CBPF-NF-025/03

Shallow lakes and their associated drainage basin

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Abstract

The explicit time dependence of the behaviour of the drainage basin of a lake is an important contribution for the lake dynamics. This information can be expressed in terms of a sum of Lorentzian functions. The coefficients of the sum of Lorentzians are extrated using results of measured and/or calculated data for the state variables describing the shallow lake West Lake, Hangzou. This procedure can be applied also to other shallow lakes like those of the North of Rio de Janeiro, Brazil, and thus generating geological information about the drainage basin of the considered lake region. We conclude emphasizing the relevance of *the time dependence* of the drainage basin.

***Key-words*:** drainage basin, dynamical model, shallow lakes, geological information, Lorentzian distribution

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1. Introduction

In the ecological literature, several previous experiments and modelling of lakes, in given specific environments, different approaches to study the behaviour of these lakes have been proposed. The central idea of this modelling effort for a given environment is a better description of the lakes, taking into account the influence of their environment and its corresponding micrometeorological behaviour. In order to present a more complete introduction to this work, let us recall some of the previous studies existent in the literature that correspond to describe some specific details of the existent lakes. As an illustration we intend now to present in this Introduction a brief summary of some of these results of experimental / modelling efforts. To be as complete as possible we decided to consider studies of lakes performed in several regions of the world. Let us start with the work about the restoration of the Lake Eymir, in Turkey (Beklioglu et al., 2003). This work is considered at the beginning of the Introduction because its content is close to our interest in future work on lakes of the Rio de Janeiro State, Brazil. These authors show that the nutrient loading in lakes is a serious threat to water quality. The restoration of that lake was successfully executed through a great reduction in the areal loading of total phosphorus and dissolved inorganic nitrogen. In spite this effort, the poor water clarity and low submerged plant coverage still persisted. Although we are not considering in our present work the existence of fishes, in the North of Brazil, fishery is very important, but for the lakes in the Rio de Janeiro state this details are not considered here. These above authors also studied the influence of some kinds of fish which perpetuate the poor water condition. A decrease in the inorganic suspended solid concentration and in chlorophyll-a concentration, reduced the fish stock which led to a better water quality.

Another subject of interest here for the role of environment in lakes is the study about the sensibility to acidic deposition in the Adirondack region of New York (Civerolo et al., 2003), where, on a period of ten years, it was analysed the ambient air and the lake water quality and discussed the complexity of the seasonality and time variation in estimating temporal trends in their experimental data.

Also, a very interesting work concerns the zooplankton metacommunity structure (Cottenie et al., 2003). Using data obtained during three years of a system of highly interconnected ponds these authors studied the influences of regional interactions over the local zooplankton communities, showing that local environmental constraints are strongly related with the zooplankton metacommunity structure.

Within this general approach one should consider some examples which apply different techniques to study the lake environment behaviour. Chen (2003) monitored reservoir water quality using remote sensing images, based on genetic programming method. Their results were better than the traditional regression method.

Steady-state mass balance models were used by Molot and Dillon (2003) to predict concentrations of chemical substances in lakes, as iron, aluminum and dissolved carbon. This method have been considered successfully despite a limitation of the model, which assumes constant the value of the mass transfer of some substances.

Different techniques have been used to study the fish contamination on lakes and marine waters by chemical products or natural toxins. Liquid chromatography with mass spectrometric detection was developed by Dahlman et al. (2003) to determine various algal and cyanobacterial toxins extracted from phytoplankton which can lead on shellfish poisoning. Page and Murphy (2003) used a geographic information system (GIS) approach to create a data base to establish the mercury (Hg) levels in many fishes from remote lakes in Canada, where the Hg quantity exceeds the recommended level for human consumption.

In conclusion of this Introduction it was clear the role of environment in the lake dynamics. We decided to present all these details to justify why we used in the present work the more general dynamic equations for the water column but including time dependent functions Q(t) which are responsible for the description of environment effects, and the importance of that was illustrated in this series of examples, in particular in what concerns micrometeorology. Although a little long we consider quite important to not restrict ourselves to simple mathematical equations but also to justify the need of the Q(t) functions in terms of experiments as presented above.

