential misunderstandings. It is very crucial to be aware of the fact that by restricting the global vacuum state to, a say double cone algebra $\mathcal{A}(\mathcal{D})$, there is no change in the values of the global vacuum expectation values

$$(\Omega_{vac}, A\Omega_{vac}) = (\Omega_{\mathrm{mod},\beta}, A\Omega_{\mathrm{mod},\beta}), \quad A \in \mathcal{A}(\mathcal{D}) \tag{22}$$

where for the standard normalization of the modular Hamiltonian[25] $\beta = 1$. This right hand side means that the vacuum expectation values, if restricted to $A \in \mathcal{A}(\mathcal{D})$, fulfill an addicional property which without the restriction to the local algebra would not hold, namely the KMS relation

$$(\Omega_{\mathrm{mod},\beta}, AB\Omega_{\mathrm{mod},\beta}) = \left(\Omega_{\mathrm{mod},\beta}, B\Delta_{\mathcal{A}(\mathcal{O})}A\Omega_{\mathrm{mod},\beta}\right) \tag{23}$$

which says that there exists a modular Hamiltonian H_{mod} with $\Delta^{-H_{\mathrm{mod}}}$, which is different from the standard translative heat bath Hamiltonian, for which the restricted vacuum is a thermal equilibrium state at a certain temperature (by analogy the Hawking temperature) in the setting of the second law of thermodynamics. In fact there is a *continuous family of modular "Hamiltonians"* which are the generators the modular unitaries $\Delta^{it}_{\mathcal{A}(\mathcal{O})}$ for $M \supset \mathcal{O} \supset \mathcal{D}$ and hence the same vacuum expectation values have to satisfy a *continuous family of KMS boundary relations.* With all this mathematical restrictions being placed via operator localization onto an innocent looking vacuum expectations value this is an extremely surprising dynamical feature which goes much beyond the kinematical change of entanglement as the result of the quantum mechanical division into measured system and environment. It is this enormous coupling of QFT to the way it is being observed which makes it apparently very far removed from what one associates with the persistency properties of a material substance. The monad description in the next section even strengthens this little known aspect of LQP.

In both cases QM as well as QFT the entanglement comes about by a different ways of looking at the system and not by changing intrinsic properties, but the *thermal entanglement* of QFT is much more spectacular than the *"cold" (information-theoretic) kind of entanglement* of QM. As we have seen the thermal aspects of vacuum expectations restricted to a fixed subalgebra is a mathematically incredibly rich object with fulfills KMS

[24]There are of course also more refined methods which respect the singular nature of the fields throughout [37].

[25]The modular Hamiltonian lead to fuzzy motions within $\mathcal{A}(\mathcal{O})$ except in case of $\mathcal{O} = W$ when the modular Hamiltonian is identical to the boost generator.

relations with respect to continuous families of modular Hamiltonians. The ontic content of these observations is quite weak; it is only when the (imagined) causal localization horizons pass to (real) event horizons through the curvature of spacetime that observers with a preferential status to the horizon emerge. Not caring about these conceptual aspects and only following a "shut up and compute" attitude[26] one may easily be drawn into a fruitless and protractive arguments as it happened (and is still happening) with the entropy/information loss issue. These problems are connected to an insufficient conceptual understanding of QFT (identifying it with some sort of relativistic QM), the role of gravity is that of a mental catalyzer only to place them into the forefront of thinking.

2.6 Building up LQP via *positioning of monads* in a Hilbert space

We have seen in the previous section that modular localization of states and algebras is an intrinsic i.e. field coordinatization independent way to formulate the kind of localization which is characteristic for QFT. It is deeply satisfying that it also has an amazing constructive power.

Definition: *(Wiesbrock, Borchers) An inclusion of standard operator algebras* $(\mathcal{A} \subset \mathcal{B}, \Omega)$ *is "modular" if* $(\mathcal{A}, \Omega)$ *and* $(\mathcal{B},\Omega)$ *are standard and* $\Delta_{\mathcal{B}}^{it}$ *acts like a compression on* $\mathcal{A}$ *i.e.* $Ad\Delta_{\mathcal{B}}^{it}\mathcal{A} \subset \mathcal{A}$*. A modular inclusion is said to be standard if in addition the relative commutant* $(\mathcal{A}' \cap \mathcal{B}, \Omega)$ *is standard. If this holds for* $t < 0$ *one speaks about a -modular inclusion.*

Modular inclusions are very different from the better known Vaughn Jones inclusions and those associated with the Doplicher-Haag-Roberts theory which characterize internal symmetries in quantum field theory. The main difference is that the characteristic property of the latter is the existence of conditional expectations which modular inclusions cannot have. The prototype of a conditional expectation in the conventional formulation of QFT which uses charge-carrying fields is the averaging over the compact internal symmetry group with its normalized Haar measure ($U(g)$ denotes the representation of the internal symmetry group)

$$\mathcal{A} = \int d\mu(g) AdU(g)\mathcal{F} \tag{24}$$
$$E : \mathcal{F} \xrightarrow{\mu} \mathcal{A}$$

i.e. the conditional expectation E projects the (charged) field algebra $\mathcal{F}$ onto the (neutral) observable algebra $\mathcal{A}$ and such inclusions which do not change the localization are therefore related to internal symmetries as opposed to spacetime symmetries. Whenever an inclusion $\mathcal{A} \subset \mathcal{B}$ has a conditional expectation E it cannot be modular. This is the consequence of a theorem of Takesaki which states that the existence of a conditional expectation between two algebras (in standard position with respect to the same vector) is equivalent to the modular group of the smaller being the restriction of that of the bigger.

[26]With such an attitude one could have assigned a parallel world for each Lorentz frame and lived happily with Lorentz frames and without Einstein's relativity principle.

Since in the above case of a genuine compression the modular group of the smaller cannot result by restriction from the bigger, there can be no E.

The notion of *modular inclusion* may be considered a generalization of the situation covered by the Takesaki theorem. The main aim is to *generate spacetime symmetry* as well as the *net of spacetime indexed algebras* which are covariant under these symmetries. This is done as follows: from the two modular groups $\Delta_{\mathcal{B}}^{it}, \Delta_{\mathcal{A}}^{it}$ one can form a unitary group $U(a)$ which together with the modular unitary group of the smaller algebra $\Delta_{\mathcal{B}}^{it}$ leads to the commutation relation $\Delta_{\mathcal{B}}^{it}U(a) = U(e^{-2\pi t}a)\Delta_{\mathcal{B}}^{it}$ which characterizes the 2-parametric translation-dilation (Anosov) group. One also obtains a system of local algebras by applying these symmetries to the relative commutant $\mathcal{A}' \cap \mathcal{B}$. From these relative commutants one may form a new algebra $\mathcal{C}$

$$\mathcal{C} \equiv \overline{\bigcup_t Ad\Delta_{\mathcal{B}}^{it}(\mathcal{A}' \cap \mathcal{B})} \tag{25}$$

In general $\mathcal{C} \subset \mathcal{B}$ and we are in a situation of a nontrivial inclusion to which the Takesaki theorem is applicable (the modular group of $\mathcal{C}$ is the restriction of that of $\mathcal{B}$) which leads to a conditional expectation $E : \mathcal{B} \rightarrow \mathcal{C}$ but of course $\mathcal{C}$ may be trivial. The most interesting situation arises if the modular inclusion is *standard*, in that case we arrive at a chiral QFT.

