# MODELING HIGHER TWIST CONTRIBUTIONS TO DEEP INELASTIC SCATTERING WITH DIQUARKS

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## ABSTRACT

The most recent detailed data on the unpolarized nucleon structure functions allow a precise determination of higher twist contributions. Quark-quark correlations induced by colour forces are expected to be a natural explanation for such effects; indeed, a quark-diquark picture of the nucleon, previously introduced in the description of several exclusive processes at intermediate  $Q^2$  values, is found to model the proton higher twist data with great accuracy. The resulting parameters are consistent with the diquark properties suggested by other experimental and theoretical analyses.

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#### Introduction

Recent data from Deep Inelastic Scattering (DIS) experiments at CERN [1,2,3] and SLAC [4] have provided precise information on the unpolarized nucleon structure function  $F_2(x, Q^2)$  and have allowed a quantitative estimate of higher twist terms, both for protons [5,6] and, in a somewhat indirect way, for neutrons [6], parametrized as

$$F_2(x,Q^2) = F_2^{LT}(x,Q^2) \left(1 + \frac{C(x)}{Q^2}\right), \qquad (1)$$

where  $F_2$  is the measured structure function,  $F_2^{LT}$  is the leading twist contribution and  $F_2^{LT}C(x)/Q^2$  is the higher twist term. The peculiar property is that the function C(x) changes sign, being negative at small x values and positive at larger ones; the transition region from negative to positive values is at  $x \simeq 0.35$  for protons and  $x \simeq 0.15$  for neutrons. Addition of a  $D(x)/Q^4$  term in Eq. (1) would equally well reproduce the data [5].

Higher twist contributions to DIS are expected to originate from quark and gluon correlations; we model them here with a quark-diquark picture of the nucleon which effectively takes into account quark-quark correlations, originated by colour forces. This model has been previously introduced and widely applied to many physical processes [7] and there is little doubt that two quark correlations are present inside nucleons and that they might play a significant rôle in some processes at moderate  $Q^2$  values, precisely the region where the higher twist effects have been observed ( $Q^2$  up to  $\simeq 10 \div 20 \text{ GeV}^2$ ).

The diquark contributions to DIS have been studied in several papers [8,9], mainly taking into account only spin 0, scalar diquarks, in simplified versions of the quark-diquark model of the nucleon. The most recent one [9] fits the full structure function  $F_2(x, Q^2)$ , by considering scalar diquarks and achieves a good agreement with the data and their  $Q^2$  dependence via the diquark form factor and a QCD-like  $Q^2$  evolution of the quark and diquark distribution functions. A most general analysis of spin 0 and spin 1 diquark contributions to DIS has been performed in Ref.[10], allowing for a vector diquark anomalous magnetic moment and for scalar-vector and vector-scalar diquark transitions; no attempt, however, was made of fitting the experimental data due to the lack of detailed information on the higher twist contributions alone.

Now such information is available [5,6] and we assume that it can be entirely explained by diquarks; in Section 2 we recall the diquark contribution to DIS and the nucleon structure functions, which we interpret as higher twist terms; in Section 3 we introduce a quark-diquark model of the nucleon, discuss its parameters and fix them by fitting the proton data on higher twist effects; we then give some conclusions.

#### 2 - Diquark contributions to DIS and higher twist

Let us assume the nucleon to be a quark-diquark state where the diquark models correlations between two quarks which, at moderate momentum transfers, might interact collectively and behave as a bound state. When probing the nucleon with the virtual photon in DIS we have then three kinds of contributions: the scattering off the single quark, the elastic scattering off the diquark, and the inelastic diquark contribution, that is the scattering off one of the quarks inside the diquark. Eventually, at large enough  $Q^2$  values, the elastic diquark contribution, weighted by form factors, vanishes and one recovers the usual pure quark results.

