

## Forty Years of the Establishment of the Induced Weak Pseudoscalar Interaction

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### ABSTRACT

The author describes his work on the muon capture by light nuclei through the interaction of muons with protons via a virtual pion exchange. Although the resulting capture rate was an order of magnitude slower than the experimental available data, the calculation showed that the pion exchange between muons and protons gives rise to an induced weak pseudoscalar coupling which has to be added to the Feynman-Gell-Mann-Sudarshan-Marshak V-A interaction.

**Key-words:** Weak interactions; Muon capture by protons; Electroweak model.

I spent the academic year 1956-1957 at the California Institute of Technology, Pasadena as a Science Research Fellow invited by Richard P. Feynman. We had previously worked together when Feynman was a Visiting Professor at the Centro Brasileiro de Pesquisas Físicas, Rio de Janeiro, during his sabbatical year 1951-1952 and in the Summer of 1953.

At Caltech I had the privilege of having several hours of discussion with him every week besides enlightning conversations with Murray Gell-Mann and B. Stech. Among the topics which were examined was the failure of Yukawa's attempt at describing the nuclear beta decay by means of the strong coupling of pions to the nucleus and their weak coupling with leptons. The fact that the electron decay of pions was not observed was the reason of this failure so that the nuclear beta decay was to be regarded as a result of the Fermi interaction between the nucleon and the electron-neutrino fields.

As, however, the muon-decay of pions was well established, it would be of interest to investigate whether the muon-capture by protons could be described by the interaction of muons with nucleons through the intermediate pion field. This mechanism could not be discarded and its calculation would have to be compared to the adopted Fermi interaction. Previous evaluations had not been successful in view of the lack of precise knowledge of the pion-nucleon vertex. In 1956 there appeared a paper by G.F. Chew and F.E. Low which developed a cut-off meson theory for describing low-energy pion-nucleon scattering and which gave a reliable value for the strong pion-nucleon coupling constant, the proton being described by an extended source function  $u(x)$ .

Although the result for the capture rate was found to be about 20 times slower as compared to the available experimental data at that time, the importance of my calculation<sup>1</sup> was the establishment of a strong interaction effect on the weak process so that the virtual pion exchange between the incoming muon and proton gave rise to an induced weak pseudoscalar interaction. This has to be added to the Feynman-Gell-Mann V-A interaction.

The well-known Puppi-Tiomno-Wheeler Triangle has now to be replaced by that indicated in Fig. 1.

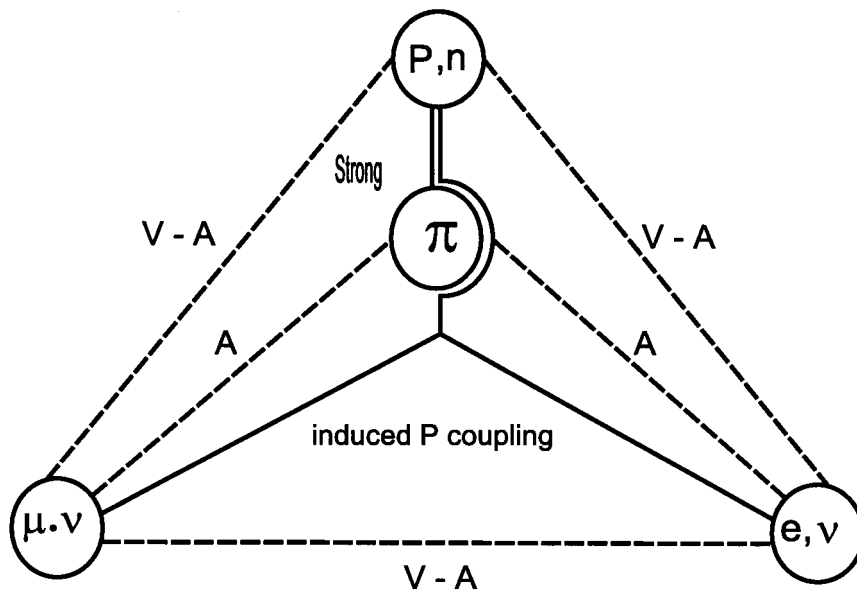


Fig. 1

This result has been taken up and further developed by Lincoln Walfenstein<sup>2</sup> of whom I include the reference to a paper<sup>3</sup> he wrote for my 70 year – Festschrift in 1988. In annex are transcribed the first few pages of papers 1 and 3.