Theorem: (Guido,Longo and Wiesbrock [56]) *Standard modular inclusions are in one-to-one correspondence with strongly additive chiral LQP.*

Here chiral LQP is a net of local algebras indexed by the intervals on a line with a Moebius-invariant vacuum vector and *strongly additive* refers to the fact that the removal of a point from an interval does not "damage" the algebra i.e. the von Neumann algebra generated by the two pieces is still the original algebra. One can show via a dualization process that there is a unique association of a chiral net on $S^1 = \dot{\mathbb{R}}$ to a strongly additive net on $\mathbb{R}$. Although in our definition of modular inclusion we have not said anything about the nature of the von Neumann algebras, it turns out that the very requirement of the inclusion being modular forces both algebras to be hyperfinite type III_1 factor algebras. The closeness to Leibniz's image of (physical) reality of originating from relations between monades (with each monade in isolation of being void of individual attributes) more than justifies our choice of name; besides that "monade" is much shorter than the somewhat long winded mathematical terminology "hyperfinite type III_1 Murray-von Neumann factor algebra". The nice aspect of chiral models is that one can pass between the operator algebra formulation and the construction of pointlike fields without having to make additional technical assumptions[27]. Another interesting constructive aspect is that the operator-algebraic setting permits to establish the existence of algebraic nets in the sense of LQP for all $c < 1$ representations of the energy-momentum tensor algebra. This is much more than the vertex algebra approach is able to do since that formal power series approach is blind against the dense domains which change with the localization regions.

The idea of placing the monade into modular positions within a common Hilbert space

[27]The group theoretic arguments which go into that theorem seem to be available for any conformal QFT.

may be generalized to more than two copies. For this purpose it is convenient to define the concept of a *modular intersection* in terms of modular inclusion.

Definition (Wiesbrock [41]): *Consider two monades A and B positioned in such a way that their intersection $A \cap B$ together with A and B are in standard position with respect to the vector $\Omega \in H$. Assume furthermore*

$$\begin{aligned} &(\mathcal{A}\cap\mathcal{B} \subset\mathcal{A}) \text{ and } (\mathcal{A}\cap\mathcal{B}\subset\mathcal{B}) \text{ are } \pm mi \\ &J_{\mathcal{A}} \lim_{t\to\mp} \Delta_{\mathcal{A}}^{it}\Delta_{\mathcal{B}}^{-it} J_{\mathcal{A}} = \lim_{t\to\mp} \Delta_{\mathcal{B}}^{it}\Delta_{\mathcal{A}}^{-it} \end{aligned} \tag{26}$$

then (A, B, Ω) is said to have the $\pm$ modular intersection property ($\pm$ mi).

It can be shown that this property is stable under taking commutants i.e. if $(\mathcal{A}, \mathcal{B}, \Omega)\pm$ mi then $(\mathcal{A}', \mathcal{B}', \Omega)$ is $\mp mi$.

The minimal number of monads needed to characterize a 2+1 dimensional QFT through their modular positioning in a joint Hilbert space is three. The relevant theorem is as follows

Theorem: (Wiesbrock [42]) *Let A_{12},A_{13} and A_{23} be three monades*[28] *which have the standardness property with respect to $\Omega \in H$. Assume furthermore that*

$$\begin{aligned} &(\mathcal{A}_{12}, \mathcal{A}_{13}, \Omega) \text{ is } -mi \\ &(\mathcal{A}_{23}, \mathcal{A}_{13}, \Omega) \text{ is } +mi \\ &(\mathcal{A}_{23}, \mathcal{A}_{12}', \Omega) \text{ is } -mi \end{aligned} \tag{27}$$

then the modular groups Δ_{12}^{it}, Δ_{13}^{it} and Δ_{23}^{it} generate the Lorentz group $SO(2,1)$.

Extending this setting by placing an additional monade $\mathcal{B}$ into a suitable position with respect to the $\mathcal{A}_{ik}$ of the theorem, one arrives at the Poincaré group $\mathcal{P}(2,1)$ [43]. The action of this Poincaré group on the four monads generates a spacetime indexed net i.e. a LQP model and all LQP have a monad presentation.

To arrive at d=3+1 LQP one needs 6 monads. The number of monads increases with the spacetime dimensions. Whereas in low spacetime dimensions the algebraic positioning is natural within the logic of modular inclusions, in higher dimensions it is presently necessary to take some additional guidance from geometry, since the number of possible modular arrangements for more than 3 monads increases.

We have presented these mathematical results and used a terminology in such a way that the relation to Leibniz philosophical view is highly visible.

Since this is not the place to give a comprehensive account but only to direct the attention of the reader to this (in my view) startling conceptual development in the heart of QFT.

Besides the radically different conceptual-philosophical outlook on what constitutes QFT, the modular setting offers new methods of construction. It turns out that for that purpose it is more convenient to start from one monad $\mathcal{A} \subset B(H)$ and assume that one knows the action of the Poincaré group via unitaries $U(a, \Lambda)$ on $\mathcal{A}$. If one interprets the monad $\mathcal{A}$ as a wedge algebra $\mathcal{A} =$ than the Poincaré action generates a net of wedge

[28] As in the case of a modular inclusion, the monad property is a consequence of the modular setting. But for the presentation it is more convenient and elegant to talk about monads from the start.

algebras $\{\mathcal{A}(W)\}_{W\in\mathcal{W}}$. A QFT is supposed to have local observables and if the double cone intersections[29] $\mathcal{A}(D)$ turn out to be trivial (multiples of the identity algebra) the net of wedge algebras does not leads to a QFT. This is comparable to the non-existence of a QFT which was to be associated via quantization to a Lagrangian. If however these intersections are nontrivial than the ontological status is much better than that we would have an existence proof which is much more than a non-converging renormalized perturbative series of which we do not know if and how it is related to a QFT. There are of course two obvious sticking points: (1) to find Poincaré-covariant generators of $\mathcal{A}(W_0)$ and (2) a method which establishes the non-triviality of intersections of wedge algebras and leads to formulas for their generating elements.

As was explained in the previous section, both problems have been solved within a class of factorizing models. Nothing is known about how to address these two points in the more general setting i.e. when the tempered PFG are not available. Perhaps one should first test a perturbative version of this program which is expected to incorporate more possibilities than the perturbation theory based on pointlike fields since wedge-localized generators are free of those ultraviolet aspects which come from pointlike localization. The dynamical input in that case would not be a Lagrangian but rather the lowest order (tree-approximation) S-matrix interpreted as the in-out formfactor of the identity operator.

There is one property of LQP which is indispensable for understanding how the quantum mechanical tensor factorization can be reconciled with modular localization: the *split property.*

Definition: *Two monads A, B are in a split position if the inclusion of monads $A \subset B'$ admits an intermediate type I factor N such that $A \subset N \subset B'$*

Split inclusions are very different from modular inclusions or inclusions with conditional expectations (Jones-DHR). The main property of a split inclusion is the existence an $\mathcal{N}$-dependent unitarily implemented isomorphism of the $\mathcal{A}, \mathcal{B}$ generated operator algebra into the tensor product algebra

$$\mathcal{A} \vee \mathcal{B} \to \mathcal{A} \otimes \mathcal{B} \subset \mathcal{N} \otimes \mathcal{N}' = B(H) \tag{28}$$

The prerequisite for this factorization in the LQP context is that the monads commute, but it is well-known that local commutativity is not sufficient, the counterexample being two double cones which touch each other at a spacelike boundary [3]. As soon as one localization region is separated from the other by a (arbitrary small) spacelike security distance, the interaction-free net satisfies the split property under very general conditions. In [44] the relevant physical property was identified in form of a phase space property. Unlike QM, the number of degrees of freedom in a finite phase space volume in QFT is not finite, but its infinity is quite mild; it is a nuclear set for free theories and this nuclearity requirement[30] is then postulated for interacting theories. The physical reason behind this nuclearity requirement is that it allows to show the existence of temperature states once one knows that a QFT exists in the vacuum representation.