2. The model

The main steps of this work are of two different types: the first one has a mostly theoretical description of lake modelling and the second concerns experimental data as measured for the lake phyto-zooplankton community and obtained on an annual time scale. As it will be made more clear later on, the experimental data for the lake community, as a function of time, is the starting point to extract information on the strength and time dependence of the drainage water.

The dynamical state equations for the description of a lake in *a given* geographical environment involve several parameters. The most frequently used are the rate equations which describe quantitatively the time dependence of the process of birth, growth and decomposition of the lake constituents, together with external forcing functions like, for example, the time dependent sun light incidence. It should be emphasized that drainage water is not considered here strictly as a *forcing function*, and we extract information from it through the observed experimental data on the lake community. Many of these *rate constants* can be measured using physico-chemical techniques, and the values to illustrate our study are taken here identical to those assumed for the Chinese West Lake by Hongping and Jianyi (2002) at their Table 2 (showed in Appendix). Clearly for eventual application to Brazilian lakes, the physico-chemical rate processes should be adapted to the local conditions and again extracted from experiment (Suzuki, 1997). In particular, shallow lakes connected to the ocean do exist in the region of the North of the Rio de Janeiro state. For application to these particular lakes, the dynamic equations adopted in the present work should be extended to include specific effects of the connection to the sea. These cases will be considered in future work.

A currently adopted hypothesis consists in assuming that the physico-chemical process *structure* of the model equations is *independent* of the geographic coordinates of a given lake. For very specific situations, e.g. the *connection to the sea*, modifications are required in the mathematical structure of the model. Specific details of some situations of the Chinese West Lake, only need to introduce *a time dependence* in some parameters associated to the geographic environment as we present in detail later on, and from these geographic parameters the time dependence of the drainage water is extracted from experiment using the coefficients of a Lorentzian expansion.

Here we consider the same assumptions of the modelling of the Chinese lake i.e., the state variables are the biomass of four species of algae, Cyanophyta, Chlorophyta, Criptophyta and Bacillariophyta that we denote $(BA1(t), BA2(t), BA3(t) \text{ and } BA4(t))$ respectively, with their respective content of phosphorous $(PA1(t), PA2(t), PA3(t) \text{ and } PA4(t))$, biomass of the zooplankton $(BZ(t))$ and its content of phosphorous $(PZ(t))$, phosphorous in detritus $(PD(t))$, phosphorous in sediment $(PE(t))$ and finally orthophosphate $(PS(t))$ in the lake water column. In the present model the existence of fishes is entirely disregarded. This hypothesis should be verified when one considers Brazilian lakes, where in some cases (as in the North of the Country) fishery is quite relevant.

The *central point* in this work concerns the distribution in *time* of the drainage water which is considered here as the main micrometeorological parameter. Drainage water *depends* *on time*, via a distribution function *Q(t)* and this quantity is described by the sum of a collection of Lorentzians. It should be emphasized that contrary to us, Hongping and Jianyi (2002) consider *Q* as a *time independent parameter*. The motivation for the Lorentzians is that the center and the width are associated to micrometeorological behaviour and thus drainage effects usually disregarded depending on time instant and time interval, are now taken into account. In fact the center of the Lorentzian define the time at which *water precipitation* occurs and the width is associated to the *time interval* of the precipitation and /or its diffusion in the soil. As it will be made clear, this is the *main difference* between Hongping and Jianyi (2002) work and the present one.

We have retrieved the available theoretical results in order to extract the time dependence of the water drainage using a software developed by Cavalcante (personal communication). It should be stressed that for other available data corresponding to lakes, in Brazil or else, our proposal can be applied to extract drainage water results. From now on, the *dynamics* defined below will be considered fixed and well defined, thus only parameters and forcing functions may be changed to describe different lakes and regions.

The state equations from which we extract information about the drainage water have the following *general form*:

$$\frac{dBA_i(t)}{dt} = \left\{A_1^{(i)}[t;PS(t),BA_i(t)] - Q_{BA_i}^{(i)}(t)/V\right\} \times BA_i(t) - BZ(t) \times A_2^{(i)}[t;BA_i(t)] \tag{1}$$

where i =1,...4 for the four algae. The main difference respect to Hongping and Jianyi (2002) concerns the time dependent drainage water as described by $Q_{BA_i}^{(i)}(t)$. These distribution functions are *strictly time dependent* and this intends to describe the contribution of the diverse micrometeorological effects.