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## References

- [1] J. Leite Lopes, *Phys. Rev.* **109**, 509 (1958).
- [2] L. Wolfenstein, *Nuovo Cimento* **8**, 882 (1958).
- [3] L. Wolfenstein, The weak pseudoscalar interaction, Leite Lopes Festschrift page 365, World Scientific, Singapore 1988.

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### Capture of Negative Muons by Light Nuclei\*

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The capture of negative muons by protons can be regarded as taking place as a result of a direct coupling or via the pi-meson field. An attempt is made to check whether the static, cut-off pseudovector coupling model of the  $\pi$ -meson-nucleon interaction can give rise to predictions on the capture of muons by nuclei in agreement with experiment. Chew theory leads to a capture rate 20 times smaller than that predicted by the direct-coupling theory. Accurate calculations can be made for the case of hydrogen, deuterium or helium, for which experimental information would be highly desirable. Comparison with the observational data for carbon, oxygen, and silicon, based on the ideal-gas model for nuclei, indicates that they are consistent with the direct-coupling theory. Meson theory gives a capture rate about 20 times too small.

#### INTRODUCTION

THE experimental measurement of the capture rate of negative muons by protons would provide an invaluable source of information on the nature of the interaction between  $\mu$  mesons and nucleons.

Owing to the fact that the decay of negative muons predominates over their nuclear absorption by nuclei with atomic number less than about 12, the experimental data run up to now to medium and heavy nuclei.<sup>1</sup> It is, however, very difficult at present to make accurate theoretical predictions of the capture rate for such nuclei because of our lack of knowledge of the nuclear wave functions.

As is well known, two alternative views are possible for describing the interaction between muons and nucleons.<sup>2</sup> The most widely accepted view prescribes a direct Fermi coupling between the nucleon and the muon-neutrino fields, in a manner analogous to beta decay. The alternative view is that of an interaction of muons and nucleons via the  $\pi$ -meson field. The first interpretation received a great deal of attention when it was discovered that, within the experimental uncertainty, the same order of magnitude of the coupling constant could account for the beta decay of neutron and  $\mu$  meson as well as for the capture of muons by nuclei. On the other hand, the alternative interpretation of the interaction of  $\mu$  mesons with nucleons through the pion field is inevitable according to current quantum-mechanical principles. Since  $\pi$  mesons decay into  $\mu$  mesons and interact strongly with nucleons, the mechanism of capture of  $\mu$  mesons by protons via virtual  $\pi$  mesons cannot be discarded unless it gives a negligible contribution to the process or some as yet unknown rule forbids it.

The recent success of the cut-off meson theory for describing low-energy  $\pi$ -meson-nucleon scattering and related processes,<sup>3</sup> led us to re-examine this problem. In this paper, we present results of calculations carried out under both interpretations, our aim being to check whether the Chew-Low meson theory would be capable of giving predictions in agreement with experiment. The calculations can be made accurately for the capture of  $\mu$  mesons by a free proton, by the deuteron and to a certain extent by the alpha particle. The Chew theory predicts a capture rate about 20 times smaller than the direct coupling theory, for acceptable values of the coupling constants. An estimate based on the Fermi gas model for nuclei spherically symmetric in spin and isotopic spin space, indicates that the available experimental data are consistent with the direct coupling theory, not with the Chew theory which predicts lifetimes 20 times larger. The conclusion will become definite when measurements for capture by hydrogen become available.