The split property for two securely causally separated algebras has a nice physical interpretation. Let $\mathcal{A} = \mathcal{A}(\mathcal{O}),\ \mathcal{B}' = \mathcal{A}(\check{\mathcal{O}}),\ \mathcal{O} \subset \check{\mathcal{O}}$. Since $\mathcal{N}$ contains $\mathcal{A}$ and is contained

[29]Double cones are the typical causally complete compact regions which can be obtained by intersecting wedges.

[30]A set of vectors is nuclear if it is contained in the range of a trace class operator.

in $\mathcal{B}$ (but without carrying the assignment of a localization between $\mathcal{O}$ and $\check{\mathcal{O}}$), one may imagine $\mathcal{N}$ as an algebra which shares the sharp localization with $\mathcal{A}(\mathcal{O})$ in $\mathcal{O}$, but its localization in the "collar" between $\mathcal{O}$ and $\check{\mathcal{O}}$ is "fuzzy" i.e. the collar subalgebra is like a "fog" which does not really occupy the collar region. This is precisely the region which is conceded to the vacuum polarization cloud in order to spread and thus avoid the infinite compression into the surface of a sharply localized monad. If we take a sequence of $\mathcal{N}$'s which approach the monad $\mathcal{A}$ the vacuum polarization clouds become infinitely large so that no direct definition of e.g. their energy or entropy is possible.

The inclusion of the tensor algebra of monads into a type I tensor product (28) looks at first sight like a déjà vu of QM tensor factorization, but there are interesting and important differences. Contrary to naive expectation the finite energy states (this includes the vacuum and particle states i.e. all states for which the Reeh-Schlieder theorem applies) of the global theory are thermalized upon restriction to $\mathcal{N}$, i.e. one finds the same KMS situation as in (23) which of course never happens in QM. Since KMS states on type I factors are Gibbs states, there exists a density matrix of the Gibbs form with a "Hamiltonian". But there is more to it; whereas for monads the modular Hamiltonian has continuous spectrum (a typical example is the Hamiltonian in the thermodynamic limit representation) and hence an ill-defined (infinite) value of energy and entropy, this is not the case for the $\mathcal{N}$-associated density matrix. So the way out is obvious: just imitates the thermodynamic limit in which a monad (the limiting KMS equilibrium situation in the standard heat bath setting) is approximated by a sequence of finite volume Gibbs states for which energy and entropy are finite and only diverge in the "monad limit". Indeed this will be the main idea or the derivation of the entropical area law in the next section.

In the above form the monad-positioning aims at characterizing LQP in Minkowski spacetime. This begs the question whether there is a generalization to curved spacetime. A very special exploratory attempt in this direction would be to investigate whether the Diff(S^1) symmetries beyond the Moebius group in chiral theories have a modular origin in terms of positioning monads relative to reference states. Since the extended chiral theories which originate from null-surface holography (and not from chiral projections of a two-dimensional conformal QFT) seem to have great constructive potential, this question may also be of practical interest. There are indications that this can be done if one relaxes on the idea of a universal vacuum reference state and allows "partial vacua" i.e. modular defined states which have geometric properties only on certain subalgebras (work in progress).

I expect that by pursuing the algebraization of QFT in CST via the positioning of monads to its limits one will learn important lessons about the true QFT/QG interface. A conservative approach which explores unknown aspects of QFT while staying firmly rooted in known principles seems to be the most promising path in the present situation.

3 Problematization of the QFT-QG interface

In the previous section we outlined a radical new way of interpreting the conceptual content of QFT which at the same time is conservative vis-à-vis the underlying physical principles and in certain cases (of simple vacuum polarization properties of wedge generators resulting in factorizing models) leads to fully intrinsic constructions of models in which the umbilical quantization cord with classical physics has been cut. We also indicated how the positioning of monads could be useful for a better future understanding of the interface between QFT in CST and QG. In this section more light will be shed on the thermal manifestations of causal localization. In particular two new results about presently hotly debated topics will be presented namely the *universal area law of localization entropy,* which shifts[31] the interface between LQP and the elusive QG, and an intrinsic definition of the *energy density in cosmological reference states* (vacuum-like states in cosmological models) in the setting of QFT in CST. This definition respects the local covariance principle and stands in contrast to estimates which were computed via level occupation within relativistic QM with a cut-off at the mass m and in this way led to unreasonably large cut-off dependent values of the cosmological "vacuum" energy proportional to m^4.

3.1 The universal area proportionality of localization entropy and consequences for the elusive QG-LQP interface

The fact that the vacuum state restricted to a local algebra $\mathcal{A}(\mathcal{O})$ becomes a KMS state at a fixed modular temperature with respect to the modular Hamiltonian of the standard pair $(\mathcal{A}(\mathcal{O}),\Omega)$ suggests to ask for the associated entropy. It turns out that localization entropy for conceptual reasons is hard to calculate directly in the bulk; it is easier to do this in its *holographic projection.*

The simplest situation for explanatory purpose is the bulk quantum matter contained in a wedge $W=\{|x_1|>x_0, x_\perp\in\mathbb{R}^2\}$ with half the lightfront as its (upper) horizon $H(W)=\{x_-=0, x_+>0,\ x_\perp\in\mathbb{R}^2\}\equiv\partial W\subset LF(W)$ where the lightfront $LF(W)$ results from the linear extension of ∂W. The net of all wedges arise from this particular one by the application of Poincaré transformations. In classical causal wave propagation a spatial regions $R\subset\mathbb{R}^3$ at a fixed time casts a *causal shadow* in the sense that data in R determine the amplitudes in the causal shadow R'' (the causal completion which is obtained by taking twice the causal complement in Minkowski spacetime) which is a double cone region subtended from R. Subregions in the horizon (i.e. partial characteristic data) cast in general no causal shadow except when they are of the semiinfinite form

$$H_a(W)=\left\{x_-=0, x_+>a>0,\ x_\perp\in\mathbb{R}^2\right\}\subset H(W)\subset LF(W) \tag{29}$$

i.e. if they are two-sided transverse and (at least) one-sided longitudinally (lightray direction) extended.

[31]This applies only to people who thought that one needs QG in order to understand the area law of black hole entropy.

The determination of a classical wave in the bulk region W from characteristic data on ∂W is a well-known problem in classical hyperbolic wave propagation[32]; it holds for the massive as well as massless classical fields, the only exception is a massless d=1+1 propagation where one needs characteristic data on both light rays (the classical analog of the conformal decomposition into two chiral components). The LQP counterpart of this property is the equality of operator algebras

$$\mathcal{A}(\partial W) = \mathcal{A}(W) \tag{30}$$

which is an established fact [55] for free fields and an "axiomatic" imposition (extended "causal shadow property") on LQP. To be sure, one finds mathematical counter examples in the form of generalized free fields [45], but they have too many degrees of freedom in order to be physically acceptable.