For the zooplankton one has the dynamics:

$$\frac{dBZ(t)}{dt} = B_1[t;BA_i(t)] \times BZ(t) - \left(Q_{BZ}(t)/V\right) \times BZ(t) \tag{2}$$

For orthophosphate and phosphorous in detritus one has respectively

$$\frac{dPS(t)}{dt} = LPS + B_2[t;PD(t),PS(t),PE(t),PA_i(t),BA_i(t)] - \left(Q_{PS}(t)/V\right) \times PS(t) \tag{3}$$

$$\frac{dPD(t)}{dt} = LPD + B_3(t; PZ(t), PD(t)) - (Q_{PD}(t)/V) \times PD(t) \tag{4}$$

where V is the lake volume. It should be noted that the *time dependent* drainage water contributions to the lake dynamics are expected to be different for each one of the lake state variables and represented as $Q(t)/V$. This difference is associated to the several substances drained which interfere in the state variables time dependence.

In these equations the functions, $A_1^{(i)}(t), A_2^{(i)}(t), B_1(t), B_2(t)$ and $B_3(t)$ are strictly non linear of the state variables and time. These functions include also the model parameters that we assume well defined experimentally once for. In Eq. (3), $PA_i(t)$ correspond to the phosphorous in the algae. These non linear functions have the general form $F(t; S_i(t), S_2(t), \ldots S_q(t); l_1, l_2, \ldots l_p)$ where the l_i are the parameters of the model and $S_q(t)$ are the state functions obtained from experiment as a function of time t. The l_i parameters are identical to those of Hongping and Jianyi (2002) but a simulation obtained changing these parameters can be made provided the time dependence of the state variables for this new situation is known and thus using the proposed procedure the new drainage results can be obtained. The detailed formulation of the functions $A_1^{(i)}(t), B_1(t), B_2(t)$ and $B_3(t)$ are presented in Honping and Jianyi (2002), with the parameters defined in its Table 2, which we expect to be adequate to the present study. Given the experimental values for time in the interval from $t = 0$ and $t = 360$ days, for the functions $A_1^{(i)}(t), B_1(t), B_2(t)$ and $B_3(t)$ and performing numerical differentiation of the available time dependence of the state variables, one can extract the corresponding drainage water $Q(t)$´s that will be fitted by eight Lorentzians. The numerical results of such a procedure will be shown in Fig.1.

Another type of dynamical equation concerns the amount of phosphorous in the four species of algae and the zooplankton. Their equation of motion read in general terms:

$$\frac{dPA_i(t)}{dt} = C_i[t; BA_i(t), PS(t), PA_i(t)] - (Q_{BA_i}^{(i)}(t)/V) \times PA_i(t) \tag{5}$$

$i = 1, \ldots 4$

$$\frac{dPZ(t)}{dt} = D[t; BA_i(t), PA_i(t), BZ] - (Q_{BZ}(t)/V) \times PZ(t) \tag{6}$$

where again different $Q_i(t)$ are used since different substances are distinctly introduced by the geological drainage distribution.

A possible work assumption, since one has no data concerning the phosphorous contained in the algae is to *assume* that $PA_i(t)$ is proportional to the algae biomass. The same hypothesis holds for $PZ(t)$ because we expect that the internal phosphorous must be proportional to the biomass in both cases.

For the case of phosphorous in the sediment one has the following equation:

$$\frac{dPE(t)}{dt} = E[t; PE(t)PS(t), PD(t), PA_i(t)] \tag{7}$$

It is important to note that contrary to the remaining equations for the lake state variables, the drainage water *does not appear explicitly* in Eq. (7). This fact seems to be reasonable since drainage affects the sediment of the lake only indirectly via the water column dynamics, and this is expected to have a *distinct time scale*.