#### 1. Capture by a Proton

The Hamiltonian of a  $\pi$ -meson field in interaction with a muon-neutrino field and an extended nucleon is the following:

$$H = H_\pi + H_\mu + H_p + H_{\pi N} + H_{\pi\mu}, \quad (1)$$

where  $H_\pi$ ,  $H_\mu$ ,  $H_p$  are the Hamiltonians of free  $\pi$  mesons, free muons, and neutrinos, respectively.

$$H_{\pi N} = F_0 \tau_\alpha \int U(x) (\sigma \cdot \nabla) \varphi_\alpha(x) d^3x \quad (2)$$

is the pseudovector interaction between  $\pi$  mesons and the extended nucleon with source  $U(x)$ , where  $F_0 = (4\pi)^{1/2} f_0 m_\pi^{-1}$  and the sum over the subscript  $\alpha = 1, 2, 3$  of isotopic spin space is implied;  $\hbar = 1$ ,  $c = 1$ .

$$H_{\pi\mu} = iG_0 \sqrt{2} \int \bar{\psi}_\mu \gamma_5 \gamma_j \frac{\partial \varphi}{\partial x_j} \psi_\mu d^3x + \text{Herm. conj.} \quad (3)$$

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<sup>1</sup> R. D. Sard and M. F. Crouch, in *Progress in Cosmic-Ray Physics* edited by J. G. Wilson (North-Holland Publishing Company, Amsterdam, 1954), Vol. 2, p. 3.

<sup>2</sup> References to earlier theoretical work can be found in the review article by L. Michel, in *Progress in Cosmic-Ray Physics* (Interscience Publishers, Inc., New York, 1952), Vol. 1, p. 125. See also reference 1.

<sup>3</sup> G. C. Wick, *Revs. Modern Phys.* 27, 339 (1955); G. F. Chew and F. E. Low, *Phys. Rev.* 101, 1570 (1956).

is the pseudovector interaction between the pion- and muon-neutrino fields. The muon-neutrino operators are labeled by the indices  $\mu$  and  $\nu$ ;  $\varphi = (\varphi_1 - i\varphi_2)/\sqrt{2}$  and  $j=0, 1, 2, 3$  is the ordinary space subscript. The representation has been taken in which  $\gamma_4 = -\gamma_0$  and  $a_j b_j = a_0 b_0 - \mathbf{a} \cdot \mathbf{b}$ . Also  $G_0 = (4\pi)^{1/2} g_0 m_\pi^{-1}$ .

The interaction (3) gives rise to the decay  $\pi \rightarrow \mu + \nu$ , in which the transition probability for pions at rest is

$$\tau_{\pi\mu}^{-1} = g_0^2 m_\pi (m_\mu/m_\pi)^2 [1 - (m_\mu/m_\pi)^2]^2. \quad (4)$$

The experimental value of  $\tau_{\pi\mu} = 2.55 \times 10^{-8}$  sec determines  $g_0^2$ :

$$g_0^2 = 0.18 \times 10^{-14}.$$

The smallness of  $g_0$  allows the treatment of  $H_{\pi\mu}$  as a small perturbation of the remaining Hamiltonian in (1).

The annihilation of a negative muon accompanied by the transformation of a proton into a neutron and the emission of an antineutrino is described by the matrix element  $\langle \bar{\nu} n | H_{\pi\mu} | \bar{\mu} p \rangle$ . The Fourier development of  $\varphi$ ,  $\psi_\mu$ , and  $\psi_\nu$ :