It is important to realize that, unlike the Cauchy propagation from a spatial region R, there is no direct relation between characteristic data on finite subregions $\mathcal{R} \subset \partial W$ on the horizon and data in the bulk (no "causal shadow"). Such a subalgebra, viewed in the spacetime ordering which is natural to the bulk, spreads over part of W in an algebraic sense which has no counterpart in the classical theory (it becomes "fuzzy").

As it stands the relation (30) contains no information about the local substructures; holography is the art of obtaining the net structure $\{\mathcal{A}(\mathcal{R})\}_{\mathcal{R}\subset LF}$ from the net structure of the bulk. The direct approach using the global type I algebras with $\mathcal{A}(LF) = \mathcal{A}(\mathbb{R}^4) = B(H)$ does not work, we need the *standardness* property of the monads with respect to the vacuum (euphemistically referred to as the *state-operator relation*[33]) in order to get a hold on the modular localization structure. It will turn out that we need all those $W's$ whose (upper) horizon lies on a fixed lightfront $\partial W \subset LF$. Anticipating the main result of algebraic lightfront holography we have

Theorem: *([47][48]) The holographic net structure $\{\mathcal{A}(\mathcal{R})\}_{\mathcal{R}\subset LF}$ with its 7+1 parametric symmetry group is obtained from the subnet of W-algebras in the bulk with $\partial W \subset LF$ with the help of forming intersections, unions and applying the 7-parametric subgroup of the W-subnet consisting of lightlike translations, W-boosts, a 3-parametric transverse symmetry and the "translations" of the Wigner little group of the lightray in LF. The 8^{th} symmetry is the chiral "rotation" which together with the lightray translation and the dilation (alias boost) represents the Moebius group and expresses the symmetry gain through holography.*

The construction proceeds in several steps. One first constructs the local net in lightray direction i.e. algebras $\mathcal{A}(I_{a,b} \times R^2)$ whose longitudinal (lightray) localization region is the interval $[a, b]$, $a, b > 0$ and whose transverse localization is as yet unresolved. This is done by intersecting LF affiliated wedge algebras. Let W again be the $x_0 - x_3$ wedge in Minkowski spacetime which is left invariant by the $x_0 - x_3$ Lorentz-boosts. Consider a family of wedges W_a which are obtained by sliding the W along the $x_+ = x_0 + x_3$ lightray

[32] Note that the connection between characteristic data on ∂W and those in the W-bulk is non-local i.e. the situation is very different from that of the Cauchy propagation from a spatial initial data where a support restriction of initial data results in a causal shadow restriction.

[33] This terminology surpresses the important fact that their are domain properties involved which change together with the localization region. The terminology among mathematical physicists is "the Reeh-Schlieder property".

by a lightlike translation $a > 0$ into itself. The set of spacetime points on LF consisting of those points on ∂W_a which are spacelike to the interior of W_b for $b > a$ is denoted by $\partial W_{a,b}$; it consists of points $x_+ \in (a,b)$ with an unlimited transverse part $x_\perp \in R^2$. These regions are two-sided transverse "slabs" on LF and their algebras are obtained from relative commutants i.e. from intersections

$$\mathcal{A}(\partial W_{a,b}) = \mathcal{A}(W_a) \cap \mathcal{A}(W_b)' \tag{31}$$

They have no spacetime interpretation in the bulk, rather they belong to the holographically LF projection. To get to intersections of finite transverse size one may "tilt" these slabs by the action of a two-parametric subgroup $\mathcal{G}_2$ of the 3-parametric Lorentz subgroup which leave the lightray invariant; this 3 parametric subgroup appears in the literature under the name "Wigner's little group" and the tilting is done with its two parametric abelian subgroup of "translations". Together with the 3 transverse symmetry the symmetry of the LF is a 7-parametric subgroup $\mathcal{G} \subset \mathcal{P}$ of the 10-parametric Poincaré group. It is easy to see that by taking intersection of ∂W_{ab}-localized algebras with their $\mathcal{G}_2$ transformed counterpart

$$\mathcal{A}(\partial W_{a,b}) \cap \mathcal{A}(g(\partial W_{a,b})),\ g \in \mathcal{G}_2 \subset \mathcal{G} \tag{32}$$

one is led to algebras with a finite transverse extension. By repeated application of intersections and LF transformations one may arrive at rather general shapes of compact regions. Note the LF is a manifold in which the concept of causal completion trivializes. By continuing with forming intersections and unions, one can get to finite convex regions $\mathcal{O}$ of a quite general shape.

An alternative method for obtaining holographically projected compactly localized subalgebras $\mathcal{A}(\mathcal{O}), \mathcal{O} \subset LF$ which does not make use of transverse symmetries consists in intersecting $\mathcal{A}(\partial W_a)$ with suitable algebras in the bulk which are localized in a tubular neighborhood of $\mathcal{O}$ [55]. This is especially useful for null-surfaces in curved spacetime.

The nontrivial question is now whether this geometrically guided approach can really be backed up by the construction of a nontrivial net of operator algebras which are indexed by those regions. Since a subregion on ∂W which either does not extend to infinity in the x_+ lightray direction or lacks the two-sided transverse extension also does not cast any causal shadow[34], one cannot base the nontriviality of algebras $A(\partial W_{a,b})$ on the causal shadow property. If this algebra would be trivial (i.e. consist of multiples of the identity), the motivation for the use of holographic projections as a means to obtain a simpler description of certain properties would break down and with it the dream of simplifying certain physical aspects via lightlike holography.

It has been customary in the algebraic approach to add those structural properties concerning intersections to the "axiomatic" list of algebraic requirements which can be derived in the absence of interactions and at least can be formulated without contradicting the presence of interaction-induced polarization clouds (which according to previous section was the intrinsic definition of interaction).

[34] In the classical setting this means that such characteristic data in contrast to data on ∂W (W arbitrary) on LF do not define a hyperbolic propagation problem in the ambient spacetime.

A glance at the holographic properties of the free field shows that lightfront holography has a nontrivial realization. To see this we construct generators of $\mathcal{A}(LF)$ directly following the formal prescription $x_- = 0$ of the old lightfront approach (using the abbreviation $x_\pm = x^0 \pm x^3,\ p_\pm = p^0 + p^3 \simeq e^{\mp\theta},\ \theta$ the momentum space rapidity):

$$\begin{aligned} &A_{LF}(x_+, x_\perp) \simeq \int \left(e^{i(p_-(\theta)x_+ + ip_\perp x_\perp} a^*(\theta, p_\perp) d\theta dp_\perp + h.c.\right) \\ &\left\langle \partial_{x_+} A_{LF}(x_+, x_\perp) \partial_{x\prime_+} A_{LF}(x'_+, x'_\perp) \right\rangle \simeq \frac{1}{(x_+ - x'_+ + i\varepsilon)^2} \cdot \delta(x_\perp - x'_\perp) \\ &\left[\partial_{x_+} A_{LF}(x_+, x_\perp), \partial_{x\prime_+} A_{LF}(x'_+, x'_\perp)\right] \simeq \delta'(x_+ - x'_+)\delta(x_\perp - x'_\perp) \end{aligned} \tag{33}$$

The justification for this formal manipulation consists in using the fact that the equivalence class of test function which have the same restriction $\tilde{f}|_{H_m}$ to the mass hyperboloid of mass m is mapped to a unique test function f_{LF} on the lightfront [46][47]. It only takes the margin of a newspaper to verify the identity $A(f) = A(\{f\}) = A_{LF}(f_{LF})$. But note also that this identity does not mean that the A_{LF} generator can be used in the bulk since the inversion involves an equivalence class and does not distinguish an individual test function in the bulk; in fact a localized test function $f(x_+, x_\perp)$ on LF has no bulk localization region whatsoever; this is how fussiness shows up in the traditional field description. It comes therefore as no surprise that the pointlike field notation on LF leads to a different singular field although A as well as A_{LF} are both living in the same Hilbert space; holography, contrary to the formation of the critical limit, of a massive theory keeps the original (massive) theory and its holographic projection in the same Hilbert space. The common wisdom that conformality implies zero mass does not apply to chiral theories because the mass spectrum cannot be computed without knowing the lightlike momentum associated with the lightlike direction opposite to LF.