The expression of the structure functions F, in the quark-diquark parton model, is then given in general by:

$$\begin{split} F(x,Q^2) &= \sum_{q} F^{(q)} + \sum_{S} F^{(S)} + \sum_{V} F^{(V)} + \\ &+ \sum_{q_S} F^{(q_S)} + \sum_{q_V} F^{(q_V)} + \sum_{S,V} F^{(S-V)} + \sum_{S,V} F^{(V-S)} , \end{split}$$
(2)

where (q) denotes the single quark contribution, (S) and (V) respectively the scalar and vector diquark ones, and  $(q_s)$   $((q_v))$  the contribution of the quark inside the scalar (vector) diquark. We have also allowed for elastic diquark contributions with a scalar-vector (S - V) or vector-scalar (V - S) transition.

Let us consider here the unpolarized structure function  $F_2(x, Q^2)$  only; the single quark contribution is the usual one,

$$F_2^{(q)}(x,Q^2) = e_q^2 x q(x,Q^2), \qquad (3)$$

where the quark density number  $q(x, Q^2)$  evolves according to perturbative QCD. The elastic diquark contributions have been computed in Ref.[10] and for  $F_2$  they read:

$$\begin{split} F_2^{(S)}(x,Q^2) &= e_S^2 \, S(x) \, x \, D_S^2 \\ F_2^{(V)}(x,Q^2) &= \frac{1}{3} e_V^2 V(x) \, x \left\{ \left[ \left( 1 + \frac{\nu}{m_N x} \right) D_1 - \frac{\nu}{m_N x} D_2 + \right. \\ &+ 2m_N \nu x \left( 1 + \frac{\nu}{2m_N x} \right) D_3 \right]^2 + 2 \left[ D_1^2 + \frac{\nu}{2m_N x} D_2^2 \right] \right\} \quad (4) \\ F_2^{(S-V)}(x,Q^2) &= \frac{1}{2} e_S^2 S(x) \, x^2 \, m_N \nu D_T^2 \\ F_2^{(V-S)}(x,Q^2) &= \frac{1}{6} e_S^2 V(x) \, x^2 \, m_N \nu D_T^2 \end{split}$$

where  $D_S$  is the scalar diquark form factor,  $D_{1,2,3}$  are form factors appearing in the most general coupling of the virtual photon to the vector diquark and  $D_T$  is the transition form factor [10]. S(x) and V(x) are, respectively, the scalar and vector diquark density numbers; as they are supposed to model non perturbative effects in a limited  $Q^2$  range we ignore their QCD evolution.

The inelastic diquark contributions are given by

$$F_2^{(q_S)} = e_{q_S}^2 x \, q_S(x, Q^2) \left(1 - D_S^2\right) \tag{5}$$

for scalar diquarks and

$$F_2^{(q_V)} = e_{q_V}^2 x \, q_V(x, Q^2) \left(1 - D_V^2\right) \tag{6}$$

for vector ones. In Eq. (5)  $D_S$  is the scalar diquark form factor and in Eq. (6)  $D_V$  is one of the vector diquark form factors, as will be explained in the sequel.

From Eqs.(2)-(6) one can obtain the full contribution of the quark-diquark nucleon model to  $F_2$ . The terms proportional to diquark form factors vanish at large  $Q^2$  values, leaving only the usual QCD contributions; at moderate  $Q^2$  values, however, they give non negligible contributions. These are the terms which we assume to model higher twists in DIS. Explicitly they are given by

$$F_{2}^{HT} = \sum_{S} e_{S}^{2} S(x) x D_{S}^{2} + \sum_{V} \frac{1}{3} e_{V}^{2} V(x) x \left\{ \left[ \left( 1 + \frac{\nu}{m_{N}x} \right) D_{1} + \frac{\nu}{m_{N}x} D_{2} + 2m_{N}\nu x \left( 1 + \frac{\nu}{2m_{N}x} \right) D_{3} \right]^{2} + 2 \left[ D_{1}^{2} + \frac{\nu}{2m_{N}x} D_{2}^{2} \right] \right\} + \frac{1}{2} \sum_{S} e_{S}^{2} S(x) x^{2} m_{N}\nu D_{T}^{2} + \frac{1}{6} \sum_{V} e_{S}^{2} V(x) x^{2} m_{N}\nu D_{T}^{2} + \frac{1}{6} \sum_{V} e_{S}^{2} V(x) x^{2} m_{N}\nu D_{T}^{2} + \frac{1}{2} \sum_{q_{S}} e_{q_{S}}^{2} x q_{s}(x, Q^{2}) D_{S}^{2} - \sum_{q_{V}} e_{q_{V}}^{2} x q_{v}(x, Q^{2}) D_{V}^{2}.$$

$$(7)$$

Notice that, depending on the actual behaviour of the form factors, one not only obtains contributions proportional to  $1/Q^2$ , but also to higher power of  $1/Q^2$ , which might be significant in the lowest  $Q^2$  range of the experimental data [5,6]. Notice also that Eq. (7) has both positive and negative contributions, so that it might yield positive or negative higher twist values, according to the different x or  $Q^2$  ranges.