$$\begin{aligned} \varphi_a &= \frac{1}{(2V)^{1/2}} \sum_{\mathbf{k}} \frac{1}{\omega_k^{1/2}} [a_a(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{x}} + b_a^*(\mathbf{k}) e^{-i\mathbf{k} \cdot \mathbf{x}}], \\ \varphi &= \frac{1}{\sqrt{2}} (\varphi_1 - i\varphi_2); \quad a(\mathbf{k}) = \frac{1}{\sqrt{2}} [a_1(\mathbf{k}) - ia_2(\mathbf{k})]; \\ &\dots \dots \dots b^*(\mathbf{k}) = \frac{1}{\sqrt{2}} (b_1^* - ia_1^*), \end{aligned}$$

$$\psi_{\mu,\nu} = \frac{1}{V^{1/2}} \sum_{\mathbf{k}} \sum_{r=1,2} [c_{r,\mu,\nu}(\mathbf{k}) u_{r,\mu,\nu} e^{i\mathbf{k} \cdot \mathbf{x}} + d_{r,\mu,\nu}^* v_{r,\mu,\nu} e^{-i\mathbf{k} \cdot \mathbf{x}}],$$

in (3) gives for the matrix element  $\langle \bar{\nu} n | H_{\pi\mu} | \bar{\mu} p \rangle$  between the initial state  $d_{r,\mu}^*(\mathbf{p}_\mu) | p \rangle \Psi_0$  and the final state  $d_{r,\nu}^*(\mathbf{p}_\nu) | n \rangle \Psi_0$  the following expression ( $\Psi_0$  is the vacuum state vector):

$$\begin{aligned} \langle \bar{\nu} n | H_{\pi\mu} | \bar{\mu} p \rangle &= (\bar{r}, \nu, M(\mathbf{k}) v_r^*) \langle n | a(\mathbf{k}) | p \rangle \\ &+ (\bar{r}, \mu, M(-\mathbf{k}) v_r^*) \langle n | b^*(-\mathbf{k}) | p \rangle. \end{aligned} \quad (5)$$

Here  $| p \rangle$  and  $| n \rangle$  are the wave functions which describe the physical proton and neutron (with their meson clouds):

$$(H_\pi + H_{\pi N}) | p \rangle = E_p | p \rangle, \quad (H_\pi + H_{\pi N}) | n \rangle = E_n | n \rangle,$$

and  $M(\mathbf{k}) = (G_0/V^{1/2}) \omega_k^{-1} \gamma_4 \mathbf{k}$ ,  $\mathbf{k} = \mathbf{p}_\mu - \mathbf{p}_\nu$ ,  $k = k_\mu \gamma_4$ .

The matrix elements  $\langle n | a(\mathbf{k}) | p \rangle$  and  $\langle n | b^*(\mathbf{k}) | p \rangle$  can be expressed in terms of the nucleon and  $\pi$ -meson variables if one considers the commutation relations:

$$[H_\pi + H_{\pi N}, a(\mathbf{k})] = -\omega_k a(\mathbf{k}) + V_a(\mathbf{k}),$$

$$[H_\pi + H_{\pi N}, b^*(\mathbf{k})] = \omega_k b^*(\mathbf{k}) + V_b(\mathbf{k}),$$

$$V_a(\mathbf{k}) = \frac{F_0 \tau^-}{(V \omega_k)^{1/2}} i(\boldsymbol{\sigma} \cdot \mathbf{k}) v^*(\mathbf{k}); \quad V_b(\mathbf{k}) = -V_a(-\mathbf{k});$$

$$v(\mathbf{k}) = \int u(\mathbf{x}) e^{i\mathbf{k} \cdot \mathbf{x}} d^3x.$$

One obtains

$$\langle n | a(\mathbf{k}) | p \rangle = \frac{\langle n | V_a(\mathbf{k}) | p \rangle}{E_n - E_p + \omega_k}, \quad \langle n | b^*(\mathbf{k}) | p \rangle = \frac{\langle n | V_b(\mathbf{k}) | p \rangle}{E_n - E_p - \omega_k}.$$