We have formulated the algebraic structure of holographic projected fields for bosonic free fields, but it should be obvious that a generalization to free Fermi fields is straightforward.

This idea of taking the holographic projection of individual bulk fields can be generalized to composites of free fields as the stress-energy tensor; the result is always of the form

$$\left[B_{LF}(x_+, x_\perp), C_{LF}(x'_+, x'_\perp)\right] = \delta(x_\perp - x'_\perp) \sum_{k=0}^{n} \delta^k(x_+ - x'_+) D_{LF}^{(k)}(x_+, x_\perp) \tag{34}$$

where the dimensions of the composites $D^{(k)}$ together with the degrees of the derivatives of the delta functions obey the standard rule of dimensional conservation; in other words the fields which feature in this extended chiral theory are *chimera between QFT and QM*, they have one leg in QFT and n-2 legs in QM with the chimeric vacuum being partially a factorizing quantum mechanical state of "nothingness" and partially the LQP vacuum which upon localization (in our case to ∂W) becomes a KMS thermal state with thermal radiation and entropy.

Lightfront holography is consistent with the fact that except for d=1+1 there are no operators which "live" on a lightray since the presence of the quantum mechanical delta function prevents such a possibility i.e. the presence of the quantum mechanical transverse extensions must be taken into account important each lightfront operator must have a nonvanishing transverse extension[35].

The algebraic method via relative commutants (31) leads to a net of observable (bosonic) algebras on LF.But for (transversely extended) chiral theories the spin is connected with the scale dimension. This is also the case for free fields in the bulk and accounts for the fact that the net on LF generates the same global algebra as the bulk. But for interacting theories with anomalous short distance dimension one should expect that the globalization of the net on LF is smaller than the original algebra, in particular

$$\overline{\bigcup_{O\subset\partial W}\mathcal{A}(\mathcal{O})} \subsetneq \mathcal{A}(\partial W) \tag{35}$$
$$H(net(\partial W)) \equiv \overline{\bigcup_{O\subset\partial W}\mathcal{A}(\mathcal{O})\Omega} \subsetneq H(\partial W) \equiv H(W)$$

where the second line defines the subspace of the algebraic holographic projection. But we know from the study of chiral observable algebras that they have many superselection sectors. In term of pointlike generators this means that there are fields obeying braid group commutation relations which extend the Hilbert space of the observables and commute relatively with the observable fields. This leads to the following conjecture about this holographic extension

Conjecture: (holographic extension) *The holographic extension restores the identity*

$$\overline{\bigcup_{O\subset\partial W}\mathcal{A}_{ext}(\mathcal{O})} = \mathcal{A}(\partial W) \tag{36}$$

Without the validity of this identity there would be no inverse holography i.e. the reconstitution of the bulk from its holographic projection. For this the knowledge of and its 7-parametric symmetry group is not sufficient. However by knowing the action of the full Poincaré group on $\mathcal{A}(\partial W) = \mathcal{A}(W)$ and taking algebraic intersections one is able to regain the full bulk net.

A supportive argument in favor of this conjecture comes from the attempt to formulate the lightfront holography directly in terms of the interacting pointlike fields. The only way to do this is to use the following mass-shell representation which in the 60s became known under the name Glaser-Lehmann-Zimmermann representation[36]

$$A(x) = \sum \frac{1}{n!}\int dx_1 ... \int dx_n \; a(x; x_1, ...x_n) : A_{in}(x_1)....A(x_n) : \tag{37}$$
$$A(x) = \sum \frac{1}{n!}\int_{H_m} dp_1 ... \int_{H_m} dp_n \; e^{ix(\sum p_i)}\tilde{a}(p_1, ...p_n) : \tilde{A}(p_1)....\tilde{A}(p_n) :$$
$$A(x)_{LF} = A(x)_{x_-=0}$$

[35]This means in particular that lower bounds [60] in chiral theories for lightlike energy densities averaged along the lightray can be extended after transverse integration to the holographic projections of higher dimensional theories.

[36]The coefficient functions in this representation are mass-shell restrictions of retarded functions which in turn obey the nonlinear system of GLZ equations [49].

The relation between the Heisenberg field and its incoming free limits is very non-local and therefore it is further away from physical intuition than the algebraic approach. The second line shows that only the mass-shell restriction of these functions matter; the momentum space integration extends over the entire mass-shell and the two components of the mass hyperboloid H_m are associated with the annihilation/creation part of the Fourier transform of the incoming field. Clearly we can take in this on-shell representation without creating any problems in addition to the already present convergence problems which such GLZ formula entail with or without this restriction. Doing this directly in the Wightman functions would lead to meaningless divergences.

In fact as a result of the close relation of the Zamolodchikov-Faddeev operators to the incoming fields, the series (21) in terms of the formfactors in factorizing models is the ideal starting point for a lightfront restriction in the sense of $x_- = 0$. It turns out that for the (massive) Ising QFT model the infinite sum which represents the dimension can be done exactly and gives the expected value $\frac{1}{16}$. The calculations is similar[37] to [50] apart from the fact the the critical limit in that work is conceptionally very different from holography; the critical limit describes another theory in the same universality class of theories in a different Hilbert space from that of the original theory. In the light of this result the previous conjecture means that in the algebraic holographic projection one needs two steps namely the construction of the observable (bosonic) net on LF and its extension by superselection sectors in order to arrive at the same algebra as that obtained by holographically projecting pointlike fields directly. This issue requires more work.

Whereas in general the passing from a LQP description in terms of algebraic nets to the more standard description in terms of generating fields is presently not possible without making additional technical assumption, the situation is much better in the presence of conformal symmetries [51]. So the conditions for passing between the algebraic setting and pointlike generating fields in holography are particularly favorite. For consistency reasons such fields must fulfill (34) if they are observable fields and corresponding braid group commutation relations in case they have anomalous scale dimension..

Now we are well-prepared to address the main point of this section: the area law for localization entropy. The universal presence of a transverse delta function without derivatives in the above holographic commutation relations indicates the absence of transverse vacuum fluctuations i.e. the holographic projection is "quantum mechanical" in the transverse direction and quantum field theoretical in lightray direction. Another way to say the same thing is to talk about a quantum mechanically extended chiral QFT.

The absence of transverse vacuum fluctuations in the holographic projection is also the consequence of the transverse factorization theorem which goes back to Borchers [52]. Let $\mathcal{A}_i \subset B(H)$, $i = 1, 2$ be two operator algebras with $[\mathcal{A}_1, U(a)\mathcal{A}_2 U(a)^*] = 0 \; \forall a$ and $U(a)$ a translation with *nonnegative* generator which fulfills the cluster factorization property (i.e. asymptotic factorization in correlation functions for infinitely large cluster separations) with respect to a unique $U(a)$-invariant state vector Ω[38]. It then follows that the two algebras tensor factorize: $\mathcal{A}_1 \vee \mathcal{A}_2 = \mathcal{A}_1 \otimes \mathcal{A}_2$. In the case at hand the tensor factorization

[37] The holographically computed dimension of the basic field in the Sine-Gordon theory also yields the same integrals as in the Babujian-Karowski critical limit calculation.