Let us now turn to a discussion of the different quantities and parameters in Eq. (7), before using it for numerical estimates.

#### 3 - Comparison between diquark contributions and higher twist data

We would like to check whether or not Eq. (7) is a good model to explain higher twist data. In order to do so we parametrize the different physical quantities appearing in it, following some suggestions on the diquark properties resulting from other applications of the model [11].

The valence quark content of the proton in the quark-diquark model [12] is given by the flavour and spin wave function:

$$|p, S_{z} = \pm 1/2 > = \pm \frac{1}{3} \left\{ \sin \Omega \left[ \sqrt{2} V_{(ud)}^{\pm 1} u^{\mp} - 2 V_{(uu)}^{\pm 1} d^{\mp} + \sqrt{2} V_{(uu)}^{0} d^{\pm} - V_{(ud)}^{0} u^{\pm} \right] \mp 3 \cos \Omega S_{(ud)} u^{\pm} \right\},$$
(8)

where  $V_{(ud)}^{\pm 1}$  stays for a (ud) vector diquark with the third component of the spin  $S_z = \pm 1, u^{\mp}$  is a *u* quark with  $S_z = \pm 1/2$  and so on. The vector and scalar diquark components have different weights, so that the probabilities of finding a vector or scalar diquark in the proton are  $\sin^2 \Omega$  and  $\cos^2 \Omega$  respectively. Eq. (8) fixes the normalization of the valence distribution functions:

$$\int_{0}^{1} S(x) dx = \cos^{2} \Omega$$

$$\int_{0}^{1} V_{(ud)}(x) dx = \frac{1}{3} \sin^{2} \Omega$$

$$\int_{0}^{1} V_{(uu)}(x) dx = \frac{2}{3} \sin^{2} \Omega$$

$$\int_{0}^{1} u_{s}(x) dx = \int_{0}^{1} d_{s}(x) dx = \cos^{2} \Omega$$

$$\int_{0}^{1} u_{V_{(uu)}}(x) dx = \frac{4}{3} \sin^{2} \Omega$$

$$\int_{0}^{1} u_{V_{(ud)}}(x) dx = \int_{0}^{1} d_{V_{(ud)}}(x) dx = \frac{1}{3} \sin^{2} \Omega,$$
(9)

so that we can define

$$S(x) = \cos^{2} \Omega f_{S}(x)$$

$$V_{(ud)}(x) = \frac{1}{3} \sin^{2} \Omega f_{V_{(ud)}}(x)$$

$$V_{(uu)}(x) = \frac{2}{3} \sin^{2} \Omega f_{V_{(uu)}}(x)$$

$$u_{s}(x) = \cos^{2} \Omega f_{u_{s}}(x)$$

$$d_{s}(x) = \cos^{2} \Omega f_{d_{s}}(x)$$

$$u_{V_{(uu)}}(x) = \frac{4}{3} \sin^{2} \Omega f_{u_{V_{(uu)}}}(x)$$

$$u_{V_{(ud)}}(x) = d_{V_{(ud)}}(x) = \frac{1}{3} \sin^{2} \Omega f_{u_{V_{(ud)}}}(x)$$
(10)

where all f's are normalized as  $\int_0^1 f \, dx = 1$ . Eqs. (7-10) then give, for a proton,