Substitution of these expressions in (5) gives

$$\langle \bar{\nu} n | H_{\pi\mu} | \bar{\mu} p \rangle = -\frac{2G_0 F_0 \omega_k}{V} \langle n | \boldsymbol{\sigma} \cdot \mathbf{p} | p \rangle \frac{(\bar{r}, \nu, \gamma_4 \not{p} v_r^*)}{p_\mu^2 - m_\pi^2}, \quad (6)$$

where  $\not{p}_\mu = (\not{p}_\mu - \not{p}_\mu \gamma_4)$  is the (four-) momentum transfer. Now the well-known theorem,

$$F_0 \langle n | \boldsymbol{\sigma} \cdot \mathbf{p} r^- | p \rangle = F \langle u_\nu | \boldsymbol{\sigma} \cdot \mathbf{p} r^- | u_p \rangle,$$

permits the replacement in our formula of  $F_0$  by the renormalized coupling constant  $F$  and the unknown matrix element  $\langle n | \boldsymbol{\sigma} \cdot \mathbf{p} r^- | p \rangle$  by the one computed with free-particle spinors  $(u_\nu | \boldsymbol{\sigma} \cdot \mathbf{p} r^- | u_p)$ .

The transition probability for capture of a negative muon by a proton is then

$$\begin{aligned} \tau_H^{-1} &= 2\pi \int \frac{1}{2} \sum_{r,\nu=1,2} \frac{1}{2} \sum_{r,\mu} |\langle \bar{\nu} n | H_{\pi\mu} | \bar{\mu} p \rangle|^2 \\ &\quad \times \frac{E_\nu^2 V}{(2\pi)^3} |\varphi(0)|^2 V d\Omega_\nu, \end{aligned}$$

where  $\frac{1}{2} \sum_{r,\nu} \frac{1}{2} \sum_{r,\mu}$  means the sum over the spins of the final neutron and neutrino and the average over the spins of the initial proton and muon,  $E_\nu$  is the energy of the neutrino, and  $|\varphi(0)|^2 = (1/\pi)(m_\mu c^2)^3$  is the probability to find the muon at the proton position (from the  $K$  orbit).

Assuming that both the proton and  $\mu$  meson are at rest gives

$$\tau_H^{-1} = 32 f^2 g_0^2 m_\pi (m_\mu/m_\pi)^2 (c^2)^2 [1 + (m_\mu/m_\pi)^2]^{-2}. \quad (7)$$

In this formula, as well as in the matrix element (6), we have replaced the Fourier transform of the source function,  $v(\mathbf{k})$ , by 1. This results from the fact that the cut-off momentum in the Chew-Low theory is of the order of the nucleon rest energy. If one assumes a Gaussian distribution for the source, it may in practice be replaced by unity.

From (4) and (7) we obtain

$$\frac{\tau_{\pi\mu}}{\tau_H} = 32 f^2 \left(\frac{m_\mu}{m_\pi}\right)^7 (c^2)^2 \left[1 - \left(\frac{m_\mu}{m_\pi}\right)^4\right]^{-2}.$$

## 2. Muon Capture by a Light Nucleus

The preceding calculations can be generalized easily when a muon is captured by a nucleus.

Let  $|\mathcal{N}\rangle$  be the wave function which describes the state of a nucleus with energy  $E$ :

$$H |\mathcal{N}\rangle = E |\mathcal{N}\rangle.$$

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$H$  is formed from the sum of the kinetic energies of the (slowly moving) nucleons and of the  $\pi$ -meson field Hamiltonian and its interaction with the nucleons. The latter is

$$\sum_{i=1}^A F_0 \tau_{\alpha}^i \int U(x_i) (\sigma_i \cdot \nabla_i) \varphi_{\alpha}(x_i) d^3x_i,$$