[38] Locality in both directions shows that the lightlike translates $\langle \Omega \, | AU(a)B | \, \Omega \rangle$ are boundary values of entire functions and the cluster property together with Liouville's theorem gives the factorization.

follows as soon as the open regions $\mathcal{O}_i$ in $\mathcal{A}(\mathcal{O}_i)$ $i = 1, 2$ have no transverse overlap. The lightlike cluster factorization is weaker (only a power law) than its better known spacelike counterpart, but as a result of the analytic properties following from the non-negative generator of lightlike translations it renders the asymptotic *factorization to be valid for all distances*. The resulting transverse factorization implies the transverse additivity of extensive quantities as energy and entropy and their behavior in lightray direction can then be calculated in terns of the associated auxiliary chiral theory. a well-known property for spacelike separations.

This result [47][48] of the transverse factorization may be summarized as follows

1. *The system of LF subalgebras* $\{\mathcal{A}(\mathcal{O})\}_{\mathcal{O}\subset LF}$ *tensor-factorizes transversely with the vacuum being free of transverse entanglement*

$$\begin{aligned} &\mathcal{A}(\mathcal{O}_1 \cup \mathcal{O}_2) = \mathcal{A}(\mathcal{O}_1) \otimes \mathcal{A}(\mathcal{O}_2),\ (\mathcal{O}_1)_\perp \cap (\mathcal{O}_2)_\perp = \emptyset \\ &\langle \Omega \left| \mathcal{A}(\mathcal{O}_1) \otimes \mathcal{A}(\mathcal{O}_2) \right| \Omega \rangle = \langle \Omega \left| \mathcal{A}(\mathcal{O}_1) \right| \Omega \rangle \langle \Omega \right| \mathcal{A}(\mathcal{O}_2) \left| \Omega \right\rangle \end{aligned} \tag{38}$$

2. *Extensive properties as entropy and energy on LF in the vacuum are proportional to the extension of the transverse area.*

3. *The area density of localization-entropy in the vacuum state for a system with sharp localization on LF diverges logarithmically*

$$s_{loc} = \lim_{\varepsilon \to 0} \frac{c}{6} \left| ln\varepsilon \right| + ... \tag{39}$$

where ε is the size of the interval of "fuzziness" of the boundary in the lightray direction which one has to allow in order for the vacuum polarization cloud to attenuate and the proportionality constant c is (at least in typical examples) the central extension parameter of the Witt-Virasoro algebra.

The following comments about these results are helpful in order to appreciate some of the physical consequences as well as extensions to more general null-surfaces.

As the volume divergence of the energy/entropy in a heat bath thermal system results from the thermodynamic limit of a sequence of boxed systems in a Gibbs states, the logarithmic divergence in the vacuum polarization attenuation distance ε plays an analogous role in the approximation of the semiinfinitely extended ∂W by sequences of algebras whose localization regions approach ∂W from the inside. In both cases the limiting algebras are monads whereas the approximands ate type I "box" algebras. In fact in the present conformal context the analogy between the standard heat bath thermodynamic limit and the limit of vanishing attenuation length for the localization-caused vacuum polarization cloud really becomes an isomorphism. This is so because long distances are conformally equivalent to short ones.

This surprising result is based on two facts [47][48]. On the one hand conformal theories come with a natural covariant discretizing "box" approximation (one which does not break all the spacetime covariances) of the thermodynamic limit since the continuous spectrum translational Hamiltonian can be obtained as a scaled limit of a sequence of discrete conformal rotational Hamiltonians associated to global type I systems. On the other hand it has been known for some time that a heat bath chiral KMS state leading to a monad representation can always be re-interpreted as the Unruh restriction of a system in an larger type I world in a vacuum state i.e. as a kind of inverse Unruh effect [53]. Both fact together lead to the above formula for the area density of entropy. In fact using the conformal invariance one can write the area density formula in the more suggestive manner by identifying ε with the conformal invariant cross-ratio of 4 points

$$\varepsilon^2 = \frac{(a_2 - a_1)(b_1 - b_2)}{(b_1 - a_1)(b_2 - a_2)} \tag{40}$$

where $a_1 < a_2 < b_2 < b_1$ so that (a_1, b_1) corresponds to the larger localization interval and (a_2, b_2) is the approximand which goes with the interpolating type I algebras.

One expects that the arguments for the absence of transverse vacuum fluctuations carry over to other null-surfaces as e.g. the upper horizon $\partial\mathcal{D}$ of the double cone $\mathcal{D}$. In this case it is not possible to obtain $\partial\mathcal{D}$ generators through test function restrictions. For zero mass free fields there is however the possibility to conformally transform the wedge into the double cone and in this way obtain the holographic generators as the conformally transformed generators of $\mathcal{A}(\partial W)$. In order to show that the resulting $\mathcal{A}(\partial\mathcal{D})$ continue to play their role even when the bulk generators cease to be conformal, one would have to prove that certain double-cone affiliated inclusions are modular inclusions. We hope to return to this interesting problem.

Since the original purpose of holography is identical to that of the ill-fated lightcone quantization, namely to achieve a simplified description of certain "short distance aspects" of QFT, the question arises if one can use holography as a tool for the classification and construction of QFTs; with other words can one make sense of *inverse holography*? Knowing the local net of the lightfront, one can only obtain part of the local substructure of the bulk, namely that part which arises from intersecting the LF-affiliated wedge algebras. The full net is reconstructible if the action of those remaining Poincaré transformations (which do not belong to the 7-parametric LF symmetry group) is known.

The presence of the Moebius group acting on the lightlike direction on null-surfaces in curved spacetime resulting from bifurcate Killing horizons [54] has been established in [56], thus paving the way for the transfer of the thermal results to QFT in CST. This is an illustration of symmetry enhancement which is one of holographies magics. The above interaction-free case with its chiral abelian current algebra structure (33) admits a much larger unitarily implemented symmetry group, namely the diffeomorphism group of the circle. However the unitary implementers (beyond the Moebius group) do not leave the vacuum invariant (and hence are not Wigner symmetries). As a result of the commutation relations (34) these Diff(S^1) symmetries are expected to appear in the holographic projection of interacting theories. These unitary symmetries act only geometrically on the holographic objects; their action on the bulk (on which they are also well-defined) is fuzzy, i.e. not describable in geometric terms. This looks like an interesting extension of the new setting of local covariance [25]

The area proportionality for localization entropy is a structural property of LQP which creates a challenging contrast with Bekenstein's are law for black hole horizons. Bekenstein's statistical mechanics reading of the area behavior of a certain quantity in classical Einstein-Hilbert like field theories has been interpreted as a clue on the interface of QFT with QG. Now we see that the main support, namely the claim that QFT cannot produce an area behavior, is not correct. There remains the question whether Bekenstein's numerical value via Hawkings's thermal results is a credible value for quantum entropy. I do not know any situation where a *classical* value remained intact after passing to the quantum theory, except for certain *quasiclassical* values in case the system is integrable. QFT gives a family of area laws with different *vacuum polarization attenuation parameters* ε and it is easy to fix this parameter in terms of the Planck length so that the value is compatible with Bekenstein's. But this amounts hardly to the localization of an interface between QFT and QG. One advantage of the present method is that instead of a cutoff which changes the model in a way which conceptually is totally beyond any control, the *attenuation length* ε is a quantity which is defined within a given QFT. Whether QG adds additional degrees of freedom which modify the contribution of localized matter remains an open problem.