3. Numerical results and conclusions

To illustrate the results of the model calculation for the drainage water, which is *time dependent* as suggested by micrometeorology, contrary to Honping and Jianyi (2002), which *assume time independent* drainage, we decided to fit its *time dependence* through a sum of Lorentzians. The numerical procedure goes as follows: we take the data presented in the figures of Honping and Jianyi (2002) for $BA_i(t)$, $i = 1,...4$, $BZ(t)$, $PS(t)$ and $PD(t)$, together with the parameters presented in their Table 2. Given these data we numerically differentiate the curves and adopt the values for the quantities *LPS* and *LPD* given by these authors. These parameters are other forms of phosphorous which are extracted from *outside* the lake. The values for these two unknown quantities are taken as those which furnish the best fit for the measured quantities. To perform the fit we adopted a collection of Lorentzian functions, a procedure that is quite usual in fitting experimental data, since one extracts the center (in time) and the time width of the drainage water functions. We are thus in position to extract the $Q_{BA_i}^{(i)}(t)$ and the $Q_M(t)$ curves (with $M = BZ, PS$ and PD) using equations Eq. (1) to Eq. (4) The so obtained functions are thus fitted using the sum of eight Lorentzians of the general form:

$$H(t)=\sum_{i=1,8} Y^{i}(t) \tag{8}$$

with

$$Y^{(i)}=Y_0^{(i)}+\frac{2A^{i}}{p}\times\frac{\Delta^{(i)}}{\left(X(t)-X^{(i)}\right)^2+\left(\Delta^{(i)}\right)^2} \tag{9}$$

The centers of the Lorentzians are expected to define the time at which water has been introduced/absorbed in the basin and the negative sign of the coefficients $A^{(i)}$ are interpreted as absorption. The widths $\Delta(i)$ describe time intervals of rain water precipitation and/or diffusion. The results of the fitting are presented in the figures. We can see in Fig. 2 that Q_{BA2}, Q_{BA3} and Q_{BA4} are almost identical, and we can note also that except in the cases of Figs. 3 and 4, only positive values for the coefficients $A^{(i)}$ of the Lorentzians do exist.

The advantage of the Lorentzian fit is to show at what time and for which time intervals the geological water dynamics for the drainage occurs. The negative terms of the expansion are interpreted as absorption processes, in particular for the zooplankton and orthophosphate processes which present large negative values indicating important absorption of these drainage elements by the lake.

Again let us emphasize the importance of separating the biochemical processes included in the non linear functions $A_1^{(i)}(t), A_2^{(i)}(t), B_1(t), B_2(t)$ and $B_3(t)$ of Eqs. (1) to (4) (including obviously the parameters) from the terms Q_i and Q_M which are fitted to the Lorentzians as previously described. This method thus shows the significance of the strong absorption processes shown in Fig. 3 and Fig. 4. These large absorption rates occur as shown in these figures, with large negative value contributions, thus indicating the relevance of the existence of time dependence of absorption in the drainage describing the dynamics of the lake, which are disregarded in some existent approaches in the literature. Also these results suggest to make geological experiments to measure the relevance of the time dependence of the dynamics of drainage water.

Acknowledgements: The authors acknowledge J. Thadeu Cavalcante for his kind help by developing a software for extracting the experimental and theoretical data plotted in figures of Hongping and Jianyi (2002) and to C. da Silva for a critical reading of the manuscript.

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Figure Captions

Figure 1 – The irregular curve corresponds to the numerical solution for $Q_{BA1}(t)$/V. The dashed lines are the eight Lorentzians distribution functions, which the sum gives the fit showed in the solid line.

Figure 2 - The irregular curve corresponds to the numerical solution for $Q_{BA_i}(t)$/V (i = 2,3,4). (These three curves are almost indistinctive). So it is shown only the eight Lorentzians distribution functions for $BA_2(t)$ (dashed lines), and the fit given by their sum (solid line).

Figure 3 - The irregular curve corresponds to the numerical solution for $Q_{BZ}(t)$/V. The dashed lines are the eight Lorentzians distribution functions, which the sum gives the fit showed in the solid line.

Figure 4 - The irregular curve corresponds to the numerical solution for $Q_{PS}(t)$/V. The dashed lines are the eight Lorentzians distribution functions, which the sum gives the fit showed in the solid line.

Figure 5 - The irregular curve corresponds to the numerical solution for $Q_{PE}(t)$/V. The dashed lines are the eight Lorentzians distribution functions, which the sum gives the fit showed in the solid line.