where the sum is extended over the nucleons and summation over the isotopic spin subscript is understood. We want the matrix elements  $\langle \mathcal{N}' | a(\mathbf{k}) | \mathcal{N} \rangle$  and  $\langle \mathcal{N}' | b^*(\mathbf{k}) | \mathcal{N} \rangle$  which occur in the expression [analogous to (5)] for the amplitude  $\langle \mathcal{N}' | H_{\pi\mu} | \mu \mathcal{N} \rangle$  of capture of a negative muon by a nucleus in state  $|\mathcal{N}\rangle$  which then goes over into state  $|\mathcal{N}'\rangle$ :

$$\langle \mathcal{N}' | H_{\pi\mu} | \mu \mathcal{N} \rangle = (\bar{v}_{r',\mu} M(\mathbf{k}) v_{r'}) \langle \mathcal{N}' | a(\mathbf{k}) | \mathcal{N} \rangle + (\bar{v}_{r',\mu} M(-\mathbf{k}) v_{r'}) \langle \mathcal{N}' | b^*(-\mathbf{k}) | \mathcal{N} \rangle.$$

They are obtained from the commutation relation,

$$[H, a(\mathbf{k})] = -\omega_k a(\mathbf{k}) + \sum_{j=1}^A V_j(\mathbf{k});$$

$$V_j(\mathbf{k}) = \frac{iF_0}{(V\omega_k)^{1/2}} (\sigma_j \cdot \mathbf{k}) \tau_j^- v_j^*(\mathbf{k}),$$

and another one with  $b^*(\mathbf{k})$ .

One obtains

$$\langle \mathcal{N}' | a(\mathbf{k}) | \mathcal{N} \rangle = \left\langle \mathcal{N}' \left| \sum_{i=1}^A V_i(\mathbf{k}) \right| \mathcal{N} \right\rangle / (E' - E + \omega_k),$$

$$\langle \mathcal{N}' | b^*(\mathbf{k}) | \mathcal{N} \rangle = \left\langle \mathcal{N}' \left| \sum_{i=1}^A V_i(\mathbf{k}) \right| \mathcal{N} \right\rangle / (E' - E - \omega_k).$$

The matrix element analogous to (6) is then

$$\langle \mathcal{N}' | H_{\pi\mu} | \mu \mathcal{N} \rangle = -\frac{2G_0 F_0 i}{V} \left\langle \mathcal{N}' \left| \sum_{i=1}^A (\sigma_i \cdot \mathbf{p}) \tau_i^- v_i^*(\mathbf{k}) \right| \mathcal{N} \right\rangle \frac{(\bar{v}_{r',\mu} \gamma_5 \not{p} v_{r'})}{p^2 - m_{\pi}^2}.$$

Here we have neglected the effect of the nuclear Coulomb field on the muon wave function which was represented by a plane wave. The formula should, however, be sufficiently accurate for light nuclei if we use the effective atomic number calculated by Wheeler.<sup>4</sup>

In the preceding expression we shall replace  $F_0$  by the renormalized Chew coupling constant  $F$ . This amounts to introducing the impulse approximation.<sup>5</sup> The interaction of a nuclear proton with the muon is complicated by the presence of other nucleons with which it interacts. However, the time during which the interaction with the muon place is small compared to the past and future history of the nucleus. We therefore

<sup>4</sup> J. A. Wheeler. *Revs. Modern Phys.* 21, 133 (1949).  
<sup>5</sup> See, for example, G. F. Chew and M. L. Goldberger, *Phys. Rev.* 57, 778 (1952).

assume that only a small error is committed if we take the proton in question as free during that time. The effect of this assumption will be to replace  $F_0$  by the effective renormalized coupling constant relative to a free nucleon. This approximation is also equivalent to assuming that a proton which absorbs a pion emitted by the muon does not exchange pions with the neighboring nucleons for a short time before and after the arrival of the virtual  $\pi$  meson.