$$\begin{split} (F_2^{HT})_p &= \frac{1}{9}\cos^2\Omega \left[ f_S(x) - \left[4f_{u_S}(x,Q^2) + f_{d_S}(x,Q^2)\right] \right] x D_S^2 + \\ &+ \frac{1}{81}\sin^2\Omega \left[ f_{V_{(ud)}}(x) + 32f_{V_{(uu)}}(x) \right] x \left\{ \left[ \left(1 + \frac{\nu}{m_N x}\right) D_1 + \right. \\ &- \frac{\nu}{m_N x} D_2 + 2m_N \nu x \left(1 + \frac{\nu}{2m_N x}\right) D_3 \right]^2 + 2 \left[ D_1^2 + \frac{\nu}{2m_N x} D_2^2 \right] \right\} + \\ &- \frac{1}{27}\sin^2\Omega \left[ 16f_{u_{V_{(uu)}}}(x,Q^2) + 5f_{u_{V_{(ud)}}}(x,Q^2) \right] x D_V^2 + \\ &+ \frac{1}{18} \left[ \cos^2\Omega f_S(x) + \frac{1}{9}\sin^2\Omega f_{V_{(ud)}}(x) \right] x^2 m_N \nu D_T^2 \,. \end{split}$$

We have adopted the following parametrization of the different distribution functions appearing in Eq. (11):

$$f_{S}(x) = N_{S} x^{\alpha_{S}} (1-x)^{\beta_{S}}$$

$$f_{V_{(ud)}}(x) = N_{V_{(ud)}} x^{\alpha_{V_{(ud)}}} (1-x)^{\beta_{V_{(ud)}}}$$

$$f_{V_{(uu)}}(x) = N_{V_{(uu)}} x^{\alpha_{V_{(uu)}}} (1-x)^{\beta_{V_{(uu)}}}$$

$$f_{u_{S}}(x) = N_{u_{S}} x^{\alpha_{u_{S}}} (1-x)^{\beta_{u_{S}}}$$

$$f_{d_{S}}(x) = N_{d_{S}} x^{\alpha_{d_{S}}} (1-x)^{\beta_{d_{S}}}$$

$$f_{u_{V_{(ud)}}}(x) = N_{u_{V_{(ud)}}} x^{\alpha_{u_{V_{(ud)}}}} (1-x)^{\beta_{u_{V_{(ud)}}}}$$

$$f_{u_{V_{(uu)}}}(x) = N_{u_{V_{(uu)}}} x^{\alpha_{u_{V_{(uu)}}}} (1-x)^{\beta_{u_{V_{(uu)}}}}$$
(12)

where the N are the proper normalization constants.

Concerning the diquark form factors we have chosen the most simple expressions which agree both with the asymptotic perturbative QCD predictions [13] and the pointlike,  $Q^2 \to 0$ , limits:

$$D_{S} = \frac{Q_{S}^{2}}{Q_{S}^{2} + Q^{2}}$$

$$D_{1} = \left(\frac{Q_{V}^{2}}{Q_{V}^{2} + Q^{2}}\right)^{2}$$

$$D_{2} = (1 + \kappa)D_{1}$$

$$D_{3} = \frac{Q^{2}}{m_{N}^{4}}D_{1}^{2}$$
(13)

$$D_V = D_1$$
$$D_T = \frac{\sqrt{Q^2}}{m_N} D_1$$

where  $\kappa$  is the vector diquark anomalous magnetic moment.

A different possible choice of the vector diquark form factors [10],  $D_1 = D_2 \sim Q^{-2}$  leads to a poorer agreement with the experimental data. The form factors  $D_3$  and  $D_T$  have to be 0 at  $Q^2 = 0$  and are expected to behave as  $Q^{-6}$  and  $Q^{-3}$  respectively at large  $Q^2$  values:  $D_3$  could actually be neglected. A most simple expression of  $D_T$  has been chosen which avoids the introduction of new parameters. The form factor  $D_V$  appearing in the inelastic vector diquark contribution has been taken to be the same as  $D_1$  and  $D_2$  with  $\kappa = 0$ .

We have then used Eq. (11), with the parametrizations given in Eqs.(12) and (13), as a model to fit the experimental data on the higher twist contributions to  $F_2^p$  [5,14]. We have neglected the small effect due to the perturbative QCD  $Q^2$  evolution of the single quark distribution functions  $f_{u_s}$ ,  $f_{d_s}$ ,  $f_{u_{V_{(ud)}}}$  and  $f_{u_{V_{(uu)}}}$ .