3.2 Vacuum fluctuations and the cosmological constant "problem"

If there is any calculation which holds the record for predicting a quantity which comes out way off the observed mark (by more than 40 orders of magnitudes), it is the estimate for the cosmological constant based on a quantum mechanical argument of filling oscillator-like levels of zero point of vacuum energy. This leads to a gigantic mismatch between quantum mechanics of free relativistic particle and the astrophysical reality; it became widely known under the name the *cosmological constant problem* and generated a lot of commotion (and also led to fantastic ideas as the *anthropic principle*).

However hardly any quantum field theorists who does not subscribe to the maxim "compute and shut up" would endorse a calculation which is quantum mechanical (since consists in simply filling the energy levels above an assumed vacuum state up to a certain cutoff mass κ which of course should be larger than the physical masses of the theory) and leads to an energy density which behaves roughly as $\rho_E \sim \kappa^4$. Such a calculation ignores the important *local covariance principle* of QFT in CST and in this way contradicts the dictum of Hollands and Wald cited at the end of the introduction. Before we pass to a description of vacuum energy which complies with this principle, it is quite amusing to mention one consequence of the above large cosmological constant which can be found in various articles.

Assuming that light at A is emitted in the outmost violet part of the spectrum, we can ask for the distance from A to B such that at B the red-shifted light has reached the borderline in the optical infrared spectrum, i.e. we are asking for the length of maximal visibility in a universe with the vacuum energy calculate in the aforementioned way. The result is that when standing straight you can hardly see the floor.

This kind of argument is often used as showing that QG is needed to resolve paradoxical situations of QFT in CST. But the above calculation has hardly anything to do with a paradoxical situation within QFT in CST which requires the intervention of a yet unknown QG.

It is instructive to look first at the problem of vacuum energy in Minkowski spacetime. The standard argument by which one defines the stress-energy tensor as a composite of a free field is well known, one starts from an associated bilocal split-point expression and take the limit after subtracting the vacuum expectation value so that the result agrees with the Wick-ordered product. The resulting stress-energy tensor has all the required properties. Its expectation values are well-defined on a dense set of states which includes the finite energy states. But contrary to its classical counterpart, there is an (unexpected at the time of its discovery [57]) problem with its boundedness from below since one can find state vectors on which the energy density $T_{00}(x)$ takes on arbitrarily large negative values.

This has of course led to worries which affected in particular those general relativists who knew about the importance of classical positivity inequalities for questions of stability. It started a flurry of investigations [59] which led to state-independent lower bounds for fixed test functions $T_{00}(f)$ as well as inequalities on subspaces of test functions. These inequalities which involve the free stress-energy tensor were then generalized to curved space time. In the presence of curvature the main problem is that the definition of $T_{\mu\nu}(x)$ is not obvious since in a generic spacetime[39] there is no vacuum like state which is distinguished by its high symmetry; and to play that split point game with an arbitrarily chosen state does not seem to be right. But what are the physical principles which could select the physical stress-energy tensor?

The answer was given some time ago by Wald [58] in the setting of free fields. The *local covariance principle* determines the correct energy-momentum tensor up to local curvature terms (whose degree depends on the spin of the free fields). In fact one can construct a basis of composite fields so that every member is a locally covariant composite of the free field such that for the Minkowski spacetime we re-obtain the simpler Wick basis. The formulation of the local covariance principle uses local isometric diffeomorphisms of the kind which already appeared in Einstein's classical formulation and this requires to consider simultaneously all QFT which share the same quantum substrate but follow different spacetime ordering principles. In other words, even if one's interest is to study QFT in a particular spacetime (Robertson-Walker for the rest of this section), one is forced to look at all globally hyperbolic spacetimes in order to find the most restrictive condition imposed by the local covariance principle.

The result is somewhat surprising in that this principle cannot be implemented by taking the coincidence limit after subtracting the expectation in *any* of the states of the theory. Rather one needs to subtract a "Hadamard parametrix" [24] i.e. a function which depends on a pair of coordinates and is defined in geometric terms; in fact in the limit of coalescence it depends only on the metric in a neighborhood of the point of coalescence. As a result the so constructed stress-energy tensor at the point x depends only on the $g_{\mu\nu}$ in an infinitesimal neighborhood of x. As in the case of Minkowski spacetime it has a finite value in physical states and the only aspect to worry about is that, as in that case $T_{00}(f)$, it is not bounded below. In that case the computation which lead to good lower bounds are somewhat more demanding [60]. Recently this idea was extended in order to determine states on free fields in the Robertson-Walker model which minimize the suitably smeared energy density [61]. In this way contact was made with an old concept of adiabatic vacua introduced by Parker [62][63]. The idea is to use such distinguished

[39] In a static (time-independent Hamiltonian) universe one could use the ground state as a distinguished reference state. But our universe is not static.

extremal states as a substitute for a ground state. The calculation which is still missing in order to arrive at the energy density in such a cosmologically selected state[40] is the calculation of the of the Hadamard parametrix in a R-W spacetime which is needed for the energy-momentum tensor. This is the only nontrivial part of such a calculation. In the cited work on energy inequalities the difficulty of computing Hadamard parametrices was avoided by studying differences of T_{00}-expectations between two states in which case the purely geometrically determined Hadamard parametrix cancels out.

Although such a computation has not been carried out at the time of writing, there can be no doubt that the result would constitute a more credible expression for cosmological energy density since it is finite without cut-off and, according to the way it was derived, it complies with the local covariance principle. Taking the version of the model in which the coupling to the curvature is absent (the "minimal" model), the result can only depend on the geometric parameters of the RW metric.

4 Concluding remarks

QFT and QM are two quantum theories which, to say it in somewhat provocative terms, apart from the notion of algebra and state have not much more in common than $\hbar$. In the present work we showed that the reason for this is that both theories are based on very different localization concepts; whereas QM leads to unrestricted Born localization, the Born (Newton-Wigner) localization in interacting QFT is restricted to asymptotic regions. On the other hand the causal propagation over finite distances is governed by modular localization which only exists in QFT. The opinion that QFT is an extension of QM is supported by the fact that the quantization formalism, in particular the functional integral representation via the classical action functional is common to both theories. But the irony behind this statement is that in QM where the existence of the Euclideanized Feynman representation can be rigorously established nobody would have the courage to base a course on it and in QFT where its measure theoretical basis does not go beyond the canonical struture (finite wave function renormalization, superrenormalizable QFT) the majority of particle physicists use it to a degree where it has become synonymous with QFT. What they really use is its metaphorical power; there is presently no other formulation where one can specify in one line which interaction one wants to consider and with a lot of artistry hindsight and consistency checks one can produce results whose correctness can be checked independent of whether they fulfills the original functional integral representation or not (for strictly renormalizable interactions the answer is negative). LQP is a reminder that even if one uses those metaphoric methods, one should not forget that there is another world which roughly speaking corresponds to the operator formulation of QM which still needs to be developed.