The transition probability is

$$\tau^{-1} = 2\pi \sum_{E'} \int \frac{1}{2} m_{\pi}^2 \left( 1 - \frac{\mathbf{p} \cdot \mathbf{p}_{\pi}}{E_{\pi} E_{\mu}} \right) \frac{E_{\pi}^2}{(2\pi)^2} \left( \frac{1}{\pi} \right) \times (Z m_{\pi} e^2)^2 \left( \frac{4G_0^2 F_0^2}{(p^2 - m_{\pi}^2)^2} \right) \langle M \rangle^2 d\Omega_{\pi},$$

where  $\sum_{E'}$  is the sum over the final nuclear states and  $\langle M \rangle^2$  is the sum over the final nuclear spins and the average over initial nuclear spin of the absolute square of

$$M = \left( \mathcal{N}' \left| \sum_{i=1}^A (\sigma_i \cdot \mathbf{p}) \tau_i^- e^{-i\mathbf{k} \cdot \mathbf{x}_i} \right| \mathcal{N} \right).$$

Here again we neglect the nucleon extension and thus replace  $v_j(\mathbf{k})$  by  $\exp(-i\mathbf{k} \cdot \mathbf{x}_j)$ .

$$\tau^{-1} = \frac{8}{\pi} g_0^2 f^2 m_{\pi} \left( \frac{m_{\mu}}{m_{\pi}} \right)^4 (Z e^2)^2 \sum_{E'} \int \frac{1}{\Omega_{\pi}} \left( 1 - \frac{\mathbf{p} \cdot \mathbf{p}_{\pi}}{E_{\pi} E_{\mu}} \right) \frac{E_{\pi}^2}{[(E_{\pi} - E_{\mu})^2 - c^2(\mathbf{p} - \mathbf{p}_{\pi})^2 m_{\pi}^2]^2} \langle M \rangle^2 d\Omega_{\pi}.$$

3. Case of the Deuteron

$\langle M \rangle^2$  can be calculated in the case of the deuteron. The ground state wave function is

$$|\mathcal{N}\rangle = \frac{1}{(2\pi)^{1/2}} \left( \frac{1}{\sqrt{2}} \right) [\psi(1)n(2) - \psi(2)n(1)] {}^3\chi_m(1,2) u_d(r),$$

where  $\psi$  and  $n$  are the proton and neutron components of the isotopic-spin wave function,  ${}^3\chi_m(1,2)$  is the triplet spin wave function, and  $u_d(r)$  is the radial function.

After capture of the muon, the deuteron transforms into a di-neutron which can have spin 0 or 1. The two possible final wave functions are thus

$$|\mathcal{N}_s'\rangle = \frac{1}{(2\pi)^{1/2}} n(1)n(2) {}^1\chi_0(1,2) u_n(r) e^{i\mathbf{P} \cdot \mathbf{R}},$$

$$|\mathcal{N}_s'\rangle = \frac{1}{(2\pi)^{1/2}} n(1)n(2) {}^3\chi_m(1,2) u_n(r) e^{i\mathbf{P} \cdot \mathbf{R}},$$

where  ${}^1\chi_0$  is the singlet spin wave function,  $\mathbf{P}$  is the center-of-mass momentum,  $\mathbf{R} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)$ ,  $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ , and

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## The Weak Pseudoscalar Interaction

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Dedicated to José Leite-Lopes  
on the occasion of his seventieth birthday

### Abstract

The Yukawa theory of the weak interaction based on pion exchange was shown by Leite-Lopes to predict a significant rate for muon capture by nuclei, but one that was an order of magnitude below experiment. Within the V-A theory pion exchange provides a strong interaction correction to the weak interaction, which has the result of reducing the rate of muon capture by about 20%. Other examples of weak pion exchange and weak pseudoscalar interactions are discussed.

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The Yukawa theory was not only a theory of the nuclear force but also of nuclear beta decay. Virtual pion exchange with a strong coupling at the  $(np)$  vertex and a very weak coupling at the  $(e\nu)$  vertex gave a model for beta decay. With the discovery that  $\pi$  decays to  $\mu\nu$  and not to  $e\nu$  this theory had to be abandoned.