The results of our best fits [15], using the MINUIT program (version 92.1) of CERNLIB, are shown in Figs. 1-4; the corresponding values of the free parameters turn out to be:

$$\cos^{2} \Omega = 0.81 \qquad Q_{S} = 1.42 \text{ GeV} \qquad Q_{V} = 1.10 \text{ GeV}$$

$$\alpha_{S} = 2.13 \qquad \beta_{S} = 18.51 \qquad \alpha_{V} = 7.93 \qquad \beta_{V} = 3.32 \qquad (14)$$

$$\beta_{u_{S}} = 5.13 \qquad \beta_{d_{S}} = 5.13 \qquad \beta_{u_{V(ud)}} = 8.41 \qquad \beta_{u_{V(uu)}} = 8.41$$

The values of  $\alpha_{u_s}$ ,  $\alpha_{d_s}$ ,  $\alpha_{u_{V(ud)}}$  and  $\alpha_{u_{V(uu)}}$  have been fixed to be -0.5 in agreement with the expected small x behaviour of the valence quark distributions. The value of  $\kappa$  which allows the best fits is  $\kappa = 0$ .

Figs. 1-4 show that our diquark parametrization of higher twist contributions is indeed very good; not only, but the diquark features, as emerging from the values of the best fit parameters, Eq. (14), are consistent with the expectations from many other studies and applications of quark-diquark models of the nucleon [7]. The scalar diquarks turn out to be more abundant ( $\cos^2 \Omega = 0.81$ ) and more pointlike ( $Q_S^2 > Q_V^2$ ) than the vector ones. Notice that a value of  $Q_S = 1.42$ GeV corresponds to a mean square radius of the scalar diquark  $\sqrt{\langle r^2 \rangle_s} \simeq 0.35$  fm, whereas  $Q_V^2 = 1.10$  GeV implies  $\sqrt{\langle r^2 \rangle_V} \simeq 0.63$  fm. Also, the average x carried by a scalar diquark, proportional to its average mass, is smaller, as expected, than the average x of a vector diquark. In Fig. 5 we plot the functions xf(x), where the f(x) are the quark and diquark distributions given in Eq. (12). The total nucleon momentum fraction carried by the diquarks amounts to 0.24.

We also point out that the independent parameters  $\beta_{u_s}$  and  $\beta_{d_s}$ , fixing respectively the large x behaviour of u and d quarks inside a scalar diquark, turn

out to be equal, as expected; the same is true also for the u quark inside (ud) and (uu) vector diquarks,  $\beta_{u_{V_{(ud)}}} = \beta_{u_{V_{(uu)}}}$ .

In conclusion we have shown how contributions to deep inelastic scattering by bound states of two quarks can accurately model the higher twist data on  $F_2^p$ ; such a model takes into account non perturbative effects – two quark correlations induced by colour forces – and allows an estimate of all the higher twist effects related to the presence of diquarks, not only the leading ones. The values of the parameters of the model which give the best description of the higher twist data are in remarkable agreement with the features of the diquarks as expected from theoretical computations and applications of the model to several other intermediate  $Q^2$  processes; this points towards an unambigous presence of strong two quark correlations inside the nucleon.

Our model could easily be applied to the computation of the higher twist contributions to  $F_2^n$  as well; we have not done it here because the experimental knowledge of the higher twist contributions to the neutron structure function is not yet as clear as for the proton [6], and a complete set of raw data on  $(F_2^{HT})_n$  is not yet available.

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## **Figure Captions**

- Fig. 1 Plot of  $(F_2^{HT})_p$  vs.  $Q^2$  for x = 0.10, 0.45 and 0.75; the solid line is obtained from Eqs. (11-13) of the text with the best fit values of the parameters given in Eq. (14); the  $\star$  denote the data [5,14]
- Fig. 2 The same as in Fig. 1 for x = 0.10, 0.225, 0.55
- Fig. 3 The same as in Fig. 1 for x = 0.14, 0.35, 0.65
- Fig. 4 The same as in Fig. 1 for x = 0.07, 0.18, 0.75
- Fig. 5 Plot of xf(x) vs. x; the quark and diquark distributions f(x) are given in Eqs. (12) and (14) of the text









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