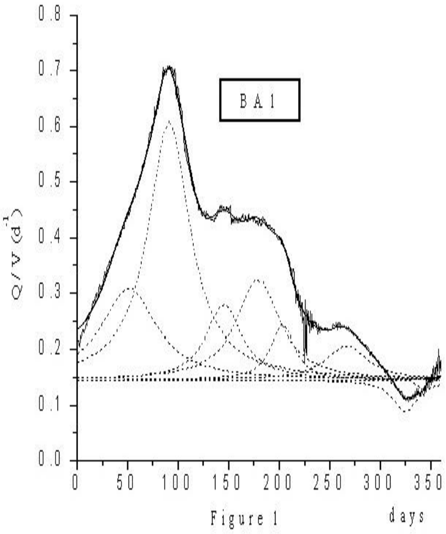

Figure 1

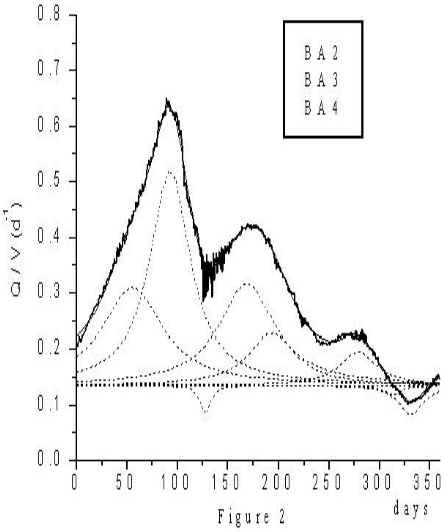

Figure 2

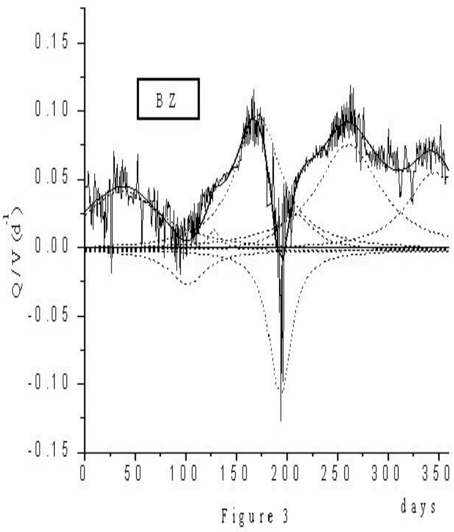

Figure 3

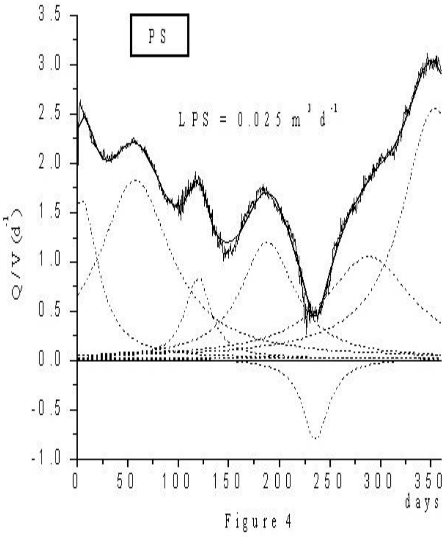

Figure 4

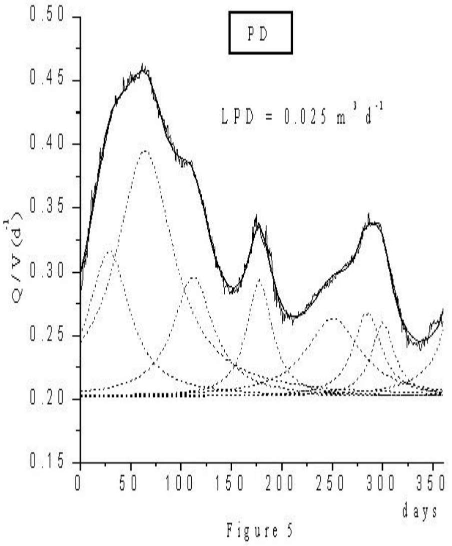

Figure 5