As a consequence entanglement in QM follows the "cold" information theoretical logic. By localizing with Born projectors one encounters both entangled and non-entangles states. On the other hand the quantum field theoretical (modular)localization is totally intrinsic since there are no pure states on monads. The physical states on the global alge-

[40] For obvious reasons we have some reservations to follow the usual parlance and use the word "vacuum state".

bra (finite particle number, in particular the vacuum) upon restriction to a local monad turn into thermal KMS states with an area proportional entropy. The entanglement from modular localization is a "hot" entanglement. The dichotomy pure-entangled does not exist with monad algebras so if we continue to use the same name "entanglement" as in QM, it has to be taken with a grain of salt. theoretics entanglement is inexorably linked to the thermal aspects of vacuum polarization resulting from modular localization i.e. there are no localized non-entangled states.

As a consequence this kind of entanglement is much more violent and leads to a breakdown of the usual an observed system/environment separation on which the modern measurement theory depends. The split property leading back to tensor-factorizing type I algebras but it does not quite present a return-ticket into the world of QM. Although on such tensor factors there exist pure product states, all the physical states on the global algebra restricted to such a tensor factor are still KMS states described by a Gibbs density matrix. There can be no problem of localization entropy with information loss because the information theoretical interpretation is only applicable to cold entropy and last not least nothing has been done (except changing the vantage point for observation) when one analyses a state from a local instead of a global algebra.

As it was clearly seen by Rob Clifton there is a lot of unfinished business between LQP and measurement theory. Even the most accommodating version of measurement theory, the so-called *modal interpretation*, runs into rough water with LQP [1]. In fact a LQP compatible theory of measurement does not yet exist, which is a strange situation in view of the fact that physicists have over more than 80 years studied the measurement theory of the less fundamental QM in great detail [64].

The most important message of the present work is that some of the problems of black hole physics, in particular the so-called information loss problem which led to what L. Susskind in a particular context once called the "30 year war" between Hawking's versus 'tHooft and Susskind's view on this problem, are more related to an insufficient appreciation of thermal manifestations of modular localization in QFT than with new physics from QG. There is certainly a contradiction with the information theoretic entropy in QM but this discrepancy is in no way different than that with the thermal aspects of states in QFT restricted to observables on null-surfaces. The explanation why these conceptual problems of QFT were first noticed in black hole physics is that only in this context of horizons fixed by curvature they loose their virtual nature and turn into in principle (astrophysically) measurable properties. Nevertheless it is amazing that despite the discovery of vacuum polarization clouds already as far back as 1934 it took this round about way via Hawking's computational proof of the thermal properties of black holes to became aware of the profound differences of localization in QFT from Born localization. Part of the explanation is that historically QFT was first accessed via Lagrangian quantization and subsequently by functional integrals i.e. by methods which, being equally valid in QM, are notoriously blind against properties resulting from modular localization. The first observance that thermal aspect appear in vacuum QFT upon restriction to wedges and lead to deep structural property of QFT [10] were made in the same decade in which Unruh proposed his Gedankenexperiment [66], but only later these studies were united [65].

For the thermal area law for localization entropy to play a physical role it is important to have a situation in which a fleeting (Gedanken) causal horizon is traded with a real event horizon which only the geometry of curvature can generate. Without this eye-opener from black hole physics, the thermal aspect of localization in QFT would probably have remained of interest only to some mathematical physicists. The point I want to stress here is that up to now these observations point more towards the large areas within QFT which despite Lagrangian quantization and functional integration remained hidden, rather than alluding towards a still elusive QG. In fact this paper was written to point out that some of the aspects (holography, entropic area law) which in the present discussions are attributed to QG are really belonging to deeper and little known areas of QFT. If the reader feels that what was presented here is not what he learned from the books, than he got the main message which is to counteract the demise of a totally unfinished project as QFT as "old particle physics" by you know who.

We also saw that vacuum polarization in form of FPG's represent an ideal intrinsic indicator for the presence/absence of interactions. In this context I mentioned some recent results about how modular localization can be used for model construction which shows that at least some interesting nontrivial low-dimensional non-canonical QFTs have a solid ontological status. These models also exhibit for the first time the property of *asymptotic completeness* and show limitation of the Lagrangian approach; i.e. they confirm the old suspicion that the world of LQP models with physical (unitary, crossing, invariant) S-matrices is much bigger than the available baptisms by Lagrangian names. There is of course some irony in the fact that instead of Chew's dream of uniqueness via the S-matrix bootstrap which he proposed against the world of many Lagrangian of QFT, the bootstrap-formfactor approach to factorizing models leads to even greater plethora of models than the Lagrangian setting.

Finally we passed to two problems of great current interest namely *localization entropy* and *cosmological energy density*. It was shown that the calculation of localization entropy is possible thanks to holography onto the lightfront. The result is an area law, and since this result questions the belief that the Bekenstein area law derived in certain classical field theories defines an interface of QG with QFT in CST, the problem of localizing the QFT-QG interface remains open.

There is also the controversial problem about degrees of freedom counting in holography; it has been claimed (or postulated) that the cardinality of degrees of freedom in the holographic projection has to be the same as in the bulk. For holography on null surfaces this is certainly not the case since knowing a local net on the lightfront is not sufficient to re-construct the bulk, even though the global lightfront algebra is equal to $B(H)$ and also $\mathcal{A}(\partial W)$=$\mathcal{A}(W)$. It is the local net structure which determines the cardinality of degrees of freedom. A completely different situation one meets in the AdS-CFT correspondence [67].

The main conclusion about the cosmological energy density was that the cut-off dependent calculations have been incorrectly attributed to QFT since they violate one of its most cherished principles namely the local covariance principle. We have indicated of how one can set up a cut-off free quantum field theoretic calculation for free fields and hope that concrete results for minimal energy states in a R-W cosmology will be available in the near future.

Since a sizable fraction of the particle physics community considers QG as the main problem of this century, some remarks of caution are in order. First the idea that gravitation may be a relic of already known quantum forces (like the van der Waals force) and therefore the necessity for quantization is dispensed has never been completely ruled out. But secondly, even if one accepts the idea of QG, one cannot fail to realize with amazement that some of the ideas, whose clarification had been placed onto the shoulders of the still elusive QG, are already taken care of by the recent gain of insights into QFT in CST. The implementation of the very nontrivial and restrictive local covariance principle implies that isometrical diffeomorphisms between manifolds lead to the quantum physical isomorphisms i.e. local quantum symmetries which make it impossible to decide by local experiments in what global world one lives; in this sense the *independence from the reference system is already implemented in QFT in CST* .

QG-physicists want a more radical version namely that (not unlike the idea of gauge invariance) a description in which those equivalence classes are compressed into one object i.e. a *diffeomorphism invariant* algebra. In a recent very interesting and still somewhat speculative paper, Brunetti and Fredenhagen show [68] that a quantum field theory which fulfills Einstein-Hilbert like quantum field equations between suitable physical states (the authors think in terms of a perturbative BRST approach) is automatically invariant under isometric diffeomorphisms which for many researchers is the epitome of QG. Hence the main argument against the perturbative approach to QG is not that it inevitably leads to violations of the *independence of the background* principle, but rather that it leads to a perturbation theory around the free spin 2 field in which the number of undetermined parameters increase with the perturbative order i.e. it is non-renormalizable[41]. It seems that claims about incompatibilities between QG with QFT are premature.

The present situation, which is characterized by often unguided speculative guesswork about what constitutes QG suggests analogies with another very speculative period in particle physics namely the speculations of how to solve the ultraviolet catastrophe of QFT. This was a period which preceded the more conservative Schwinger-Tomonaga-Feynman-Dyson renormalized QED theory. Could history of particle physics repeat itself?

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