However there exists the analog of the beta decay interaction involving muons, which manifests itself in the process  $\mu^- + p \rightarrow n + \nu$ . In 1957, Leite-Lopes [1] applied the Yukawa theory to this process. A reliable calculation was possible because a good value of the strong pion-nucleon coupling was available from a variety of experiments using the Chew-Low theory or dispersion methods. The result was found to be about 20 times too low, although the experimental data at the time (mainly muon capture in carbon) was quite limited.

It was clear that muon capture should be described by the universal V-A interaction that also explained nuclear beta decay and muon decay. This interaction involved couplings between the  $(e\nu)$ ,  $(\mu\nu)$ , and  $(np)$  currents. It occurred to a number of people [2, 3], including me, that one must consider strong interaction corrections to the  $(np)$  current. The most obvious of these was the renormalization of the axial-vector coupling constant  $g_A$ . No one knew how to calculate that at the time, but the answer was known experimentally to be 1.18 (since experimentally renormalized to 1.25).

There was, however, another obvious strong interaction effect, namely the virtual pion exchange. This gave an effective or induced pseudoscalar interaction; that is, an interaction of a form that simply didn't exist before the strong corrections. (An example of this sort more familiar to our younger colleagues is the  $O_5$  or  $O_8$  operator in kaon decay induced by the QCD penguin graph [4].) The Yukawa weak interaction had to be added to the V-A interaction. Furthermore although Leite-Lopes had shown it was 20 times too small by itself, it interfered with the axial contribution to muon capture and so proved to be quite significant.

To analyze the effect of the induced pseudoscalar interaction I had to calculate the sign of the interference term. This is somewhat tricky because one must determine the sign of the  $(\pi\mu\nu)$  coupling in terms of  $g_A$ . After considerable effort I convinced

myself that the effect was to reduce the muon capture rate by about 20%. Around the same time Gell-Mann [5] gave an explicit formulation of the weak magnetism effect, which when applied to muon capture, increased the rate by about 20%. Thus the strong corrections actually changed the spin-averaged capture rate only a little. As more accurate experiments on the muon capture process were made, particularly for muon capture in hydrogen, it became customary to analyze the experiment in terms of the value of the pseudoscalar coupling  $g_p$ , assuming the other couplings were known. Within sizeable experimental uncertainty the value of  $g_p$  agreed with the theory [6]. In fact there really is very little uncertainty in the value of  $g_p$  from the pion exchange other than a small correction in extrapolating from the pion pole to the physical value of momentum transfer. If  $g_p$  were found to be significantly different from the Leite-Lopes value, it would have to be a sign of new physics.

It is possible to get even closer to the pion pole through the process of radiative muon capture. The detailed analysis of Manacher [7, 8] showed that for high energy photons from  $\mu^- + p \rightarrow n + \nu + \gamma$  the dominant contribution was due to  $g_p$ . So far radiative capture has been studied only in complex nuclei, in which case it is hard to disentangle the nuclear physics. The possibility of studying radiative muon capture in hydrogen is still being considered [9].

The weak pion exchange also plays an important role in discussions of parity violation in nuclei. The weak nuclear force calculated directly from the (V-A) theory describes a delta-function interaction. Because of the repulsion at short distances between nucleons, nuclear forces of long range are particularly important. The longest range nucleon-nucleon interaction is that arising from pion exchange. In the case of the weak nuclear force, this is derived from one strong nucleon-nucleon-pion vertex and one that is weak. This is again quite analogous to the Leite-Lopes calculation, except in this case there is no direct measure of the weak vertex. Interestingly in the (V-A) theory with only charged currents, this weak vertex is proportional to  $\sin^2 \theta_C$  where  $\theta_C$  is the Cabibbo angle. With the discovery of neutral currents, the calculated value of this weak vertex was much larger. Thus the contribution of neutral currents to parity violation in nuclear physics was expected to be important. Many exper-